Probability&RV Assignment-08

Anuradha U-ee21resch01008

Download Latex code from

https://github.com/Anuradha−Uggi/Assignments−AI5002−Probability−and−Random−Variables/blob/main/Prob_ass08/rvsp_8.tex

Download Python code from

https://github.com/Anuradha−Uggi/Assignments−AI5002−Probability−and−Random−Variables/blob/main/Prob_ass08/rvsp_8.py

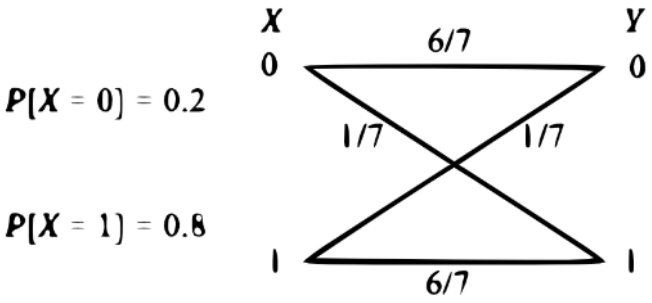

Fig. 1.

I. QUESTION(GATE-Q17)

The input X to the binary Symmetric Channel(BSC) shown in fig.1 is '1' with probability 0.8. The cross-over probability is $\frac{1}{7}$. if the received bit Y=0,the conditional probability that '1' was transmitted is.....

II. SOLUTION

Given

$$\Pr[Y=0|X=0] = \Pr[Y=1|X=1] = \frac{6}{7} \quad (1)$$

$$\Pr[Y=0|X=1) = \Pr[Y=1|X=0] = \frac{1}{7} \quad (2)$$

we know that

$$\Pr[X \cap Y] = \Pr[Y \cap X] \quad (3)$$

Above equation can also be written as

$$\Pr[X|Y]\Pr[Y] = \Pr[Y|X]\Pr[X] \quad (4)$$

Therefore

$$\Pr[X=1|Y=0] = \frac{\Pr[Y=0|X=1]\Pr[X=1]}{\Pr[Y=0]} \quad (5)$$

From the given data

$$\Pr[Y=0] = \Pr[Y=0|X=0]\Pr[X=0] + \Pr[Y=0|X=1]\Pr[X=1] \quad (6)$$

$$\Pr[Y=0] = \frac{6}{7}\times 0.2 + \frac{1}{7}\times 0.8 = \frac{2}{7} \quad (7)$$

we have

1) $\Pr[Y=0|X=1] = \frac{1}{7}$
2) $\Pr[X=1] = 0.8$
3) $\Pr[Y=0] = \frac{2}{7}$

Substituting above values in equation (5) results

$$\Pr[X=1|Y=0] = \frac{0.8}{2} = 0.4 \quad (8)$$

III. CONCLUSION

probability that X=1 is transmitted given that Y=0 is received is

$$\Pr[X=1|Y=0] = 0.4 \quad (9)$$