Practice Problems

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1 Solved Out Problems

Q. The function $y = x^n$ is even or odd.

Solution:

If n = Even Integer

$f(x) = f(-x)$, therefore, we can say that x raised to the power of even integers will be even functions

If n = Odd Integer

$f(x) = -f(-x)$, therefore, we can say that x raised to the power of odd integers will be odd functions

Q. $A = \begin{bmatrix} 1+2+3+4+\ldots n & 1^2+2^2+3^2+4^2+\ldots n^2 \\ 1^3+2^3+3^3+4^3+\ldots n^3 & 1+2+3+4+\ldots n \end{bmatrix}$, find A^{-1}.

Solution:

$$\Rightarrow A = \begin{bmatrix} \frac{n\times(n+1)}{2} & \frac{n\times(n+1)\times(2n+1)}{6} \\ (\frac{n\times(n+1)}{2})^2 & \frac{n\times(n+1)}{2} \end{bmatrix}$$

$$\Rightarrow Adj(A) = \begin{bmatrix} \frac{n\times(n+1)}{2} & -(\frac{n\times(n+1)}{2})^2 \\ -\frac{n\times(n+1)\times(2n+1)}{6} & \frac{n\times(n+1)}{2} \end{bmatrix}^T$$

$$= \begin{bmatrix} \frac{n\times(n+1)}{2} & -\frac{n\times(n+1)\times(2n+1)}{6} \\ -(\frac{n\times(n+1)}{2})^2 & \frac{n\times(n+1)}{2} \end{bmatrix}$$

$$\Rightarrow |A| = \left(\frac{n\times(n+1)}{2}\right)^2 - \left(\frac{n\times(n+1)}{2}\right)^2 \times \frac{n\times(n+1)\times(2n+1)}{6}$$

$$\Rightarrow |A| = \left(\frac{n\times(n+1)}{2}\right)^2 (1 - \frac{n\times(n+1)\times(2n+1)}{6})$$

$$\Rightarrow A^{-1} = \frac{Adj(A)}{|A|}$$

$$= \frac{1}{\left(\frac{n\times(n+1)}{2}\right)^2 (1 - \frac{n\times(n+1)\times(2n+1)}{6})} \begin{bmatrix} \frac{n\times(n+1)}{2} & -\frac{n\times(n+1)\times(2n+1)}{6} \\ -(\frac{n\times(n+1)}{2})^2 & \frac{n\times(n+1)}{2} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{Adj(A)}{|A|}$$

$$= \begin{bmatrix} \frac{\frac{n\times(n+1)}{2}}{\left(\frac{n\times(n+1)}{2}\right)^2 (1-\frac{n\times(n+1)\times(2n+1)}{6})} & \frac{-\frac{n\times(n+1)\times(2n+1)}{6}}{\left(\frac{n\times(n+1)}{2}\right)^2 (1-\frac{n\times(n+1)\times(2n+1)}{6})} \\ \frac{-(\frac{n\times(n+1)}{2})^2}{\left(\frac{n\times(n+1)}{2}\right)^2 (1-\frac{n\times(n+1)\times(2n+1)}{6})} & \frac{\frac{n\times(n+1)}{2}}{\left(\frac{n\times(n+1)}{2}\right)^2 (1-\frac{n\times(n+1)\times(2n+1)}{6})} \end{bmatrix}$$

Q. $A = \begin{bmatrix} 1+2+3+4+\ldots n & 1^2+2^2+3^2+4^2+\ldots n^2 \\ 1^3+2^3+3^3+4^3+\ldots n^3 & 1+2+3+4+\ldots n \end{bmatrix}$, find $|A^{-1}|$

Solution:

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{\left(\frac{n\times(n+1)}{2}\right)^2 (1-\frac{n\times(n+1)\times(2n+1)}{6})}$$

Q. Find the range and domain of $y = \frac{x^2+5x+6}{x^2+8x+12}$

Solution:

For finding range, we will have to evaluate the possible values of y.

For finding domain, we will have to evaluate the possible values of x.

$$y = \frac{x^2+5x+6}{x^2+8x+12}$$

$$y = \frac{x^2+3x+2x+6}{x^2+6x+2x+12} = \frac{x(x+3)+2(x+3)}{x(x+6)+2(x+6)} = \frac{\cancel{(x+2)}\times(x+3)}{\cancel{(x+2)}\times(x+6)} = \frac{x+3}{x+6}$$

So, the critical values will be -3 and -6.

Plotting them on the number line,

we can see, that on putting values greater than -3, in the equation y = f(x), the value of the equation turns out to be positive.

On putting values between -3 and -6, in the equation y = f(x), the value of the equation turns out to be negative.

On putting values less than -6, in the equation y = f(x), the value of the equation turns out to be positive.

So, the values of x, for which the equation is positive is

$(-\infty, -6) \cup [-3, \infty)$

$$y = \frac{x+3}{x+6}$$

$$\Rightarrow y = \frac{x+3}{x+6}$$

$\Rightarrow y(x + 6) = x + 3$

$\Rightarrow yx + 6y = x + 3$

$\Rightarrow yx - x = 3 - 6y$

$\Rightarrow x = \dfrac{3 - 6y}{y - 1}$

Critical Points are ½ and 1.

Plotting them on the number line,

we can see, that on putting values greater than 1, in the equation x = f(y), the value of the equation turns out to be negative.

On putting values between ½ and 1, in the equation x = f(y), the value of the equation turns out to be positive.

On putting values less than ½ , in the equation x = f(y), the value of the equation turns out to be negative.

So, the values of y, for which the equation is positive is

$[\frac{1}{2}, 1)$

So, Domain will be
$(-\infty, -6) \cup [-3, \infty)$

and range will be

$[\frac{1}{2}, 1)$

2 Try it Yourself Problems

Q. The function, $y = \frac{d^n}{dx^n}(sin\ x)$, is even or odd?

Q. $A = \begin{bmatrix} 1 + 2 + 3 + 4 + \ldots n & 1^2 + 2^2 + 3^2 + 4^2 + \ldots n^2 \\ 1^3 + 2^3 + 3^3 + 4^3 + \ldots n^3 & 1 + 2 + 3 + 4 + \ldots n \end{bmatrix}^2$, find A^{-1} and $|A^{-1}|$.

Q. Find the range and domain of $y = \sum_{i=1}^{n} \frac{1}{log_i\, n}$