CS5800: Algorithms — Virgil Pavlu

Homework 8

Name:
Collaborators:

Instructions:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- Please review the grading policy outlined in the course information page.
- You must also write down with whom you worked on the assignment. If this changes from problem to problem, then you should write down this information separately with each problem.
- Problem numbers (like Exercise 3.1-1) are corresponding to CLRS 3^{rd} edition. While the 2^{nd} edition has similar problems with similar numbers, the actual exercises and their solutions are different, so make sure you are using the 3^{rd} edition.

1. 16.3-3

Solution: For each node x in the heap, let $d_i(x)$ be the depth of x in D_i . Define $\phi(D_i) = \sum_{x \in D_i} k(d_i(x))$ where k is a constant such that each INSERT or EXTRACT-MIN operation takes at most $k \ln n$ time. Initially, the heap has no items, which means that the sum is over an empty set, and so $\phi(D_0) = 0$. We always have $\phi(D_i) \geq 0$, as required. After an INSERT, the sum changes only by an amount equal to the depth of the new last node of the heap, which is $\lfloor \lg n_i \rfloor$

Thus, the change in potential due to an INSERT is $k(1 + \lfloor \lg n_i \rfloor)$, and so the amortized cost is $O(\lg n_i)$ + $O(\lg n_i) = O(\lg n_i) = O(\lg n)$. After an EXTRACT-MIN, the sum changes by the negative of the depth of the old last node in the heap, and so the potential decreases by $k(1 + \lfloor \lg n_{i-1} \rfloor)$. The amortized cost is at most $k \lg n_{i-1} - k(1 + \lfloor \lg n_{i-1} \rfloor) = O(1)$ $\lceil x \rceil$