Probability&RV Assignment-01

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I. Problem(9.3)

random variable $z = n1 - n2$, where $n1, n2 \sim N(0,1)$ prove that z is a Gaussian random variable and find mean and variance of z.comment it.

II. Solution

Let x=n1 and y=n2 then z=x-y
Moment Generating Function (MGF)is given by

$$M_t(s) = E[e^{-st}] \tag{1}$$

pdf of x is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \tag{2}$$

MGF of x is
$M_x(s) = \int_{-\infty}^{\infty} e^{-sx} f(x) dx$
above equation represents Fourier Transform of f(x),we know that the F.T of $f_X(x)$ is

$$M_x(s) = e^{-s\mu_x} e^{-\frac{s^2\sigma_x^2}{2}} \tag{3}$$

similarly pdf of y is

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}} \tag{4}$$

MGF of $f_Y(y)$

$$M_y(s) = e^{-s\mu_y} e^{-\frac{s^2\sigma_y^2}{2}} \tag{5}$$

MGF of z can be written as
$M_z(s) = E[e^{-(x+y)s}] = M_x(s) \times M_y(s)$
by substituting eq(3) and eq(5) in $M_z(s)$

$$M_z(s) = e^{-s(\mu_x+\mu_y)} e^{-\frac{s^2(\sigma_x^2+\sigma_y^2)}{2}} \tag{6}$$

$$\therefore f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(z-(\mu_x+\mu_y))^2}{2(\sigma_x^2+\sigma_y^2)}} \tag{7}$$

lets substitute values $\mu_x = \mu_y = 0$ and $\sigma_x^2 = \sigma_y^2 = 1$ in above equations

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{8}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \tag{9}$$

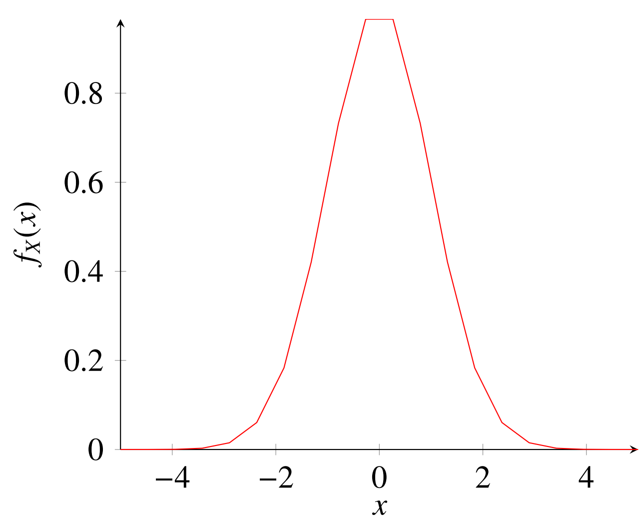
$$M_x(s) = e^{-\frac{s^2}{2}} \tag{10}$$

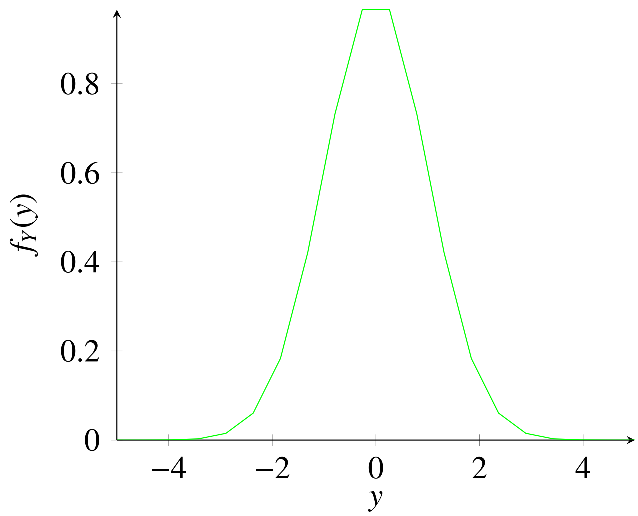
$$M_y(s) = e^{-\frac{s^2}{2}} \tag{11}$$

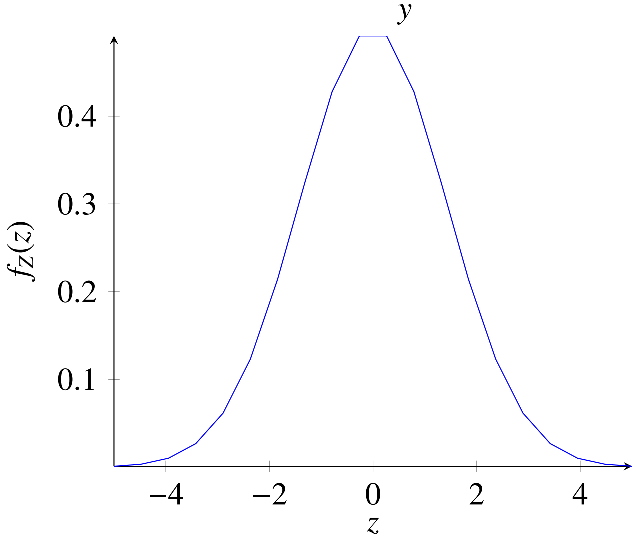
$$M_z(s) = e^{-s^2} \tag{12}$$

finally

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{4}} \tag{13}$$







III. Conclusion

Random variable which is either sum or difference of two standard normal variables is also a normal variable.
$\mu_z = \mu_x + \mu_y = 0$ and $\sigma_z^2 = \sigma_x^2 + \sigma_y^2 = 2$