Chaotic Maps in 1 - Dimension

1. Logistic Map

Logistic maps are quadratic polynomial maps, which are often cited as typical examples of how complex and chaotic behavior can occur from very simple nonlinear dynamic equations. Increase. Mathematically, a logistic map is written as

$$x_{k+1} = rx_k(1 - x_k)$$

where,

x_k represents the $k'th$ term in the Recurrence

r represents the rate of change

This rate of change can be of two types

- Reproduction: $x_{k+1} \geq x_k \forall k \geq 0$
- Starvation: $x_{k+1} < x_k \forall k \geq 0$

In usual cases, we are ought to take r in the range $[0, 4]$, which surely indicates that the x_k will remain bounded between $[0, 1]$, else we are supposed to get negative population instances.

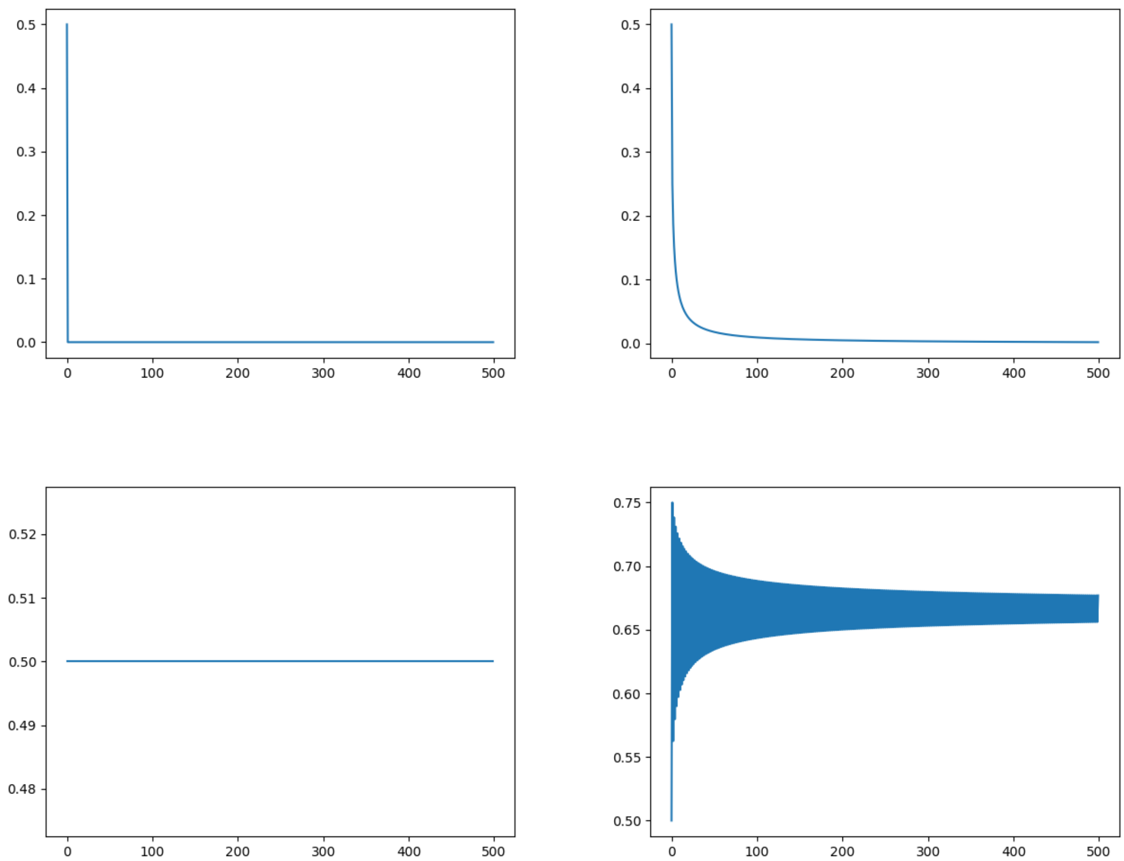

Fig. 1. Graphical Plot of the Logistic Map for $r = 0, 1, 2, 3$ respectively, in row major manner

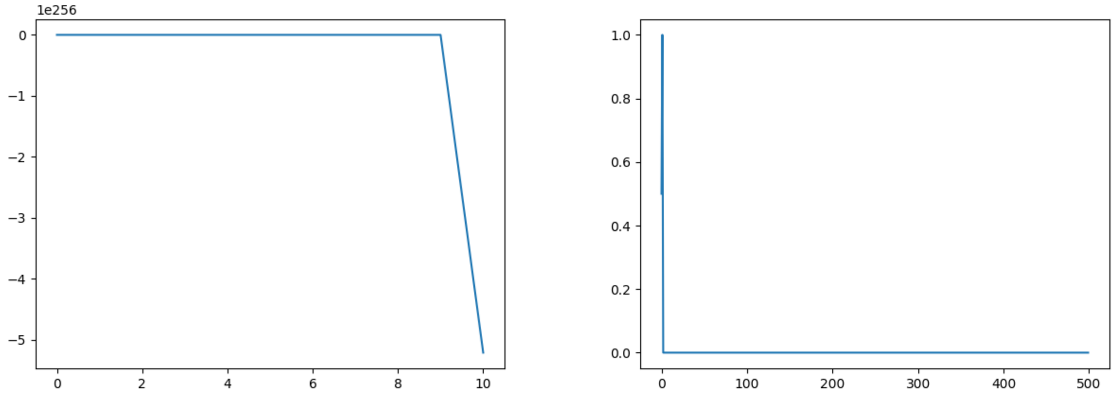

Fig. 2. Graphical Plot of the Logistic Map for $r = 4, 5$ respectively, in row major manner

The bifurcation diagram[1] will be

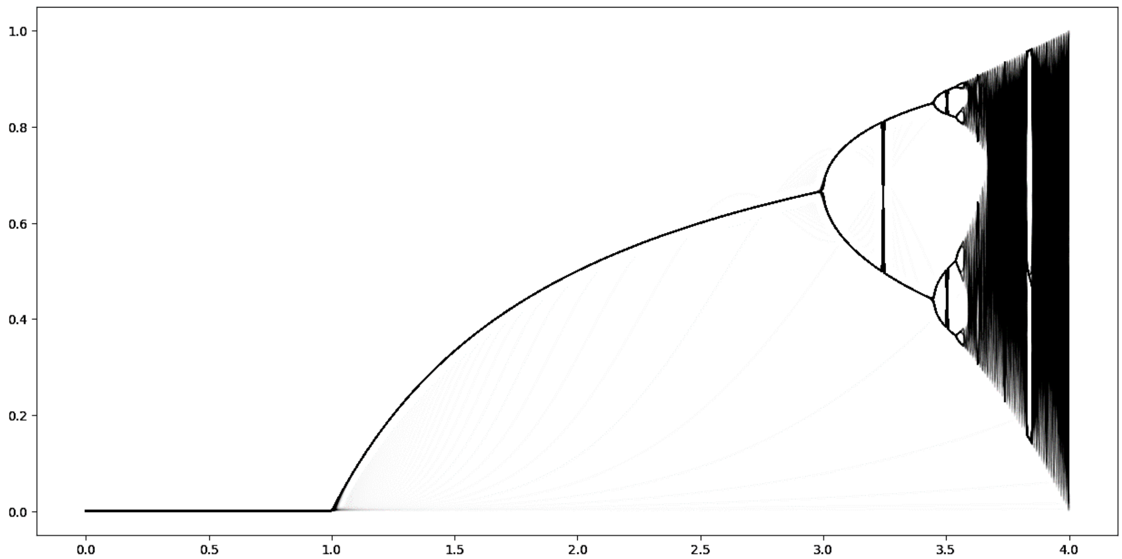

Fig. 3. Bifurcation Plot of the Logistic Map

2. Gauss Map

Gauss Map is a non-linear iteration mapping of real numbers to the real number interval given by the Gaussian function.

$$x_{k+1} = e^{-\alpha(x_k)^2} + \beta$$

where,

α, β are real valued parameters

In usual cases, we are ought to take β in the range $[-1, 1]$

[1] It denotes the stable state values

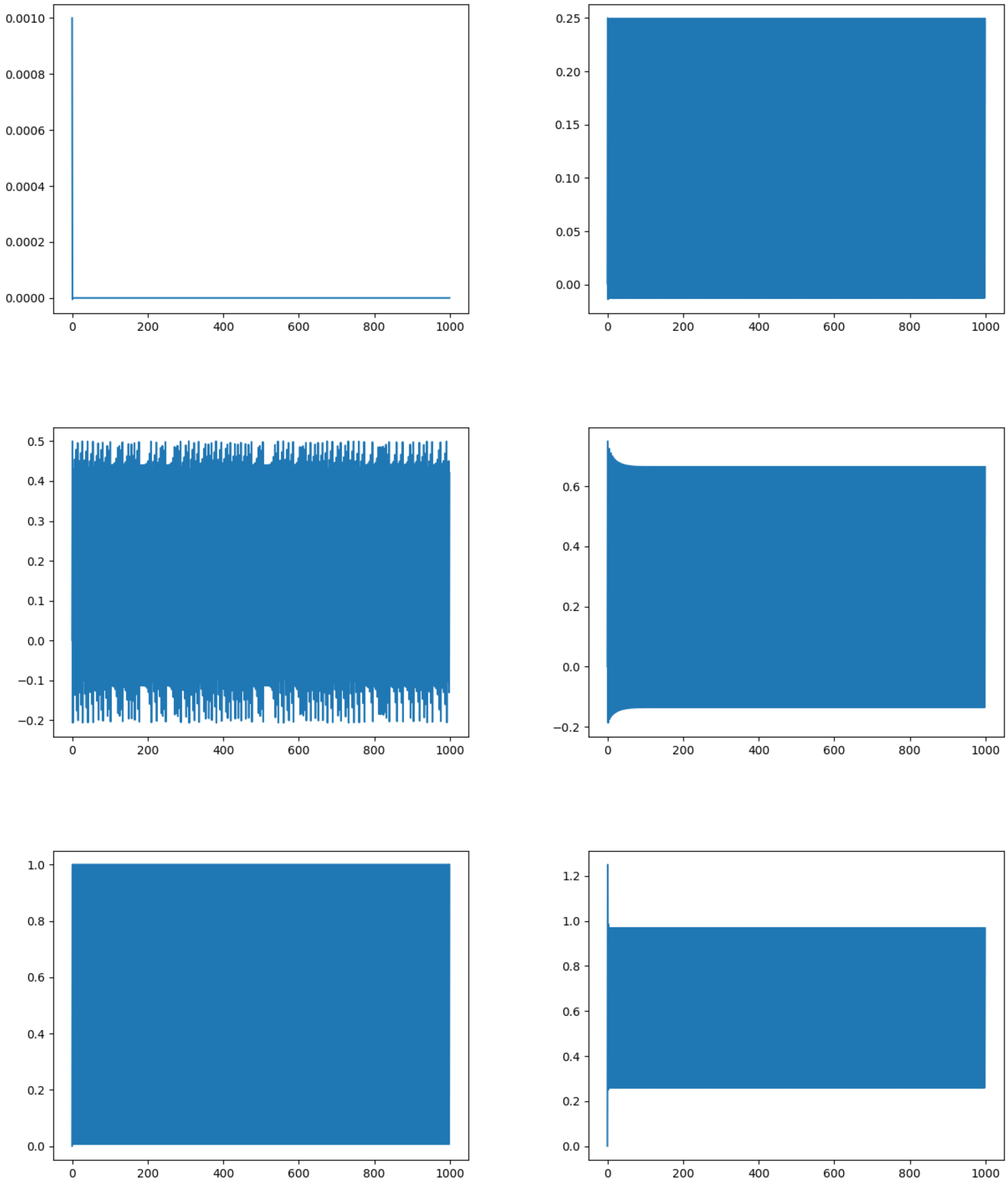

Fig. 4. Graphical Plot of the Gauss Map for $\beta = -1,\ -0.75,\ -0.50,\ -0.25, 0, 0.25$ and $\alpha = 4.9$ respectively, in row major manner

The bifurcation diagram will be

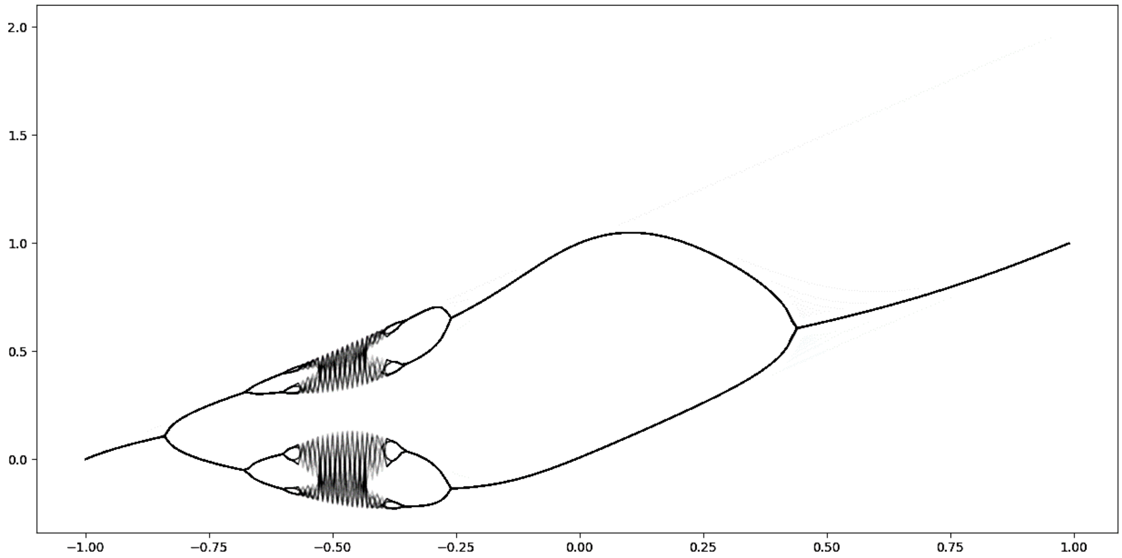

Fig. 5. Bifurcation Plot of the Gauss Map with $\alpha = 4.90$

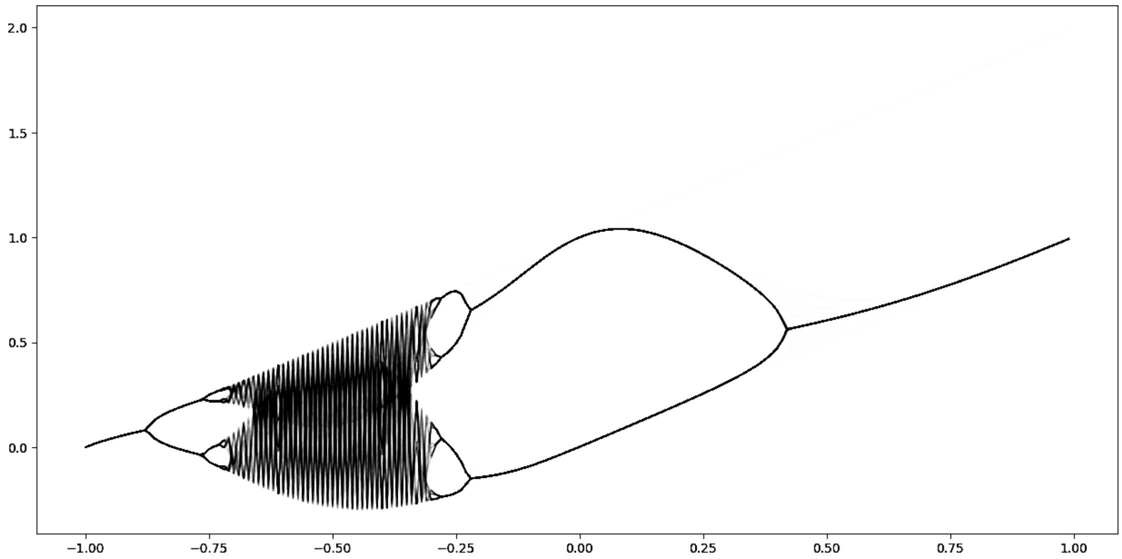

Fig. 6. Bifurcation Plot of the Gauss Map with $\alpha = 6.20$

3. Circle Map

A family of pie charts is a function (or endomorphism) of a circle relative to itself. It is mathematically easier to think of a point on a circle as a point on a solid line that is modulo 2π should be interpreted and represents the angle at which the point is in the circle. Even if the modulo is obtained with a value other than 2π, the result represents an angle, but normalized so that the entire range is $[0, 2\pi]$ can be expressed. With this in mind, the family of circle maps is given as follows

$$\theta_{k+1} = \theta_k + \Omega + \frac{K}{2\pi} sin(2\pi\theta_k)$$

where,

K, Ω are real valued parameters

In usual cases, we are ought to take K in the range $[0, 4\pi]$

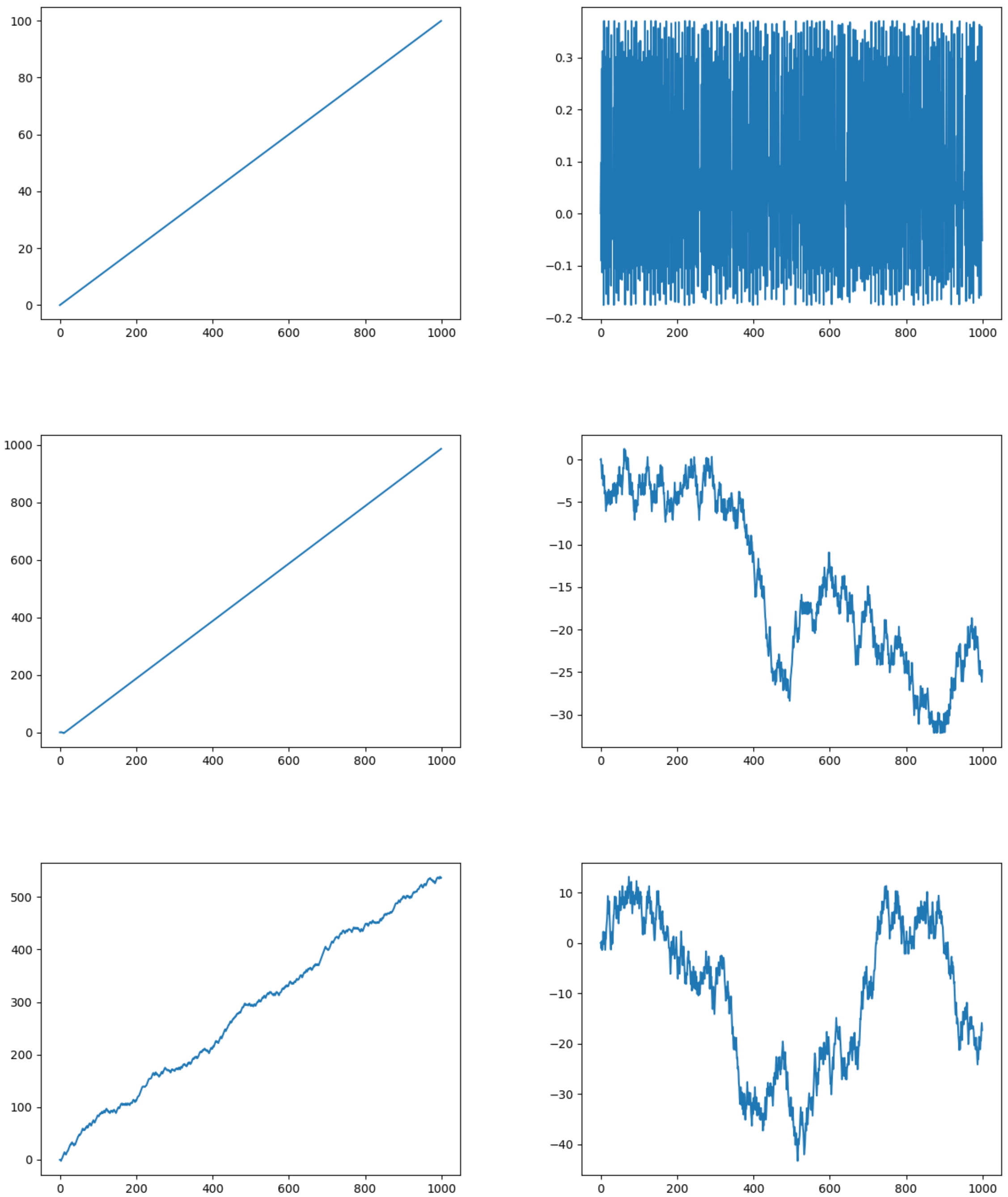

Fig. 7. Graphical Plot of the Gauss Map for $K = 0, \pi, 2\pi, 3\pi, 4\pi^2, 5\pi$ and $\omega = 0.1$ respectively, in row major manner

The bifurcation diagram will be

[2] Devil's Staircase

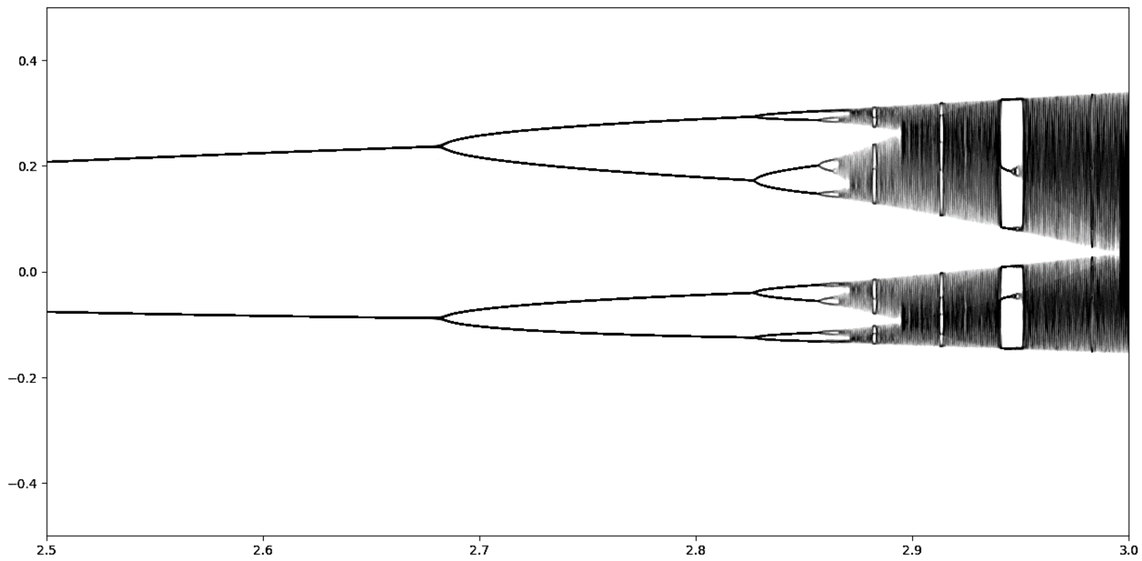

Fig. 8. Bifurcation Plot of the Circular Map with $\omega = 0.1$