COORDINATE GEOMETRY

Distance Formula

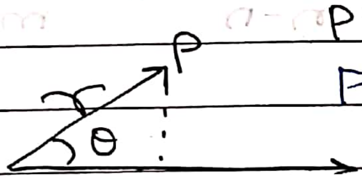
$A \equiv (x_1, y_1)$ $\quad$ $B \equiv (x_2, y_2)$

$$\boxed{|\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

Polar coordinates

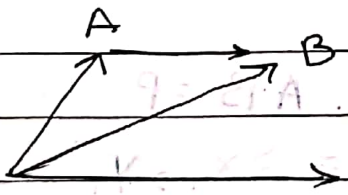
$P \equiv (r, \theta) = (r\cos\theta, r\sin\theta)$

Polar $\quad$ cartesian



Distance Formula in Polar form

eg) $A\left(2, \frac{\pi}{3}\right)$ $\quad$ $B\left(3, \frac{\pi}{6}\right)$



$$|AB| = |\vec{A} - \vec{B}|$$

$$= \sqrt{2^2 + 3^2 - 2\times 2\times 3\cos\frac{\pi}{6}}$$

$A(r_1, \alpha)$ $\quad$ $B(r_2, \beta)$

$$\boxed{|AB| = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\alpha-\beta)}}$$

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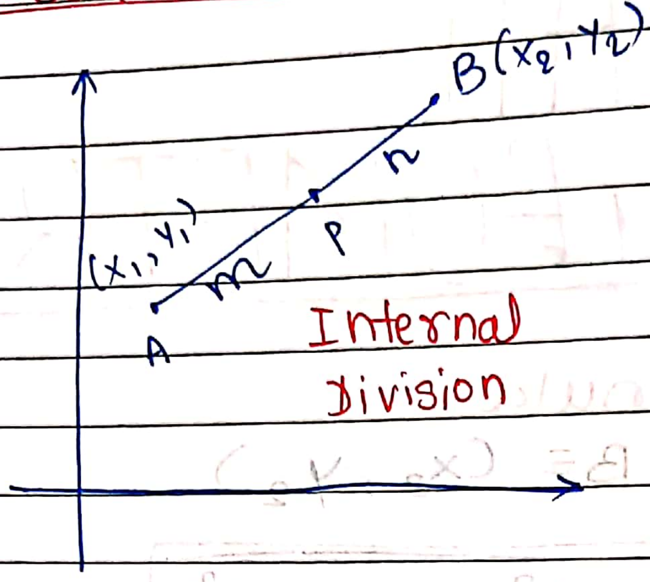
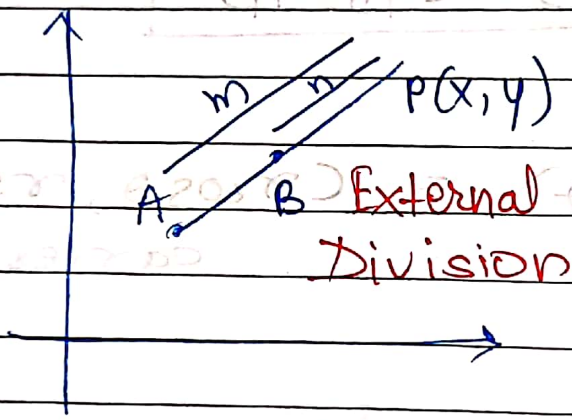
Section formula



$$P = \left\{ \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right\}$$



$$P = \left\{ \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right\}$$

$A(x_1, y_1)$ M $B(x_2, y_2)$

$$M = \left\{ \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right\}$$

Q) (x_1, y_1) $P(\alpha, \beta)$ $(x_2 y_2)$

A B

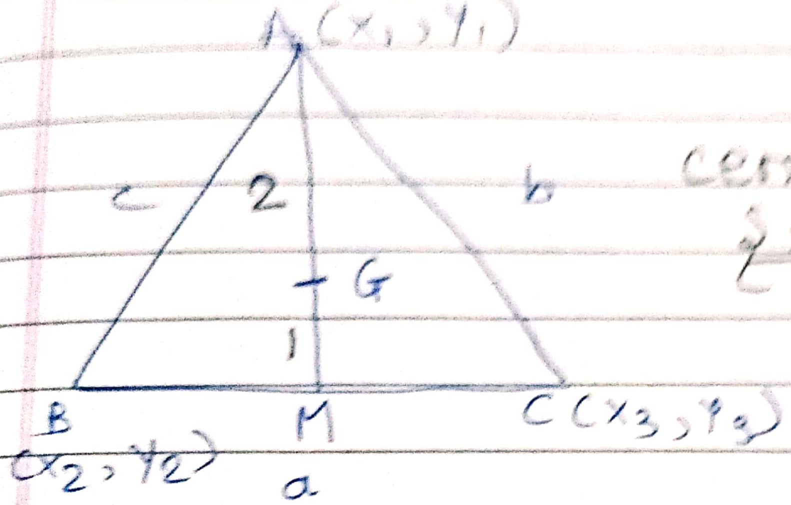
Mid pt. AB = P

$A = \{x_1, y_1\}$

$P\cancel{B} = \{\alpha, \beta\}$

$$B = \{2\alpha - x_1, 2\beta - y_1\}$$

centroid



centroid:

$$\left\{ \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right\}$$

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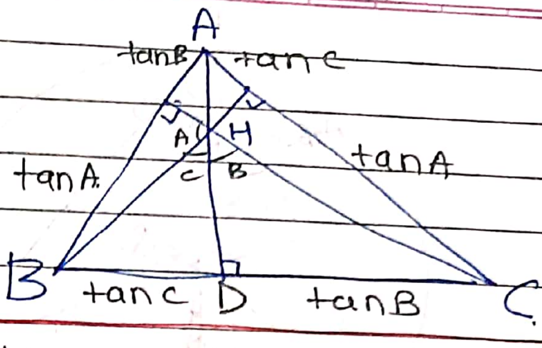
ORTHOCENTRE



$$\boxed{H = \left\{ \frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C} \right\}}$$

$BD = AD/\tan B$

$CD = AD/\tan C$

$\Rightarrow BD : CD = \tan C : \tan B$

$AE = c\cos A \Rightarrow AH = \frac{AE}{\sin C} = \frac{c}{\sin C} \cos A$

$= \boxed{2R\cos A} \rightarrow$ Learn.

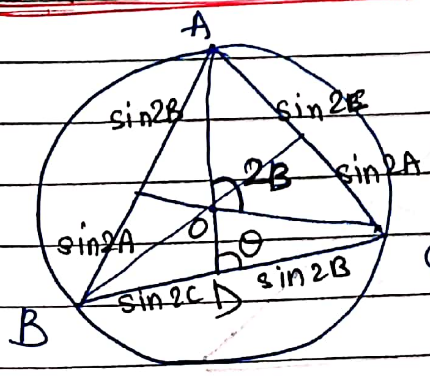
$\Rightarrow BH = 2R\cos B$

$\Rightarrow \boxed{HD = 2R\cos B\cos C} \rightarrow$ learn

$\frac{AD}{HD} = \frac{\cos A}{\cos B\cos C} = \frac{\sin A}{\cos B\cos C\tan A} = \frac{\sin(B+C)}{\cos B\cos C\tan A}$

$\Rightarrow \boxed{\frac{AD}{HD} = \frac{\tan B + \tan C}{\tan A}}$

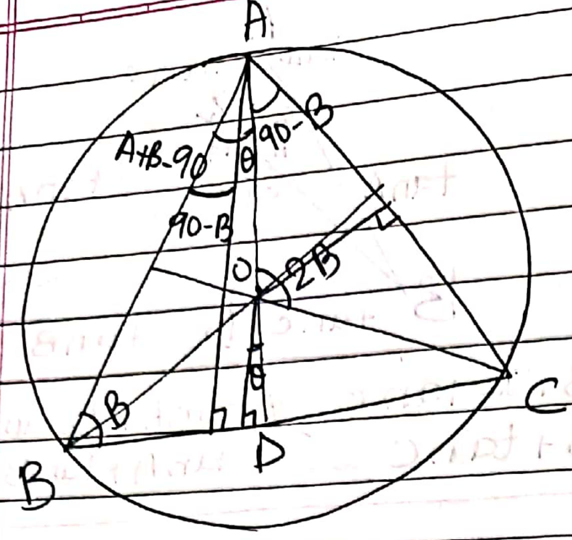
CIRCUMCENTRE



$$\boxed{O = \left\{ \frac{\Sigma \sin 2A\, x_i}{\Sigma \sin 2A}, \frac{\Sigma \sin 2A\, y_i}{\Sigma \sin 2A} \right\}}$$

$\frac{BD}{\sin 2C} = \frac{R}{\sin\theta} = \frac{CD}{\sin 2B}$

$\Rightarrow \frac{BD}{CD} = \frac{\sin 2C}{\sin 2B}$



$$\theta = A+B-90-90+B$$
$$= A+2B-180$$
$$= A+2B-A-B-C$$
$$= \boxed{B-C}$$

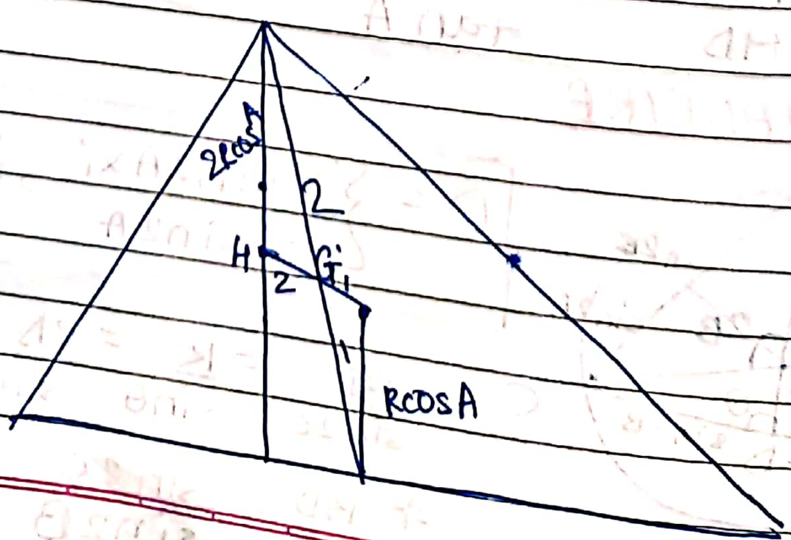
$$OD = \frac{R\cos A}{\cos\theta} \qquad (R = AO)$$

$$\Rightarrow \frac{AO}{OD} = \frac{\cos(B-C) \times 2\sin A}{\cos A \times 2\sin A}$$

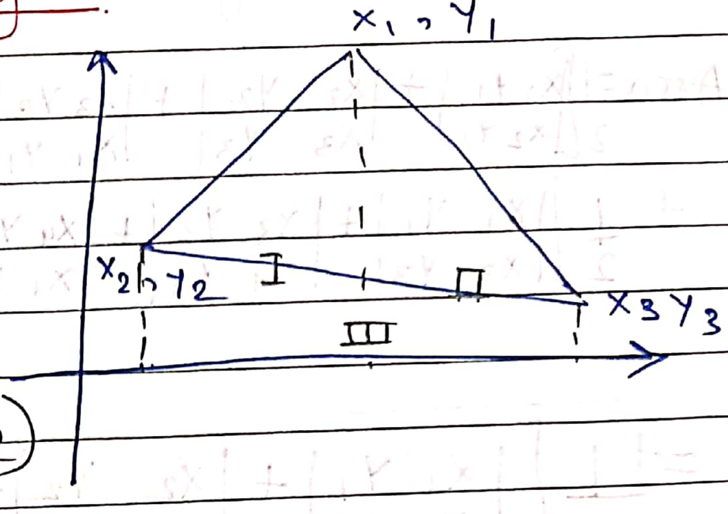
$$= \frac{2\sin(B+C)\cos(B-C)}{2\sin A\cos A}$$

$$\boxed{\frac{AO}{OD} = \frac{\sin 2B + \sin 2C}{\sin 2A}}$$

Euler's Line :



Area of Triangle



$$\mathrm{I} + \mathrm{II} - \mathrm{III}$$

$$= \frac{1}{2}(y_2+y_1)(x_1-x_2)$$

$$+ \frac{1}{2}(y_1+y_3)(x_3-x_1)$$

$$\cancel{4} - \frac{1}{2}(y_2+y_3)(x_3-x_2)$$

$$= \boxed{\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]}$$

$$= \boxed{\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$= \boxed{\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{1}{2}\left[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_1-x_1y_3)\right]$$

$$= \boxed{\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right]}$$

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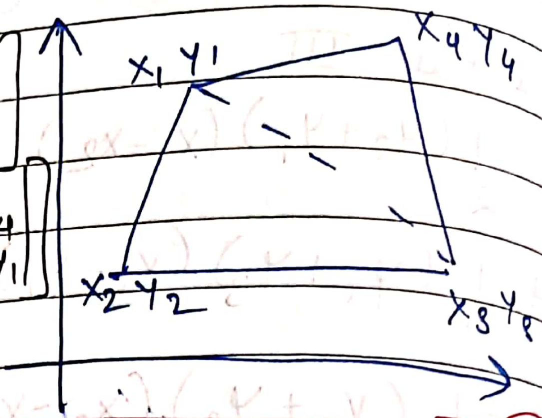
→ Area of Quadrilateral

$$\text{Area} = \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right]$$

$$+ \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}\right]$$

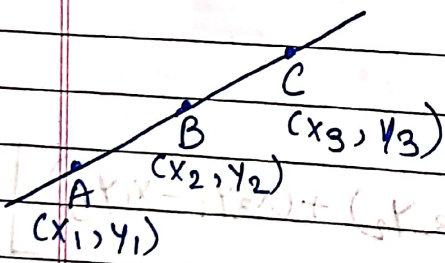


$$\boxed{= \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}\right]}$$

→ Area of n-sided polygon

$$\boxed{= \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \dots + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right]}$$

Collinearity of Points:



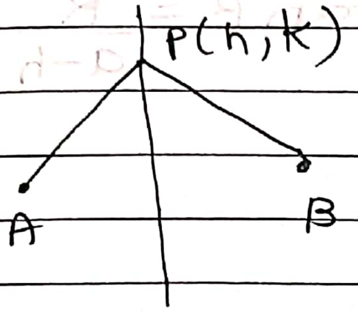
$$\rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$\rightarrow AB + BC = CA$

→ Pair of any two slopes are ~~&~~ equal then they are in a straight line.

Locus of a Point:

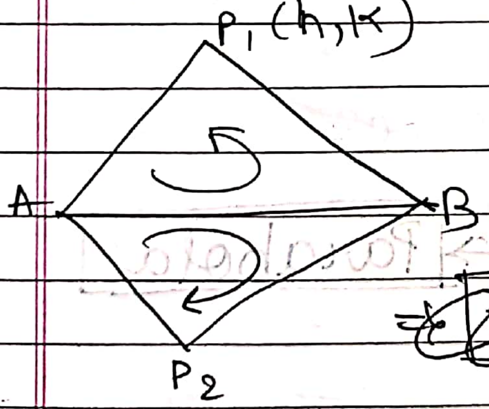
① $A = \{2, 0\}$, $B = \{4, 7\}$
find Locus of a pt. such that it is equidistant from A & B.



$$PA^2 = PB^2$$
$$(h-2)^2 + k^2 = (h-4)^2 + (k-7)^2$$
$$4h + 14k = 61$$

$\Rightarrow \boxed{4x + 16y = 61}$ → Eqn of Locus of all such pts.

② $A = \{2, 0\}$, $B = \{4, 7\}$
Find locus of a pt. such that area of $\Delta PAB = 10$.



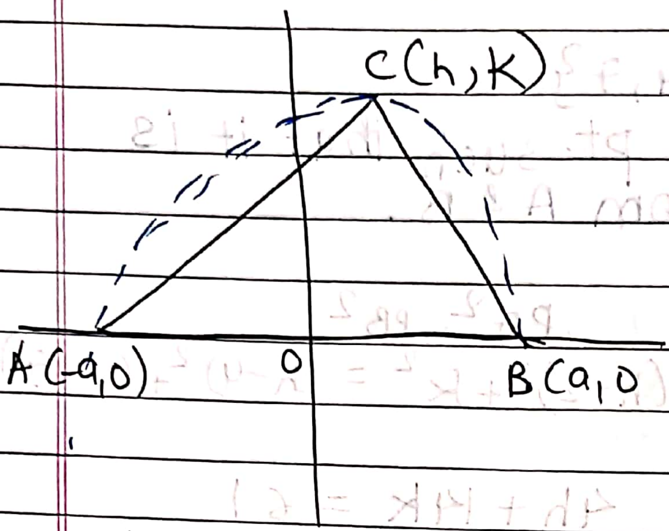
$$-20, 20 = h(-7) + 2k + 14$$
$$\Rightarrow 2k - 7h - 6 = 0$$
$$\text{or } 2k - 7h + 34 = 0$$

$\Rightarrow$ ~~Eqn 8 & 2~~ $\Rightarrow$ combined eqn $= (2k - 7h - 6)(2k - 7h + 34) = 0$

$\Rightarrow$

$\Rightarrow \boxed{(2y - 7x - 6)(2y - 7x + 34) = 0}$

Q) find locus of vertex C of a ΔABC when A, B are fixed, $\tan A + \tan B = p$.



$\tan A = \frac{K}{h+a}$

$\tan B = \frac{K}{a-h}$

$\frac{K}{h+a} + \frac{K}{a-h} = p$

$K\left(\frac{2a}{a^2-h^2}\right) = p$

$\Rightarrow 2aK = pa^2 - ph^2$

$\Rightarrow 2ay = p(a^2 - x^2)$

$\Rightarrow \boxed{y = \frac{pa}{2} - \frac{px^2}{2}} \rightarrow \boxed{\text{Parabola}}$

Q) Locus of a pt. s.t. sum of its dist. from (-1,0) & (1,0) = 3.

M-I)

$$\sqrt{(h+1)^2+k^2} + \sqrt{(h-1)^2+k^2} = 3$$

$$(h+1)^2+k^2 = (h-1)^2+k^2+9-6\sqrt{(h-1)^2+k^2}$$

$$\Rightarrow 6\sqrt{(h-1)^2+k^2} = 9-4h$$

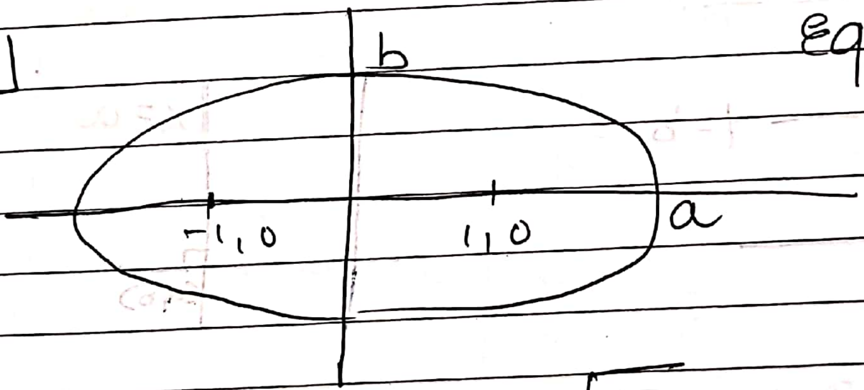
$$\Rightarrow 36(h^2-2h+1)+36k^2 = 81+16h^2-72h$$

$$\Rightarrow 36k^2+20h^2 = 45$$

$$\Rightarrow \frac{4}{5}k^2+\frac{4}{9}h^2 = 1$$

$$\Rightarrow \boxed{\frac{4}{9}x^2+\frac{4}{5}y^2 = 1}$$

M-II)



eq^n of ellipse

$$= \frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

$$\Rightarrow a = \frac{3}{2}, \quad b = \sqrt{\frac{5}{4}}$$

$$\Rightarrow \boxed{\frac{4}{9}x^2+\frac{4}{5}y^2 = 1}$$

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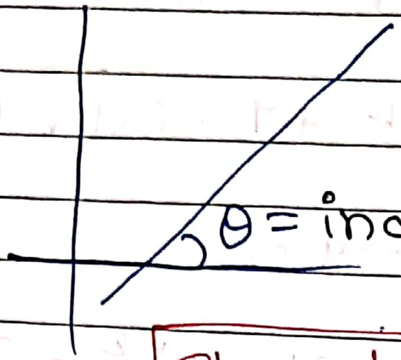
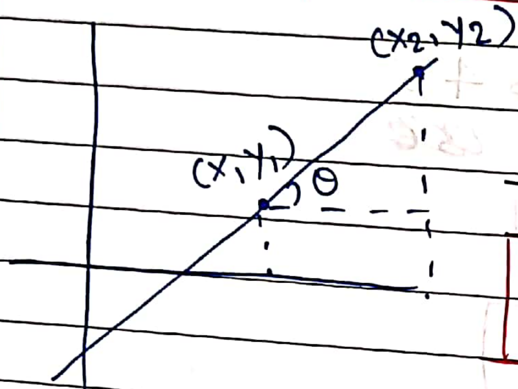
SLOPE OF A LINE



θ = inclination of the line

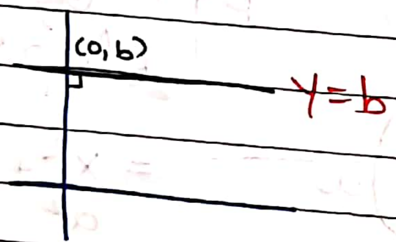
$$\boxed{\text{Slope} = \tan\theta}$$

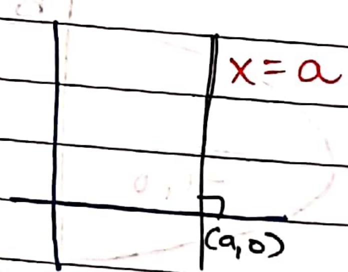


Slope

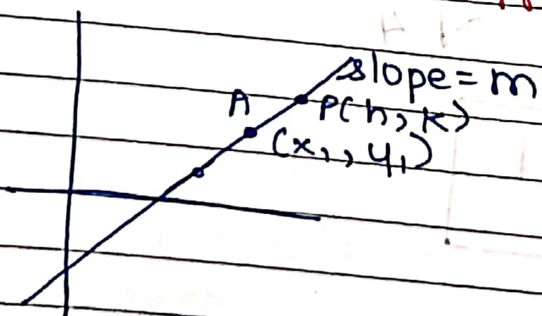
$$\boxed{m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}}$$

→ Some Lines





→ Slope Point form of a line



$$\frac{k - y_1}{h - x_1} = m$$

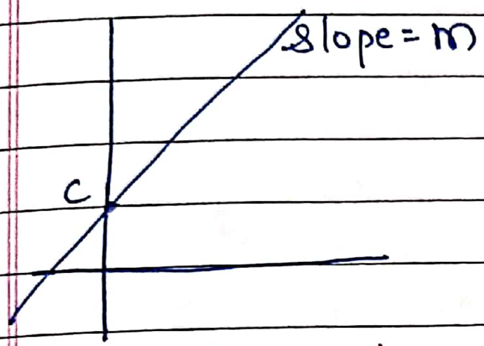
$$\boxed{y - y_1 = m(x - x_1)}$$

→ Two Points form

$A = \{x_1, y_1\}$ $B = \{x_2, y_2\}$

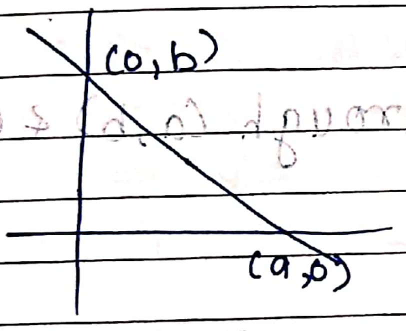
$$\boxed{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$

→ Slope intercept form :



$$\boxed{y = mx + c}$$

→ Double intercept form

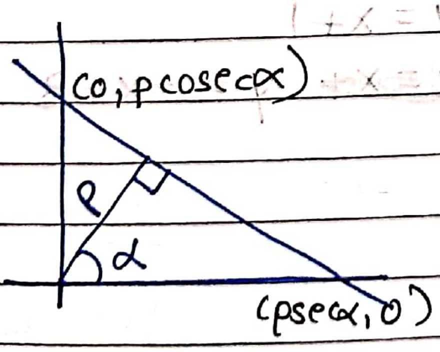


$$y - b = \left(\frac{-b}{a}\right)(x - 0)$$

$$\Rightarrow \frac{y}{b} - 1 = -\frac{x}{a}$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

→ Normal form

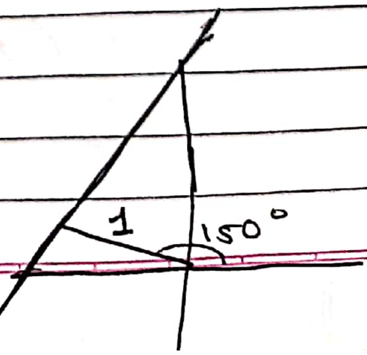


$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

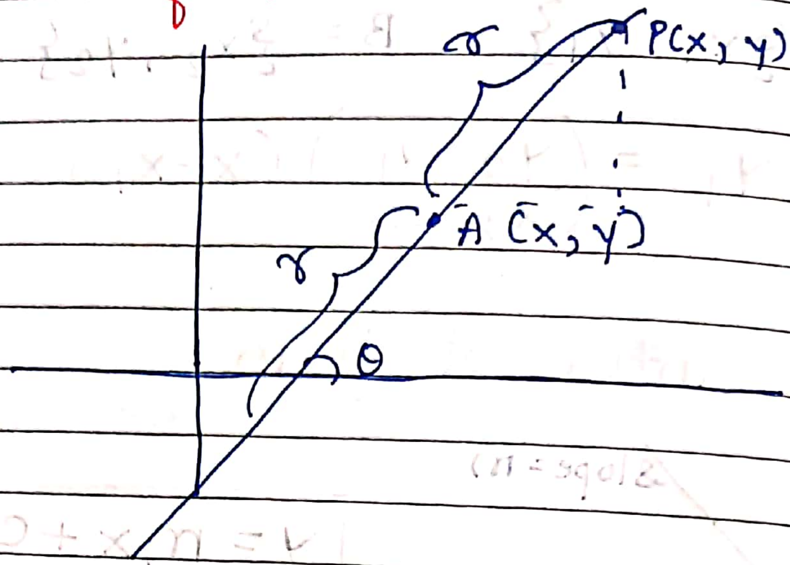
eg) $x\cos 30° - y\sin 30° = -1$

$\Rightarrow -x\cos 30 + y\sin 30 = 1$

$\Rightarrow x\cos 150° + y\sin 150° = 1$



→ Parametric form.



$x = x_1 + r\cos\theta$

$y = y_1 + r\sin\theta$

$$\boxed{\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r}$$

Q) Eqn of line passing through (a, b) & (b, a).

$\boxed{x + y = a + b}$

Q) Eqn of line passing through (1, 2) & equally inclined to both axis.

$2 = 1 + c \rightarrow c = 1 \Rightarrow y = x + 1$

$2 = -1 + c \rightarrow c = 3 \Rightarrow y = -x + 3$

$\Rightarrow$ Lines $\rightarrow$ $\boxed{x + y = 3}$ $\boxed{y - x = 1}$

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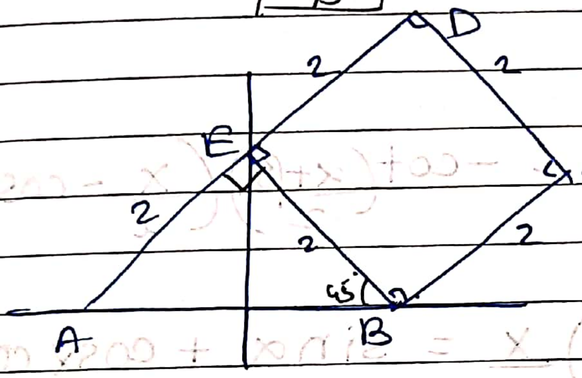
→ General form of line

$$\boxed{ax + by + c = 0}$$

Slope = $\boxed{\frac{-a}{b}}$

intercept = $\boxed{\frac{-c}{b}}$

Q)



AD = ?
DC = ?
BC = ?
BE = ?

AD ⇒ $y = x + \sqrt{2}$
DC ⇒ $y = -x + 3\sqrt{2}$
BC ⇒ $y = x - \sqrt{2}$
BE ⇒ $y = -x + \sqrt{2}$

Some Special Lines

1. $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ (chord of parabola).

$$y - 2at_1 = \frac{2}{t_1 + t_2}(x - at_1^2)$$

$$y(t_1 + t_2) = 2x - 2at_1^2 + 2at_1^2 + 2at_1t_2$$

$$\Rightarrow \boxed{y(t_1 + t_2) = 2(x + at_1t_2)}$$

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(2) $(a\cos\alpha, b\sin\alpha)$ $(a\cos\beta, b\sin\beta)$ (chord ellipse)

$\cancel{(y-a\cos\alpha)} =$

$(y-b\sin\alpha) = \frac{b(\sin\beta-\sin\alpha)}{a(\cos\beta-\cos\alpha)}(x-a\cos\alpha)$

$y-b\sin\alpha = -\frac{b}{a}\cot\left(\frac{\alpha+\beta}{2}\right)(x-a\cos\alpha)$

$\Rightarrow \frac{y}{b} - \sin\alpha = -\cot\left(\frac{\alpha+\beta}{2}\right)\left(\frac{x}{a}-\cos\alpha\right)$

$\frac{y}{b} + \cot\left(\frac{\alpha+\beta}{2}\right)\frac{x}{a} = \sin\alpha + \cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)$

$= \frac{\sin\alpha\sin\left(\frac{\alpha+\beta}{2}\right)+\cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}$

$\Rightarrow \boxed{\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)}$

(3) $(a\sec\theta_1, b\tan\theta_1)$ $(a\sec\theta_2, b\tan\theta_2)$ (Hyperbola)

$\left(y-\frac{b\sin\theta_1}{\cos\theta_1}\right) = \frac{b}{a}\left(\frac{\tan\theta_2-\tan\theta_1}{\sec\theta_2-\sec\theta_1}\right)\left(x-\frac{a}{\cos\theta_1}\right)$

$\left(y-\frac{b\sin\theta_1}{\cos\theta_1}\right) = \frac{b}{a}\left(\frac{\sin\theta_2\cos\theta_1-\sin\theta_1\cos\theta_2}{\cos\theta_1-\cos\theta_2}\right)\left(x-\frac{a}{\cos\theta_1}\right)$

$\frac{y\cos\theta_1}{b} - \sin\theta_1 = \frac{\sin(\theta_2-\theta_1)}{\cos\theta_1-\cos\theta_2}\left(\frac{x\cos\theta_1}{a}-1\right)$

$$\Rightarrow \frac{y}{b}\cos\theta_1 - \sin\theta_1 = \frac{\sin(\theta_2-\theta_1)}{2\sin\left(\frac{\theta_2-\theta_1}{2}\right)\sin\left(\frac{\theta_2+\theta_1}{2}\right)}\left(\frac{x}{a}\cos\theta_1 - 1\right)$$

$$\Rightarrow \frac{y}{b}\cos\theta_1 - \sin\theta_1 = \cot\left(\frac{\theta_1+\theta_2}{2}\right)\left(\frac{x}{a}\cos\theta_1 - 1\right)$$

$$\Rightarrow \cancel{\frac{y}{b}}\ \cot\left(\frac{\theta_1+\theta_2}{2}\right)\frac{x}{a}\cos\theta_1 - \frac{y}{b}\cos\theta_1$$

$$= \cot\left(\frac{\theta_1+\theta_2}{2}\right) - \sin\theta_1$$

$$= \cos\left(\frac{\theta_1+\theta_2}{2}\right) - \sin\theta_1 \sin(\theta_1+\theta_2)$$

$$\Rightarrow \frac{y}{b} - \frac{x}{a}\frac{\cos(\theta_2-\theta)/2}{\sin(\frac{\theta_1+\theta_2}{2})} = \frac{\sin\theta_1}{\cos\theta_2} - \frac{\cos(\frac{\theta_2-\theta_1}{2})}{\cos\theta_1 \sin(\frac{\theta_1+\theta_2}{2})}$$

$$\Rightarrow \frac{y}{b} - \frac{x}{a}\frac{\cos(\frac{\theta_2-\theta_1}{2})}{\sin(\frac{\theta_1+\theta_2}{2})} = \frac{\sin\theta_1 \sin(\frac{\theta_1+\theta_2}{2}) - \cos(\frac{\theta_2-\theta_1}{2})}{\cos\theta_1 \sin(\frac{\theta_1+\theta_2}{2})}$$

$$\Rightarrow \frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \frac{2\cos\left(\frac{\theta_1-\theta_2}{2}\right) - 2\sin\theta_1 \sin\left(\frac{\theta_1+\theta_2}{2}\right)}{2\cos\theta_1}$$

$$= \frac{2\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \left[\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \cos\left(\frac{3\theta_1}{2}+\frac{\theta_2}{2}\right)\right]}{2\cos\theta_1}$$

$$= \frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right) + \cos\left(\frac{3\theta_1+\theta_2}{2}\right)}{2\cos\theta_1}$$

$$= \frac{2\cos\theta_1 \cos\left(\frac{\theta_1+\theta_2}{2}\right)}{2\cos\theta_1}$$

$$\Rightarrow \boxed{\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1+\theta_2}{2}\right)}$$

(4) $\left(at_1, \frac{a}{t_1}\right)\left(at_2, \frac{a}{t_2}\right)$ Rectangular Hyperbola

$$\rightarrow \left(y - \frac{a}{t_1}\right) = \frac{\left(\frac{a}{t_2} - \frac{a}{t_1}\right)}{(at_2 - at_1)}(x - at_1)$$

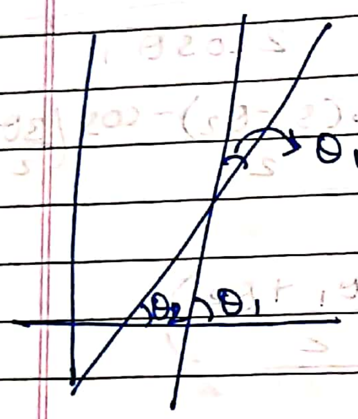
$$\Rightarrow \left(y - \frac{a}{t_1}\right) = \frac{(at_1 - x)}{t_1 t_2}$$

$$\left(y-\frac{a}{t_1}\right) = \frac{(at_1 - x)}{t_1 t_2} = \frac{a}{t_2} - \frac{x}{t_1 t_2}$$

$$\Rightarrow y + \frac{x}{t_1 t_2} = \frac{a}{t_2} + \frac{a}{t_1} = \frac{a(t_1+t_2)}{t_1 t_2}$$

$$\Rightarrow \boxed{y = \frac{-x}{t_1 t_2} + \frac{a(t_1+t_2)}{t_1 t_2}}$$

Angle b/w lines



$$\boxed{\tan(\theta_1 - \theta_2) = \frac{m_1 - m_2}{1 + m_1 m_2}}$$

Both lines $\parallel$ $\Rightarrow$ $\boxed{m_1 = m_2}$

Both lines $\perp$ $\Rightarrow$ $\boxed{m_1 m_2 = -1}$

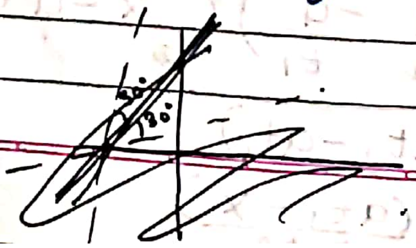
$ax + by + c = 0$

Line $\parallel$ to it $\Rightarrow$ $\boxed{ax + by + k = 0}$

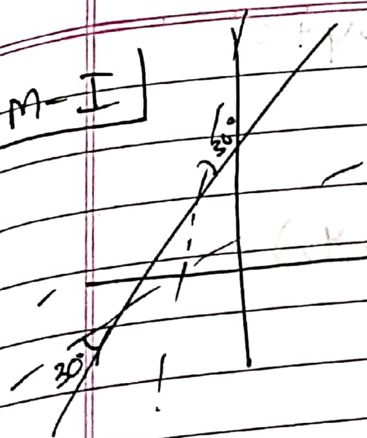
Line $\perp$ to it $\Rightarrow$ $\boxed{bx - ay = k}$

$\boxed{ay - bx = k'}$

Q) Find eqn of the line passing thr. $(1,1)$ which makes an angle of 30° with $y = 2x + 5$.



M-I



$$\pm \tan 30^\circ = \frac{2 - m_2}{1 + 2m_2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2 - m_2}{1 + 2m_2}$$

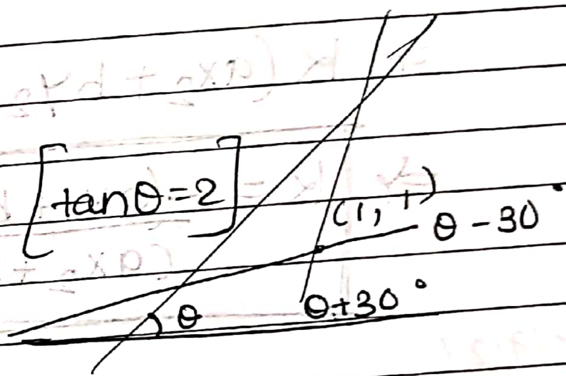
$$\Rightarrow 1 + 2m_2 = 2\sqrt{3} - m_2\sqrt{3}$$

$$\Rightarrow \boxed{m_2 = \frac{2\sqrt{3} - 1}{2 + \sqrt{3}}}$$

$$\frac{2m_2 + 1}{\cancel{\sqrt{3}}} = \sqrt{3}m_2 - 2\sqrt{3} \Rightarrow \boxed{m_2 = \frac{2\sqrt{3} + 1}{\sqrt{3} - 2}}$$

M-II

$$y - 1 = \tan(\theta \pm 30^\circ)(x - 1) \qquad \boxed{\tan\theta = 2}$$



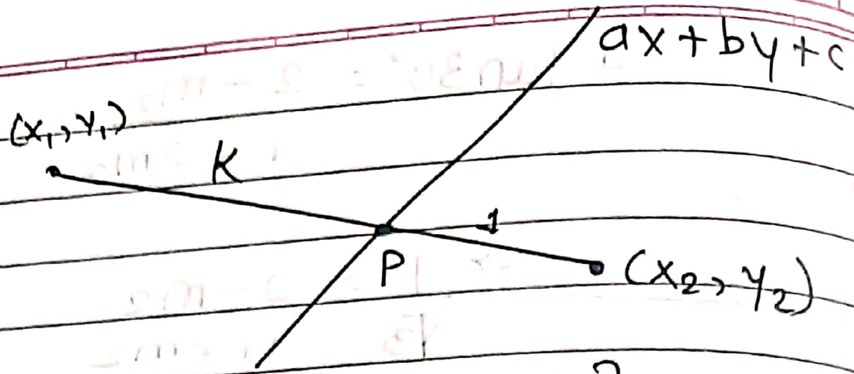
$$y - 1 = \frac{2 \pm \frac{1}{\sqrt{3}}}{1 \mp \frac{2}{\sqrt{3}}}(x - 1)$$

$$\Rightarrow \boxed{(y - 1) = \left(\frac{2\sqrt{3} \pm 1}{\sqrt{3} \mp 2}\right)(x - 1)}$$

Q) Find the ratio in which $x + 2y + 3 = 0$ divides the join of $(1, 1)$, $(-3, -2)$

Sol^n: General for line $\rightarrow ax + by + c = 0$
& points (x_1, y_1) & (x_2, y_2)

$\longrightarrow$



$$P = \left\{ \frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right\}$$

that point lies on $ax+by+c$

$$a\left(\frac{kx_2 + x_1}{k+1}\right) + b\left(\frac{ky_2 + y_1}{k+1}\right) + c = 0$$

$$\Rightarrow k(ax_2 + by_2 + c) + (ax_1 + by_1 + c) = 0$$

$$\Rightarrow \boxed{k = \frac{-(ax_1 + by_1 + c)}{(ax_2 + by_2 + c)}}$$

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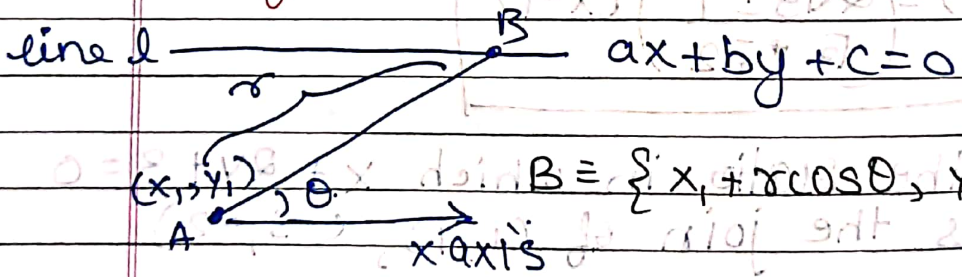
Distance of a point from a line in a given direction :-



$$B \equiv \{x_1 + r\cos\theta, y_1 + r\sin\theta\}$$

$$\Rightarrow ax_1 + ar\cos\theta + by_1 + br\sin\theta + c = 0$$

$$\Rightarrow \boxed{r = \frac{-(ax_1 + by_1 + c)}{a\cos\theta + b\sin\theta}}$$

Learn

Q) Distance of (1, 2) from $x+y+7=0$ along the line $y=\sqrt{3}x$

$y=\sqrt{3}x$

$\theta=60^\circ$

$$r=\frac{10}{\frac{\sqrt{3}}{2}+\frac{1}{2}}=\boxed{\frac{20}{1+\sqrt{3}}}$$

Q) Distance of the line $x+y=7$ from (1, 2) along the line $y=2x+5$

$y=2x+5$

$\tan\theta=2$ & $\sin\theta=\frac{2}{\sqrt{5}}$, $\cos\theta=\frac{1}{\sqrt{5}}$

$$r=\frac{-(-4)}{\frac{3}{\sqrt{5}}}=\frac{4\sqrt{5}}{3}=\boxed{\frac{4\sqrt{5}}{3}}$$

Q) A pt. $P\equiv(1,2)$ is moved 3 units along the line $y+3x=4$ so that its y ordinate is increasing. Find coordinate of the point.

Q) $AB \equiv x+2y+5=0$

Find eq^n of two lines that intersect AB at point so that dist. of that pt. from origin is 6.

$$6 = \frac{-5}{\cos\theta + 2\sin\theta}$$

$$2\sin\theta + \cos\theta + \frac{5}{6} = 0$$

$$2t+1 = \frac{-5}{6}\sec^2$$

$$4t^2+4t+1 = \frac{25}{36}(t^2+1)$$

$$4t^2 - \frac{25t^2}{36} + 4t + \frac{11}{36} = 0$$

$$119t^2 + 144t + 11 = 0$$

$$t = \frac{-144 \pm 10\sqrt{155}}{2 \times 119}$$

Q) A line thr. $A(-5,-4)$ meets $x+3y+2=0$, $2x+y+4=0$ & $x-y-5=0$ at B, C, D resp. $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$

Find eq^n of line.

$$AB = \frac{-5-12+2}{\cos\theta + 3\sin\theta} = \frac{-15}{\cos\theta + 3\sin\theta}$$

$$AC = \frac{-10-4+4}{2\cos\theta + \sin\theta} = \frac{-10}{2\cos\theta + \sin\theta}$$

$$AD = \frac{-5+4-5}{\cos\theta - \sin\theta} = \frac{-6}{\cos\theta - \sin\theta}$$

$\Rightarrow$ Put in eq^n

$\Rightarrow \cos^2\theta + 9\sin^2\theta + 6\sin\theta\cos\theta$
$+ 4\cos^2\theta + \sin^2\theta + 4\cos\theta\sin\theta$
$= \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

$5 + 5\sin^2\theta + 10\sin\theta\cos\theta$
$= 1 - 2\cos\theta\sin\theta$

$\Rightarrow 5\sin^2\theta + 12\sin\theta\cos\theta + 4 = 0$
$\Rightarrow 5t^2 + 12t + 4t^2 + 4 = 0$
$\Rightarrow 9t^2 + 12t + 4 = 0$
$\Rightarrow t = -12 \pm \sqrt{}$
$\Rightarrow (3t+2)^2 = 0$

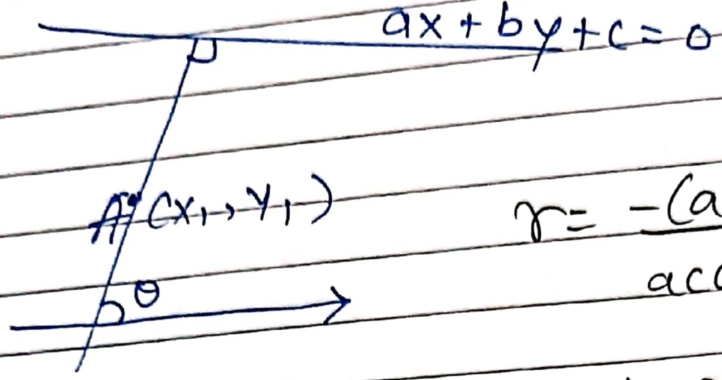
$\Rightarrow \tan\theta = \frac{-2}{3}$

$\Rightarrow y + 4 = \frac{-2}{3}(x+5)$

$\Rightarrow 3y + 12 = -2x - 10$

$\Rightarrow \boxed{2x + 3y + 22 = 0}$

Perpendicular distance

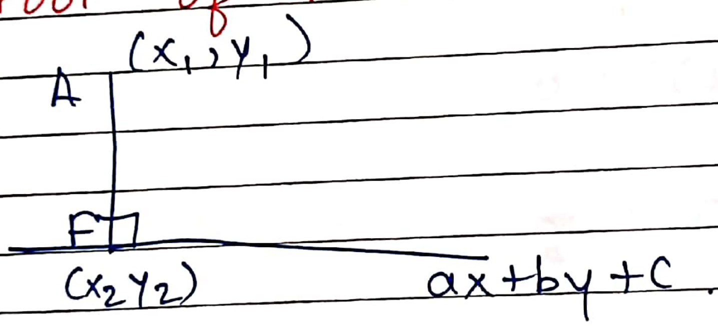


$$r = \frac{-(ax_1 + by_1 + c)}{a\cos\theta + b\sin\theta}$$

r least $\Rightarrow$ $a\cos\theta + b\sin\theta$ max.

$$\Rightarrow \boxed{AB_{\perp} = \left| \frac{(ax_1 + by_1 + c)}{\sqrt{a^2+b^2}} \right|}$$ Learn.

→ Foot of ⊥.



$$\left(\frac{y_2 - y_1}{x_2 - x_1}\right)\left(\frac{+a}{b}\right) = +1$$

$$\Rightarrow \frac{y_2 - y_1}{b} \frac{\times b}{\times b} = \frac{x_2 - x_1}{a} \frac{\times a}{\times a}$$ & add (Thrm. on = ratios)

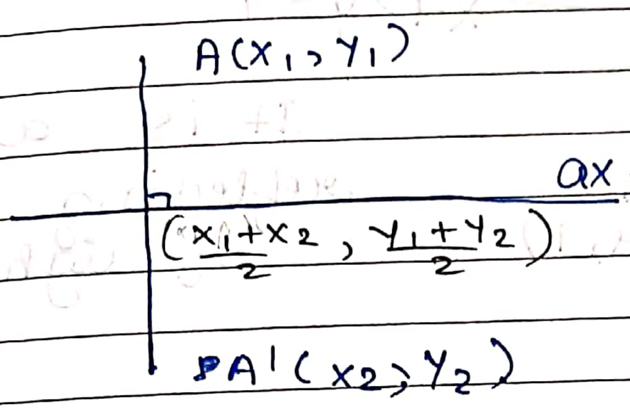
$$= \frac{b(y_2 - y_1) + a(x_2 - x_1)}{a^2+b^2}$$

~~$\frac{y_2 - y_1}{b}$~~

$$= -\frac{(ax_1 + by_1 + c)}{a^2+b^2}$$

$$\Rightarrow \boxed{\frac{y_2 - y_1}{b} = \frac{x_2 - x_1}{a} = \frac{-(ax_1 + by_1 + c)}{a^2+b^2}}$$ Learn

Image of (x_1, y_1) in $ax+by+c=0$



$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{a(x_2 + x_1 - 2x_1) + b(y_2 + y_1 - 2y_1)}{a^2 + b^2}$$

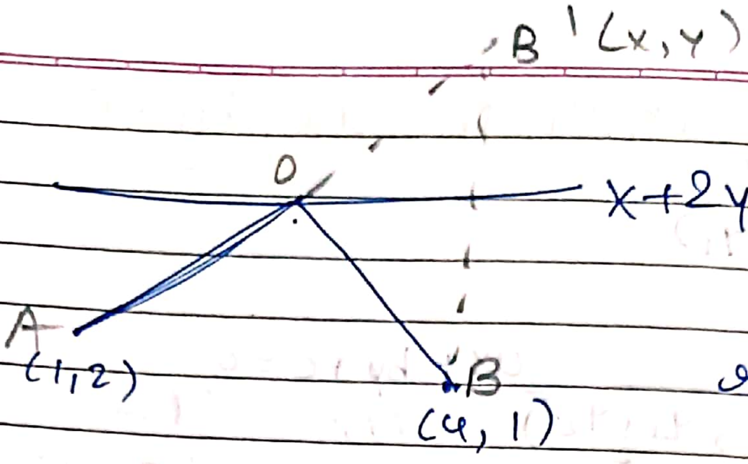
$$= \frac{-2(ax_1 + by_1) + a(x_1 + x_2) + b(y_1 + y_2)}{a^2 + b^2}$$

$$= \frac{-2(ax_1 + by_1) - 2c}{a^2 + b^2}$$

$$= \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\Rightarrow \boxed{\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}}$$ Learn

Q)



It is a reflecting beam of light

B' should lie on AO.

$$\frac{x-4}{1} = \frac{y-1}{2} = \frac{-2(-3)}{\sqrt{5}} = \frac{6}{\sqrt{5}}$$

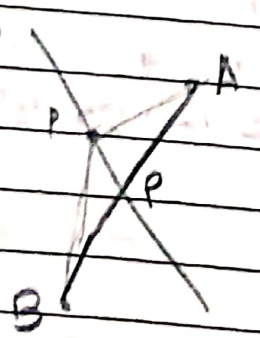
$x = \frac{6}{\sqrt{5}} + 4, \; y = \frac{12}{\sqrt{5}} + 1$

$\Rightarrow$ slope of $AB' = \dfrac{\left(\frac{12}{\sqrt{5}} + 1 - 2\right)}{\left(\frac{6}{\sqrt{5}} + 4 - 1\right)} = \boxed{\dfrac{12-\sqrt{5}}{6+3\sqrt{5}}}$

Q) Point P is on line $2x+y+3=0$
Min. value of PA + PB if

(i) $A \equiv (0,0)$, $B \equiv \{-3,-2\}$
(ii) $A \equiv (0,0)$, $B \equiv \{1,1\}$

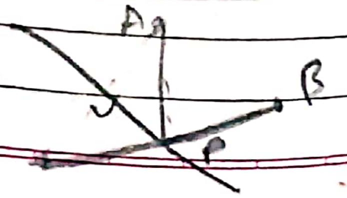
(i)



(ii) $\because PA = PA'$

$\Rightarrow (PA+PB)_{min} = (PA'+PB)_{min}$

$= \boxed{A'B}$



$\Rightarrow$ Min $PA+PB = \boxed{AB}$

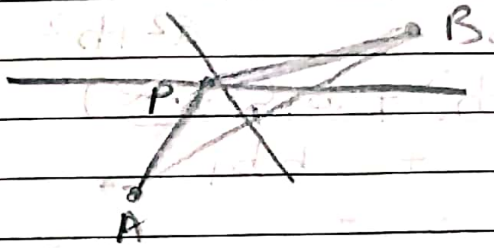
Q) P is on $x+2y+7=0$

min. of $|PA-PB|$

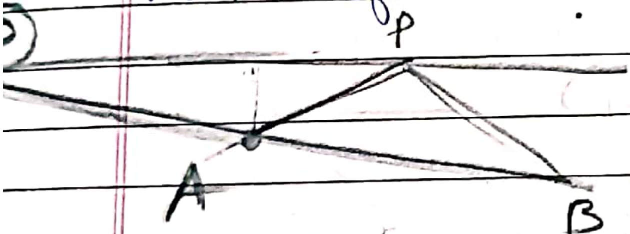
$A=(0,0)$ (i) $B=(1,1)$ (ii) $B=(-2,-4)$

$PA=PB \Rightarrow 0$ for min $|PA-PB|$

& P is the point of intersection of $x+2y+7=0$ & $\perp$ bisector of AB



Max. of $|PA-PB|$ (for same points)



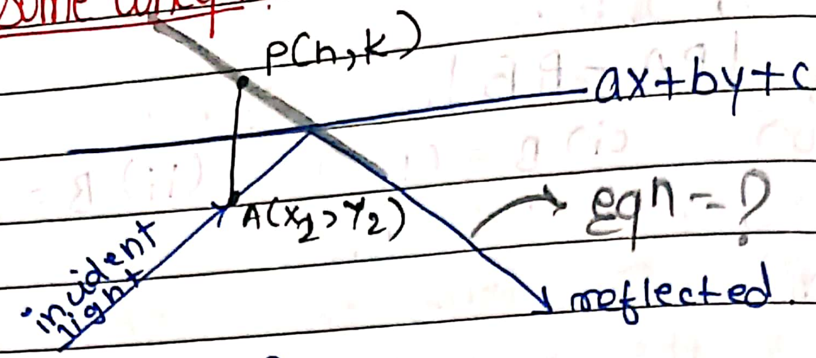
$PA-PB < AB$

$\Rightarrow Max|PA-PB| = AB$ when

P is $\cap$ of line & AB

for P when A & B are on opp. sides of line,
take image of A' / B' & then do same as in above case
Here Max $|PA-PB| = A'B$ or AB'

Some Concepts



$a_1x+b_1y+c_1=0$

$$\frac{x_2-h}{a}=\frac{y_2-k}{b}=\frac{-2(ah+bk+c)}{a^2+b^2}$$

$$=\frac{a_1(x_2-h)+b_1(y_2-k)}{aa_1+bb_1}$$

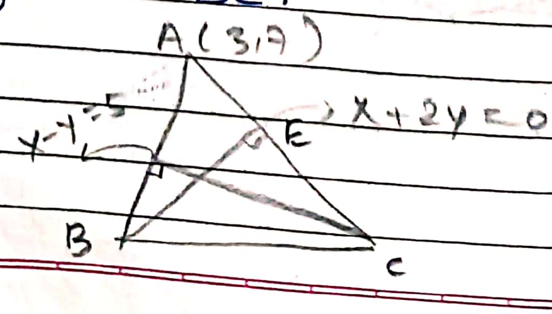
$$=\frac{a_1x_2+b_1y_2-(a_1h+b_1k)}{aa_1+bb_1}$$

$$=\frac{-(a_1h+b_1k+c_1)}{aa_1+bb_1}$$

$$\Rightarrow \boxed{(a^2+b^2)(a_1x+b_1y+c_1)=2(aa_1+bb_1)(ax+by+c)}$$

Learn

Q) A is (3,7), altitude through $B \ni x+2y=0$ & alt. thr. $C \ni x-y=5$. Then find eqn of side BC.



AB $eq^n = x + y = k$

$\Rightarrow k = 10$

$\Rightarrow x + y = 10$ (AB)

$\Rightarrow$ ~~Ref~~ $B = AB \cap BE$

$\Rightarrow B = (20, -10)$

$\Rightarrow H = \left(\frac{10}{3}, \frac{-5}{3}\right)$

$\Rightarrow$ slope AH $= \frac{7 + \frac{5}{3}}{3 - \frac{10}{3}} = \frac{26}{-1} = -26$

From this find slope of BC

& then you know slope of BC & coordinates of point B

then you can find eq^n of BC.

Q) In previous Q, instead of altitudes they are angle bisectors.

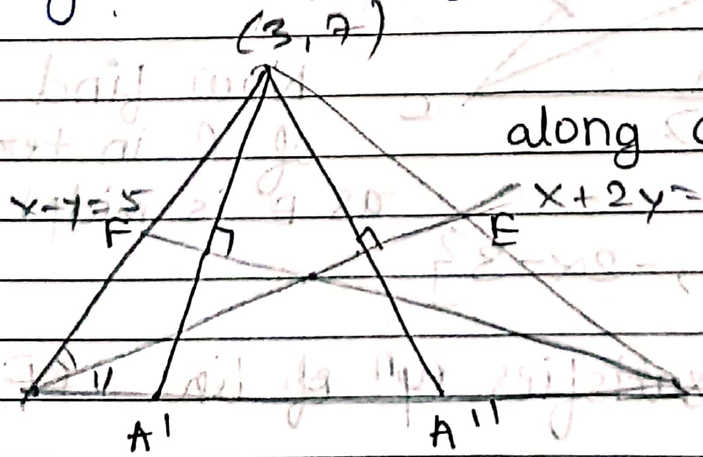


Image of A along CF lies of BC & along BE also lies on BC.

from A' & A'' find eq^n of BC.

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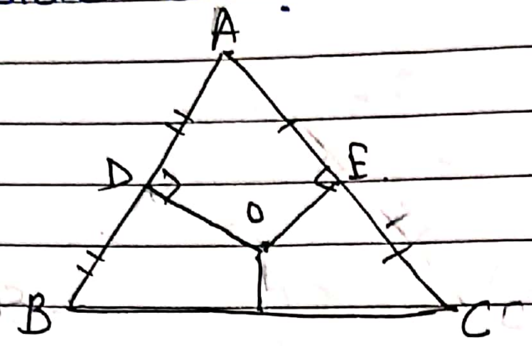
Q) $A \equiv \{1, 6\}$ $\perp$ bisector of AB & AC is $x+y=-1$ & $y = 2x-1$. Then eqn. of side BC = ?

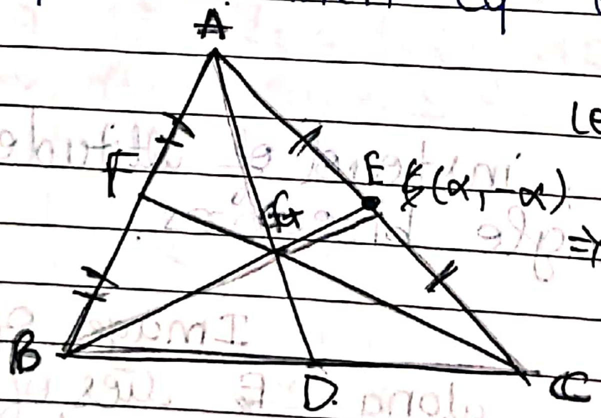


B is image of A about OD
& C is image of A about OE.

So we can find B & C as we know A & eq^ns of line OD & OE.

& then using coordinates of B & C we can find eq^n of line BC.

Q) $A \equiv \{1, 3\}$
Medians through B & C are $x+y=0$ & $y = 2x$. Then eq^n of side BC.



let x coordinate of E = α
$E(\alpha, -\alpha)$
$\Rightarrow$ y coordinate = $-\alpha$

Now find coordinates of C in terms of α as E is midpt. of AC

$\Rightarrow C \equiv \{2\alpha - 1, -2\alpha - 3\}$

Now this satisfies eq^n of line CF.

$\Rightarrow \alpha = \frac{-1}{6}$

G can be found out by intersection of BE & CF.

Here, $G \equiv \{0,0\}$

We know A & G & $AG : GD = 2:1$

so we can find D.

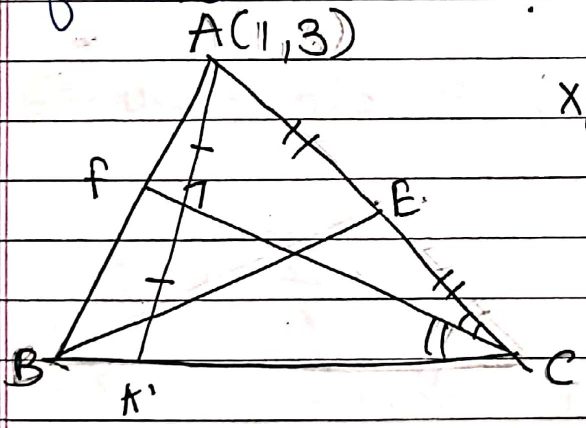
$D = \left\{\frac{-1}{2}, \frac{-3}{2}\right\}$

we have D & C.

so eq^n of line BC can be found out

8) $A \equiv \{1,3\}$

Medians thr. B & $\angle$ bisector thr. C are $x+y=0$ & $y=2x$. Then eq^n of side BC = ?



X_{co} E $= \alpha$

then Y_{co} E $= -\alpha$.

Then C can be found in terms of α.

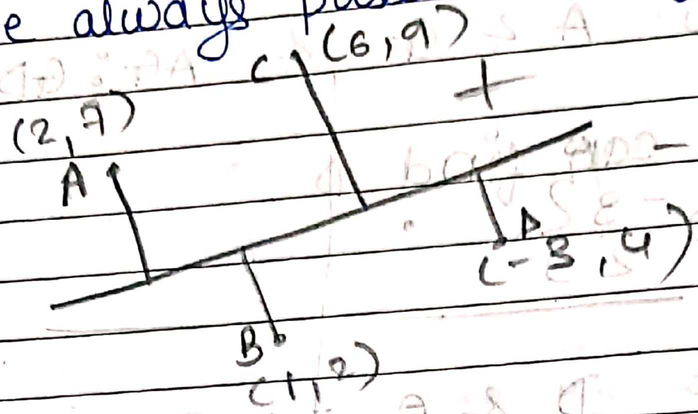
These coordinates of C satisfy eq^n of FC.

Then C can be found.

Now Image of A about FC lies on line BC (A')

$\therefore$ we have A' & C, Eq^n of line BC can be found.

Q) A variable line is such that sum of algebraic distance from $(1,2)$ $(-3,4)$ $(2,7)$ & $(6,9)$ is zero. Then the line always passes through?



Let line $y = mx + c \rightarrow y - mx - c = 0$

$A_\perp = \dfrac{-(7 - 2m - c)}{\sqrt{1+m^2}}$ $\qquad C_\perp = \dfrac{-(9 - 6m - c)}{\sqrt{1+m^2}}$

$B_\perp = \dfrac{-(2 - m - c)}{\sqrt{1+m^2}}$ $\qquad D_\perp = \dfrac{-(4 + 3m - c)}{\sqrt{1+m^2}}$

$0 = -7 + 2m + c - 2 + m + c - 9 + 6m + c - 4 - 3m + c$

$0 = -22 + 6m + 4c$

$\Rightarrow 11 = 3m + 2c$ $\qquad \left\{\dfrac{3}{2}, \dfrac{11}{2}\right\}$ ✓

$\Rightarrow \dfrac{11}{2} = \dfrac{3}{2}m + c$

$y = mx + c$

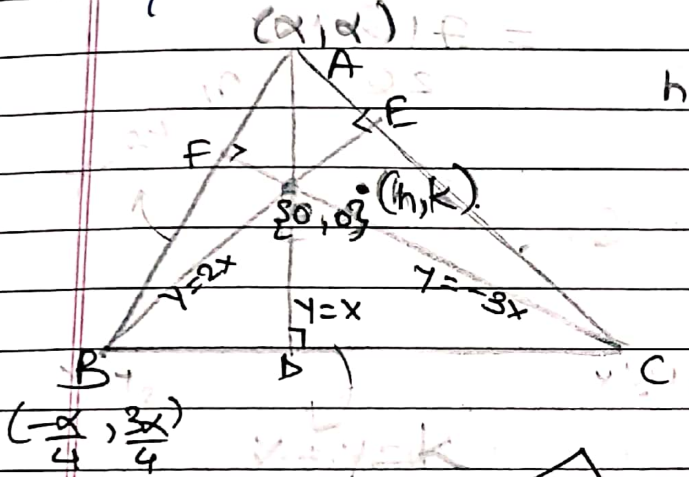
$\Rightarrow$ It will always pass thr. $\left\{\dfrac{3}{2}, \dfrac{11}{2}\right\}$

This line will actually always pass through average of all points.

$\Rightarrow \dfrac{3}{2} = \dfrac{1-3+2+6}{4}$ & $\dfrac{11}{2} = \dfrac{2+4+7+9}{4}$

⇒ If sum of algebraic distances of n points from a variable line is K, then the line will be tangent to the circle with radius $\boxed{\frac{K}{n}}$ & centre $(\underbrace{\bar{x}, \bar{y}}_{\text{average}})$

Q) Altitudes of a $\triangle$ are $y = x$, $y = 2x$, $y = -3x$. Then locus of centroid is?



$h = \frac{x_1 + x_2 + x_3}{3}$ $k = \frac{y_1 + y_2 + y_3}{3}$

Let → $A \equiv \{\alpha, \alpha\}$

IMP. Step

$AF \perp CF$

slope of CF = −3

⇒ slope of $AF = \frac{1}{3}$ $(-1 \div -3)$

If $B = \{x_1, y_1\}$

$\Rightarrow (y_1 - \alpha) = \frac{1}{3}(x_1 - \alpha)$

$\Rightarrow \cancel{y_1}^{-3x_1} - \alpha = \frac{x_1}{3} - \frac{\alpha}{3} \Rightarrow \frac{-8x_1}{\cancel{3}} = \frac{2\alpha}{\cancel{3}}$

$\Rightarrow \boxed{x_1 = \frac{-\alpha}{4}} \Rightarrow \boxed{y_1 = \frac{3\alpha}{4}}$

Similarly find C ⇒ $C = \left\{\frac{-3}{5}\alpha, \frac{9}{5}\alpha\right\}$

Scanned by CamScanner

$\Rightarrow$ If centroid $= \{h, k\}$

$$\Rightarrow 3h = \alpha - \frac{\alpha}{4} - \frac{3\alpha}{5} = \frac{20\alpha - 5\alpha - 12\alpha}{20}$$

$$= \frac{3\alpha}{20}$$

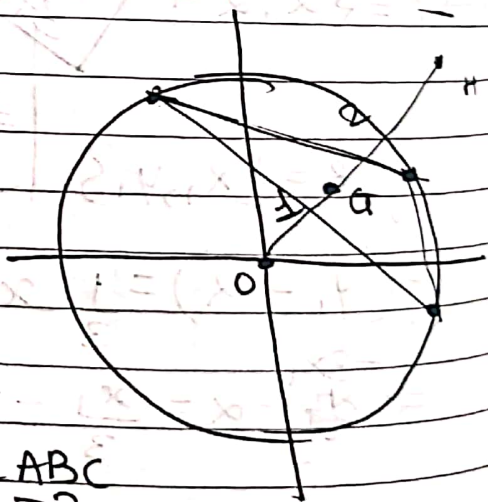
$$\& \quad 3k = \alpha + \frac{3\alpha}{4} + \frac{9\alpha}{5} = \frac{20\alpha + 15\alpha + 36\alpha}{20}$$

$$= \frac{71\alpha}{20}$$

$$\Rightarrow \frac{h}{k} = \frac{3}{71}$$

$$\Rightarrow \boxed{y = \frac{71x}{3}}$$

Q) $A(3, 4)$ $B(5\cos\alpha, -5\sin\alpha)$ $C(5\sin\beta, -5\cos\beta)$
$H \equiv ?$



let origin $= O$
$\Rightarrow AO = 5$
$BO = 5$
$CO = 5$
$\Rightarrow$ all 3 points lie on a circle of rad. 5
$\Rightarrow O \rightarrow$ circumcentre of ABC
& G (centroid) $= \{\bar{x}, \bar{y}\}$
$OG : OH = 1 : 3$

$\Rightarrow OH \rightarrow 3\bar{x}, 3\bar{y}$ $\Rightarrow H = \{3 + 5\cos\alpha + 5\sin\beta,$
$4 - 5\sin\alpha - 5\cos\beta\}$

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Q) $A(1+\sqrt{2}, 1+\sqrt{2})$ $B(0, \sqrt{3}+1)$ $C\{0, 1-\sqrt{3}\}$
$H = ?$

In this question the centre of circle has shifted to (1, 1) & radius is 2 then just shift entire figure on Origin, find H & add 1 to each coordinate of H.

Q) $A\left(2, \frac{1}{2}\right)$ $B\left(3, \frac{1}{3}\right)$ $C \equiv \left(\frac{1}{4}, 4\right)$

$H = ?$

slope of $AB = \dfrac{\frac{1}{3} - \frac{1}{2}}{3-2} = \dfrac{-1}{6}$

$\Rightarrow$ slope of $\perp$ from C to AB $= 6$

$\Rightarrow$ Eqn of $C_{\perp} \Rightarrow (y-4) = 6\left(x - \frac{1}{4}\right)$

$\Rightarrow y - 4 = 6x - \frac{3}{2}$

$\Rightarrow y - 6x - \frac{5}{2} = 0$

slope of $AC = \dfrac{4-\frac{1}{2}}{\frac{1}{4}-2} = \dfrac{\frac{\cancel{7}}{2}}{-\frac{\cancel{7}}{4}} = -2$

$\Rightarrow$ slope of $\perp$ from B to AC $= \frac{1}{2}$

$\Rightarrow$ Eqn of $B_{\perp} \Rightarrow \left(y - \frac{1}{3}\right) = \frac{1}{2}(x-3)$

$\Rightarrow y - 1/3 = \frac{x}{2} - \frac{3}{2}$

$\Rightarrow 2y - x + \frac{7}{3} = 0$

$\frac{-2}{3} + 3$

$\Rightarrow$

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$\Rightarrow H = C_{\perp} \cap B_{\perp} = \left\{ \frac{-2}{3}, \frac{-3}{2} \right\}$

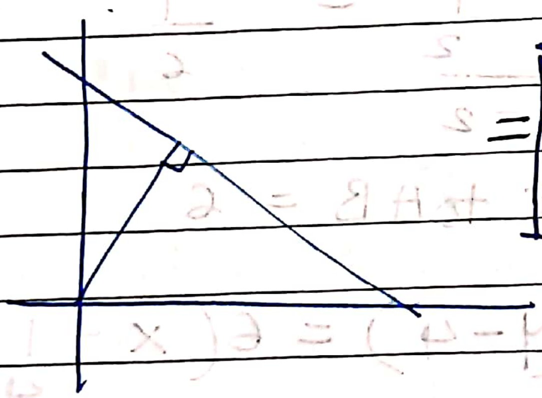
Theory

If $A = \left\{ t_1, \frac{1}{t_1} \right\}$ $B = \left\{ t_2, \frac{1}{t_2} \right\}$ $C = \left\{ t_3, \ldots \right.$

$\Rightarrow \boxed{H = \left\{ \frac{-1}{t_1 t_2 t_3}, -t_1 t_2 t_3 \right\}}$

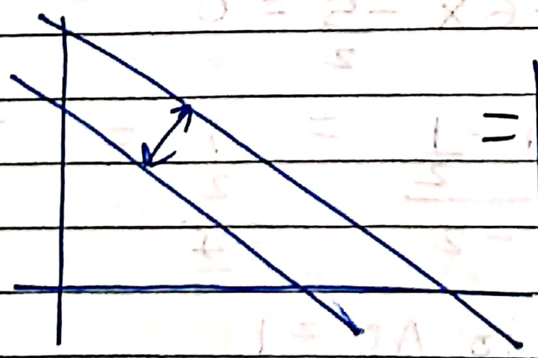
~~Distance b/w parallel lines~~

Distance from origin.



$$= \boxed{\frac{|c|}{\sqrt{a^2+b^2}}}$$

Distance ~~from~~ b/w parallel lines.



$$= \boxed{\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}}$$

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DETERMINANTS

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$$

Co factor of $a_1 = (b_2 c_3 - c_2 b_3) = A_1$

Cofactor of $b_1 = -(a_2 c_3 - a_3 c_2) = B_1$

Cofactor of $c_1 = (a_2 b_3 - b_2 a_3) = C_1$

Cofactor of $b_2 = (a_1 c_3 - a_3 c_1) = B_2$

⋮

& so on

Det. :- $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \Delta^{n-1}$

$\because$ Here $n = 3$

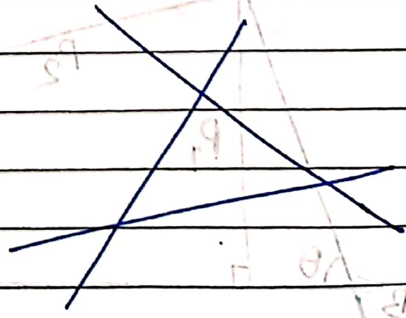
$= \boxed{\Delta^2}$

Area formed by 3 lines

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

$a_3 x + b_3 y + c_3 = 0$



$$\boxed{\text{Area} = \frac{\Delta^2}{|2 C_1 C_2 C_3|}}$$

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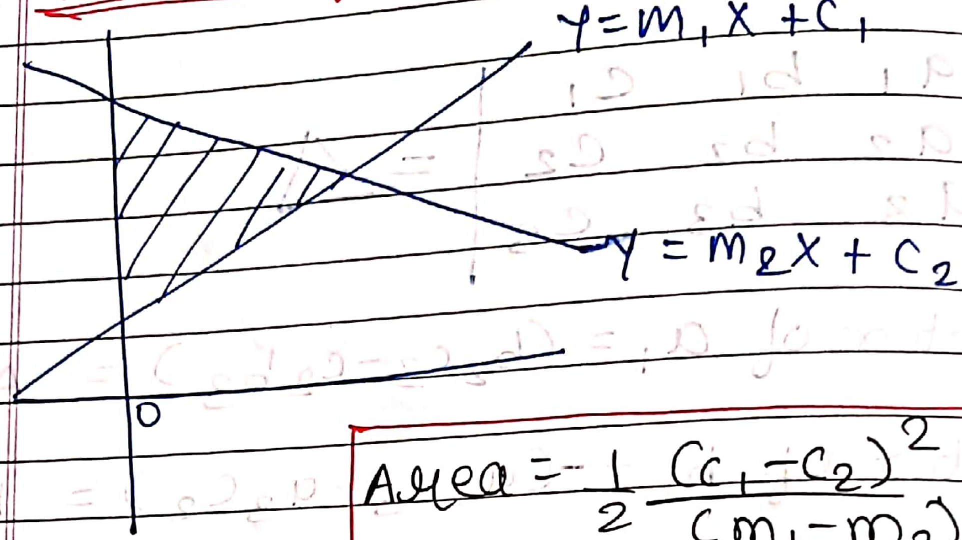
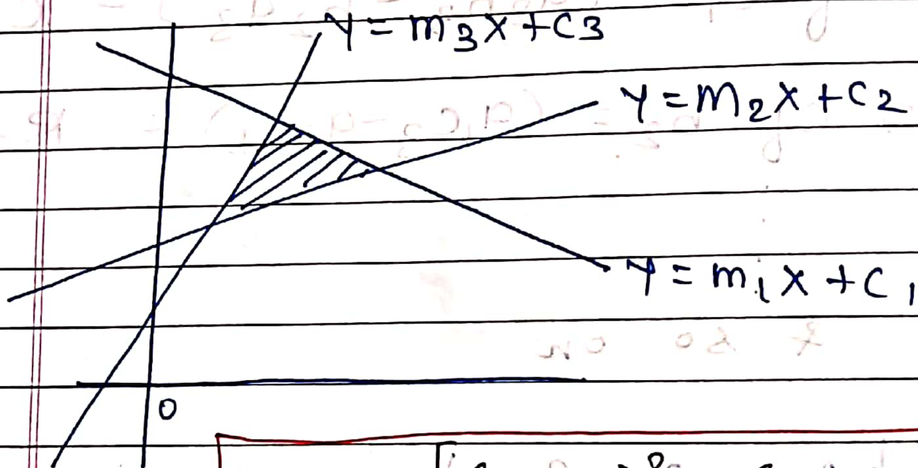
Area of a △

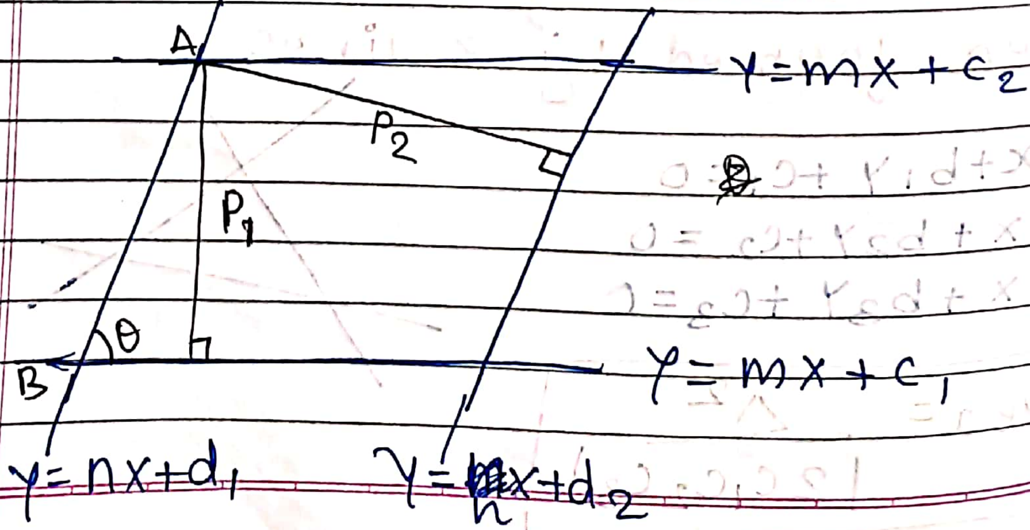


$$\boxed{\text{Area} = \frac{1}{2}\frac{(c_1-c_2)^2}{(m_1-m_2)}}$$



$$\boxed{\text{Area} = \frac{1}{2}\left[\frac{(c_1-c_2)^2}{(m_1-m_2)} + \frac{(c_2-c_3)^2}{(m_2-m_3)} - \frac{(c_3-c_1)^2}{m_1-m_3}\right]}$$

Area of a parallelogram



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→ **Method**

P_1 & P_2 can be found as they are $\perp$ distances b/w parallel lines.

tan θ can be found ~~from~~ as slopes of lines forming θ are know.

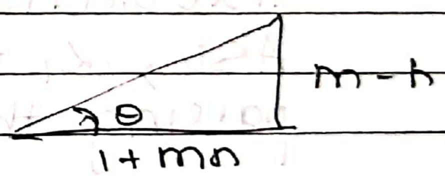
So $AB = \dfrac{P_1}{\sin\theta}$

$$\Rightarrow \text{Area} = \boxed{\dfrac{P_1 P_2}{\sin\theta}}$$

→ **Derivation**

$$P_1 = \frac{|c_1 - c_2|}{\sqrt{m^2+1}} \qquad P_2 = \frac{|d_1 - d_2|}{\sqrt{n^2+1}}$$

$$\tan\theta = \frac{m-n}{1+mn}$$



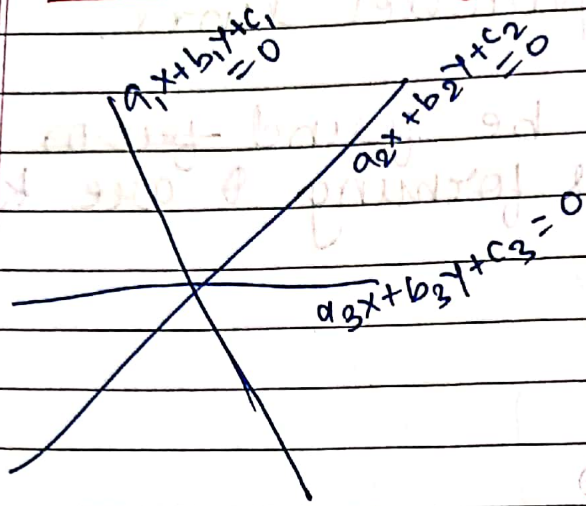
$$\Rightarrow \sin\theta = \frac{m-n}{\sqrt{(m^2+1)(n^2+1)}}$$

$$m^2 + n^2 - 2mn + 1 + m^2n^2 + 2mn = \sqrt{(m^2+1)(n^2+1)}$$

$$\Rightarrow \boxed{\text{Area} = \frac{|c_1 - c_2|\,|d_1 - d_2|}{|m-n|}}$$

Learn

Concurrency of Lines



for concurrency

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Family of Lines / Pencil of Lines

Concept: If L_1 & L_2 are two lines, then $(L_1 + \lambda L_2 = 0)$ will pass through intersection of L_1 & L_2.
Also, $L_1 + \lambda L_2$ can be any line passing thr. $L_1 \cap L_2$ except L_2.

$\longrightarrow$ $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ be two lines.

then $(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0$ is the eq^n of family of lines which passes thr. their intersection.

& this line is Linear, which is

$(a_1+\lambda a_2)x + (b_1+\lambda b_2)y + (c_1+\lambda c_2)=0$

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Q) Find eqⁿ of line passing thr. intersection of $x+2y=5$ & $2x-3y=4$ and equally inclined to the coordinate axis.

$x+2y-5=0$ $\quad$ $2x-3y-4=0$

$2x-3y-4+\lambda x+2\lambda y-5\lambda=0$

$y(2\lambda-3)+x(2+\lambda)-4-5\lambda=0$

$\Rightarrow \dfrac{-2-\lambda}{2\lambda-3}=1,-1$

$\Rightarrow \dfrac{2+\lambda}{3-2\lambda}=1$ $\qquad$ $\dfrac{2+\lambda}{2\lambda-3}=1$

$2+\lambda=3-2\lambda$ $\qquad$ $2+\lambda=2\lambda-3$

$\lambda=\dfrac{1}{3}$ $\qquad$ $\lambda=5$

$\Rightarrow$ Eqⁿ $\rightarrow$ $\dfrac{7}{3}x-\dfrac{7}{3}y-\dfrac{17}{3}=0$

$\Rightarrow \boxed{7x-7y-17=0}$

$\boxed{7x+7y-29=0}$

Never do this

Find intersection point & write eqⁿ.

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Q) Find eq^n of line passing thr. intersection of $x+2y=5$ & $2x-3y=4$ & which is at max. distance from $(3,5)$.

Steps → find intersection of the given lines
Find lines passing thr. the intersection & $(3,5)$.
Then you can find slope of line $\perp$ to new line & this more newer line should also pass thr. that intersection.
This most newest line is the req. line.

Ans → intersection: $\left(\frac{23}{7}, \frac{6}{7}\right)$

line → $$\left(y-5\right) = \frac{\left(5-\frac{6}{7}\right)}{\left(3-\frac{23}{7}\right)}\left(x-3\right)$$

$$\Rightarrow y-5 = \frac{29}{-2}(x-3)$$

$$\Rightarrow 10-2y = 29x-87$$

$$\Rightarrow 29x+2y-97=0$$

$$\text{slope} = \frac{-29}{2}$$

$$\Rightarrow \text{slope of } \perp = \frac{2}{29}$$

$$\Rightarrow eq^n \text{ of req. line} \Rightarrow \boxed{\left(y-\frac{6}{7}\right) = \frac{2}{29}\left(x-\frac{23}{7}\right)}$$

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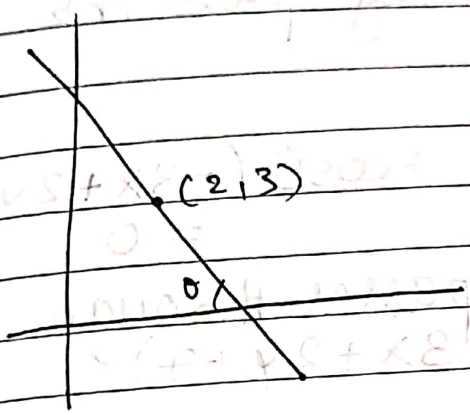
Q) $(2x-3y+5)+\lambda(x+2y-8)=0$
Find λ s.t. the line cuts +ve x & y axis & encloses min. area b/w coordinate axis.

intersection of given lines = $(2,3)$



Let eq^n of line $\Rightarrow \frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{2}{a}+\frac{3}{b}=1$

$\Rightarrow$ Applying AM GM

$$\frac{1}{2} \geq \sqrt{\frac{6}{ab}}$$

$\Rightarrow ab \geq 24$

$\Rightarrow$ min. area $= \underline{\underline{12}}$

& for this

$\frac{2}{a}=\frac{3}{b} \Rightarrow \frac{b}{a}=\frac{3}{2}=\tan\theta$

$\Rightarrow$ we need a line with slope $-\frac{3}{2}$ passing through $(2,3)$

$\Rightarrow$ Eq^n of line $\Rightarrow (y-3)=-\frac{3}{2}(x-2)$

$\Rightarrow 2y-6=6-3x$

$\Rightarrow \boxed{3x+2y-12=0}$

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~~Q) (5cosθ − 3~~

Q) $(5\sin\theta - 3\cos\theta)x + (2\cos\theta - 4\sin\theta)y + 7\cos\theta - 5\sin\theta = 0$

This line will always pass thr which pt.?

$\Rightarrow \sin\theta(5x - 4y - 5) + \cos\theta(-3x + 2y + 7) = 0$

⇒ This line always passes through $(5x-4y-5)$ & $(-3x+2y+7)$

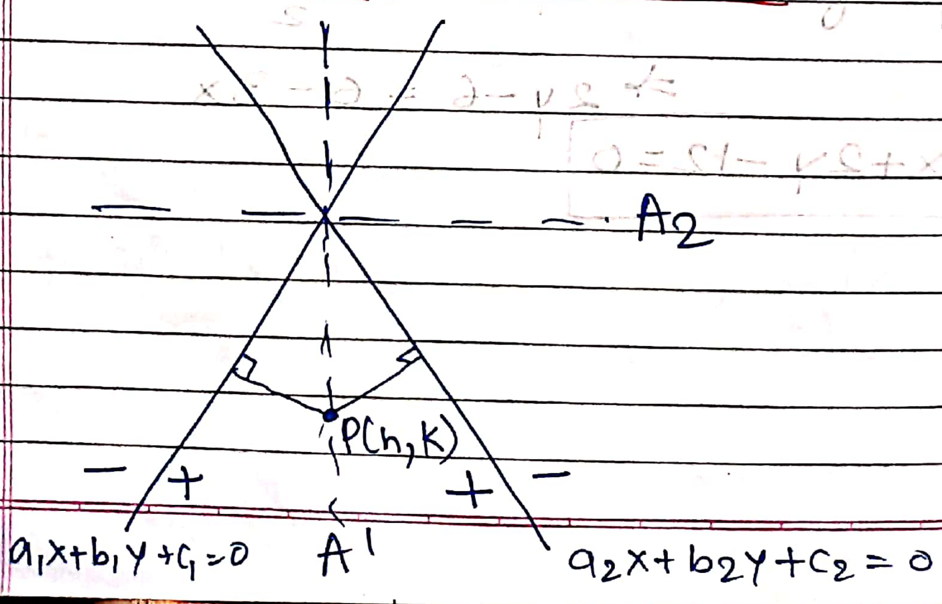
Find it //

Q) $3a + 4b + 5c = 0$
$ax + by + c = 0$ will always pass thr.?

$\rightarrow \frac{3a}{5} + \frac{4}{5}b + c = 0$

⇒ $ax + by + c$ always passes thr $\left(\frac{3}{5}, \frac{4}{5}\right)$

ANGLE BISECTOR



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$$\frac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}} = \frac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$$

A_1 : $$\boxed{\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}}$$

A_2 : $$\boxed{\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{-(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}}$$

To find where origin lies.

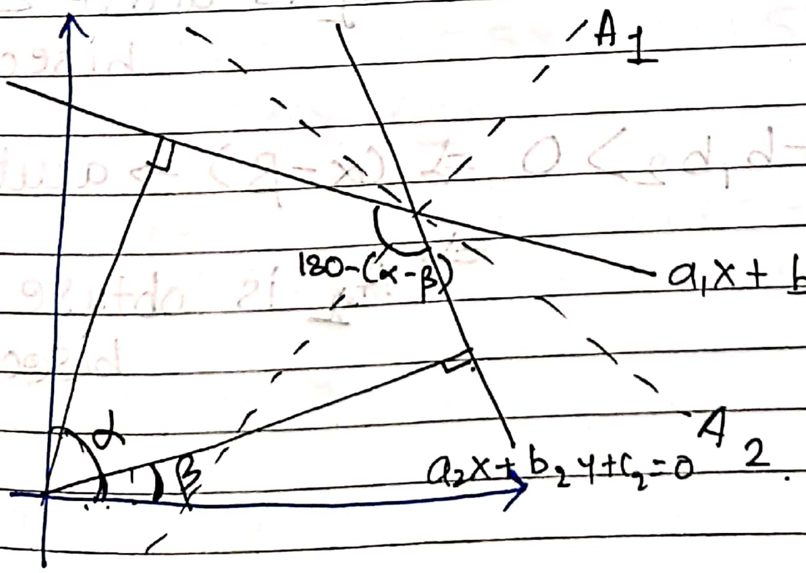
first make $\boxed{c_1 \cdot c_2 > 0}$

then origin lies in $\boxed{A_1}$

To find where $P(\alpha, \beta)$ lies.

if after putting (α, β) both distances are of same sign then P lies in A_1, if of opposite sign the P lies in A_2.

Acute Angle Bisector



make $c_1, c_2 \longrightarrow +ve$
& take to one side.

$-a_1x - b_1y = c_1$ $\qquad$ $-a_2x - b_2y = c_2$.

$$\frac{-a_1x}{\sqrt{a_1^2+b_1^2}} - \frac{b_1 y}{\sqrt{a_1^2+b_1^2}} = \frac{c_1}{\sqrt{a_1^2+b_1^2}}$$ & similar for a_2, b_2, c_2

$$\cos\alpha = \frac{-a_1}{\sqrt{a_1^2+b_1^2}} \qquad \sin\alpha = \frac{-b_1}{\sqrt{a_1^2+b_1^2}}$$

$$\cos\beta = \frac{-a_2}{\sqrt{a_2^2+b_2^2}} \qquad \sin\beta = \frac{-b_2}{\sqrt{a_2^2+b_2^2}}$$

$$\Rightarrow \boxed{\cos(\alpha-\beta) = \frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2+b_1^2} \times \sqrt{a_2^2+b_2^2}}}$$

☆ if $a_1a_2 + b_1b_2 < 0 \Rightarrow (\alpha-\beta) \rightarrow$ obtuse

$\Rightarrow 180-(\alpha-\beta) \rightarrow$ acute

$\Rightarrow A_1$ is acute $\angle$ bisector

☆ if $a_1a_2 + b_1b_2 > 0 \Rightarrow (\alpha-\beta) \rightarrow$ acute

$\Rightarrow A_2$ is obtuse $\angle$ bisector

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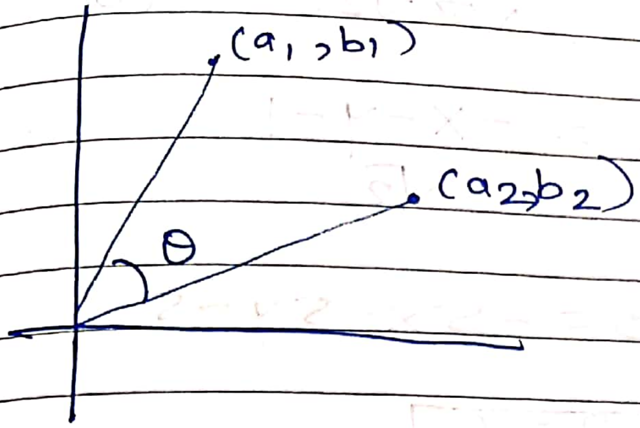
Concept



$$\boxed{\cos\theta = \frac{a_1 a_2 + b_1 b_2}{\sqrt{a_1^2+b_1^2}\ \sqrt{a_2^2+b_2^2}}}$$

Q) $L_1 \equiv 7x - y = 3 \qquad L_2 \equiv x + y + 1 = 0$

Find eqn of angle bisector:

(a) origin lies in the same quadrant.

(b) (2, -5) lies in same quadrant.

(c) obtuse angle bisector.

(a) $-7x + y + 3 = 0 \qquad x + y + 1 = 0$

$$\frac{-7x + y + 3}{\cancel{\sqrt{50}}\ 5} = \frac{x + y + 1}{\cancel{\sqrt{2}}}$$

$\Rightarrow -7x + y + 3 = 5x + 5y + 5$

$\Rightarrow \boxed{12x + 4y + 2 = 0}$

(b) $7x - y - 3 \quad (2, -5) = 14 + 5 - 3 > 0$

$x + y + 1 \quad (2, -5) = 2 - 5 + 1 < 0$

$$\Rightarrow \frac{7x - y - 3}{5} = -x - y - 1$$

$\Rightarrow 7x - y - 3 = -5x - 5y - 5$

$\Rightarrow \boxed{12x + 4y + 2 = 0}$

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(c) $7x - y = 3$, $-x - y = 1$

* Make c_1, c_2 +ve

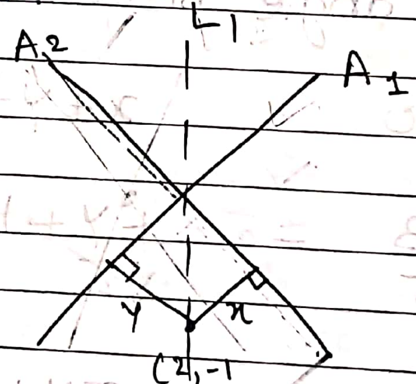
(i) $\dfrac{a_1a_2 + b_1b_2}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}} = \dfrac{-7+1}{\sqrt{50}\cdot\sqrt{2}} = \dfrac{-6}{10} < 0$

$\Rightarrow A_2 \quad \dfrac{-7x+y+3}{\sqrt{50}} = \dfrac{-x-y-1}{\sqrt{2}}$

$\Rightarrow -7x + y + 3 = -5x - 5y - 5$

$\Rightarrow \boxed{2x - 6y - 8 = 0}$

Q) Eqn of angle bisectors of L_1 & L_2 are $7x - 3y + 8 = 0$ & $3x + 7y - 2 = 0$. $(2,-1)$ lies on L_1 then eqn of acute bisector is



If $x > y$ then $A_2 \to$ obtuse, $A_1 \to$ acute & vice versa

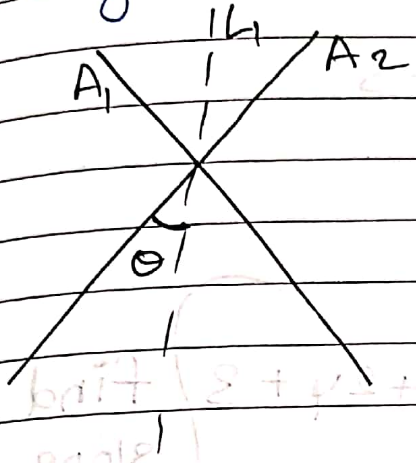
(i) $7(2) - 3(-1) + 8 = 25$

$3(2) + 7(-1) - 2 = |-3| = 3$

$\because 3 < 25$

$\Rightarrow \boxed{3x + 7y - 2 = 0}$ is acute bisector.

Q) eqn of $\angle$ bisectors of L_1 & L_2 are $x - 7y - 8 = 0$ & $7x + y + 3 = 0$ slope of L_1 is $8/7$ then acute angle bisector is ?



→ if $|\tan\theta| < 1$ then A_2 is acute $\angle$ bisector.

$$\tan\theta = \frac{\frac{8}{7} - \frac{1}{7}}{1 + \frac{8}{49}} < 1.$$

$\Rightarrow A_2 \rightarrow$ acute $\angle$ bisector.

$\Rightarrow$ $x - 7y - 8 = 0$ is acute $\angle$ bisector.

Q) Two equal sides of an isosceles $\triangle$ are along $x + 2y = 3$ & $4x - 2y = 3$. Third side passes through $(3, 4)$ then the eqn of third side is —.

method: find the slopes of 2 $\angle$ bisectors passing thr. vertex. These will be slopes of req. possible 2 sides of $\triangle$.

We have slope, we have point

$\Rightarrow$ we can find eqn.

Solution

$L_1 \equiv x+2y=3 \qquad L_2 \equiv 4x-2y=3$

$$\frac{x+2y-3}{\sqrt{5}} = \frac{4x-2y-3}{2\sqrt{5}}$$

$\Rightarrow 2x+4y-6 = 4x-2y-3$

$\Rightarrow 2x-6y+3=0$

$\Rightarrow \text{slope} = \boxed{\frac{1}{3}}$

$(x+2y-3)2 = -4x+2y+3$

$\Rightarrow 6x+2y-3=0$

$\Rightarrow \text{slope} = \boxed{-3}$

find 2nd slope by $\boxed{m_1 m_2 = -1}$

⇒ third side can be

① $(y-4) = \frac{1}{3}(x-3)$

$\Rightarrow 3y-12 = x-3$

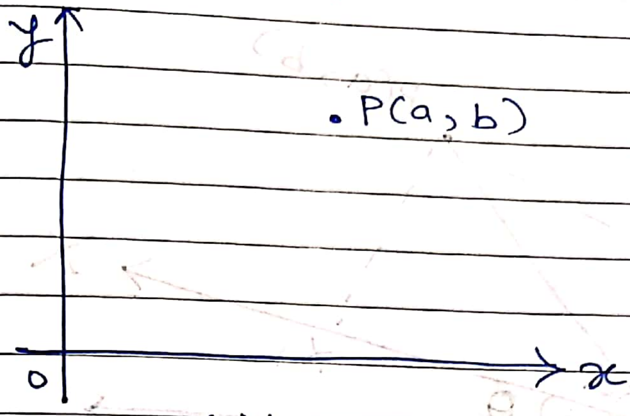
$\Rightarrow \boxed{3y-x-9=0}$

② $(y-4) = -3(x-3)$

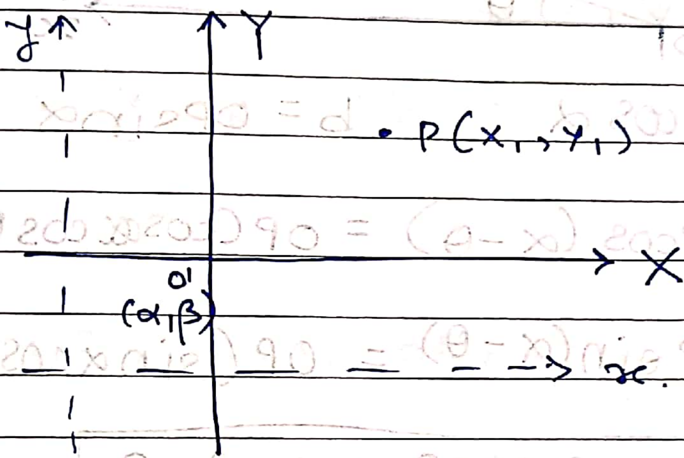
$\Rightarrow \boxed{y+3x-13=0}$

Change of Axis

originally



new



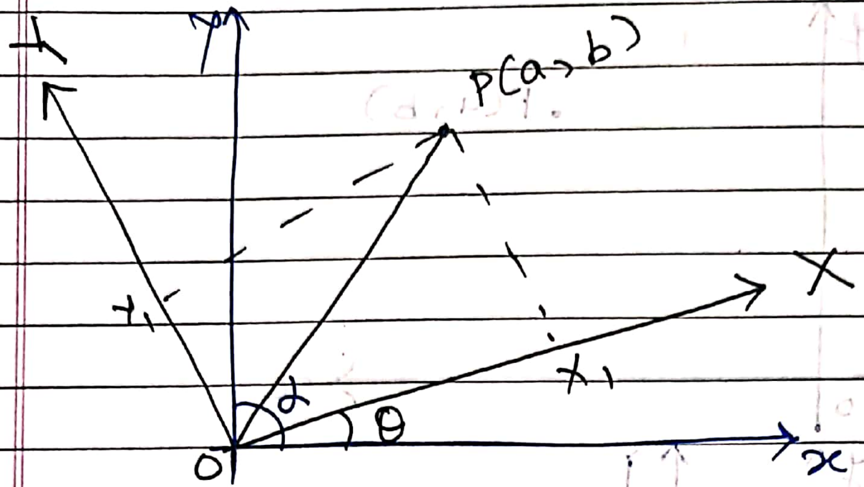
$$X = a - \alpha \qquad Y = b - \beta$$

General

$$\boxed{X = x - \alpha} \qquad \boxed{Y = y - \beta}$$

$$\boxed{x = X + \alpha} \qquad \boxed{y = Y + \beta}$$

Rotation of Axis



$a = OP\cos\alpha$, $b = OP\sin\alpha$

$X_1 = OP\cos(\alpha - \theta) = OP(\cos\alpha\cos\theta - \sin\alpha\sin\theta)$

$Y_1 = OP\sin(\alpha - \theta) = OP(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$

$$\Rightarrow \boxed{\begin{aligned} X_1 &= a\cos\theta + b\sin\theta \\ Y_1 &= b\cos\theta - a\sin\theta \end{aligned}}$$

General

$$P(x, y) \xrightarrow{\text{Rotate by } \theta \circlearrowleft} P(X, Y)$$

$$\boxed{\begin{aligned} X &= x\cos\theta + y\sin\theta \\ Y &= -x\sin\theta + y\cos\theta \end{aligned}}$$

Shortcut for Remembering

	x	y
X	$\cos\theta$	$\sin\theta$
Y	$-\sin\theta$	$\cos\theta$

Ex) If X, Y given, find x

$\Rightarrow x = X\cos\theta - Y\sin\theta$.

Concept

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

Q) Through what $\angle$ the axes must be rotated so that the term containing XY is eliminated from 2nd deg. gen. eqn.

Let $\angle = \theta$.

$\Rightarrow x = X\cos\theta - Y\sin\theta$

$y = X\sin\theta + Y\cos\theta$.

Put this in main eqn. & just find coeff. of XY, that coeff. $= 0$.

$a(X^2\cos^2\theta + Y^2\sin^2\theta - 2XY\cos\theta\sin\theta)$
$+ b(X^2\sin^2\theta + Y^2\cos^2\theta + 2XY\sin\theta\cos\theta)$
$+ 2h(X^2\sin\theta\cos\theta - Y^2\sin\theta\cos\theta + XY\cos^2\theta$
$- XY\sin^2\theta)$

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~~$= \sin\theta\cos\theta(-2aXY + 2bXY + 2hX^2$~~

$\Rightarrow$ Coeff. of $XY = -2a\sin\theta\cos\theta + 2b\sin\theta\cos\theta + 2h(\cos^2\theta - \sin^2\theta)$

$$0 = (b-a)\sin 2\theta + 2h\cos 2\theta$$

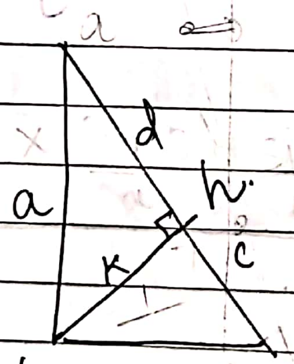
$$(a-b)\sin 2\theta = 2h\cos 2\theta$$

$$\Rightarrow \boxed{\cot 2\theta = \frac{a-b}{2h}}$$ Learn.

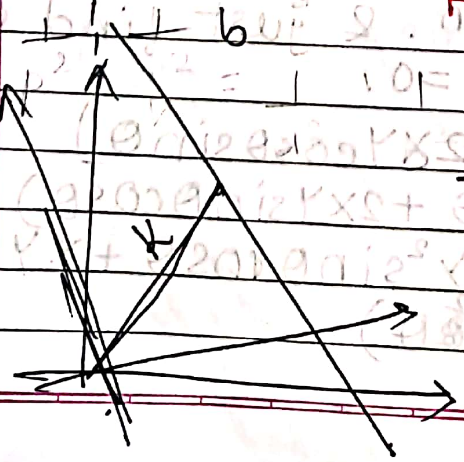
Q) Line L has intercepts a & b on coordinate axis. after rotation, intercepts = p & q.

M-I) $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}$

M-II)



$$\boxed{\begin{aligned} a^2 &= dh \\ b^2 &= ch \\ k^2 &= cd \\ \frac{1}{k^2} &= \frac{1}{a^2}+\frac{1}{b^2} \end{aligned}}$$



$$\frac{1}{k^2} = \boxed{\frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}}$$

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Q) find the acute ∠ bisectors of $5x+7y=0$ & $7x-8y=5$.

method

shift $5x+7y=0$ parallely such that it passes thr. (1,0)

⇒ that line ≡ $5x+7y=5$.

find acute ∠ bisector of $7x-8y=5$ & $5x+7y=5$.

then once you find the slope of that ∠ bisector you can find actual req. ∠ bisector.

solution

$$-5x-7y+5=0 \qquad 8y-7x+5=0$$

$$a_1a_2+b_1b_2 = 35-56 < 0$$

$$\frac{-5x-7y+5}{\sqrt{74}} = \pm\frac{8y-7x+5}{\sqrt{113}}$$

~~⇒ from this~~ ⇒ we can directly take the -ve wala eqⁿ.

$$\frac{-5x-7y}{\sqrt{74}} = \frac{-7x+8y+5}{\sqrt{113}}$$

$$\Rightarrow \boxed{\frac{-5x-7y}{\sqrt{74}} = -\frac{7x+8y+5}{\sqrt{113}}}$$ This will be the required angle bisector,

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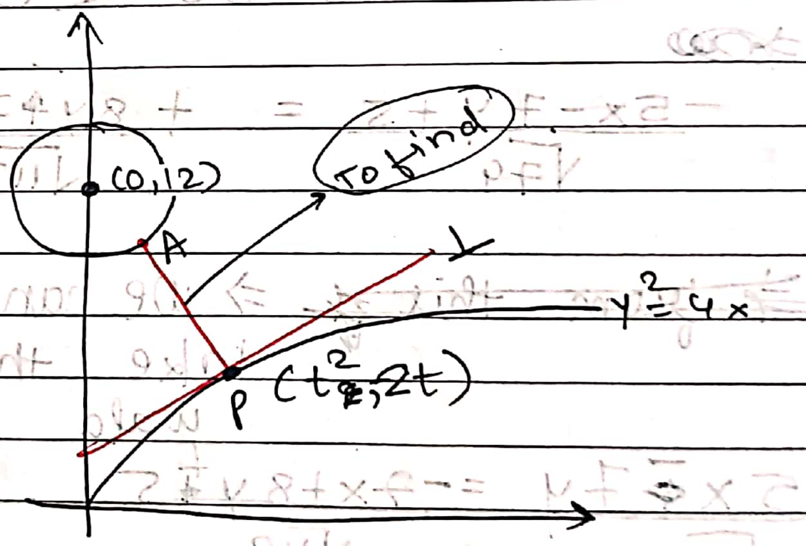
Q) The minimum value $[(x_1-x_2)^2+(12-\sqrt{1-x_1^2}-\sqrt{4x_2})^2]^{1/2}$ for all permissible values of x_1 & x_2 is equal to $a\sqrt{b}-c$ where $a,b,c \in N$ then find value of $a+b-c$.

The expression given resembles the distance between two points

$\Rightarrow$ we have to find minimum dist. b/w two points on two diff. curves

$y = 12-\sqrt{1-x_1^2}$ $\qquad$ $y_2 = \sqrt{4x_2}$

$\Rightarrow x_1^2+(y_1-12)^2 = 1$ $\qquad$ $y_2^2 = 4x_2$



$t^2, 2t$

differentiating $\Rightarrow$ slope $= \frac{1}{t}$ $(\perp)$

$\Rightarrow$ slope of $\overset{\text{to find}}{\cancel{}} = -t$

$\Rightarrow (y-2t) = -t(x-t^2)$

$\Rightarrow y+tx = 2t+t^3$

$\Rightarrow 12 = 2t+t^3$

$\Rightarrow \boxed{t=2}$

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$\Rightarrow$ point $= (4,4)$

$\Rightarrow$ ~~ne~~ dist. $AP = [\text{dist.}(0,12)(4,4)] - 1$ radius.

$= \sqrt{80} - 1$

$= 4\sqrt{5} - 1$

$= a\sqrt{b} - c$

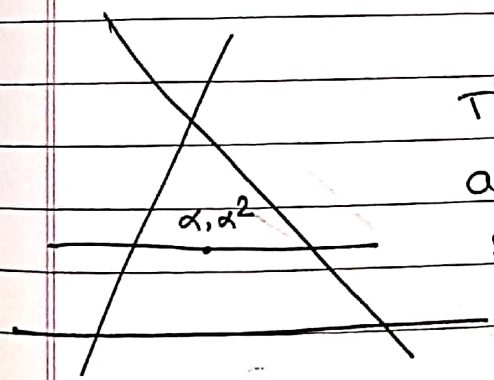
$\Rightarrow a+b-c$

$= 4+5-1$

$= \boxed{8}$

Q) Determine value of α such that (α, α^2) lie inside the Δ formed by lines

$$2x+3y-1=0$$
$$x+2y-3=0$$
$$5x-6y-1=0$$



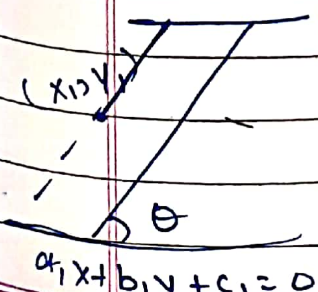
Take distance of point from any 2 sides along the 3rd side.

The distance should be of opp. sign $\Rightarrow$ their product should be less than zero.

Distance of point from line along dirxn of another line.

$\tan\theta = \frac{-a_1}{b_1} \Rightarrow \sin\theta = \frac{a_1}{\sqrt{a_1^2+b_1^2}}$

$\cos\theta = \frac{b_1}{\sqrt{a_1^2+b_1^2}}$



dist. $= -\frac{(ax_1+by_1+c)}{a\cos\theta + b\sin\theta}$

$= \frac{-(ax+by+c)(\sqrt{a_1^2+b_1^2})}{ba_1 - ab_1}$

$= \boxed{\frac{-(ax+by+c)}{\begin{vmatrix} a_1 & b_1 \\ a & b \end{vmatrix}}}$

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Second degree general Eqn.

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

⇒ It represents P.O.S.L. when

$$abc+2fgh-af^2-bg^2-ch^2=0$$

or
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Q) find k if $x^2-5xy+4y^2+x+ky-2=0$ repr. P.O.S.L.

→ $1\times4\times(-2)+2\left(\frac{k}{2}\right)\left(\frac{1}{2}\right)\left(\frac{-5}{2}\right)-1\times\frac{k^2}{4}-4\times\frac{1}{4}-(-2)\left(\frac{25}{4}\right)=0$

⇒ $-8-\frac{5k}{4}-\frac{k^2}{4}-1+\frac{50}{4}=0$

⇒ $\frac{k^2}{4}+\frac{5k}{4}+\frac{14}{4}=0$

⇒ $k^2+5k-14=0$

⇒ $\boxed{k=2,-7}$

consider $k=-2$, $x^2-5xy+4y^2+x+2y-2=0$

$(x-4y+c_1)(x-y+c_2)=0$

⇒ $c_1+c_2=1$

$c_1+4c_2=-2$

⇒ $\boxed{c_2=-1,\ c_1=2}$

$1+1$

Factorisation of Gen. 2nd deg. eqn.

let factors = $(y - m_1x - c_1)(y - m_2x - c_2)$

$\Rightarrow y^2 - (m_1+m_2)xy + m_1m_2x^2$

$+ (c_1m_2 + c_2m_1)x - (c_1+c_2)y + c_1c_2 = 0$

Now compare coefficients.

~~atop~~ $\angle$ b/w the two factors line

$$\Rightarrow \boxed{\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}}$$

Q) Find lines. $6y^2 - xy - x^2 + 30y + 36 = 0$

$(3y + x + c_1)(2y - x + c_2)$

$\Rightarrow c_2 - c_1 = 0 \Rightarrow c_1 = c_2$

$3c_2 + 2c_1 = 30$

$\Rightarrow c_1 = c_2 = 6$

$\Rightarrow$ lines = $(3y + x + 6)(2y - x + 6)$

Q) $kxy - 8x + 9y - 12 = 0 \longrightarrow$ P.O.S.L

Then find lines & k.

Divide by -12

$1 - \frac{3}{4}y + \frac{2}{3}x - \frac{1}{2}xy$ $\quad\Rightarrow ab = \frac{1}{2} = \frac{k}{12}$

$(1-a)(1+b)$ $\quad\Rightarrow k = \frac{1}{2} \times 12 = \boxed{6}$

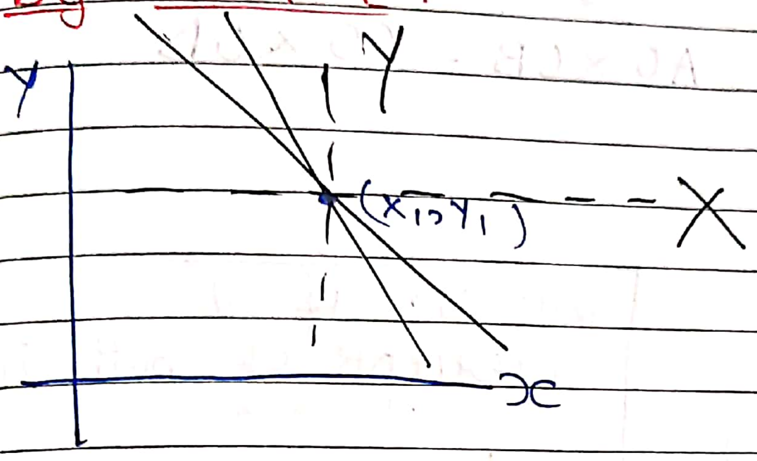
$= 1 - a + b - ab$

$\Rightarrow$ lines = $\boxed{\left(1 - \frac{3}{4}y\right)\left(1 + \frac{2}{3}x\right)}$

→ concept.

→ $\boxed{\dfrac{a}{m_1 m_2} = \dfrac{b}{1} = \dfrac{2h}{-(m_1+m_2)} = \dfrac{2g}{-(c_1 m_2 + c_2 m_1)} = \dfrac{2f}{-(c_1+c_2)} = \dfrac{c}{c_1 c_2}}$

→ Point of intersection of lines repr. by S.G.E.



$x = X + x_1$

$y = Y + y_1$

$P(x) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Rightarrow a(X+x_1)^2 + 2h(X+x_1)(Y+y_1) + b(Y+y_1)^2 + 2g(X+x_1) + 2f(Y+y_1) + c = 0$

but these will be homogenous at new origin.
no term of X & Y should exist

coeff of

$X \rightarrow 2ax_1 + 2hy_1 + 2g$

$Y \rightarrow 2hx_1 + 2by_1 + 2f$

$\Rightarrow \boxed{ax_1 + hy_1 + g = 0 \quad \& \quad hx_1 + by_1 + f = 0}$

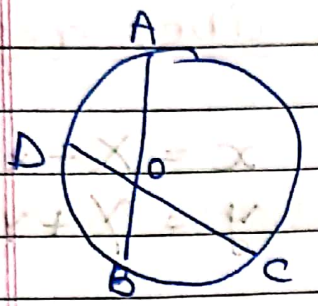
M-II $\quad \begin{vmatrix} a & h & g \\ h & b & f \end{vmatrix} \begin{matrix} = 0 \\ = 0 \end{matrix} \quad \Rightarrow \quad \boxed{x_1 = \dfrac{hf - bg}{ab - h^2}}$ $\boxed{y_1 = \dfrac{gh - af}{ab - h^2}}$ Learn

M-III $\quad \dfrac{\partial P}{\partial x} = 2ax + 2hy + 2g = 0$

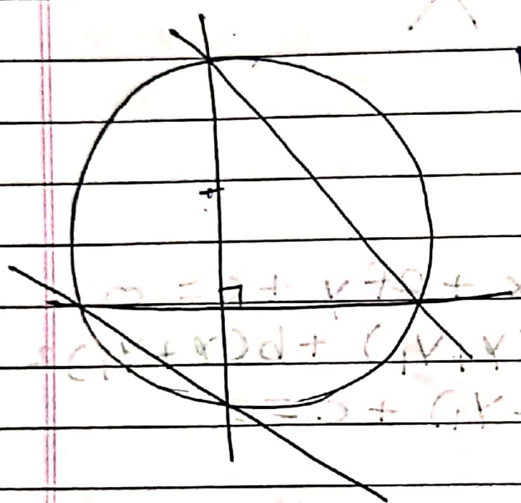
$\partial P / \partial y = 0$

Condition for two lines cutting coordinate axis at concyclic points.

We know that,



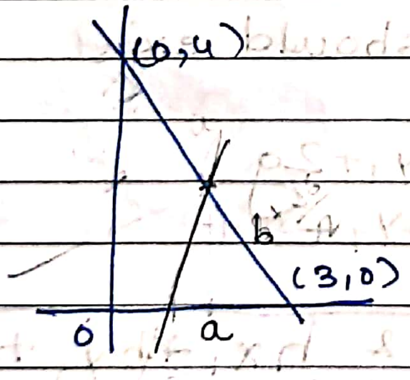
$AO \times OB = CO \times OD$



product of Y intercepts of both lines

= Product of x intercepts of both lines.

Q)



Eqn of line that divides the $\triangle$ into 2 parts of equal perimeter & equal areas.

$a+b = \frac{1}{2} \times \text{perimeter}$ $a+b = 6$

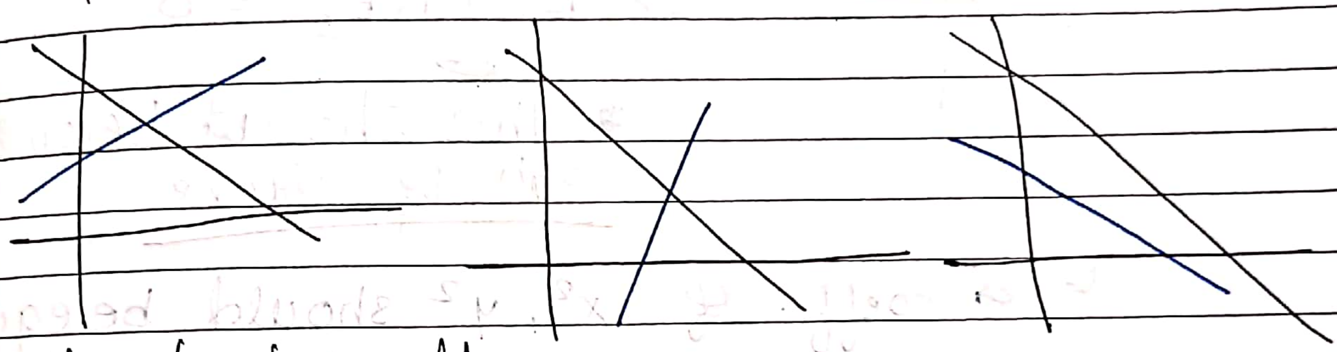
& area = 3

$\Rightarrow \frac{1}{2} \times a \times b \times \sin 53^\circ = 3 \Rightarrow ab = 7.5$

$\Rightarrow$ a, b are roots of eq^n
$x^2 - 6x + 7 \cdot 5 = 0$

so for those values of a, b
you can find coordinates of
those intersections and then
you can find the eqn of line.

3 possible cases $\longrightarrow$



check for all.

Q) $ax^3 + bx^2y + cxy^2 + dy^3 = 0$. Two of
Them form complementary $\angle$s from
x axis in anticlockwise.
Find relation b/w a, b, c, d.

$\rightarrow$ divide eqn by x^3

$\Rightarrow dm^3 + cm^2 + bm + a = 0$ $\quad$ $\because \tan 90 = \frac{m_1 + m_2}{1 - m_1 m_2}$
& $m_1 m_2 = 1$

$\Rightarrow m_3 = -\frac{a}{d}$

$\Rightarrow d\left(\frac{-a^3}{d^3}\right) + c\left(\frac{a^2}{d^2}\right) + \frac{b(-a)}{d} + a = 0$

$\Rightarrow \boxed{-a^3 + a^2c - abd + ad^2 = 0}$

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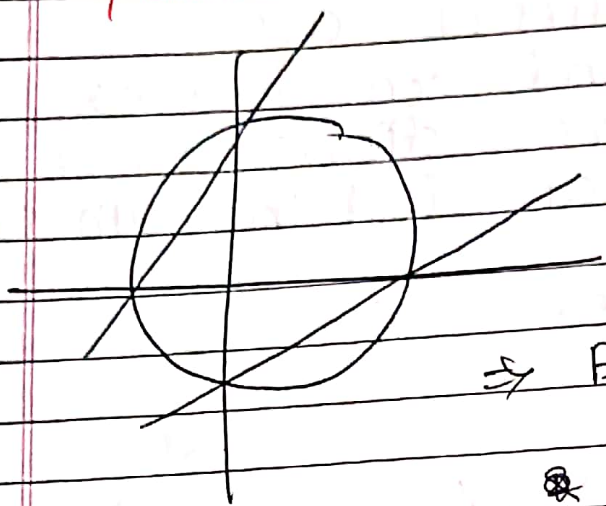
Pair of straight lines cutting coordinate axes at concyclic points

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$



Those 4 points are P.O.I with $xy = 0$

$\Rightarrow E_1 + KE_2 = 0$

this should become eqn of circle

$\Rightarrow$ coeff. of x^2, y^2 should be equal.

$\Rightarrow \boxed{a = b}$

& radius should exist.

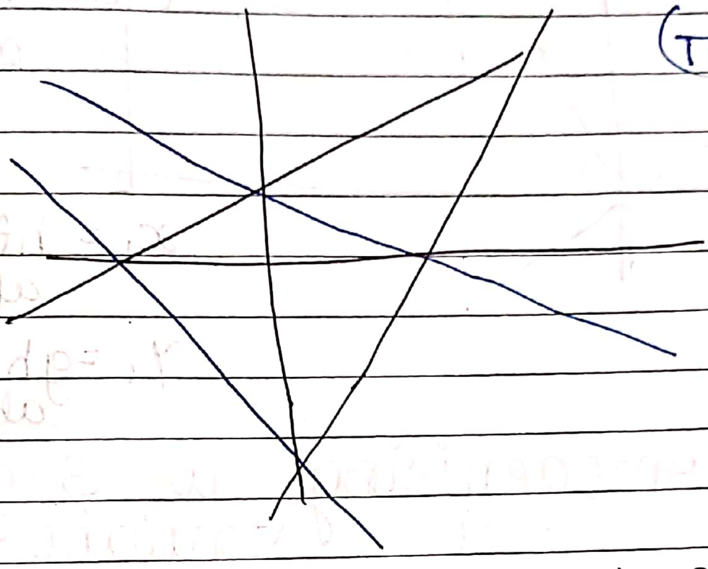
$\Rightarrow \sqrt{\left(\frac{g}{a}\right)^2 + \left(\frac{f}{a}\right)^2 - \frac{c}{a}} = \text{radius}$

$\Rightarrow \boxed{g^2 + f^2 - ac > 0}$

CONCEPT

A P.O.S.L. is given.
Another P.O.S.L cutting coordinates axes at the same points as original P.O.S.L is to be found.

(To be found)



Method

$\Rightarrow$ $E_1 + KE_2$ should still represent P.O.S.L.

$E_2 = xy = 0$

$\Rightarrow ax^2 + 2\left(h+\frac{k}{2}\right)xy + by^2 + 2gx + 2fy + c = 0$

$\Rightarrow abc + 2fg\left(h+\frac{k}{2}\right) - bg^2 - c\left(h+\frac{k}{2}\right)^2 - af^2 = 0$

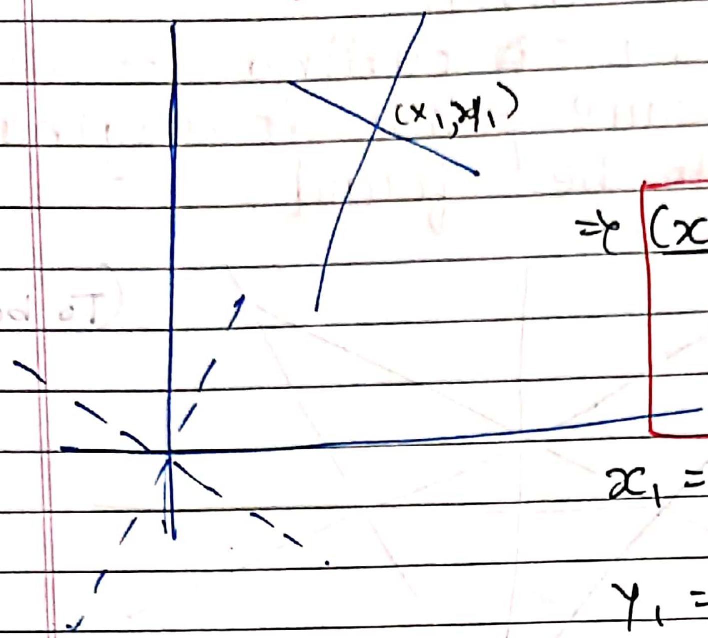
$\Rightarrow \underbrace{abc + 2fgh - af^2 - bg^2 - ch^2}_{=0} + fgk - \frac{ck^2}{4} - chk = 0$

$\Rightarrow fgk - \frac{ck^2}{4} - chk = 0$

$\Rightarrow fg - \frac{ck}{4} - ch = 0$

$\Rightarrow \boxed{k = \frac{4}{c}(fg - ch)}$

Pair of Angle Bisector



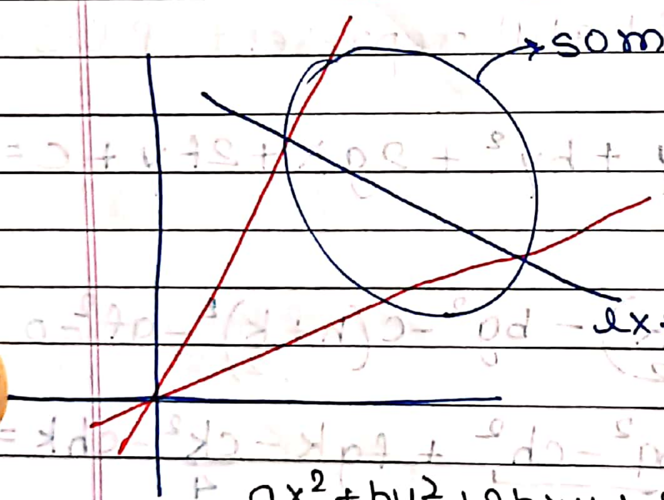
$$\frac{x^2-y^2}{a-b} = \frac{xy}{h}$$

$$\Rightarrow \boxed{\frac{(x-x_1)^2-(y-y_1)^2}{(a-b)} = \frac{(x-x_1)(y-y_1)}{h}}$$

$$x_1 = \frac{hf-bg}{ab-h^2}$$

$$y_1 = \frac{gh-af}{ab-h^2}$$

Homogenising a S.G.E using a straight line



$$ax^2+by^2+2hxy+2gx+2fy+c=0$$

$$\Rightarrow \boxed{ax^2+2hxy+by^2+(2gx+2fy)\left(\frac{lx+my}{n}\right) + c\left(\frac{lx+my}{n}\right)^2 = 0}$$

This is eqn of P.O.S.L. through origin & passing through intersection of 2nd deg. curve & straight line

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→ If in a Q, a chord $\overset{\text{of curve}}{}$ is making $\angle\theta$ at origin, it is clear case of homogenisation.

Q) P.T. str. lines joining origins to P.O.I of $x-y=2$ & $5x^2+12xy-8y^2+8x-4y+12=0$ make equal $\angle$ with axis.

→ $5x^2+12xy-8y^2+(8x-4y)\left(\frac{x-y}{2}\right)$

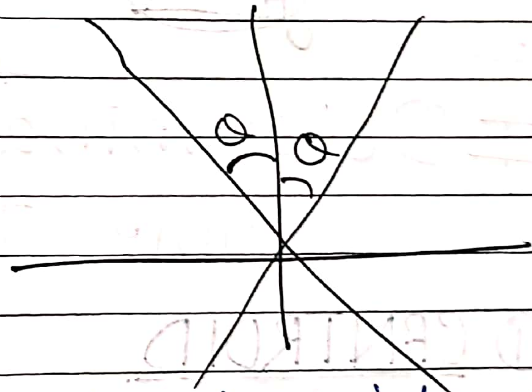
$+12\left(\frac{x-y}{2}\right)^2$

$= 5x^2+12xy-8y^2+4x^2-4xy-2xy+2y^2$

$\cancel{+3x^2+6y^2}$ $3x^2+3y^2-6xy=0$

$= 12x^2-3y^2+0$

$\Rightarrow y=\pm 2x$



Q) ST Ls joining origin to pt. of int. of line $kx+hy=2kh$.

curve : $(x-h)^2+(y-k)^2=c^2$

make $90°$ at O. then $h^2+k^2=\lambda c^2$

$\cancel{x^2+y^2-2hx-2yk+h^2+k^2-c^2=0}$

$\cancel{x^2+y^2-2(hx+yk)\frac{1}{2}\left(\frac{x}{h}+\frac{y}{k}\right)+\frac{h^2+k^2-c^2}{4}\left(\frac{x}{h}+\frac{y}{k}\right)^2}$

$\cancel{-\frac{hxy}{k}-\frac{xyk}{h}+\frac{h^2}{4}\frac{x^2}{h^2}+\frac{k^2x^2}{4h^2}-\frac{c^2x^2}{4h^2}}$

$$x^2+y^2-(2hx+2ky)\frac{(kx+hy)}{2hk}+(h^2+k^2-c^2)\left(\frac{kx+hy}{2hk}\right)^2$$

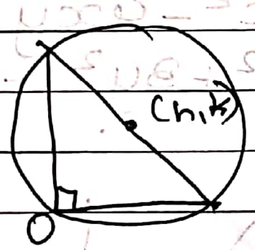
$$x^2 \to 1-1+\frac{h^2+k^2-c^2}{4h^2}$$

$$y^2 \to \frac{h^2+k^2-c^2}{4k^2}$$

$\because 90^\circ$

$$\frac{h^2+k^2-c^2}{4h^2}+\frac{h^2+k^2-c^2}{4k^2}=0$$

$$\Rightarrow h^2+k^2=c^2 \Rightarrow \boxed{\lambda=1}$$



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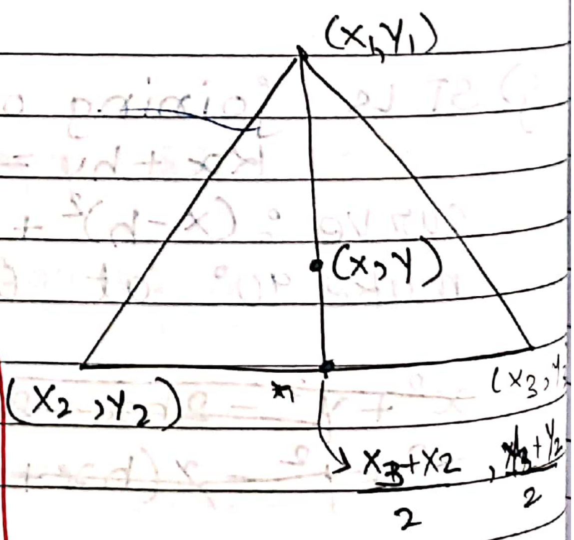
Determinant forms of Some Imp. Lines

① MEDIAN

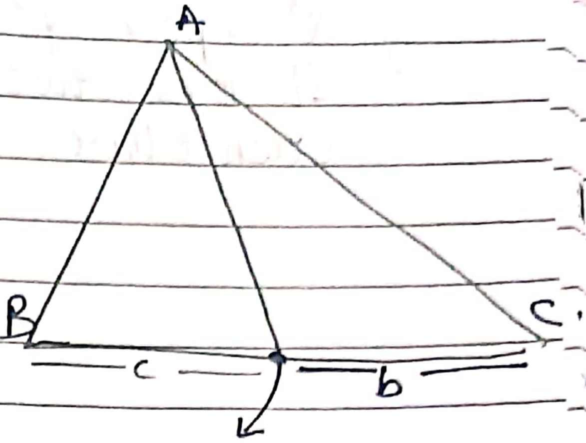
$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix} = 0$$



$$\Rightarrow \boxed{\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0}$$

(1) ANGLE BISECTOR

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \dfrac{cx_3+bx_2}{b+c} & \dfrac{cy_3+by_2}{b+c} & 1 \end{vmatrix} = 0$$



$$\left\{ \frac{cx_3+bx_2}{c+b}, \frac{cy_3+by_2}{c+b} \right\}$$

$$\Rightarrow \boxed{c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0}$$

Distance b/w a parallel pair of straight lines.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

first check if it is P.O.S.L.

then check if parallel.

$\longrightarrow \boxed{h^2 = ab} \Rightarrow \tan\theta = 0 \Rightarrow$ parallel

now dist. $= \boxed{2\sqrt{\dfrac{g^2-ac}{a(a+b)}}}$ or $\boxed{2\sqrt{\dfrac{f^2-bc}{b(a+b)}}}$

If $\underline{g^2=ac}$ or $\underline{f^2=bc}$

then lines are coincident.