

SPARSITY MODELLING

LOW RANK MODELLING

$$\begin{bmatrix} \mathbb{D}_{1,1} & \cdots & \mathbb{D}_{1,t} \\ \vdots & \ddots & \vdots \\ \mathbb{D}_{m,1} & \cdots & \mathbb{D}_{m,t} \end{bmatrix} : \text{Basis Matrix}$$

$\lambda$ : Regularization Parameter

Optimization Function

$$\min(\|\mathbb{X} - \mathbb{D}\mathbb{Z}\|^2 + \lambda\|\mathbb{Z}\|_1)$$

Hyperspectral Image  $\begin{bmatrix} \mathbb{X}_{1,1} & \cdots & \mathbb{X}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbb{X}_{m,1} & \cdots & \mathbb{X}_{m,n} \end{bmatrix}$

$\begin{bmatrix} \mathbb{D}_{1,1} & \cdots & \mathbb{D}_{1,t} \\ \vdots & \ddots & \vdots \\ \mathbb{D}_{m,1} & \cdots & \mathbb{D}_{m,t} \end{bmatrix}, \lambda$

Sparse Coefficient Matrix  $\begin{bmatrix} \mathbb{Z}_{1,1} & \cdots & \mathbb{Z}_{1,m} \\ \vdots & \ddots & \vdots \\ \mathbb{Z}_{n,1} & \cdots & \mathbb{Z}_{n,m} \end{bmatrix}$

$$\begin{bmatrix} \mathbb{D}_{1,1} & \cdots & \mathbb{D}_{1,t} \\ \vdots & \ddots & \vdots \\ \mathbb{D}_{m,1} & \cdots & \mathbb{D}_{m,t} \end{bmatrix} \times \begin{bmatrix} \mathbb{Z}_{1,1} & \cdots & \mathbb{Z}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbb{Z}_{t,1} & \cdots & \mathbb{Z}_{t,n} \end{bmatrix}$$

$\mathbb{X}_{\text{sparse-modelling}}$

H

Y

R

E

S

Hyperspectral Image  $\begin{bmatrix} \mathbb{X}_{1,1} & \cdots & \mathbb{X}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbb{X}_{m,1} & \cdots & \mathbb{X}_{m,n} \end{bmatrix}$

$\begin{bmatrix} [\mathbb{U}_{i,j}]_{m \times r} \\ [\mathbb{S}_{i,j}]_{r \times r} \\ [\mathbb{V}_{i,j}]_{n \times r} \end{bmatrix}$

$\mathbb{X} = \mathbb{U} \times \mathbb{S} \times \mathbb{V}^T$

PCA / SVD

Low-rank  $\begin{bmatrix} \mathbb{U}_{1,1} & \cdots & \mathbb{U}_{1,r} \\ \vdots & \ddots & \vdots \\ \mathbb{U}_{m,1} & \cdots & \mathbb{U}_{m,r} \end{bmatrix} \times \begin{bmatrix} \mathbb{S}_{1,1} & \cdots & \mathbb{S}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbb{S}_{r,1} & \cdots & \mathbb{S}_{r,n} \end{bmatrix}$

$m$ : Number of Spectral Bands in HSI

$n$ : Number of Image Pixels in HSI

$r$ : Number of Low Rank Components

$$\mathbb{X}_{\text{restored}} = \mathbb{X}_{\text{low-rank}} \times \mathbb{Z}_{\text{sparse}}$$