# Probability&RV Assignment-08

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## I. QUESTION(UGC NET 2019,Q-108)

Suppose $X_i$=X$_1$,X$_2$,....,X$_n$ are i.i.d
Uniform $(\theta,2\theta)$, $\theta > 0$. Let X$_{(1)}$=min{X$_1$,X$_2$,...,X$_n$}
and X$_{(n)}$=max{X$_1$,....,X$_n$}.then which of the
following statements are correct.

1) ( X$_{(1)}$,X$_{(n)}$) is jointly sufficient and complete for
   $\theta$
2) ( X$_{(1)}$,X$_{(n)}$) is jointly sufficient but not complete
   for $\theta$
3) $\frac{X_{(n)}}{2}$ is maximum likelihood estimate for $\theta$
4) X$_{(1)}$ is maximum likelihood estimate for $\theta$

## II. BASIC DEFINITIONS

**Sufficient Statistic:**
Given X i.i.d Data conditioned on an unknown
parameter $\theta$, T(X) is called sufficient statistic for $\theta$
if its values contains all the information needed to
compute any estimate of the parameter (Maximum
likelihood estimate). according to Fisher−Neymen
Factorization PDF is

$$f(X; \theta) = h(X)S(\theta, T(X)) \tag{1}$$

where h(X) is a constant and S($\theta$,T(X)) is a function
through which $\theta$ will interact to X only through
T(X).

**Statistic Completeness:**

T(X) is said to be complete for $\theta$ if for every
measurable function g; if

$$E_\theta(g(T)) = 0 \tag{2}$$

for all $\theta$ then

$$P_\theta(g(T) = 0) = 1 \tag{3}$$

for all $\theta$.

## III. SOLUTION

Given

$$X_{(1)} = min\{X_1, X_2, ...., X_n\} = min\{X_i\} \tag{4}$$

$$X_{(n)} = max\{X_1, X_2, ...., X_n\} = max\{X_i\} \tag{5}$$

$$X_i = \{X_1, X_2, ...., X_n\} \sim U(\theta, 2\theta) \tag{6}$$

$$P(X_i) = \frac{1}{\theta} \tag{7}$$

Statistic

$$T(X) = (min\{X\}, max\{X\}) \tag{8}$$

X are i.i.d so Likelihood is given by

$$f_X(x) = \frac{1}{\theta^n} 1_{\{\theta \leq X_i \leq 2\theta\}} \tag{9}$$

equation (7) can be split into

$$f_X(x) = \frac{1}{\theta^n} 1_{\{\theta \leq min\{X_i\}} 1_{max\{X_i\} \leq 2\theta} \tag{10}$$

from equation (8)

$$h(X) = 1 \tag{11}$$

which is constant and

$$S_{(\theta,2\theta)}(X) = \frac{1}{\theta^n} \tag{12}$$

which is function of only $\theta$.
therefore $T(min\{X_i\}, max\{X_i\})$ is jointly sufficient to
define $\theta$ thus Sufficient statistic.

Let

$$g(T) = max\{X_i\} - max\{X_i\} \tag{13}$$

$$E[g(T)] = E[max\{X_i\} - min\{X_i\}] = \int (max\{X_i\} - min\{X_i\}) \frac{1}{\theta^n} dx$$
(14)

from equation (12) its clear that

$$max\{X_i\} - min\{X_i\} \neq 0 \qquad (15)$$

for all $\theta$ therefore

$$P(\ max\{X_i\} - min\{X_i\} = 0)\ \neq 1 \qquad (16)$$

therefore $T(X_{(n)}, X_{(1)})$ is Jointly sufficient but not complete for $\theta$.

**Maximum Likelihood Estimate(MLE) :**

Likelihood can be written as

$$f_\theta(X_1, X_2, .., X_n) = \frac{1}{\theta^n} I\left(\frac{max\{X_i\}}{2} \leq \theta \leq min\{X_i\}\right)$$
(17)

the MLE is the statistic that maximizes the liklihood.from equation (7) liklihood is a decreasing function of $\theta$.therefore MLE of $\theta$ is

$$\theta = \frac{X_{(n)}}{2} = \frac{max\{X_i\}}{2} \qquad (18)$$

## IV. **CONCLUTION**

From above observations option (2) and option (3) holds.