IIT Kanpur

Game theory case study

Game theoretic analysis of Brexit referendum

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Abstract

After more than four decades of (mostly) happy marriage, the U.K. is approaching an in-out vote on its membership of the 28-country European Union. Over the years, the decision making process of the EU has changed from unanimity to majority decision. With the option of Withdrawal from the EU made available to member states in the Lisbon treaty, the member states have regained some negotiation power with the EU. Here we look at the model that admits bargaining between the country opting for withdrawal and the EU and look at how compensation offers depend on bargaining costs and number of bargaining rounds.

1. Introduction

The European union (EU) is a political-economic union of its 28 member states. Since its inception, some of its major economic goals is to create a European single market through unrestricted free trade across boundaries of its member states and liberal trade policies helping free movement of labour and capital in the area covered by EU. Other than market integration it also promotes monetary integration that reduces exchange rate risks and allows free movement of the common currency Euro. Its social cohesion fund has boosted the growth of its poor members like Ireland and Portugal by reinvesting in them to reduce regional disparties and thus aiming to create a level playing field for its member states to contribute equivalently economically. It also advocates to protect human rights and thus has power to make labour regulations that protects the interests of people of EU.

UK has been a part of EU (formerly EEC) since 1973, and has been one of the most powerful states of the EU. This is evident from the fact that UK had been the second highest contributor to EU budget in year 2013 only after Germany. However, in his 2015 election campaign, UK's current prime minister promised that he would renegotiate the terms of UK's membership

in the EU and should it be necessary may hold a referendum regarding the withdrawal of UK from the EU. Such a step has come into realisation due to the following concerns of the Uk:

- Political Sovereignty: EU has always worked hard to bring the people of the union closer while UK feels continued interference by EU diminishes its sovereignty. For the same reason, PM Cameron has said that he would resist any attempts made towards the European army and want to free British police from EU interference.

- Immigration: Freedom of movement is one of the most important principle of EU that allows citizens of member states to move freely in the EU countries. Although British PM supports adding of new states in EU, he advocates that immigrants from these states should be restricted of certain benefits until they are residents for some considerable time. This, according to Eurosceptic, will ease of pressure from public services like schools and hospitals as well as create more jobs for the native population.

- Economy Independence: As mentioned above, Britain has been one of the net contributors to the budget of the EU. But it pays more budget than it actually receives. According to BBC, The Conservatives want to free business from red tape and "excessive interference" from Brussels and to provide access to new markets through "turbo charging" free trade deals with America and Asia. They argue that EU membership is hampering UK's trade ties outside the bloc and that they would have better economic relations with trading giants like China had they not been a part of EU.

It is due the following concerns that Brexit supporters have gained force. Economically, They feel that UK will save a lot of money as it will no longer have to pay the fee in form of annual contribution to the annual budget of EU. Also, pro-exit supporters feel that, UK being a important centre for commerce, other member states will have to keep the ongoing ties with the Britain and Britain will be free from labour regulations like 48 hour week for work. At the same time they think that this will allow UK to free itself from the suffocating experience it has had with the EU and create more jobs for its citizens as well as rapidly increase its growth.

However, many people are apprehensive of this decision and the future of UK and EU if this disintegration take place. Many suggest that withdrawal

of one of the most powerful members of EU may start a wave of anti-EU membership among other member states and EU would attempt to make life difficult for the UK by not allowing any special privileges or trade ties with them just to prevent such breakaways. It will also make UK a less attractive location for foreign investment as it is a important location for the same as it provides access to the EU markets.

In the following section, we look at an the model presented by Susanne Lechner and Renate Ohr(2011) when a country not happy with decision taken by EU decides whether or not to leave EU using game theory. We will follow by looking at the interaction between EU and country using finite and infinite rounds of bargaining with bargaining costs , which will be followed by the conclusion, discussion of the solution obtained using sequential bargaining and introduction of Nash bargaining solution and properties to be satisfied by it .

2. Methodology

2.1. The Model

The model presented here is taken from [1]. Let us denote the event, over which difference persists between EU and a country say B, be denoted by E. Let the benefit received by all other countries B_i except B be given by U_{EU}. Let us denote the utility of country B by U_B. Since we want to solve a situation when event E does not favour member country B and benefits all the other member states, we can assume:

$$dU_B(E) < 0 \tag{1a}$$
$$dU_{EU}(E) > 0 \tag{1b}$$
$$|dU_{EU}(E)| > |dU_B(E)| \tag{1c}$$

This means that the event E provides more utility to all the other member states and hence to EU than to country B. If such a decision is made, a compensation v should be offered to the country B (can be monetary or political) by the EU, thus,

$$dU_B(v) > 0 \tag{2a}$$
$$dU_{EU}(v) < 0 \tag{2b}$$
$$dU_B(v) = |dU_{EU}(v)| \tag{2c}$$

Here, we assume that whatever loss is made by EU due to this compensation is gained by B. The decision(E) made at t=0 determines the utility of the two players, EU and B as follows:

$$U_B(t = 1) = U_B(t = 0) + dU_B(v) + dU_B(E) \tag{3a}$$
$$U_{EU}(t = 1) = U_{EU}(t = 0) + dU_{EU}(v) + dU_{EU}(E) \tag{3b}$$

where $U_B(t = 1)$ and $U_{EU}(t = 1)$ denotes the overall benefit of EU membership for B and for other member states at t=1 while $U_B(t = 0)$ and $U_{EU}(t = 0)$ denotes the overall benefit of EU membership for B and for other member states at t=0 respectively.

Let us denote the utility of B and EU, if B withdraws from EU at t=1, as follows:

$$U_B(t = 1) = U_B(-EU) \tag{4a}$$
$$U_{EU}(t = 1) = U_{EU}(t = 0) + dU_{EU}(E) + dU_{EU}(-B) \tag{4b}$$

where $U_B(-EU)$ is the overall benefit of B after leaving EU and $U_B(-EU) < U_B(t = 0)$ as only then B would have been the part of EU at t=0 in the first place. Also, $dU_{EU}(-B) < 0$ as the payoff of all the other nations would reduce if B withdraws from the EU as then there would be less area available to trade as well as there will be loss of attraction due to disintegration.

In the next section, we look at the finite sequential bargaining solution to the bargaining game between the two players with bargaining costs, once we define the various aspects and assumptions of the game formally.

2.2. Finite Bargaining Case

In this subsection, we admit bargaining between B and EU as it is a more realistic scenario than no bargaining. Here we assume the following:

- Both Players are rational.

- Common knowledge: Both players have complete information with respect to changes in utility of other player due to the decision.

- Assume that the game is played sequentially and EU proposes first. Though, this is not a valid assumption realistically, as any one of the player can make the first offer. But for this case we assume EU proposes first.

- Number of bargaining rounds are finite.

We start with a very simple game where EU offers a compensation v and B has an option to either accept it or reject it. Here we assume that if B accepts it, then B stays in the EU, accepts decision E and accepts compensation of v for the loss caused due to the same.However, if B rejects the offer, we assume that B leaves the EU (we do not assume any cost of leaving at the moment as it will be a constant and will not qualitatively affect the model results). A graphical version of the same is shown below:
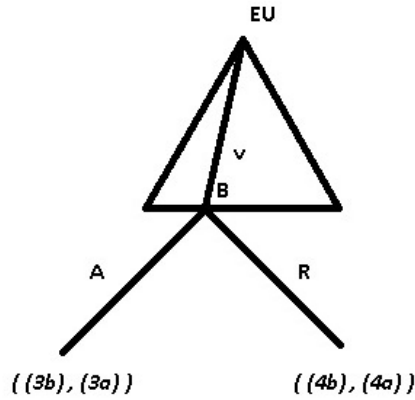

Figure 1: Game where both players play once and EU plays first

Since only one round of bargaining is discussed above we have no bargaining costs involved in this game. Without loss of generality, the game will give same outcome as far as preference order of the players remain the same. For this reason this game can be written same in the following form too.
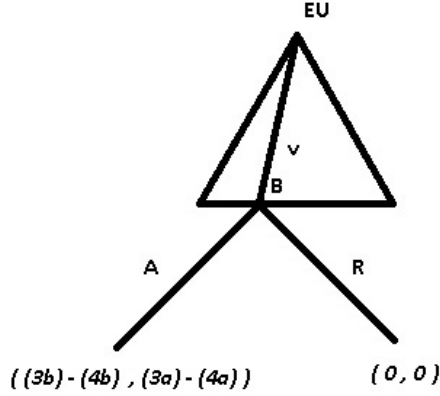
Figure 2: A game equivalent to Game 1

Note- When we write (3a), (4a), (3b) or (4b) in the graph above or anywhere below we mean only RHS of that equation unless explicitly stated that we are referring to complete equation.

The solution of the above mentioned game can be obtained as follows. B accepts offer if payoff of B from accepting the offer is greater than 0 and will be indifferent between accepting and rejecting if (3a)=(4a) i.e. its utility from leaving the union will be same as staying in it, accepting the decision E and accepting the compensation v* such that such equality holds. How much should be offered by EU. We should note that EU offers only till that v such that (3b) < (4b)i.e. EU offers only if it is better of by paying B a compensation v to retain it than the scenario in which B withdraws from the EU. B accepts any offer such that its utility becomes greater than or equal to zero. Hence, if EU offers a amount marginally more than the amount v* obtained when (3a)=(4a) and B will accept it.Hence, solution to this game is: EU offers v* and B accepts all offers such that its utility is non negative i.e.

$$U_B(-EU) = U_B(t=0) + dU_B(v*) + dU_B(E) \qquad (5a)$$
$$|dU_{EU}(v*)| < |dU_{EU}(-B)| \qquad (5b)$$

Let us look at a version of this game for a finite t rounds of bargaining, where alternative proposal are made from the two parties with EU making the first proposal. Let us assume the bargaining cost of EU and B are represented by δ_1 and δ_2 respectively. We should note that both $\delta_1, \delta_2 \in (0, 1)$.
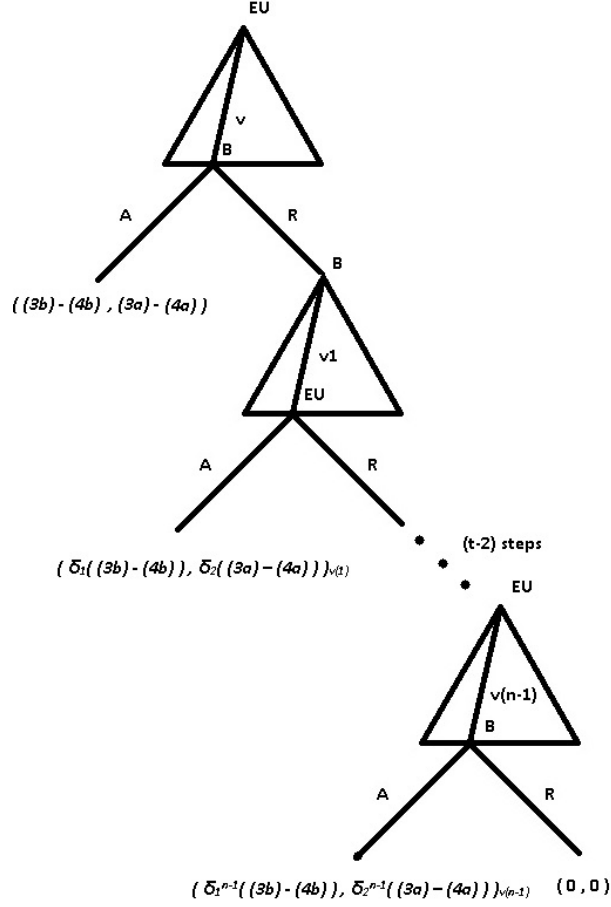
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Figure 3: A t-round bargaining game with bargaining cost δ_1 and δ_2 respectively

This is a typical bargaining problem where the surplus M is $(3b) - (4b) + (3a) - (4a)$ which has to be divided among the two players EU and B. Without loss of generality, we can normalise this surplus to 1 and look at the simplified game as a bargaining solution where surplus of 1 has to be divided among the two players. This can be done for qualitative analysis of how the payoff would depend on the bargaining cost of two players. The figure of the simplified game is as shown below
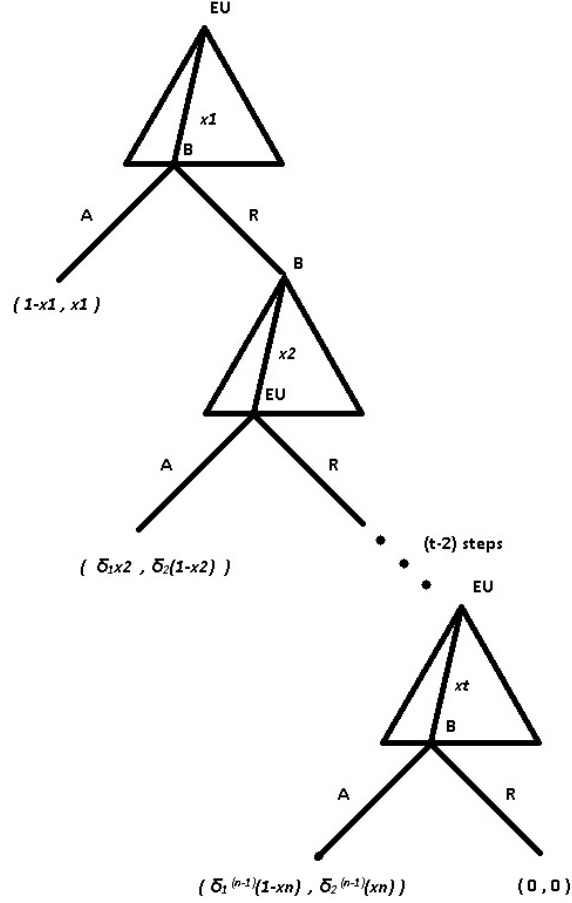
Figure 4: A simplified version of t-round bargaining game with bargaining cost δ_1 and δ_2 respectively

In the above figure, we assume that EU makes the first decision and period for which bargaining goes is odd i.e. t=1,3,5,...

We had already seen in class, case when **t=1**.When t=1, B's utility from rejecting the offer will be 0 and there B will accept any offer greater than or equal to 0 (as the offer is continuous). EU can maximise its profits by minimising its offer to B. As 0 is the minimum offer that EU can give that B can accept, solution will be: *EU offers 0 and B accepts all.* Similarly, we can solve **2 round bargaining problem** using backward induction. Starting from back, the first subgame we encounter has the same structure as game

with t=1 with just a slight modification, that the two players have exchanged their positions and maximum payoff that a player can now get is δ_2 (due to bargaining costs involved to acknowledge the fact that time is money!). Therefore, solution to this subgame will be, B offers 0 and EU accepts all offers. The maximum payoff that B gets now is δ_2. Now, in the next subgame, EU has to offer B at least δ_2 as otherwise B being rational will figure out that if it rejects this offer, it can keep all of δ_2 for itself and offer EU nothing. So the SPNE of this game will be: *EU offers δ_2 and B accepts all offers such that its payoff is greater than δ_2 and reject otherwise. If B rejects the offer then B offers nothing to EU and EU accepts all the offers.*

Similarly, for any general finite t, the payoff of the two players are as follows:

When t=2m (even)

$$U_1(t) = (1 - \delta_2)(1 + (\delta_1\delta_2) + (\delta_1\delta_2)^2 + \cdots + (\delta_1\delta_2)^{(m-1)})$$
$$= (1 - \delta_2)\left[\frac{1 - (\delta_1\delta_2)^m}{1 - (\delta_1\delta_2)}\right] \tag{6}$$
$$U_2(t) = 1 - U_1(t)$$

When t=2m+1(odd)

$$U_1(t) = (1 - \delta_2)(1 + (\delta_1\delta_2) + (\delta_1\delta_2)^2 + \cdots + (\delta_1\delta_2)^{(m-1)}) + (\delta_1\delta_2)^m$$
$$= (1 - \delta_2)\left[\frac{1 - (\delta_1\delta_2)^m}{1 - (\delta_1\delta_2)}\right] + (\delta_1\delta_2)^m \tag{7}$$
$$U_2(t) = 1 - U_1(t)$$

Clearly, as $m \to \infty$, the following model reduces to a **Rubinstein's Model**.

3. Conclusion

From the above game, we come to the following conclusion: If the bargaining rounds are finite then the player who makes the last offer wins if there are no bargaining costs involved. However, realistically, Bargaining cost do occur as Utility of the players will begin decreasing if the bargaining process goes on for a long time. Ultimately, the game will be governed as Rubinstein model and thus the outcome will depend on the patience level denoted by $\delta's$(higher delta implying higher level of patience). So, if EU wants the decision to be taken quickly, it will have to pay more compensation to UK. While

if UK wants the decision to be taken quickly, which is actually the case as UK has preponed the referendum from 2017 to June 2016, it will provide EU with more bargaining power and hence it will result in lower payoff. All this analysis, also, has a very strong assumption that EU is the first mover. If we want to waive off this assumption we can try finding the Nash Bargaining solution, if we know how the utilities of EU and UK depend on the compensation. The Nash solution will be found to satisfy the following properties: Pareto Optimality, Independence of scale, unit and irrelevant alternatives and thus would provide a axiomatic bargaining solution to the problem.

What happens in England, will be out to the world on 23 June 2016. However Game theory tells us that it would be better if UK postpones the referendum or has enough rounds of bargaining to get a better economic compensation or political concession.

4. References

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