Probability&RV Assignment-08

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Download Latex code from

https://github.com/Anuradha−Uggi/Assignments−AI5002−Probability−and−Random−Variables/blob/main/Prob_ass08/rvsp_8.tex

Download Python code from

https://github.com/Anuradha−Uggi/Assignments−AI5002−Probability−and−Random−Variables/blob/main/Prob_ass08/rvsp_8.py

I. QUESTION(GATE-Q17)

The input X to the binary Symmetric Channel(BSC) shown in fig.1 is '1' with probability 0.8. The cross-over probability is $\frac{1}{7}$. if the received bit Y=0,the conditional probability that '1' was transmitted is.....

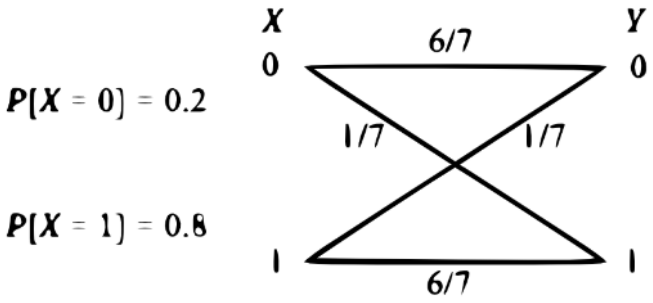

Fig. 1.

II. SOLUTION

Given

$$P(Y = 0/X = 0) = P(Y = 1/X = 1) = \frac{6}{7} \quad (1)$$

$$P(Y = 0/X = 1) = P(Y = 1/X = 0) = \frac{1}{7} \quad (2)$$

we know that

$$P(X \cap Y) = P(Y \cap X) \quad (3)$$

Above equation can also be written as

$$P(X/Y)P(Y) = P(Y/X)P(X) \quad (4)$$

Therefore

$$P(X = 1/Y = 0) = \frac{P(Y = 0/X = 1)P(X = 1)}{P(Y = 0)} \quad (5)$$

From the given data

$$P(Y = 0) = P(Y = 0/X = 0)P(X = 0)+P(Y = 0/X = 1)P(X = 1) \quad (6)$$

$$P(Y = 0) = \frac{6}{7} \times 0.2 + \frac{1}{7} \times 0.8 = \frac{2}{7} \quad (7)$$

we have

1) $P(Y = 0/X = 1) = \frac{1}{7}$
2) $P(X = 1) = 0.8$
3) $P(Y = 0) = \frac{2}{7}$

Substituting above values in equation (5) results

$$P(X = 1/Y = 0) = \frac{0.8}{2} = 0.4 \quad (8)$$

III. CONCLUSION

probability that X=1 is transmitted given that Y=0 is received is

$$P(X = 1/Y = 0) = 0.4 \quad (9)$$