Mensuration

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1 Introduction

What does 'mensuration' mean? It means measurements. By the context of the word, we can say that we are going to have a lot of measurements to be done in this Chapter.

Before diving into some crap, we will look after some basic definitions,

- Rectilinear Figure: A figure developed by some collection of straight lines. Example: Triangles, Quadrilaterals, etc.
- Closed Figure: A rectilinear figure which has no free ends. All Polygons with n sides are Closed Figure, $\forall n \geq 3 \ \& \ n \in \mathbb{Z}^+$

NOTE: Family of Triangles[1] is a 3 - sided polygon. Family of Quadrilaterals[2] is a 4 - sided polygon, Family of Circles[3] is an ∞ - sided polygon.

- Perimeter: Length of the boundary of a simple closed figure.
- Area is the measure of the region bounded by a plane figure.

2 Triangle

A Triangle is a polygon with 3 edges and 3 vertices.
As mentioned above, the family of triangles includes,

- Equilateral Triangle
- Isosceles Triangle
- Scalene Triangle
- Acute Angled Triangle
- Obtuse Angled Triangle
- Right Angled Triangle

2.1 Equilateral Triangles

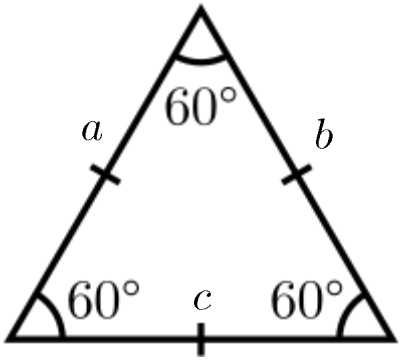


Fig. 1: Equilateral Triangle

The properties of Equilateral Triangles are:

- It is a Regular Polygon with 3 sides

[1] Family of Triangles includes Equilateral Triangles, Isosceles Triangles, Scalene Triangles, Acute Angled Triangles, Obtuse Angled Triangles, Right Angled Triangles.
[2] Family of Quadrilaterals includes Squares, Rectangles, Trapeziums, Rhombus, Parallelograms, etc.
[3] Family of Circles includes Semi-Circles, Quadrant of a Circle, etc.

- All the 3 sides are equal, say a.
- All the 3 angles are equal, say α, with each measuring 60^0.
- Perimeter of an Equilateral Triangle is $3 \times a$.

PROOF:

We know, in Equilateral Triangles, all the 3 sides are equal.

So, $a = b = c$

Now, by definition of the Perimeter, we know,

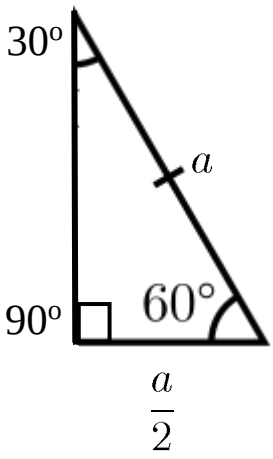
$Perimeter(P) = Sum\ of\ all\ sides$

$Perimeter(P) = a + b + c$

$Perimeter(P) = 3 \times a$ (As $a = b = c$)

- Area of an Equilateral Triangle is $\frac{\sqrt{3}}{4}a^2$.

PROOF:



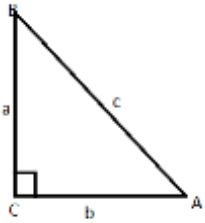
Area of half triangle[4] = $\frac{1}{2} \times \frac{a}{2} \times height$

Now, by Pythagoras Theorem[5], $a^2 = height^2 + \left(\frac{a}{2}\right)^2$

$$\Rightarrow height^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = a^2\left(1 - \frac{1}{4}\right) = \frac{3}{4}a^2$$

$$\Rightarrow height = \sqrt{\frac{3}{4}a^2} = \frac{\sqrt{3}}{2}a$$

[4] Area of a triangle (general case whose base b and height h is given) = $\frac{1}{2} \times b \times h$



[5] In a right-angled triangle, $c^2 = a^2 + b^2$

Area of half triangle = $\frac{1}{2} \times \frac{a}{2} \times \frac{\sqrt{3}}{2} a$

Area of half triangle = $\frac{\sqrt{3}}{8} a^2$

Area of full triangle = $2 \times \frac{\sqrt{3}}{8} a^2 = \frac{\sqrt{3}}{4} a^2$

2.2 Isosceles Triangles

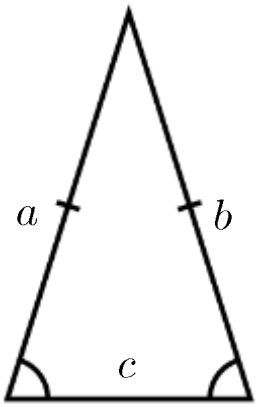


Fig. 2: Isosceles Triangle

The properties of Isosceles Triangles are:

- It is a Regular Polygon with 3 sides
- 2 of the 3 sides are equal, say $a = c$.
- 2 the 3 angles are equal, say $\alpha = \gamma$.
- Perimeter of an Isosceles Triangle is $2 \times a + b$.

PROOF:

We know, in Equilateral Triangles, 2 of the 3 sides are equal.

So, $a = c$

Now, by definition of the Perimeter, we know,

$Perimeter(P) = Sum\ of\ all\ sides$

$Perimeter(P) = a + b + c$

$Perimeter(P) = 2 \times a + b$ (As $a = c$)

- Area of an Isosceles Triangle is $\frac{b}{4} \sqrt{4a^2 - b^2}$.

PROOF:

2.3 Scalene Triangles

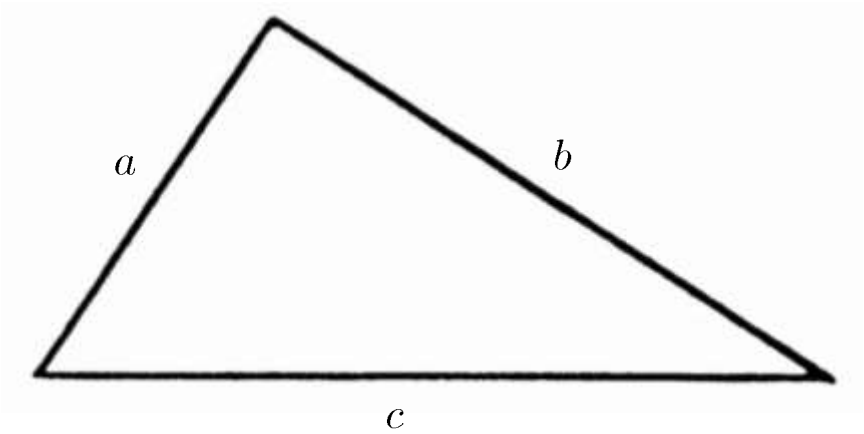


Fig. 3: Scalene Triangle

The properties of Scalene Triangles are:

- It is a Regular Polygon with 3 sides
- All of the 3 sides are unequal, say $a \neq b \neq c$.
- All the 3 angles are equal, say $\alpha \neq \beta \neq \gamma$.
- Perimeter of a Scalene Triangle is $a + b + c$.

PROOF:

Now, by definition of the Perimeter, we know,

$Perimeter(P) = Sum\ of\ all\ sides$

$Perimeter(P) = a + b + c$ (As $a \neq b \neq c$)

- Area of a Scalene Triangle is $\sqrt{s\ s-a\ \ s-b\ \ s-c}$ as $s = (\frac{a+b+c}{2})$ {Heron's Formula}

3 Circles

A circle is a shape consisting of all points in a plane that are at a given distance from a given point, the center (O) equivalently it is the curve traced out by a point that moves in a plane so that its distance from a given point is constant, which is known as radius (R).

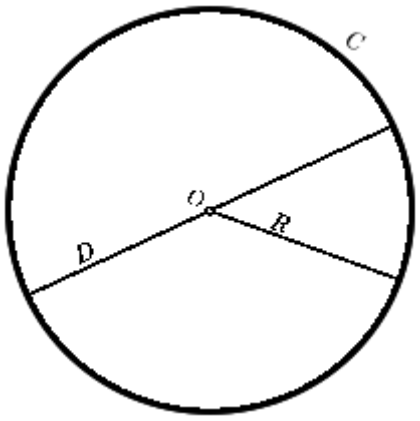


Fig. 4: Circle

The properties of Circle are:

- The distance from the centre to any point on the circle is always constant, which is known as radius of the circle.
- The Diameter (D) of the circle is the longest chord[6] of the circle.

- The Circumference (C) of the circle of radius R is $2 \times \pi \times R$, where $\pi(Pi^{7}) = {}^{22}/_{7}$.

- The Area (A) of the circle of radius R is $\pi \times R^2$, where $\pi = {}^{22}/_{7}$.

3 Quadrilaterals

A Quadrilaterals is a polygon with 4 edges and 4 vertices.
The family of Quadrilaterals includes,

- Rectangle
- Square
- Rhombus
- Trapezium
- Parallelogram
- Kite

3.1 Rectangle

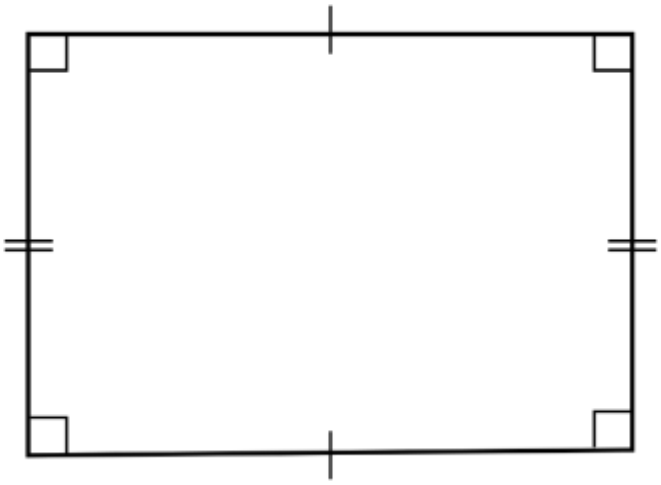


Fig. 5: Rectangle

The properties of Rectangle are:

- The opposite sides are parallel and equal to each other
- All the angles are right angled in nature.
- The diagonals of a rectangle are equal in length and intersect at a certain point present inside the rectangle.
- The Area (A) of a rectangle whose length is l units, and breadth is b units is $l \times b$ square units.

[6] It is a line segment joining two points on the circumference of the circle.
[7] It is a mathematical constant, which is nearly equal to 3.14159…It is the ratio of circumference to diameter of any circle. It is also known as Archimedes' Constant,

- The Perimeter (P) of a rectangle whose length is l units, and breadth is b units is $2 \times (l + b)$ units.

3.2 Square

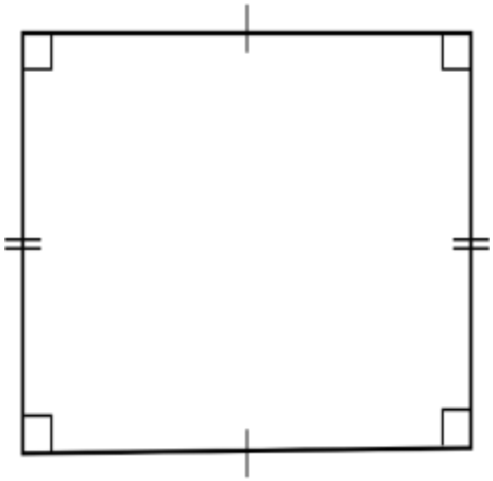


Fig. 6: Square

The properties of Square are:

- All the sides of a square are equal in length.
- All the angles are right angled in nature.
- The diagonals of a rectangle are equal in length and intersect at a certain point present inside the square.
- The Area (A) of a square with side length l units is $l \times l = l^2$ square units.
- The Perimeter (P) of a square with side length l units is $4 \times l$ units.

3.3 $||^{gm}$

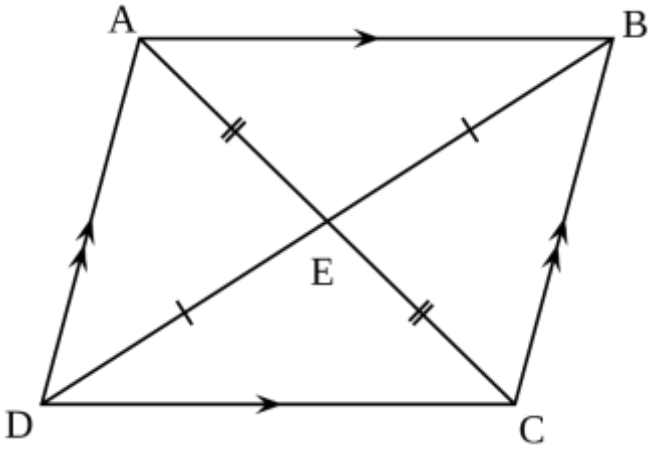


Fig. 7: $||^{gm}$

The properties of parallelogram are:

- Two pairs of opposite sides are parallel and equal in size.
- Diagonals of a $||^{gm}$ bisects each other.
- Two pairs of opposite angles are equal in measure.
- The Area (A) of a $||^{gm}$ with base length b units and height h units is $b \times h$ square units.

3.4 Rhombus

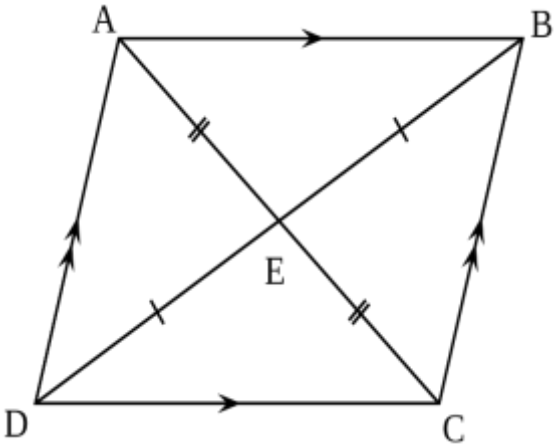


Fig. 8: Rhombus

The properties of rhombus are:

- All the sides of a rhombus are equal in size.
- Diagonals of a rhombus bisects each other at right angle.
- Two pairs of opposite angles are equal in measure.
- The Area (A) of a rhombus with diagonal lengths d_1 and d_2 is $\frac{1}{2}\ d_1 \times d_2$.