Lecture-21-22
Course: Applied Data Science

Fuzzy Time Series Forecasting

By

Dr. Sibarama Panigrahi

Senior Member, IEEE

Assistant Professor, Department of Computer Sc. & Engineering

National Institute of Technology, Rourkela, Odisha, 769008, India

Mobile No.: +91-7377302566

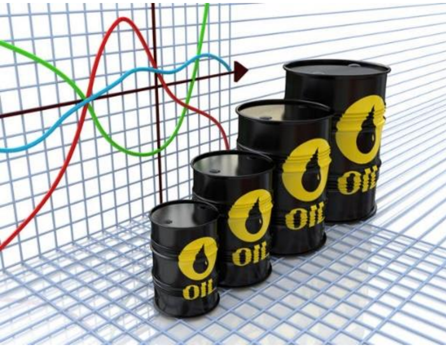
Email: panigrahis[at]nitrkl[dot]ac[dot]in

panigrahi[dot]sibarama[at]gmail[dot]com

Outlines

- Introduction and Motivation
- **Fuzzy Time Series Forecasting using Optimized Deep Learning Models**
  - Ignoring Membership Values
    - Traditional Fuzzy Set
  - Considering Membership Values
    - Traditional Fuzzy Set
    - Intuitionistic Fuzzy Set
    - Hesitant Fuzzy Set
    - Neutrosophic Fuzzy Set
- References

MOTIVATION



Crude Oil

Stock Price

Retail Industry

Internet Traffic

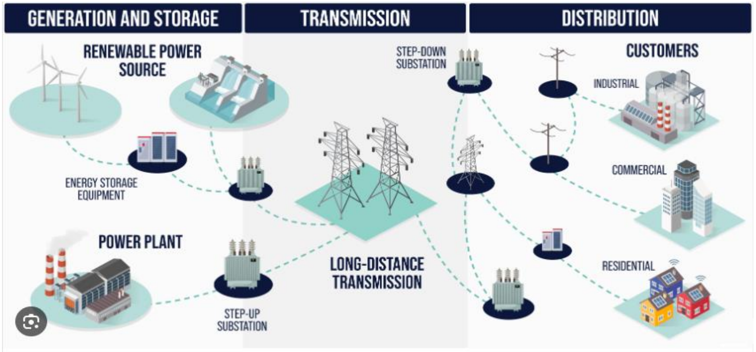
Electricity Price

Call Volume

Flood

Earthquake

MOTIVATION



Electricity Load Forecasting

Air Quality Index Forecasting

Rainfall Forecasting

Streamflow Forecasting

Agricultural Product Price Forecasting

Seed Demand Forecasting

Wind Speed Forecasting

Temperature Forecasting

:

:

:

INTRODUCTION

- Time Series
  - A (crisp) time series is a set of observations of the same variable measured sequentially through time.

y_1 | y_2 | y_3 | …. | y_n

- Fuzzy Time Series
  - A fuzzy time series for a crisp time series is a set of linguistic terms $f_1(t)$, $f_2(t)$,…, $f_n(t)$ which is obtained by fuzzifying the crisp time series y.

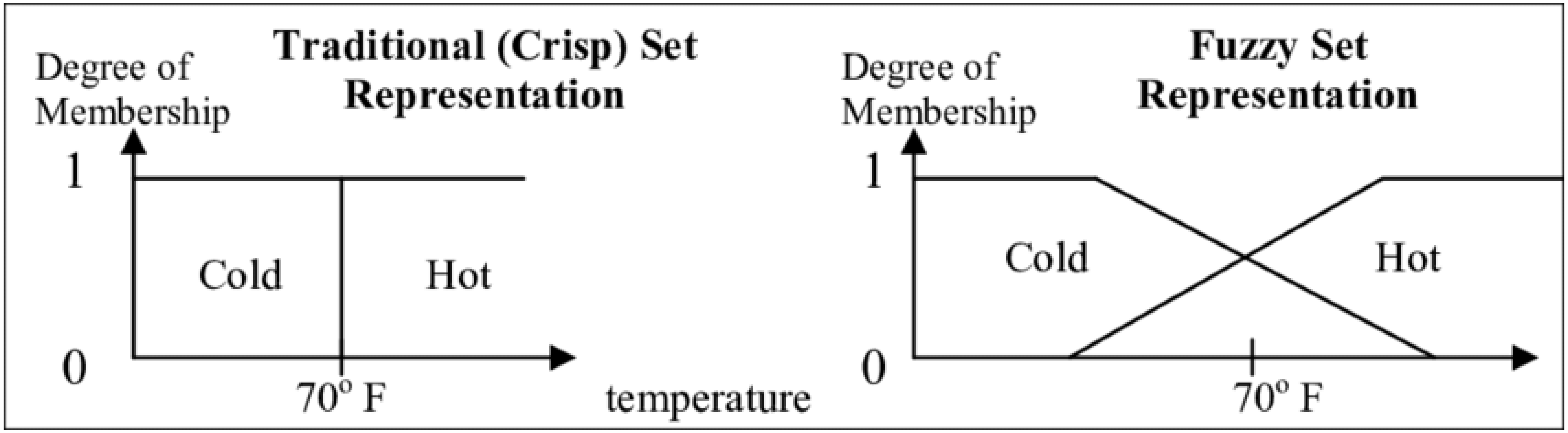
INTRODUCTION

- **Crisp Time Series Forecasting:** Crisp Time series forecasting **(TSF)** is the *process of predicting* the *future outcomes* of a phenomenon by ***systematically analyzing its past observations***.

- **Fuzzy Time Series Forecasting:** Fuzzy Time series forecasting **(TSF)** is the *process of predicting* the *future outcomes* of a phenomenon by ***systematically analyzing its past observations in*** **Fuzzy domain**.

INTRODUCTION

- **Conventional set theory** rests on the notion of a *crisp boundary between which elements are members and non-members of a particular set.*
- **Fuzzy set theory** expands the notion of purely crisp sets by *assigning membership degrees* to set elements so the transition from membership to non-membership is gradual rather than abrupt.



INTRODUCTION

- **The Universe of Discourse**: All elements in a set are taken from a *universe of discourse* or *universe set* that contains all the elements that can be taken into consideration when the set is formed.

INTRODUCTION

- **Fuzzy Subsets**:
  - A fuzzy subset *A* in *U* is characterized by a ***membership function*** **(characteristic function)** that *maps each element in A with a real number in the unit interval.*
  - Expressed as μ_A: $U \rightarrow [0,1]$ where the value $\mu_A(x)$ is called the degree of membership of the element x in the fuzzy set A.

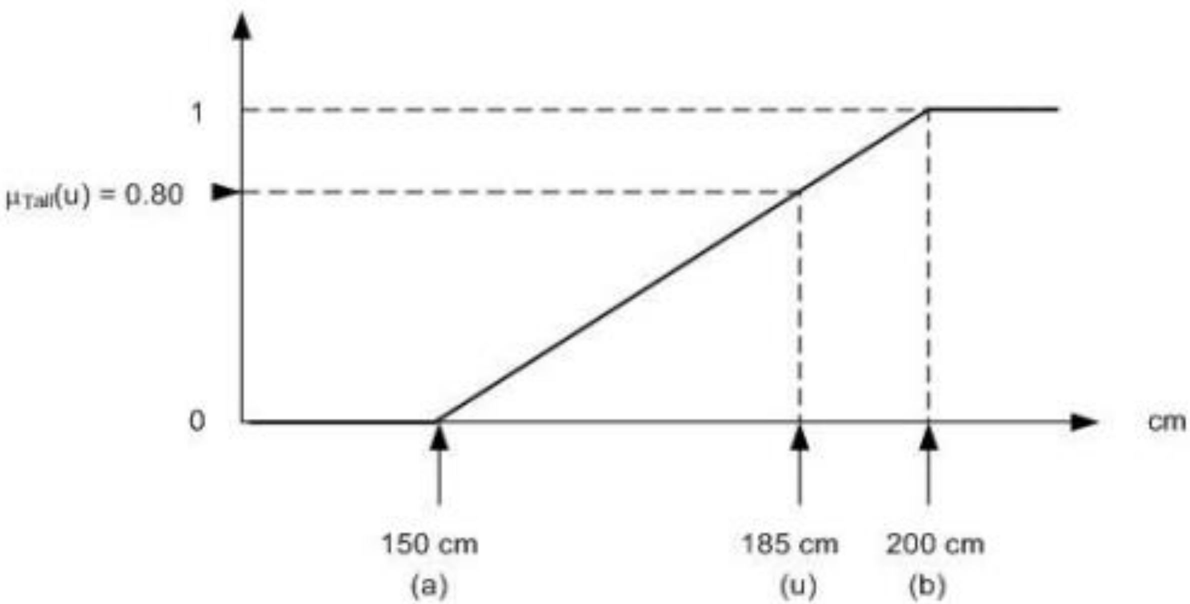
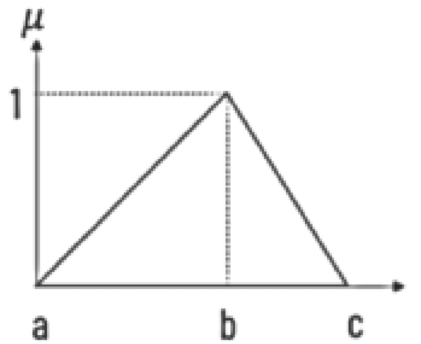


Figure 1. An example of a membership function for the fuzzy set Tall.

INTRODUCTION

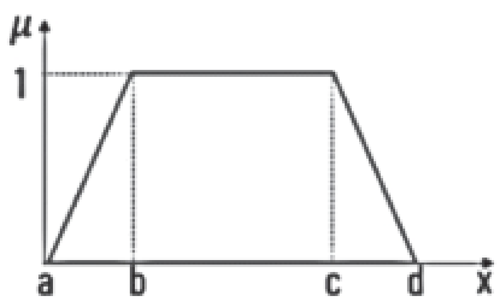
- **Membership Function:**



Triangular membership function

$$\mu_{triangle}(x; a, b, c) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a \le x \le b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0, & c \le x \end{cases}$$

$$= max(min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0)$$



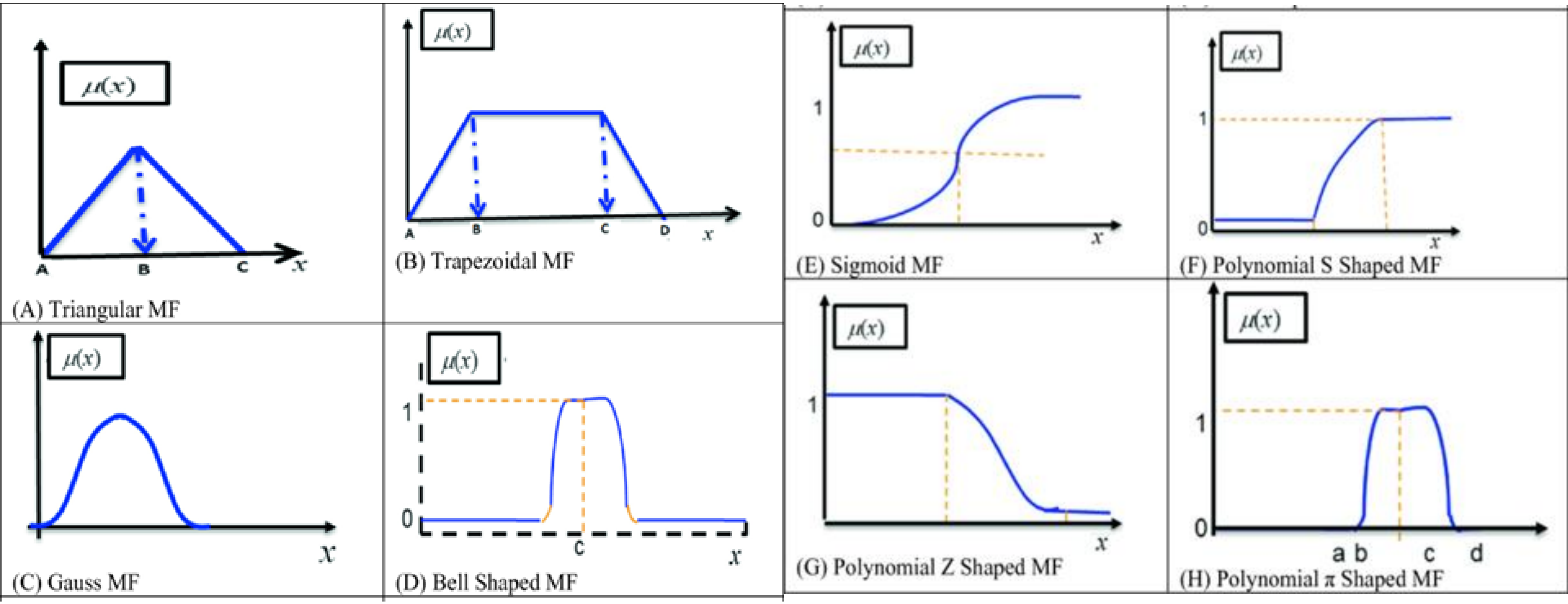
Trapezoidal membership function

$$\mu_{triangle}(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \\ 0, & d \le x \end{cases}$$

$$= max(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0)$$

INTRODUCTION

Membership Functions:



INTRODUCTION

- **Linguistic Variables**:
  - Linguistic variables take words or sentences as values, as opposed to an algebraic variable which takes numbers as values.
  - Temperature = {5, 12, 15, 22, 34, 38, 44}
  - **Temperature using Linguistic Variables = {Very Cold, Cold, Cold, Medium, hot, hot, Very hot}**

Linguistic Variable	Interval
Very Cold	0-10
Cold	11-20
Normal	21-30
Hot	31-40
Very Hot	41-50

INTRODUCTION

- **Fuzzification**: Fuzzification is the process of transforming a crisp set to a fuzzy set

- **Defuzzification:** Defuzzification is the process of reducing a fuzzy set into a crisp set.
  - **Max-membership principle**: A=0.3/10+0.45/12+0.6/15+0.9/17.
    Then Z=max (A)=17.
  - **Centroid method** :

$$Z=\frac{\sum_{i=1}^{n}\mu_A(x_i)x_i}{\sum_{i=1}^{n}\mu_A(x_i)}. \quad Z=\frac{(0.3\cdot 10)+(0.45\cdot 12)+(0.6\cdot 15)+(0.9\cdot 17)}{0.3+0.45+0.6+0.9}=14.53$$

  - **Mean-Max method**: A=0.3/10+0.45/12+0.9/15+0.9/17

$$Z = \frac{15+17}{2} = 6$$

INTRODUCTION

- **Intuition behind Fuzzy Time Series Forecasting:**

Temperature Time Series: 5, 11, 17, 23, 29, 32, 42, 45
Universe of Discourse: [5-5 - 45+5] = [0-50]

Temperature Time Series: 5, 11, 17, 23, 29, 28, 32, 45
Fuzzy Time Series: 0, 1, 1, 2, 2, 2, 3, 4

Identify the Order of the fuzzy TSF Model: (Let 2)

Convert the fuzzy time series into Fuzzy Logical Relationships
0, 1 → 1
1, 1 → 2
1, 2 → 2
2, 2 → 2
2, 2 → 3 2,2 →2,3
2, 3 → 4

Linguistic Variable	Interval	Interval Index	Mid-Point
Very Cold	0-10	0	5
Cold	11-20	1	15
Normal	21-30	2	25
Hot	31-40	3	35
Very Hot	41-50	4	45

Model the Fuzzy Logical Relationships using Optimized DL Models and Forecast the fuzzy future values.

Fuzzified Forecasts: 2, 3, 3, 4, 4
Defuzzify Forecasts: 25, 35, 35, 45, 45

INTRODUCTION

- **Aggregation:** The purpose of aggregation is to aggregate pieces of data in a desirable way in order to reach a conclusion or final decision.

Operator	Equation
The arithmetic mean	$\frac{1}{n}\sum_{i=1}^{n} x_i$
Weighted arithmetic mean	$\sum_{i=1}^{n} w_i \cdot x_i$ where $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$
Geometric mean	$\left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}}$
Harmonic mean	$\frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$
Quadratic mean	$\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$
Median	Sort the arguments in ascending order. If the number of arguments n is odd, then the middle value is selected. If n is even, then take the mean of the middle pair.
Min and max	$\min(x_1, \ldots, x_n)$ $\max(x_1, \ldots, x_n)$

INTRODUCTION

- **Fuzzy Time Series and its Definitions:**

Definition 1: Fuzzy Time Series

Let $Y(t)(t = \ldots, 0,1,2,\ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_i(t)(i=1,2,\ldots)$ are defined. If $F(t)$ is a collection of $f_i(t)(i=1,2,\ldots)$, then $F(t)$ is called a fuzzy time series on $Y(t)(t = \ldots, 0,1,2,\ldots)$.

Definition 2: Fuzzy Relation

If there exists a fuzzy relationship $R(t-1,t)$, such that $F(t) = F(t-1) \times R(t-1,t)$, where $\times$ represents an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ is denoted by

$$F(t-1) \rightarrow F(t).$$

INTRODUCTION

- **Fuzzy Time Series and its Definitions:**

Definition 3: *N*-Order Fuzzy Relations

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2),\ldots,F(t-n)$, then this fuzzy relationship is represented by

$$F(t-n),\ldots,F(t-2),\ F(t-1)\rightarrow F(t),$$

and is called an n-order fuzzy time series.

Definition 4: Time-Invariant Fuzzy Time Series

Suppose $F(t)$ is caused by $F(t-1)$ only and is denoted by $F(t-1)\rightarrow F(t)$, then there is a fuzzy relationship between $F(t)$ and $F(t-1)$ which is expressed as the equation:

$$F(t)=F(t-1)\times R(t-1,t).$$

The relation R is referred to as a first order model of $F(t)$. If $R(t-1,t)$ is independent of time t, that is, for different times t_1 and t_2, $R(t_1,t_1-1)=R(t_2,t_2-1)$, then $F(t)$ is called a time-invariant fuzzy time series. Otherwise it is called a time-variant fuzzy time series.

INTRODUCTION

- **Fuzzy Time Series and its Definitions:**

Definition 5: Fuzzy Relationship Group (FLRG)

Relationships with the same fuzzy set on the left hand side can be further grouped into a relationship group. Relationship groups are also referred to as fuzzy logical relationship groups or FLRG 's in short. Suppose there are relationships such that

$$A_i \rightarrow A_{j1,}$$
$$A_i \rightarrow A_{j2,}$$
$$\cdots$$
$$A_i \rightarrow A_{jn,}$$

then they can be grouped into a relationship group as follows:

$$A_i \rightarrow A_{j1}, A_{j2}, \ldots, A_{jn}.$$

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**
  - **Steps:**

1. Partition the universe of discourse into equally lengthy intervals.
2. Define fuzzy sets on the universe of discourse.
3. Fuzzify historical data.
4. Identify fuzzy relationships (FLR's).
5. Establish fuzzy relationship groups (FLRG's).
6. Defuzzify the forecasted output.

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

.

Year	Student enrollments	Year	Student enrollments
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

Historical student enrollments 1971 - 1992, at Alabama University.

Step-1

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 1: Define the universe of discourse and partition it into equally lengthy intervals

The universe of discourse U is defined as $[D_{min} - D_{1,} D_{max} - D_2]$ where D_{min} and D_{max} are the minimum and maximum historical enrollment, respectively. From table 4, we get $D_{min} = 13055$ and $D_{max} = 19337$. The variables D_1 and D_2 are just two positive numbers, properly chosen by the user. If we let $D_1 = 55$ and $D_2 = 663$, we get $U = [13000, 20000]$. Chen used seven intervals which is the same number used in most cases observed in literature. Dividing U into seven evenly lengthy intervals u_1, u_2, u_3, u_4, u_5, u_6 and u_7, we get $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$ and $u_7 = [19000, 20000]$.

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 2: Define fuzzy sets on the universe of discourse

Assume $A_1, A_2, \ldots, A_k$ to be fuzzy sets which are linguistic values of the linguistic variable 'enrollments'. Then the fuzzy sets $A_1, A_2, \ldots, A_k$ are defined on the universe of discourse as

$$A_1 = a_{11}/u_1 + a_{12}/u_2 + \ldots + a_{1m}/u_m,$$
$$A_2 = a_{21}/u_1 + a_{22}/u_2 + \ldots + a_{2m}/u_m,$$
$$\vdots$$
$$A_k = a_{k1}/u_1 + a_{k2}/u_2 + \ldots + a_{km}/u_m,$$

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$$
$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7$$
$$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7$$
$$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7$$
$$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_7$$
$$A_7 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7.$$

Step-3 Fuzzify

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 3: Fuzzify historical data

.

Year	Actual enrollment	Interval	Fuzzified enrollment
1971	13055	[13000, 14000]	A_1
1972	13563	[13000, 14000]	A_1
1973	13867	[13000, 14000]	A_1
1974	14696	[14000, 15000]	A_2
1975	15460	[15000, 16000]	A_3
1976	15311	[15000, 16000]	A_3
1977	15603	[15000, 16000]	A_3
1978	15861	[15000, 16000]	A_3
1979	16807	[16000, 17000]	A_4
1980	16919	[16000, 17000]	A_4
1981	16388	[16000, 17000]	A_4
1982	15433	[15000, 16000]	A_3
1983	15497	[15000, 16000]	A_3
1984	15145	[15000, 16000]	A_3
1985	15163	[15000, 16000]	A_3
1986	15984	[15000, 16000]	A_3
1987	16859	[16000, 17000]	A_4
1988	18150	[18000, 19000]	A_6
1989	18970	[18000, 19000]	A_6
1990	19328	[19000, 20000]	A_7
1991	19337	[19000, 20000]	A_7
1992	18876	[18000, 19000]	A_6

Fuzzified historical enrollments.

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 4: Identify fuzzy relationships

$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	$A_2 \rightarrow A_3$	$A_3 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_6$
$A_6 \rightarrow A_6$	$A_6 \rightarrow A_7$	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_6$

Note: Look at the Distinctness

.

Year	Actual enrollment	Interval	Fuzzified enrollment
1971	13055	[13000, 14000]	A_1
1972	13563	[13000, 14000]	A_1
1973	13867	[13000, 14000]	A_1
1974	14696	[14000, 15000]	A_2
1975	15460	[15000, 16000]	A_3
1976	15311	[15000, 16000]	A_3
1977	15603	[15000, 16000]	A_3
1978	15861	[15000, 16000]	A_3
1979	16807	[16000, 17000]	A_4
1980	16919	[16000, 17000]	A_4
1981	16388	[16000, 17000]	A_4
1982	15433	[15000, 16000]	A_3
1983	15497	[15000, 16000]	A_3
1984	15145	[15000, 16000]	A_3
1985	15163	[15000, 16000]	A_3
1986	15984	[15000, 16000]	A_3
1987	16859	[16000, 17000]	A_4
1988	18150	[18000, 19000]	A_6
1989	18970	[18000, 19000]	A_6
1990	19328	[19000, 20000]	A_7
1991	19337	[19000, 20000]	A_7
1992	18876	[18000, 19000]	A_6

Fuzzified historical enrollments.

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 5: Establish fuzzy relationship groups (FLRG's)

If the same fuzzy set is related to more than one set, the right hand sides are merged. We refer to this process as the establishment of FLRG's. For example, we see from table 6 that A_1 is related to itself and to A_2. This provides the following FLRG: $A_1 \rightarrow A_1, A_2$. A complete overview of the relationship groups obtained from table 6 is shown table 7.

Group 1:	$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	
Group 2:	$A_2 \rightarrow A_3$		
Group 3:	$A_3 \rightarrow A_3$	$A_3 \rightarrow A_4$	
Group 4:	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_6$
Group 5:	$A_6 \rightarrow A_6$	$A_6 \rightarrow A_7$	
Group 6:	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_6$	

FLRG's.

$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	$A_2 \rightarrow A_3$	$A_3 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_4 \rightarrow A_6$
$A_6 \rightarrow A_6$	$A_6 \rightarrow A_7$	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_6$

Fuzzy set relationships.

INTRODUCTION

•

Year	Actual enrollment	Forecasted enrollment	FLRG's	Interval midpoints
1971	13055		$A_1 \rightarrow A_1, A_2$	13500; 14500
1972	13563	14000	$A_1 \rightarrow A_1, A_2$	13500; 14500
1973	13867	14000	$A_1 \rightarrow A_1, A_2$	13500; 14500
1974	14696	14000	$A_2 \rightarrow A_3$	15500
1975	15460	15500	$A_3 \rightarrow A_3, A_4$	15500; 16500
1976	15311	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1977	15603	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1978	15861	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1979	16807	16000	$A_4 \rightarrow A_3, A_4, A_6$	15500; 16500; 18500
1980	16919	16833	$A_4 \rightarrow A_3, A_4, A_6$	15500; 16500; 18500
1981	16388	16833	$A_4 \rightarrow A_3, A_4, A_6$	15500; 16500; 18500
1982	15433	16833	$A_3 \rightarrow A_3, A_4$	15500; 16500
1983	15497	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1984	15145	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1985	15163	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1986	15984	16000	$A_3 \rightarrow A_3, A_4$	15500; 16500
1987	16859	16000	$A_4 \rightarrow A_3, A_4, A_6$	15500; 16500; 18500
1988	18150	16833	$A_6 \rightarrow A_6, A_7$	18500; 19500
1989	18970	19000	$A_6 \rightarrow A_6, A_7$	18500; 19500
1990	19328	19000	$A_7 \rightarrow A_6, A_7$	18500; 19500
1991	19337	19000	$A_7 \rightarrow A_6, A_7$	18500; 19500
1992	18876	19000		

Forecasted enrollments for the period 1972 - 1992.

Defuzzify Rules

INTRODUCTION

- **Fuzzy Time Series Forecasting Model**

Step 6: Defuzzify the forecasted output

Assume the fuzzified enrollment of $F(t-1)$ is A_j, then forecasted output of $F(t)$ is determined according to the following principles:

1. If there exists a one-to-one relationship in the relationship group of A_j, say $A_j \to A_k$, and the highest degree of belongingness of A_k occurs at interval u_k, then the forecasted output of $F(t)$ equals the midpoint of u_k.

2. If A_j is empty, i.e. $A_j \to \emptyset$, and the interval where A_j has the highest degree of belongingness is u_j, then the forecasted output equals the midpoint of u_j.

3. If there exists a one-to-many relationship in the relationship group of A_j, say $A_j \to A_1, A_2, \ldots, A_n$, and the highest degrees of belongingness occurs at set $u_1, u_2, \ldots, u_n$, then the forecasted output is computed as the average of the midpoints $m_1, m_2, \ldots, m_n$ of $u_1, u_2, \ldots, u_n$. This equation can be expressed as:

$$\frac{m_1 + m_2 + \ldots + m_n}{n}.$$

FUZZY TIME SERIES FORECASTING USING OPTIMIZED DL MODELS

The universe of the discourse of the time series will be determined which will be divided into a number of intervals.

↓

Fuzzify the Time Series by exchanging each observation of the time series by the index of the belonging interval of the observation.

↓

The order of the fuzzy TSF model will be determined by using the autocorrelation and partial autocorrelation function of the fuzzy time series (FTS)

↓

The Fuzzy Logical Relationships (FLRs) will be established and modeled using the aforementioned ML/DL models.

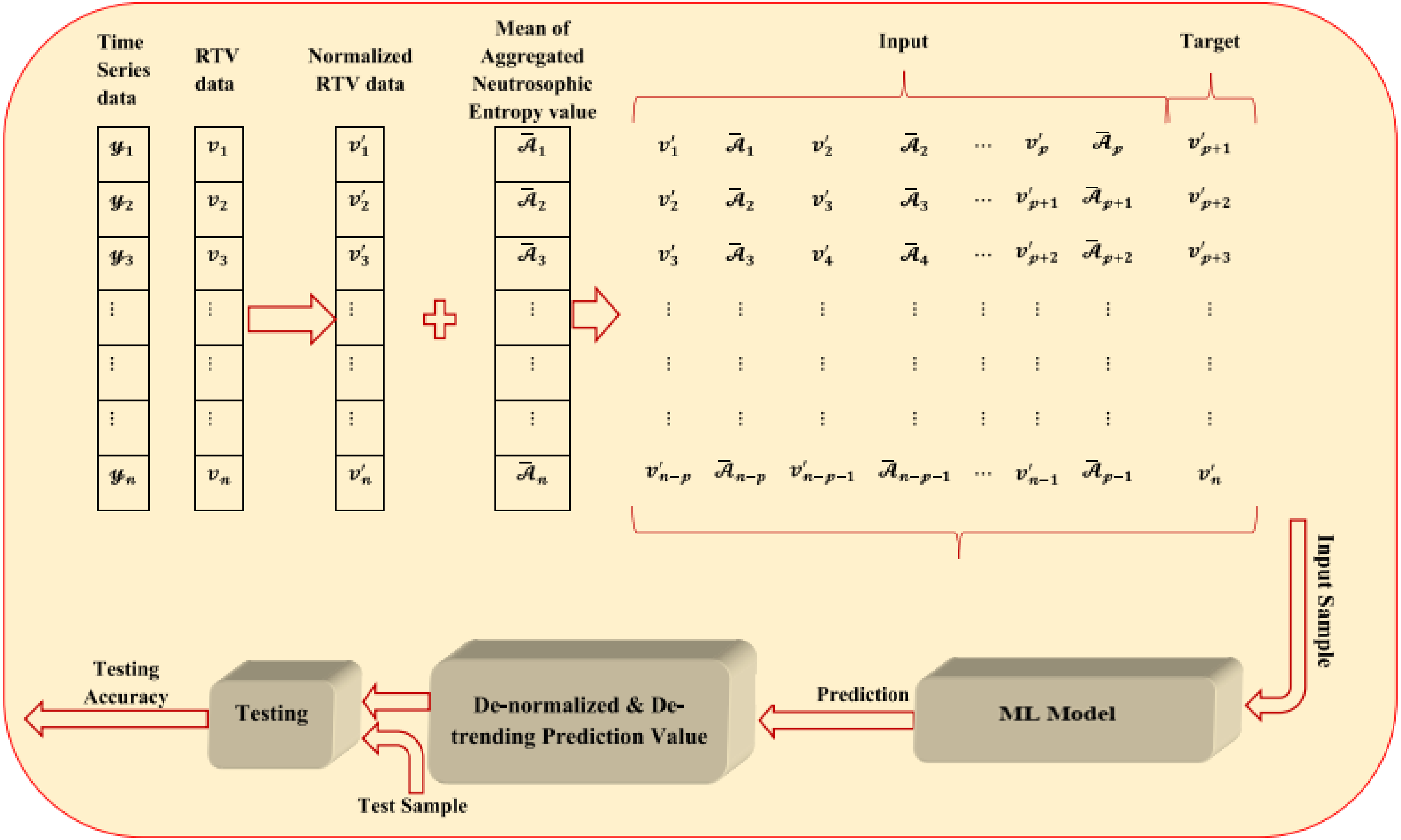
↓

Defuzzify and measure the forecasting accuracy.

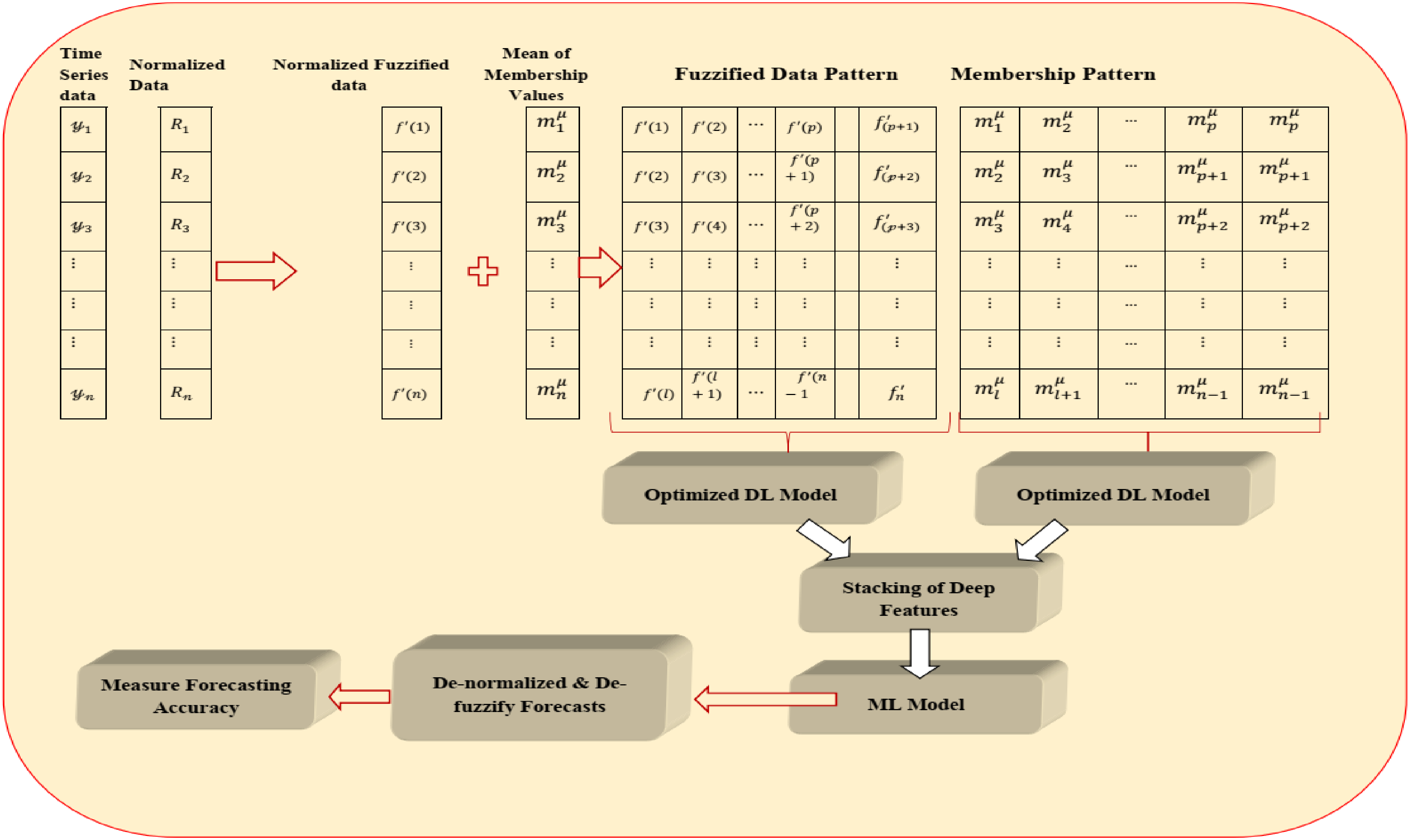
Fuzzy Time Series Forecasting using Optimized Deep Learning Models

- **Key Factors affecting the efficiency of Fuzzy Time Series Forecasting Methods:**
  - **Partitioning of Universe of Discourse**
    - **Equal-Length Partitioning Methods**
      - » **Average-Based Methods**
      - » **Modified Average-Based Methods**
      - » **Ratio-Based Methods**
    - **Variable-Length Partitioning Methods**
      - » **Cumulative Probability Distribution Approach**
      - » **Using Swarm and Evolutionary Algorithms**
      - » **Using Fuzzy-C-Means Clustering**
  - **Modelling of Fuzzy Logical Relationships**
    - **Using Rule based methods**
    - **Using Optimized ML/DL models**
    - **Using Hybrid Models**
  - **Fuzzification and Defuzzification Methods**
  - **Type of Fuzzy Set**
  - **Consideration or Non-Consideration of Membership Values**
  - **Type of Membership Function used.**

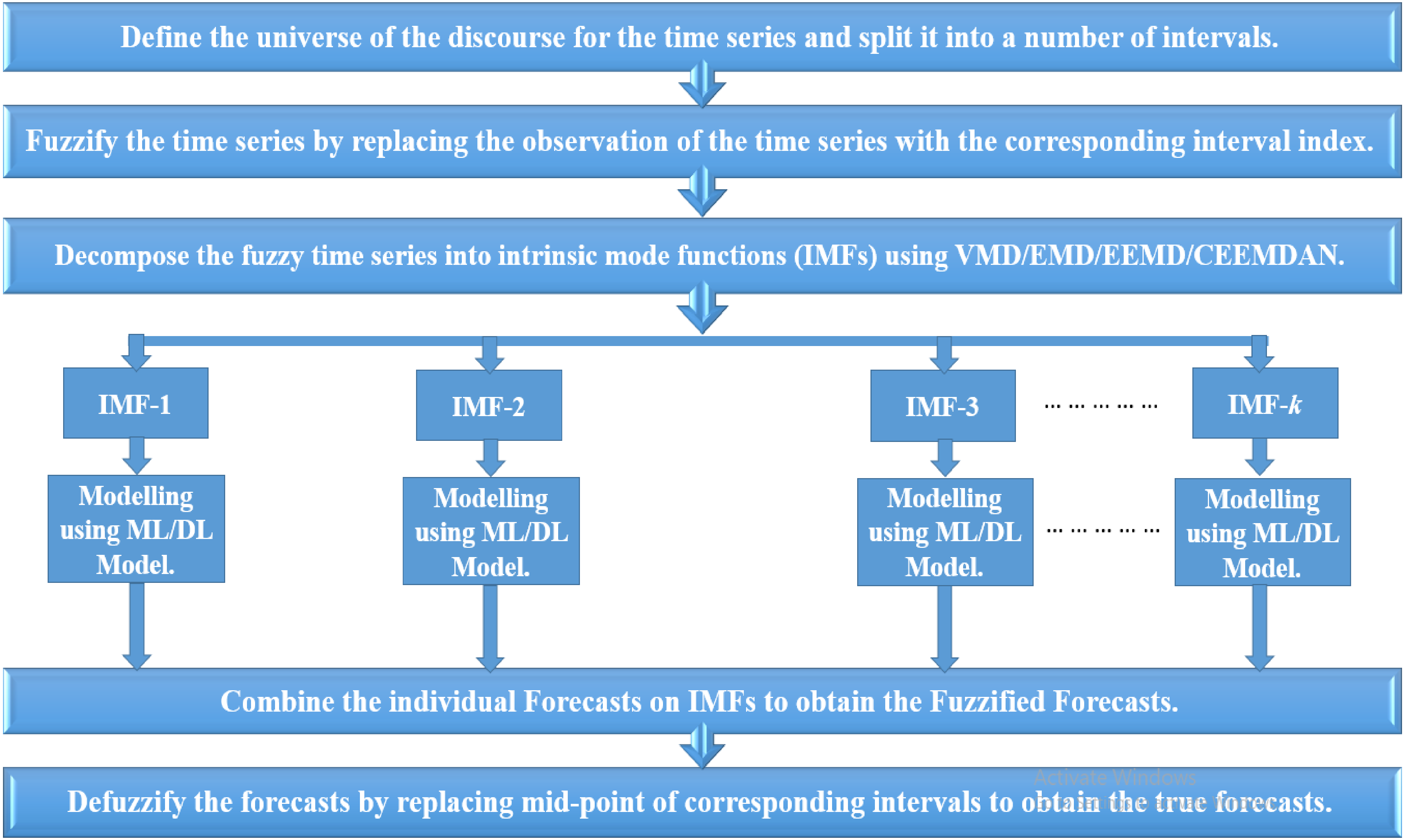
Fuzzy Time Series Forecasting using Optimized Deep Learning Models



Fuzzy Time Series Forecasting using Optimized Deep Learning Models



Fuzzy Time Series Forecasting using Decomposition Based Hybrid Model

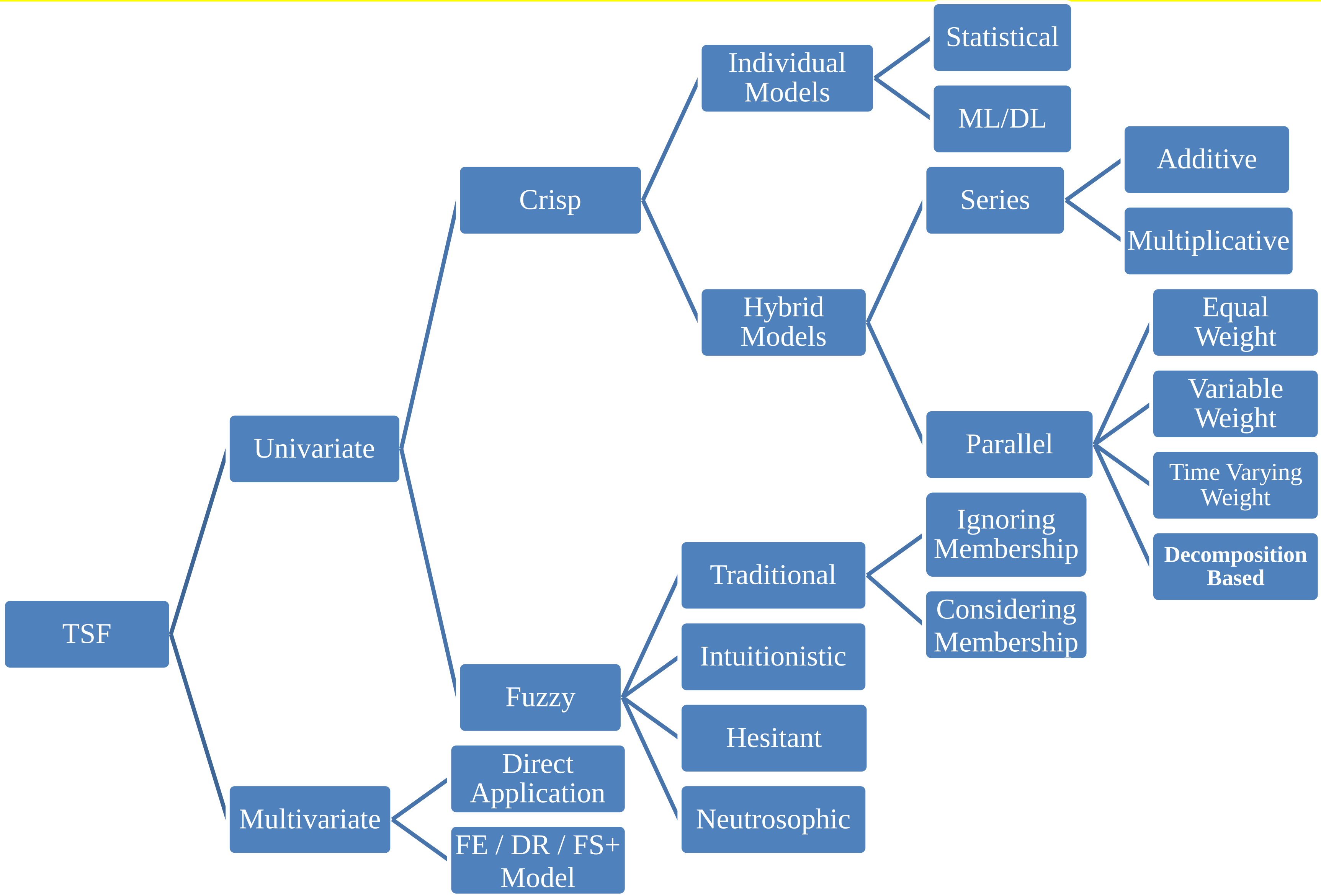


Fuzzy Time Series Forecasting using Optimized Deep Learning Models

Fuzzy Time Series Forecasting using different fuzzy sets:

- **Traditional Fuzzy Set** (Considers only membership value)
- **Intuitionistic Fuzzy Set** (Considers membership and non-membership value)
- **Hesitant Fuzzy Set** (Considers dependent membership, non-membership and hesitation value)
- **Neutrosophic Fuzzy Set** (Considers independent membership, non-membership and hesitation value)
- :
- :

Time Series Forecasting (TSF) Techniques [Classification]



Published Papers

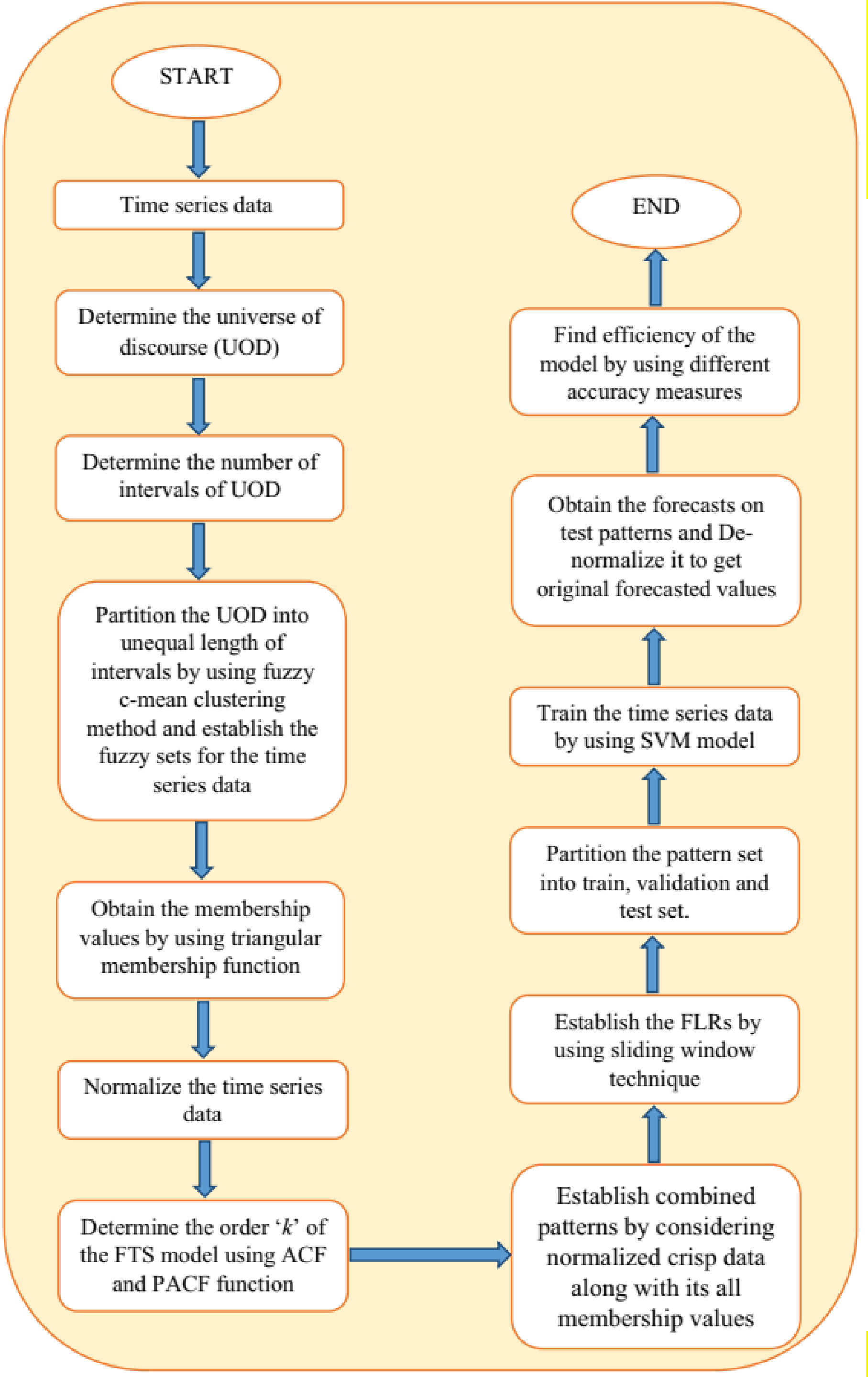
1. **Sibarama Panigrahi**, and Himansu Sekhar Behera. "A study on leading machine learning techniques for high order fuzzy time series forecasting." Engineering Applications of Artificial Intelligence 87 (2020): 103245. **[Elsevier, IF: 7.80, SCI]**

Algorithm 1: FTSF-DBN, FTSF-LSTM and FTSF-SVM Methodology

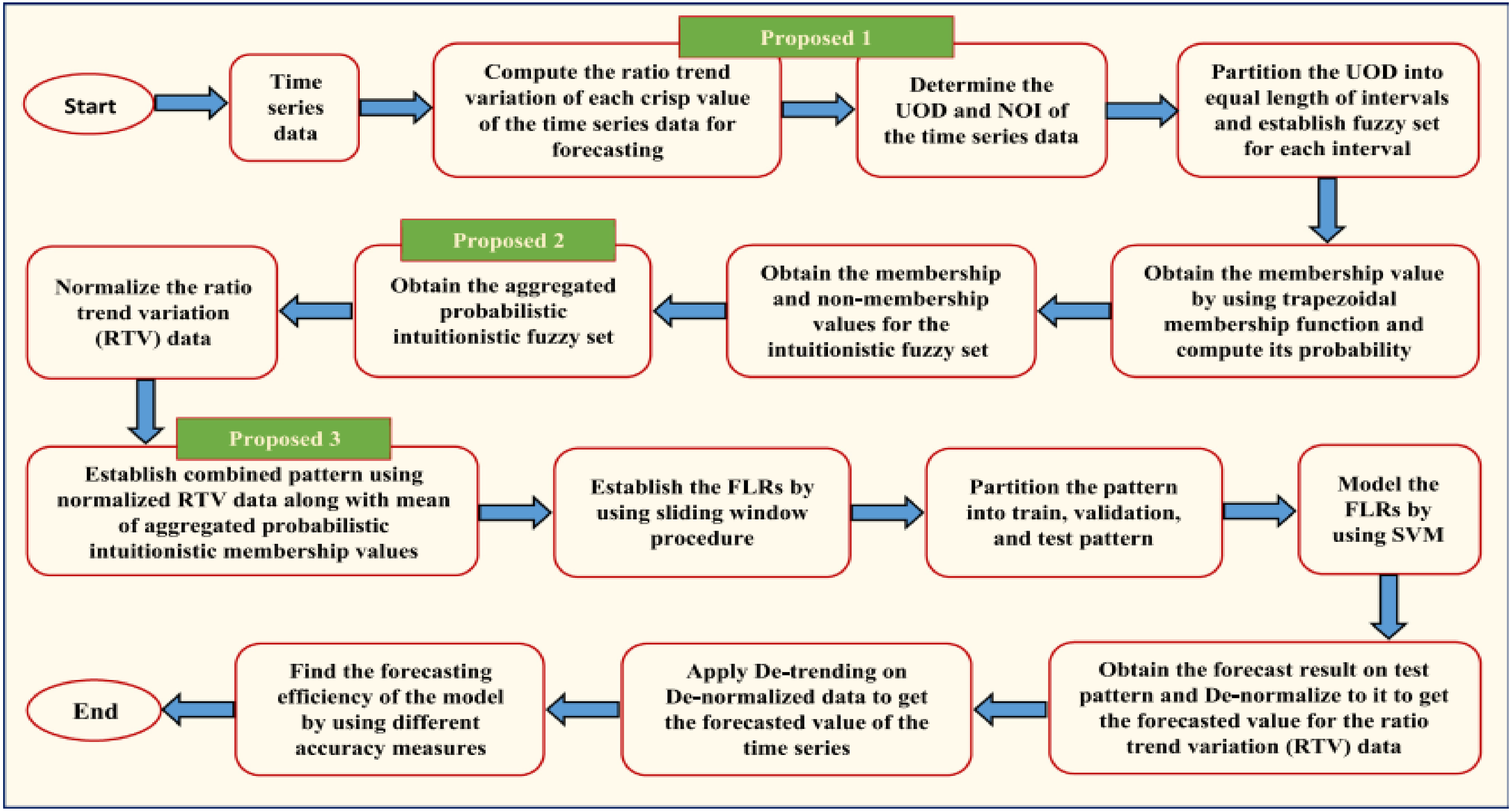
```
Input the time series (y), length of train set (l_tr), length of validation set (l_v)
//Define the universe of discourse U = [U_l  U_h] and partition U into equal length intervals
D = 0.2 × (max(y_{l_tr+l_tv}) − min(y_{l_tr+l_tv}))
U_l = min(y_{l_tr+l_tv}) − D
U_h = max(y_{l_tr+l_tv}) + D
Sort the time series
Delete the duplicate elements from the sorted time series (y')
Compute the absolute first differenced series (y'_diff)
Compute the half of the average of y'_diff and set it to length
Compute the base for round off
Compute the length of interval l by rounding off the length by base
Using l as length of interval, divide the universe of discourse U = {u_1, u_2, u_3 ... u_w} into q
equal length intervals with w = (U_h − U_l)/l
Compute the mid-point m_i of the intervals u_i.
// Fuzzify the time series data
for j=1 to n
    if y_j belongs to the partition u_i
        f_j(t) = i
    end if
end for
// Establish fuzzy logical relationships using DBN, LSTM or SVM
Normalize the fuzzified time series f(t) to obtain f'(t).
Determine the order k of fuzzy TSF model
Transform the f'(t) into a n-k patterns using sliding window protocol
Divide the patterns into train, validation and test patterns
Using the train and validation patterns determine the parameters of DBN, SVM or LSTM
Obtain the forecasts f̂'(t) using the optimized parameters of DBN, SVM or LSTM
Denormalize and round off f̂'(t) to obtain f̂(t)
// Defuzzify and measure the forecast accuracy
for each element f̂_i(t) in f̂(t)
    if f̂_i(t) < 1
        ŷ_i = m_1
    else if f̂_i(t) > q
        ŷ_i = m_q
    else
        ŷ_i = m_{f̂_i(t)}
    end if
end for
Compute the forecasting accuracy using y and ŷ
```

Published Papers

2. Radha Mohan Pattanayak, **Sibarama Panigrahi,** and H. S. Behera. "High-order fuzzy time series forecasting by using membership values along with Data and Support Vector Machine." Arabian Journal for Science and Engineering 45.12 (2020): 10311-10325. ISSN: 2191-4281 **[Springer, IF:2.81, SCI]**

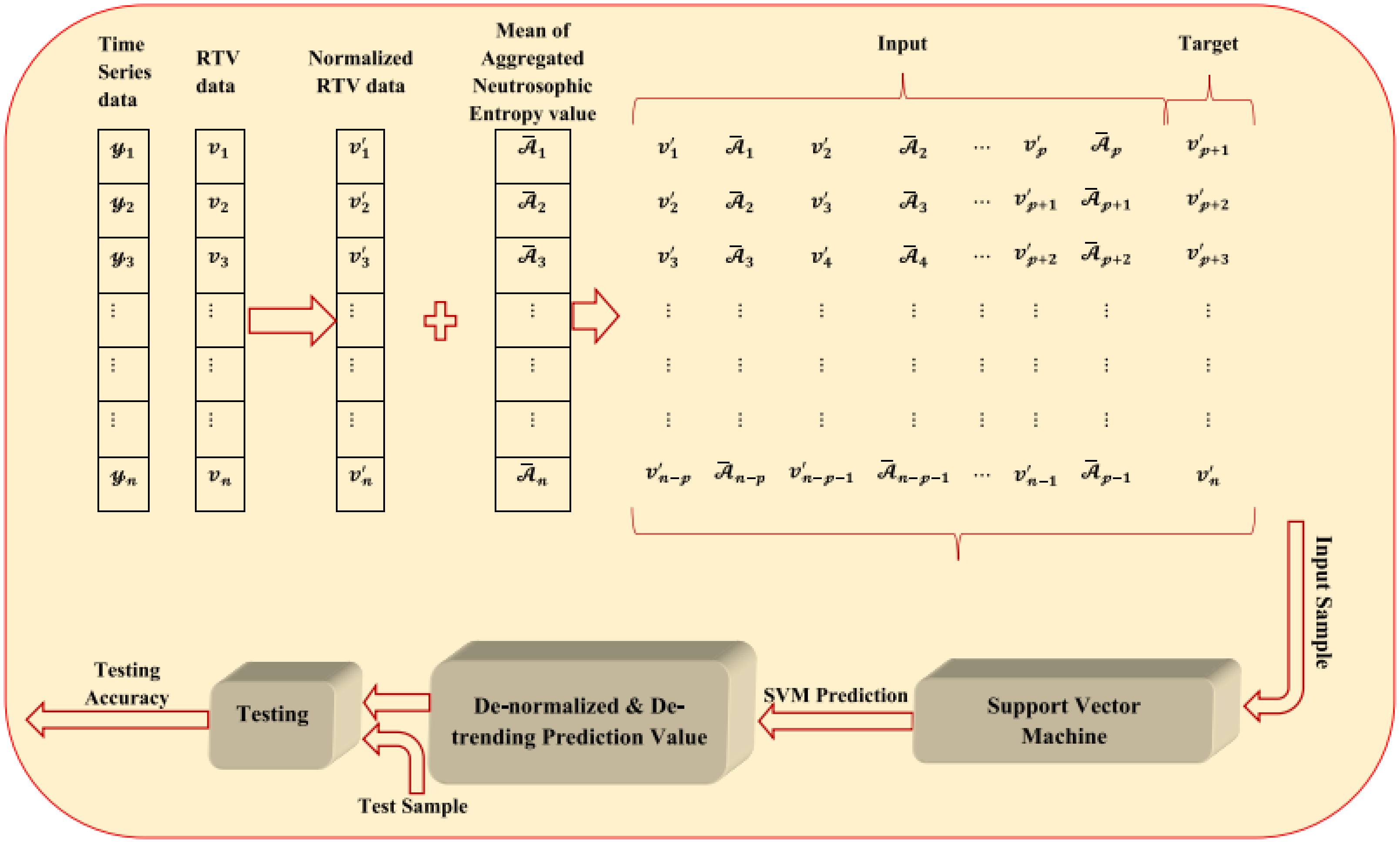


Published Papers



3. Radha Mohan Pattanayak, H. S. Behera, and **Sibarama Panigrahi**. "A novel probabilistic intuitionistic fuzzy set based model for high order fuzzy time series forecasting." Engineering Applications of Artificial Intelligence 99 (2021): 104136. ISSN: 0952-1976 **[Elsevier, IF:7.80, SCI]**

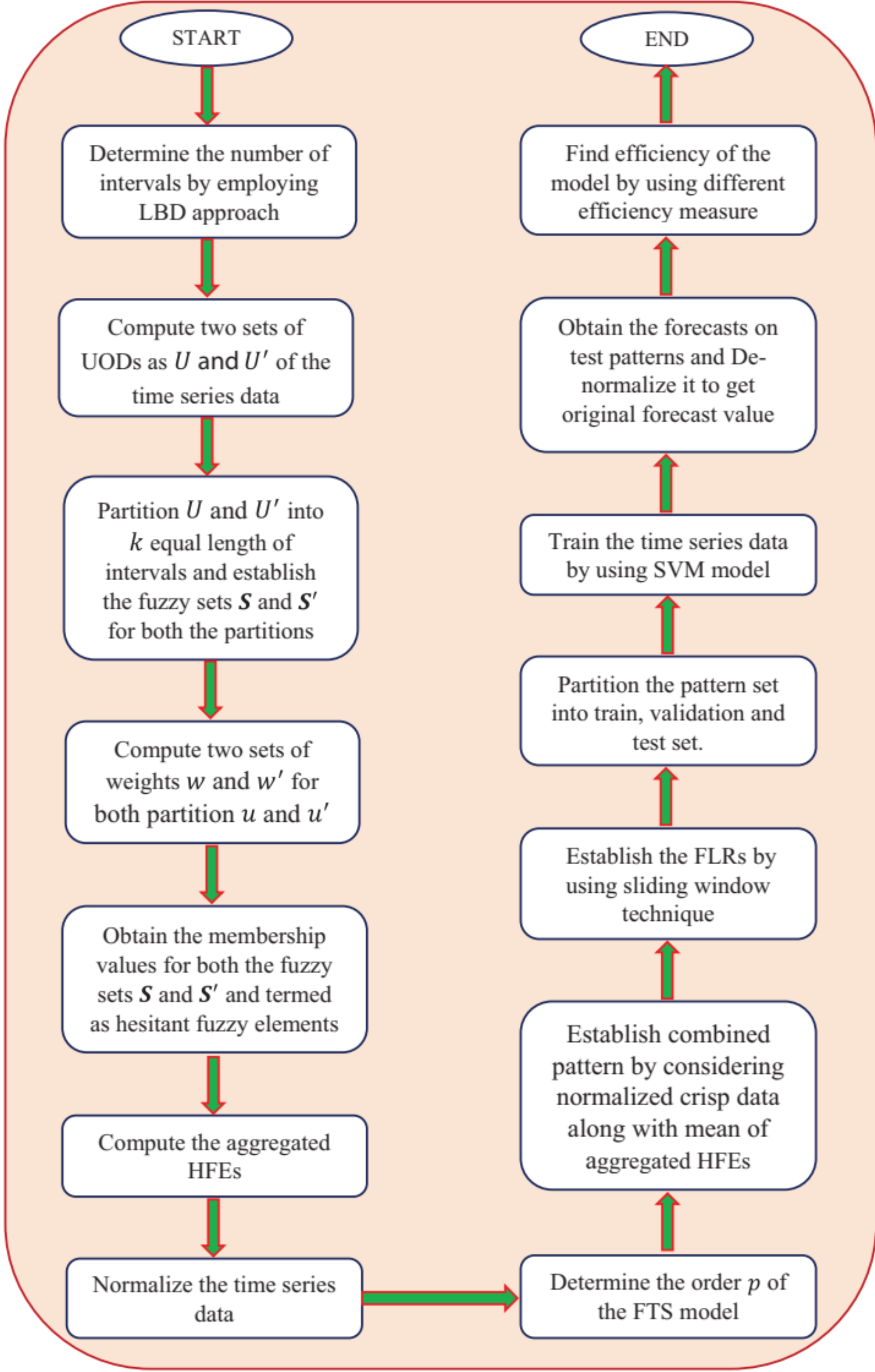
Published Papers



4. Radha Mohan Pattanayak, H. S. Behera, and **Sibarama Panigrahi**. "A non-probabilistic neutrosophic entropy-based method for high-order fuzzy time-series forecasting." Arabian Journal for Science and Engineering vol. 47, no. 2 (2022): 1399-1421. **[Springer, IF:2.81, SCI]**

Published Papers

5. Radha Mohan Pattanayak, Himansu Sekhar Behera, and **Sibarama Panigrahi**. "A Novel High Order Hesitant Fuzzy Time Series Forecasting by using mean Aggregated Membership value with Support Vector Machine." Information Sciences (2023). **[Elsevier, IF: 8.01, SCI]**



RELATED PUBLICATIONS[FUZZY TIME SERIES FORECASTING]

1. **Sibarama Panigrahi**, and Himansu Sekhar Behera. "A study on leading machine learning techniques for high order fuzzy time series forecasting." Engineering Applications of Artificial Intelligence 87 (2020): 103245. **[Elsevier, IF: 7.80, SCI]**
2. Radha Mohan Pattanayak, **Sibarama Panigrahi,** and H. S. Behera. "High-order fuzzy time series forecasting by using membership values along with Data and Support Vector Machine." Arabian Journal for Science and Engineering 45.12 (2020): 10311-10325. ISSN: 2191-4281 **[Springer, IF:2.81, SCI]**
3. Radha Mohan Pattanayak, H. S. Behera, and **Sibarama Panigrahi**. "A novel probabilistic intuitionistic fuzzy set based model for high order fuzzy time series forecasting." Engineering Applications of Artificial Intelligence 99 (2021): 104136. ISSN: 0952-1976 **[Elsevier, IF:7.80, SCI]**
4. Radha Mohan Pattanayak, H. S. Behera, and **Sibarama Panigrahi**. "A non-probabilistic neutrosophic entropy-based method for high-order fuzzy time-series forecasting." Arabian Journal for Science and Engineering vol. 47, no. 2 (2022): 1399-1421. **[Springer, IF:2.81, SCI]**
5. Radha Mohan Pattanayak, Himansu Sekhar Behera, and **Sibarama Panigrahi**. "A Novel High Order Hesitant Fuzzy Time Series Forecasting by using mean Aggregated Membership value with Support Vector Machine." Information Sciences (2023). **[Elsevier, IF: 8.01, SCI]**
6. Sushree Subhaprada Pradhan, **Sibarama Panigrahi***. “A study and development of high-order fuzzy time series forecasting methods for air quality index forecasting.” Journal of Forecasting (2024). **[Wiley, IF: 3.40, SSCI].**

For Your Valuable Time.