Probability&RV Assignment-01

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download python code from

https://github.com/Anuradha−Uggi/Assignments−AI5002−Probability−and−Random−Variables/blob/main/Prob_ass01/rvsp.py

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I. Problem(9.3)

random variable $z = n_1 - n_2$, where $n_1, n_2 \sim N(0,1)$ prove that z is a Gaussian random variable and find mean and variance of z.comment it.

II. Solution

Moment Generating Function (MGF)is given by

$$M_t(s) = E[e^{-st}] \tag{1}$$

pdf of n_1 is

$$f_{N_1}(n_1) = \frac{1}{\sqrt{2\pi}\sigma_{n_1}} e^{-\frac{(n_1-\mu_{n_1})^2}{2\sigma_{n_1}^2}} \tag{2}$$

MGF of n_1 is
$M_{n_1}(s) = \int_{-\infty}^{\infty} e^{-sn_1} f(n_1) dn_1$
above equation represents Laplace Transform of f(n1),we know that the L.T of $f_{N_1}(n_1)$ is

$$M_{n_1}(s) = e^{-s\mu_{n_1}} e^{-\frac{s^2\sigma_{n_1}^2}{2}} \tag{3}$$

similarly pdf of n_2 is

$$f_{N_2}(n_2) = \frac{1}{\sqrt{2\pi}\sigma_{n_2}} e^{-\frac{(n_2-\mu_{n_2})^2}{2\sigma_{n_2}^2}} \tag{4}$$

MGF of $f_{N_2}(n_2)$

$$M_{n_2}(s) = e^{-s\mu_{n_2}} e^{-\frac{s^2\sigma_{n_2}^2}{2}} \tag{5}$$

MGF of z can be written as
$M_z(s) = E[e^{-(n_1+n_2)s}] = M_{n_1}(s) \times M_{n_2}(s)$
by substituting eq(3) and eq(5) in $M_z(s)$

$$M_z(s) = e^{-s(\mu_{n_1}+\mu_{n_2})} e^{-\frac{s^2(\sigma_{n_1}^2+\sigma_{n_2}^2)}{2}} \tag{6}$$

$$\therefore f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(z-(\mu_{n_1}+\mu_{n_2}))^2}{2(\sigma_{n_1}^2+\sigma_{n_2}^2)}} \tag{7}$$

lets substitute values $\mu_{n_1} = \mu_{n_2} = 0$ and $\sigma_{n_1}^2 = \sigma_{n_2}^2 = 1$ in above equations

$$f_{N_1}(n_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{n_1^2}{2}} \tag{8}$$

$$f_{N_2}(n_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{n_2^2}{2}} \tag{9}$$

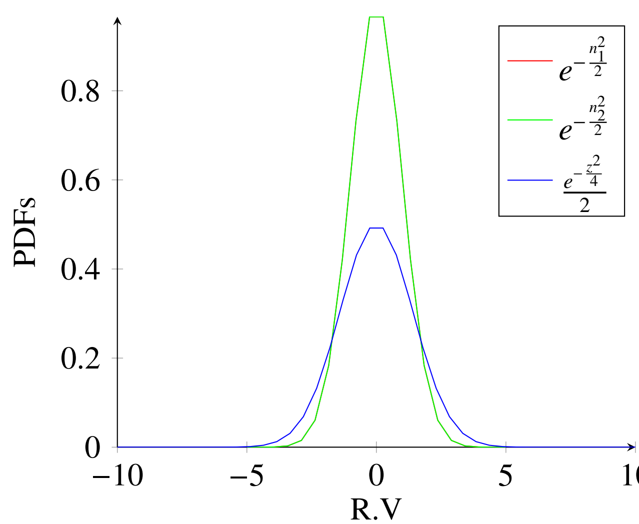
$$M_{n_1}(s) = e^{-\frac{s^2}{2}} \tag{10}$$

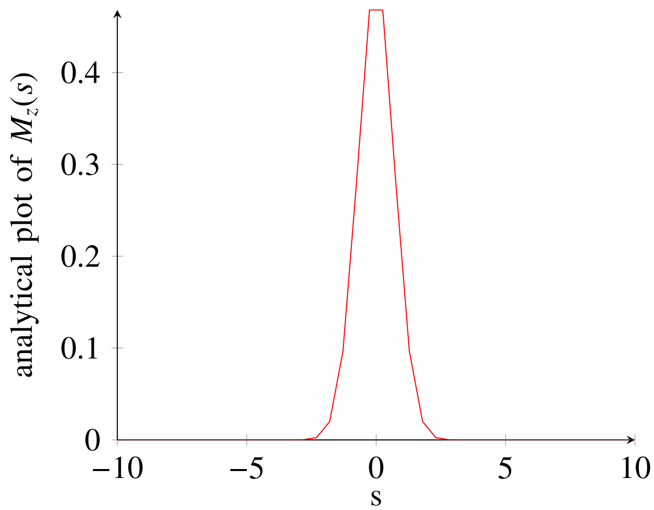
$$M_{n_2}(s) = e^{-\frac{s^2}{2}} \tag{11}$$

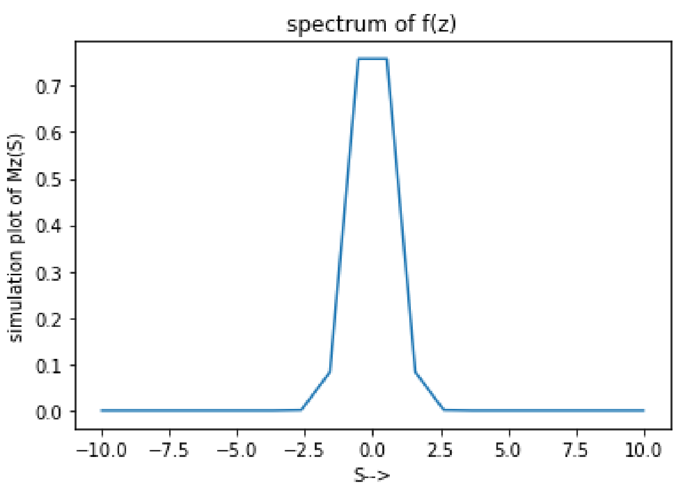
$$M_z(s) = e^{-s^2} \tag{12}$$

finally

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{4}} \tag{13}$$







III. Conclusion

Random variable which is either sum or difference of two standard normal variables is also a normal variable.

$\mu_z = \mu_{n_1} + \mu_{n_2} = 0$ and $\sigma_z^2 = \sigma_{n_1}^2 + \sigma_{n_2}^2 = 2$