DELHI UNIVERSITY

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BSC HONS PHYSICS(SEM V)

EPR Paradox

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Abstract

The EPR paradox proposed by Einstein Podolsky Rosen, appears when measurement results of some properties of two distantly-entangled particles are correlated in a way that cannot be explained classically, and apparently violate locality.For example, when we observe the position of an object, we affect its momentum uncontrollably. Thus we cannot determine both position and momentum precisely. A similar situation arises for the simultaneous determination of energy and time. The resolution of the paradox depends on one's interpretation of quantum mechanics. Explanations from quantum mechanics remain common place today, but they fail to explain the EPR paradox totally in a way that be accepted by the whole community. Here we present a simple discussion on the EPR Paradox.

Contents

1 History

In 1935, Einstein, Podolsky and Rosen published a famous paper which questioned the completeness of quantum theory, arguing that conventional quantum measurement theory leads to unreasonable conclusions. Known today as the "EPR paradox," the thought experiment was meant to demonstrate the innate conceptual difficulties of quantum theory. Their argument was later expressed in a somewhat simpler form by Bohm. Einstein then struggled unsuccessfully to find a theory that could better comply with his idea of locality. Since his death, experiments analogous to the one described in the EPR paper have been carried out that have confirmed that physical probabilities, as predicted by quantum theory, do exhibit the phenomena of Bell-inequality violations that are considered to invalidate EPR's preferred *local hidden-variables* type of explanation for the correlations to which EPR first drew attention. The paper did help deepen our understanding of quantum mechanics by exposing the fundamentally non-classical characteristics of the measurement process.

2 Stern Gerlach Experiment

If e^- is in the state

$$\Psi = a\left|\phi_{up}\right\rangle + b\left|\phi_{down}\right\rangle$$

and we are apply a magnetic field in z-direction,

$$B_z = B_o + \beta \cdot z$$

Energy associated with spin angular momentum and $\vec{B \cdot z}$

$$\begin{aligned} E &= -\mu_o \vec{S} \cdot \vec{B} \\ E &= \mu_o \vec{S_z}(B_o + \beta \cdot z) \\ &= -\frac{\hbar}{2}\mu B \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} C(z) & 0 \\ 0 & -C(z) \end{pmatrix} \end{aligned}$$

Thus,

$$\begin{aligned} E\left|\phi_{up}\right\rangle &= C(z)\left|\phi_{up}\right\rangle \\ E\left|\phi_{down}\right\rangle &= -C(z)\left|\phi_{down}\right\rangle \end{aligned}$$

Thus,

$$\begin{aligned} \Psi(t) &= ae^{-i\frac{E_{up}t}{\hbar}}\left|\phi_{up}\right\rangle + be^{-i\frac{E_{down}t}{\hbar}}\left|\phi\right\rangle_{down} \\ \Psi(t) &= ae^{-i\frac{\mu_o B_o t}{2}}e^{-i\frac{\mu_o \beta \cdot zt}{2}}\left|\phi_{up}\right\rangle + be^{-i\frac{-\mu_o B_o t}{2}}e^{-i\frac{-\mu_o \beta \cdot zt}{2}}\left|\phi_{down}\right\rangle \end{aligned}$$

This shows that spin up e^- are shifted up and down spin e^- are shifted down, meaning that we have correlated the position to spin state of e^-

The e^-s are still in superposition of both up spin and down spin state, both having 50% probability to occur.

Now when we measure the spin state of electron and if it comes out to be up spin then all the electrons will be in $\left|+\frac{1}{2}\right\rangle$ state. **We say that the e^-1 are ENTANGLED**

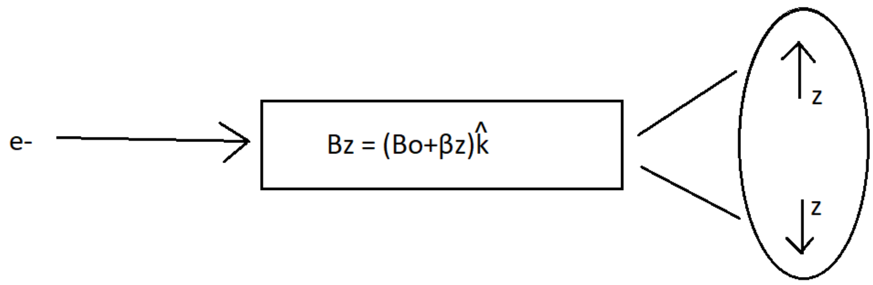

Figure 1

3 The Paradox

If we have two electrons we keep it at some distance apart. We measure one of the electron's spin state and let's say we get $\left|+\frac{1}{2}\right\rangle$ state (of S_z). And suppose we measure spin of the other electron ut along x-direction i.e. S_x and let's say it come out to be $\left|-\frac{1}{2}\right\rangle$. Then for this electron we know,

$$S_x = -\frac{1}{2}$$

$$S_z = +\frac{1}{2}$$

(by entanglement) But,

$$[S_x, S_z] \neq 0$$

i.e. we can't measure S_x and S_z simultaneously

4 Local Realism and Determinism

In a complete theory, if an observable of a system can be predicted with certainty for some measurement made out of a casual contact with the system then this observable should have a real or a definite value. This is how EPR defined local realism. EPR saw that an entangled state (9.1) was ideal for revealing the absence of local realism in quantum mechanics. At that time modern optical sources were unknown so the example visualised in their thought experiment was contrived but nonetheless a valid entangled state. It involved a pair of particles, A and B, in an eigenstate of two compatible observables: these being their total momentum $P = p_1 + p_2$, and their separation $\Delta x = x_1 + x_2$. EPR argued that if a measurement of momentum of A is made when A and B are out of casual contact the momentum of B can be predicted $(P - p_1)$ with certainty. Alternatively, by measuring the position of A when A and B are out of casual contact the location of B $(x_1 + \Delta x)$ can be predicted with certainty. Thus both the momentum and coordinates of B should have real or definite values in any complete theory. However quantum mechanics requires that the product of the precision in determining the momentum and position of B must not exceed $\hbar$. The inference that the authors drew was that *quantum mechanics is incomplete.*

Quantum mechanics successfully predicts the correlation of measurements at space-like separation on the components of an entangled system. Einstein had doubts about this phenomenon and as it clearly violates the requirement that a physical theory should be local. That is to say, measurements made on one component of a system should not affect another component if they are out of casual contact.

5 Hidden Variables Theory

Bohm and Einstein supported an alternative approach called the hidden-variables theory, which suggested that quantum mechanics was incomplete. In this viewpoint, there had to be some aspect of quantum mechanics that wasn't immediately obvious but which needed to be added into the theory to explain this sort of non-local effect. To illustrate, Let us consider that we have two envelopes that each contain money. Now somehow we know that one of them contains a 5 bill and the other contains a 10 bill. If we open one envelope and it contains a 5 bill, then we know for sure that the other envelope contains the 10 bill. The problem with this analogy is that quantum mechanics definitely doesn't appear to work this way. In the case of the money, each envelope contains a specific bill, even if we never get around to looking in them.

Hidden variable theories can explain why same-axis measurements always yield opposite results without any violation of locality: A measurement of one electron doesn't affect the other but merely reveals the preexisting value of a hidden variable.

Bell proved that we can rule out local hidden variable theories, and indeed rule out locality altogether, by measuring entangled particles' spins along different axes.

6 Bell's Theorem

Einstein Podolsky and Rosen did not doubted the correctness of Quantum Mechanics but they only claimed that it is an incomplete description of physical reality. According to them, in addition to the wave function Ψ , to characterise the state of a system fully we need some other quantity called *Hidden Variable*, λ. It is called Hidden Variable since we have no idea how to calculate or measure it. In 1964 J.S. Bell proved that any local hidden variable theory is incompatible with quantum mechanics i.e. no theory that satisfies the conditions imposed can reproduce the probabilistic predictions of quantum mechanics under all circumstances.

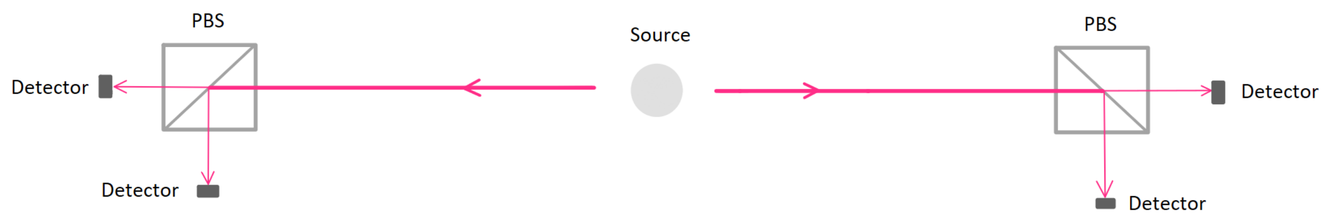

Figure 2 :Outline of generic apparatus to test for violations of Bell's inequalities. PBS indicates a polarising beam splitter.

Suppose there are a set of hidden local variables that determine the outcomes of all measurement in any correlation experiment involving entangled states. For simplicity the set of local hidden variables can be coalesced into a single one, λ. This would have some distribution $\rho(\lambda)$ such that $\int \rho(\lambda)$ is unity. In the case considered here measurements are made of the polarisation state of two fully entangled photons with parallel polarisation.The arrangement is shown in Figure 2. Photons travelling from the source in opposite directions pass through polarising beam splitters and fall thereafter on photodetectors that record all the photons without noise. In each arm the beam splitters and detectors could be rotated freely, independently of the other arm to any selected azimuthal angle around the line of flight from the source. This makes it possible to set the orientations of the polarisation required for transmission and reflection differently for the two photons. Let the azimuthal angles be set to α and α' for the left-hand path and β and β' for the right-hand paths, respectively. Also, we denote detection of a reflected photon by -1 and detection of a transmitted photon by $+1$. Then the outcomes predicted by the local hidden variables are precise and limited to these values

$$A(\lambda, a) = \pm 1 \ , \ B(\lambda, b) = \pm 1$$

and a correlation between them

$$E(\alpha, \beta) = \langle A(\alpha) B(\beta) \rangle \tag{1}$$

where $A(\alpha)$ and $B(\beta)$ are the outcomes measured by the detectors and $A(\alpha) = \pm 1$ and $B(\beta) = \pm 1$. So, within the hidden variable model the above correlation can be written as

$$E(\alpha, \beta) = \int A(\alpha, \lambda) B(\beta, \lambda) d\lambda$$

Here we see that $A(\alpha, \lambda)$ depends only on α and therefore cannot get influenced by a measurement performed on the second particle. Similarly $B(\beta, \lambda)$ depends only on β and is thus not influenced by a measurement made on the first particle. So, the locality assumption remains intact. Now, lets consider the quantity

$$\begin{aligned} |E(\alpha, \beta) - E(\alpha, \beta')| &= |\int A(\alpha, \lambda)[B(\beta, \lambda) - B(\beta', \lambda)] \rho(\lambda) d\lambda| \\ &\leq \int |A(\alpha, \lambda)[B(\beta, \lambda) - B(\beta', \lambda)]| \rho(\lambda) d\lambda \\ &\leq \int |B(\beta, \lambda) - B(\beta', \lambda)| \rho(\lambda) d\lambda \end{aligned} \tag{2}$$

Similarly

$$|E(\alpha', \beta) + E(\alpha', \beta')| \leq \int |B(\beta, \lambda) + B(\beta', \lambda)| \rho(\lambda) d\lambda \tag{3}$$

On adding equation 2 and 3, we get

$$|E(\alpha, \beta) - E(\alpha, \beta')| + |E(\alpha', \beta) + E(\alpha', \beta')| \leq \int [|B(\beta, \lambda) - B(\beta', \lambda)| + |B(\beta, \lambda) + B(\beta', \lambda)|] \rho(\lambda) d\lambda$$

$\because B(\beta, \lambda)$ and $B(\beta', \lambda)$ takes only the values ± 1

$$\implies |B(\beta, \lambda) - B(\beta', \lambda)| + |B(\beta, \lambda) + B(\beta', \lambda)| = 2 \tag{4}$$

and $\because \int \rho(\lambda) = 1$, we get

$$S \equiv |E(\alpha, \beta) - E(\alpha, \beta')| + |E(\alpha', \beta) + E(\alpha', \beta')| \leq 2 \tag{5}$$

This is the *Bell's inequality.*

For Quantum theory, the correlation given in 1 can be written as

$$\begin{aligned} E(\alpha, \beta) &= (+1) \times P_{+1} + (-1) \times P_{-1} \\ &= P(+, \alpha, +, \beta) + P(-, \alpha, -, \beta) - P(-, \alpha, +, \beta) - P(+, \alpha, -, \beta) \\ &= P(\alpha, \beta) + P(\alpha + \pi/2, \beta + \pi/2) - P(\alpha + \pi/2, \beta) + P(\alpha, \beta + \pi/2) \end{aligned}$$

The predictions given by quantum mechanics for identical polarisation outcomes left and right with the choice (α, β) is simply given by Malus' law, $[cos^2(\alpha = \beta)]$, and the prediction for orthogonal polarizations must be $[sin^2(\alpha = \beta)]$. Thus the correlator presented by quantum mechanics is

$$E(\alpha, \beta) = cos2(\alpha - \beta) \tag{6}$$

Now, for $= 0, \beta = \frac{3\pi}{8}, \alpha' = -\frac{\pi}{4}$ and $\beta' = \frac{\pi}{8}$, we get $|S| = 2\sqrt{2} > 2$. This *violates the Bell's inequality*. So, there cannot be any hidden variables.

So the conclusion is that any hidden variable theory must be nonlocal. we can have hidden variables, but if you do, then particles must exchange this hidden information at speeds faster than light.This is unsatisfying, because under relativity theory, nonlocality implies travel backwards through time.

However some physicists argue that it may be possible to construct a local theory that does not respect certain assumptions in the derivation of Bell's Inequality. The subject is not yet closed, and may yet provide more interesting insights into the subtleties of quantum mechanics.

7 Current Status

The EPR paper brought attention to the phenomena of quantum entanglement, but it did not ultimately provide a valid case against the Copenhagen interpretation of quantum mechanics. Before that paper, most physicists viewed a measurement as a physical disturbance inflicted directly on the measured system: one shines light onto an electron to determine its position, but this disturbs the electron and produces uncertainties. The EPR paradox shows that a "measurement" can be performed on a particle without disturbing it directly, by performing a measurement on a distant entangled particle. Today, the EPR paper is widely viewed as a misstep by Einstein.

8 Summary

The paradox in EPR is that - one, it suggests that a measurement can be performed on a particle without disturbing it directly, by performing a measurement on a distant entangled particle, thus violating uncertainity. Second, it suggests that information between two entangled particles can be transferred at a speed greater than speed of light thus violating special relativity.

But, Then Bell's Theorem assumes that certain "elements of reality" exist, that is the hidden variables. It also assumes that the interactions are local, that is, a setting of the hidden variables takes place when the particles are created together and once they separate, there can no longer be any adjustment of the parameters. These assumptions lead to predictions that are violated by quantum predictions and by experiments, meaning that the assumptions are invalid.

9 Appendix

9.1 Quantum Entanglement

Quantum Entanglement is a phenomenon in which, when two or more particles link up in a certain way, no matter how far apart they might be i.e., they share a common, unified quantum state. So making a measurement on one of the entangled particles would immediately cause a change or collapse of the wave function for the other particles. But before we made measurements on any of the particles the system was in a superposition of all the possible states it could be found in when we were to make a measurement on that system.

Einstein called this phenomenon *a spooky action at a distance*. This phenomenon didn't fit quite well with Einstein, Podolsky and Rosen, as the idea that in an entangled state, if we were to make a measurement on one of the entangled particles then we would have instantaneous knowledge about the other particle thereby causing a collapse of the wave function of this quantum system, meant instantaneous communication because we could know something about a particle that was far away instantaneously, without even having to wait for life to come to that particle. This brought up the EPR paradox.

9.2 Bell's Theorem contd..

In light of Bell's theorem, the experiments thus performed establish that our world is non-local. The proof of Bell's theorem is obtained by combining the EPR argument and Bell's inequality theorem.

As an alternate approach to understand Bell's theorem we can use electron spin concept as follows

Bell's Inequality

$$N(A\bar{B}) + N(B\bar{C}) \geq N(A\bar{C})$$

Now suppose we have two electrons which are entangled as shown below:

$$\Psi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Now we compute the probability of,

(i) We get $|\uparrow\rangle$ state, $\theta = 0$ then we will subsequently get $|\downarrow\rangle$ state at B measured at an angle $\theta = P(\uparrow_\theta, \downarrow_\theta)$

(ii) We get $|\uparrow\rangle$ state at θ, then we get subsequently $|\downarrow\rangle$ at B measured at angle 2θ

(iii) We get $|\uparrow\rangle$ state at 0, then we get subsequently $|\downarrow\rangle$ at B measured at angle 2θ

And we know,

$$|\downarrow_\theta\rangle = cos\left(\frac{\theta}{2}\right)|\downarrow\rangle + isin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

So for the above set up we get following Bell's Inequality,

$$P(\uparrow_\theta, \downarrow_\theta) + P(\uparrow_\theta, \downarrow_{2\theta})(\uparrow_\theta, \downarrow_\theta)$$

$$sin^2\left(\frac{\theta}{2}\right) + sin^2\left(\frac{\theta}{2}\right) \geq sin^2(\theta)$$

So for $\theta << 1$:

$$\left(\frac{\theta}{2}\right)^2 + \left(\frac{\theta}{2}\right)^2 \geq (\theta)^2$$
$$\left(\frac{\theta^2}{2}\right) \geq (\theta)^2$$

Which is obviously wrong.

- So if we make the EPR set up experimently, we get **violation** of Bell's Inequality and this is actually proven by Alain Aspect.
- This implies that there are no classical definite configurations underlying the probability of quantum mechanics. It is impossible to build a classical theory with hidden variables that are randomly distributed such that we can reproduce the predictions of quantum mechanics.

10 Contribution

Each member of the team contributed in every topic and helped each other.

- Anurag Das
  - Stern Gerlach Experiment
  - The Paradox
  - Bell's Theorem
- Abinash Tahbildar
  - History
  - Hidden Variables
  - Bell's Theorem
  - Current status
- Prabhujyoti Singh
  - Local Realism and Determinism
  - Bell's Theorem
  - Quantum Entanglement

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