Image Segmentation using Dirichlet Process Mixture Models

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Abstract

Image Segmentation - a fundamental task in computer vision and image processing. Here we look at

- Unconstrained Dirichlet process mixture (MDP) model for image segmentation
- Markov random field (MRF) constrained MDP to introduce spatial coupling for coherent segments
- Compare this techniques with other popular segmentation technique like K- means and Mean shift algorithm
- Performance evaluation of MDP-MRF model on IBSR dataset
- Discuss distance dependent CRP as an alternative to MDP-MRF model for clustering

Introduction

- One of the core issue with image segmentation is the choice of number of segments
- MDP provides a Bayesian framework for clustering problems by treating number of clusters as a random variable
- MRF takes local dependence of cluster allocation in account and thus provide spatial coherence in the image

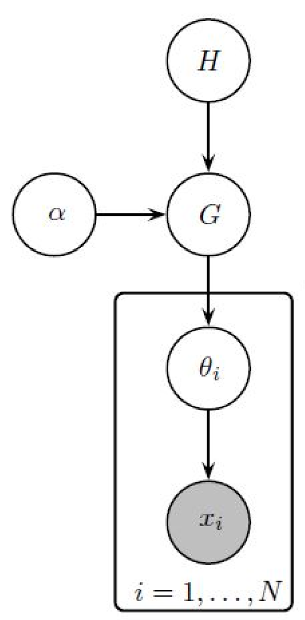


Figure 1: Graphical model of MDP

Mathematical Details

Generative Story of the model

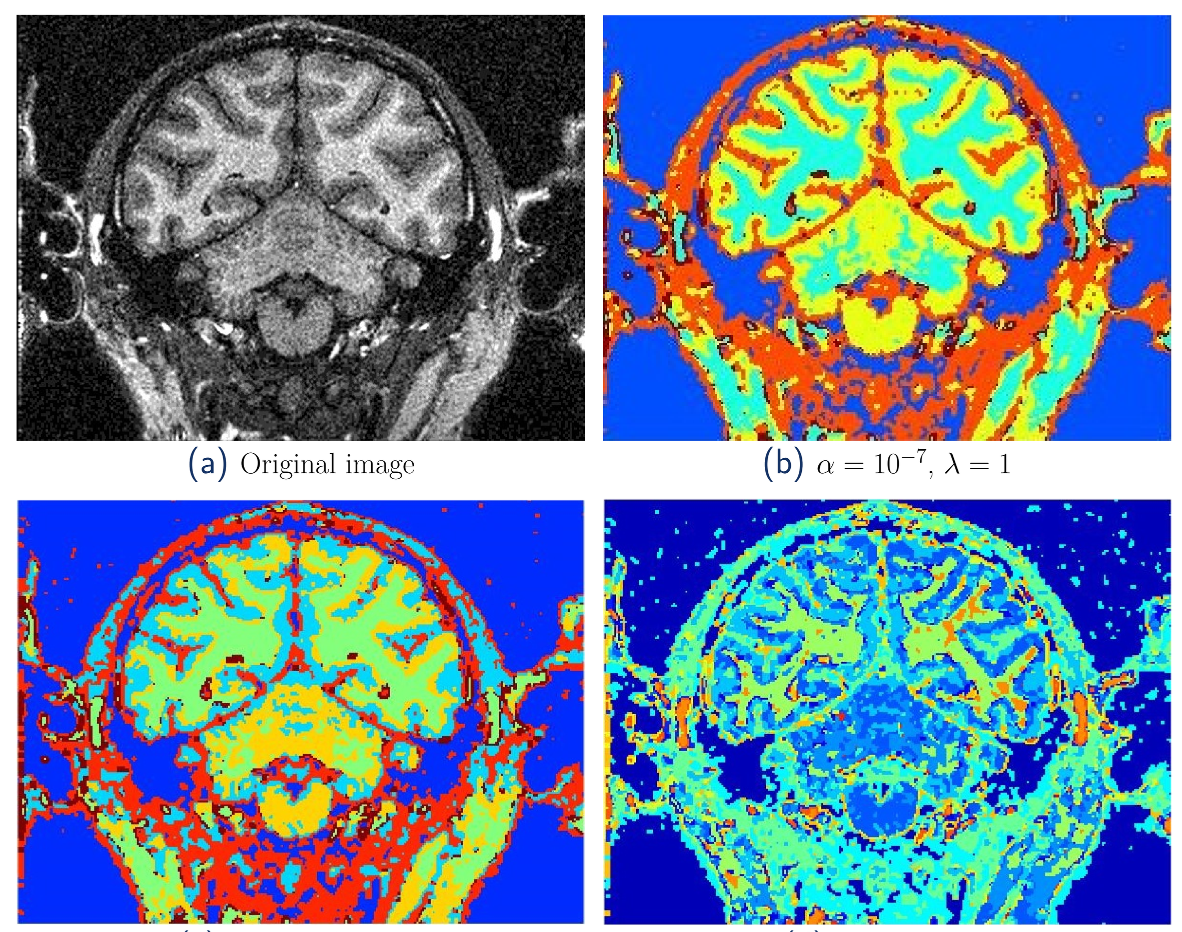
$$\begin{aligned} G &\sim DP(\alpha, H) \\ \theta_i | G &\sim M(\theta_i|\theta_{-i})G(\theta_i) \\ \mathbf{h_i} &\sim Mult(h_i|\theta_i) \end{aligned} \tag{1}$$

- M is the MRF contribution to the prior and is of form $M(\theta_i|\theta_{-i}) \propto exp(-H(\theta_i|\theta_{-i}))$ where $H(\theta_i|\theta_{-i})$ is the cost function defined on the neighborhood graph defined as follows

$$H(\theta_i|\theta_{-i}) = \Sigma_{l \in \delta(i)} \delta_{\theta_i, \theta_l}$$

Analysis of MDP-MRF parameters

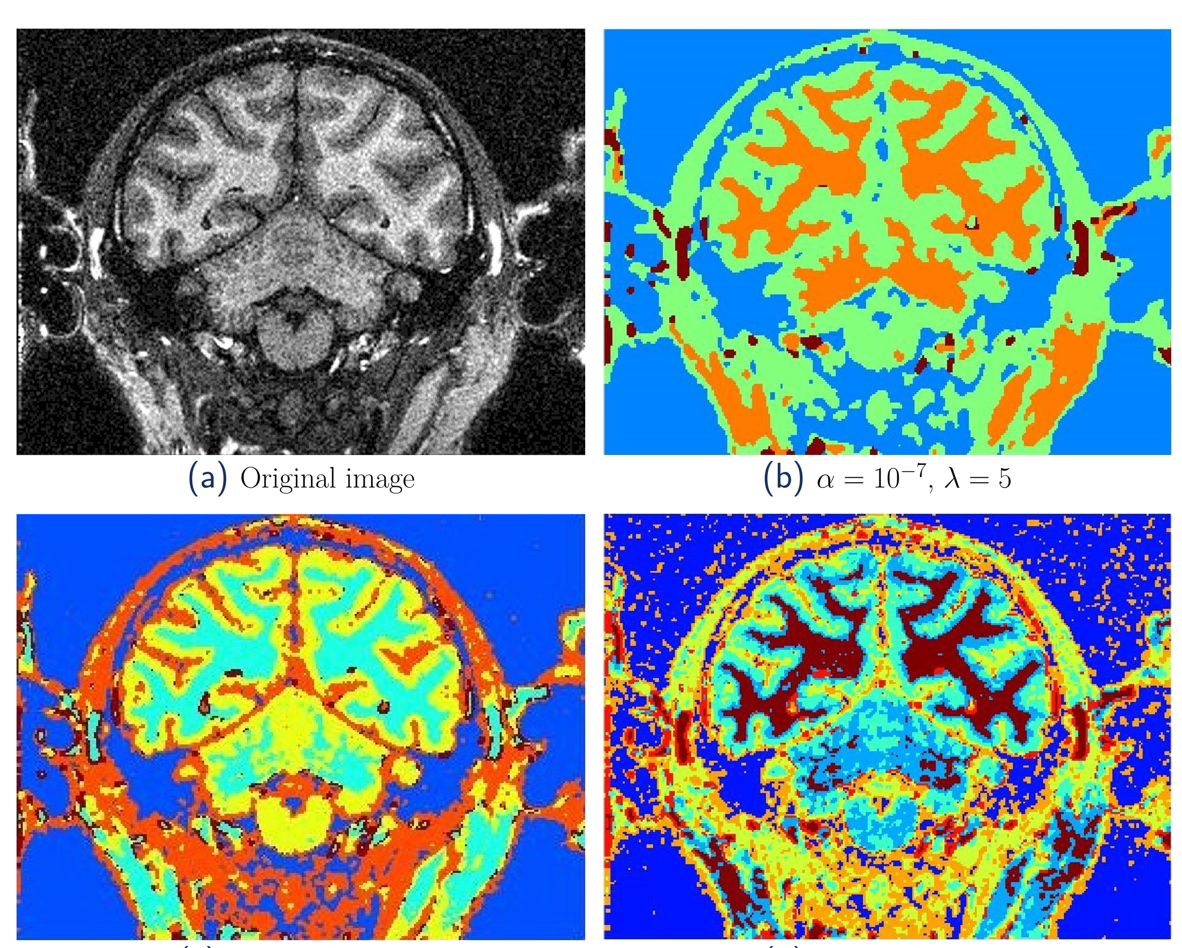
As α increases, the number of clusters also increase leading to over-segmented and granular type image. MR images are difficult to segment because the noisy and spatially incoherent structure triggers the number of segments to increase even further.



(a) Original image (b) $\alpha = 10^{-7}, \lambda = 1$
(c) $\alpha = 10^{-4}, \lambda = 1$ (d) $\alpha = 1, \lambda = 1$

Figure 2: The following images shows the original image (top-left) and segmented images with α and λ mentioned below the images. Increasing the value of α leads to more clusters and therefore over-segmentation.

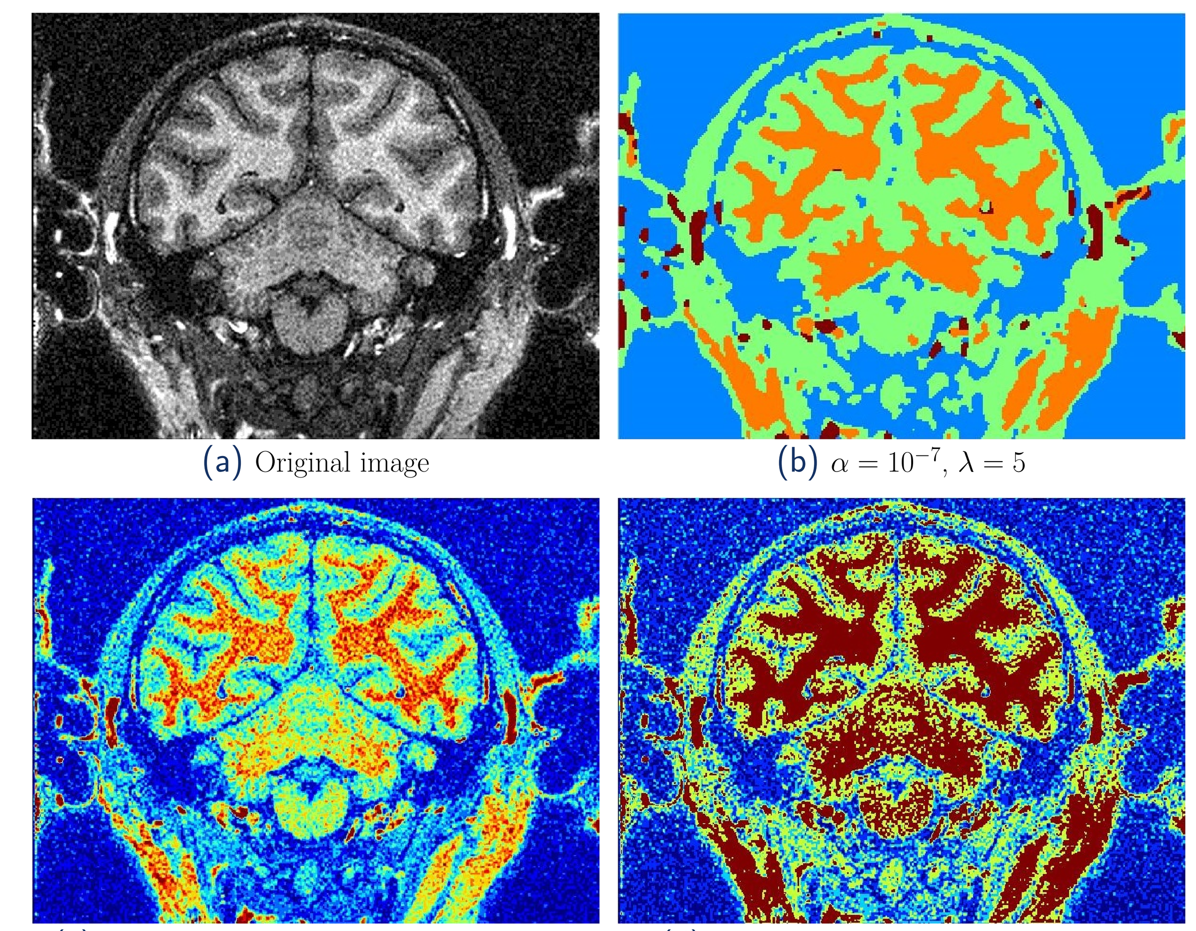
Decreasing λ results in poorer spatial coherency in resultant image as low cost of smoothing parameter leads to less influence of neighbors assignment.



(a) Original image (b) $\alpha = 10^{-7}, \lambda = 5$
(c) $\alpha = 10^{-7}, \lambda = 1$ (d) $\alpha = 10^{-7}, \lambda = 0$

Figure 3: The following images shows the original image (topleft) and segmented images with α and λ mentioned below the images. Increasing the value of λ leads to strong smoothing and ensure local smoothness while doing image segmentation.

Comparison with conventional Image segmentation algorithms



(a) Original image (b) $\alpha = 10^{-7}, \lambda = 5$
(c) Mean shift; Kernel bandwidth bw = 0.2 (d) K-means; Number of clusters K = 5

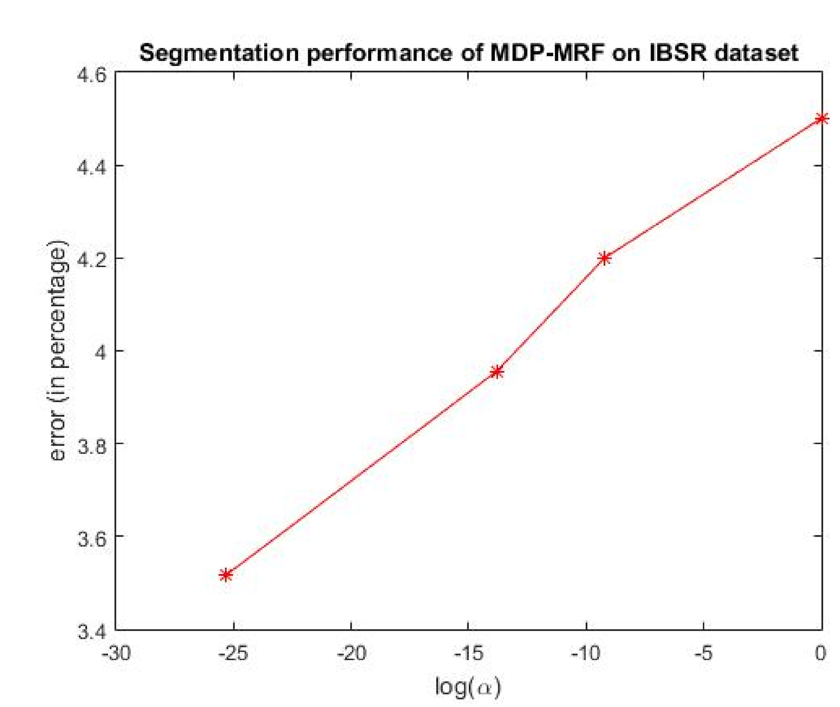
Data Set Description

Performance of the algorithm was evaluated with real MR images consisting of 15 different individual slices of a single patient's data provided by the Internet Brain Segmentation Repository (IBSR), center for Morphometric Analysis at Massachusetts General Hospital and was tested with its ground truth classification performed by trained investigators using a semi- automated intensity contour mapping algorithm.

Comparison Metric and Results

To compare the ground truth classification image with the algorithm generated segmented image, we use the following technique. Let us say that the two cluster configuration of n points is denoted by $\mathcal{C}$ and $\mathcal{C}'$. If n_{11} denotes the pair of points that lie in the same cluster in both the configuration and n_{00} denotes the pair of points that lie in different cluster in the two configurations. Then similarity index is evaluated as

$$S = \frac{(n_{11} + n_{00})}{\binom{n}{2}}$$



Distance dependent CRP

Image segmentation data is not exchangeable. Each datum tends to cluster with other data close to it externally. Distance dependent Chinese restaurant process is a random seating assignment of the customers depends on the distances between them.Some key points regarding the same are as follows:

- It represents the partition with customer assignments, rather than table assignments in the tradition CRP.
- It independently draws the customer assignments conditioned on the distance measurements

$$P(c_i = j|D, \alpha) = \begin{cases} f(d_{ij}) & : i \neq j \\ \alpha & : i = j \end{cases}$$

where f(d) is a decay function of distance.

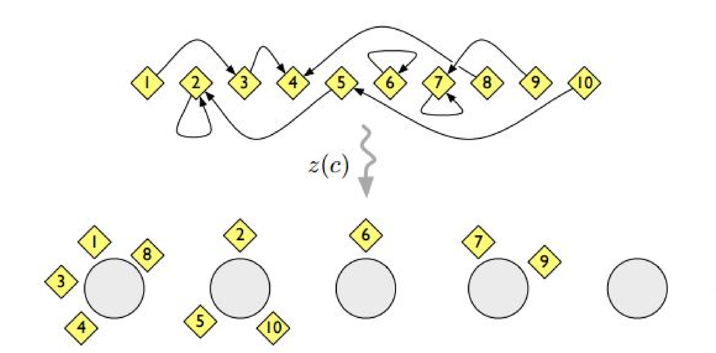


Figure 4: Illustration of distance dependent CRP.An edge in the first graph shows the customer assignment of a particular customer which can be used to get the table assignment of customers. *Pic courtesy:Blei and Frazier*

Conclusion

- The choice of control parameter α is critical. It affects the number of clusters that will be formed
- MRF cost parameter λ decides the extent of dependence of a site on its neighbours in the neighborhood graph
- ddCRP can be an alternative to MDP-MRF model as it models customer assignments using decay function of distance

References

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