7/4/22

Page No.
Date

CENTRE OF MASS

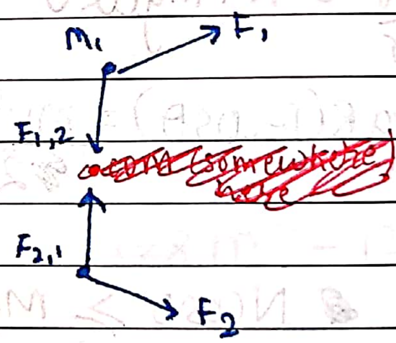
$F_{net} = ma \longrightarrow a = acc^n$ of centre of mass

$$\boxed{a_{cm} = \frac{F_{net}}{M}}$$ $F_{net} \rightarrow$ Net external force; $M \rightarrow$ Total mass of system.

C.O.M is a hypothetical point at which the total mass is assumed to be concentrated & it behaves as if total force is acting on it.

Note: Internal forces have no effect on the motion of COM.

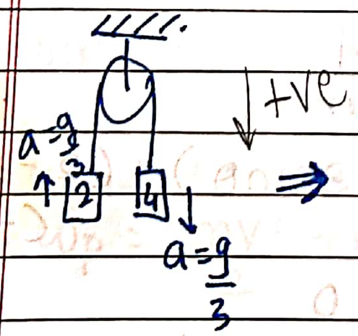
Q> Find a_{cm} of m_1 & m_2.



$$\rightarrow \vec{a}_{cm} = \frac{\vec{F}_1 + \vec{F}_{12} + \vec{F}_{21} + \vec{F}_2}{m_1 + m_2}$$

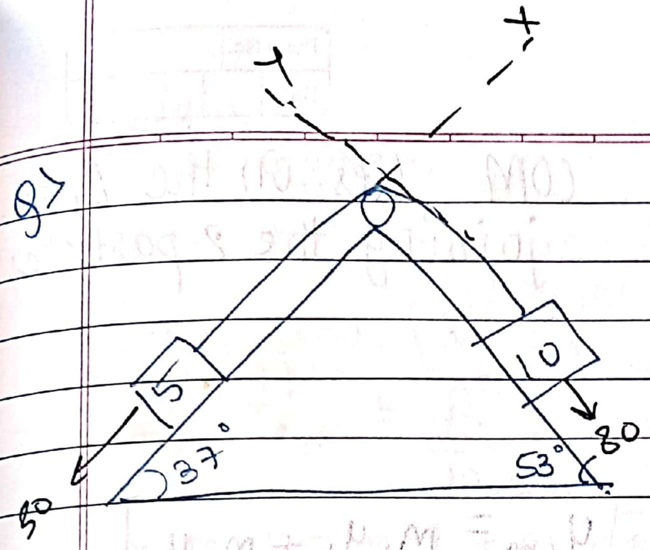
$$\boxed{\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}} \quad \text{or} \quad \boxed{\vec{a}_{cm} = \frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2}}$$

Q>



$$\Rightarrow \vec{a}_{cm} = \frac{\frac{4g}{3} - \frac{2g}{3}}{6} = \frac{20}{18} = \frac{10}{9} \ m/s^2$$

Q) Find $|\vec{a}_{cm}|$ of (5+10 kg).



$$a = \frac{80-30}{15} = \frac{10}{3}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2}$$

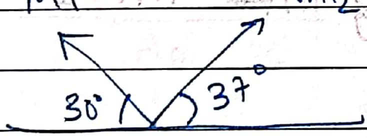
$$= \frac{5\left(\frac{10}{3}\hat{i}\right) + 10\left(\frac{10}{3}\right)\hat{j}}{15}$$

$$= \frac{50}{3\times15}\left(\hat{i}+2\hat{j}\right)$$

$$= \frac{10}{9}\left(\hat{i}+2\hat{j}\right)$$

$$|\vec{a}_{cm}| = \frac{10}{9}\sqrt{1+4} = \boxed{\frac{10\sqrt{5}}{9}\ m/s^2}$$

Q) m_1 m_2 a_{cm} of (m_1+m_2)



$$\rightarrow \vec{a}_{cm} = \frac{m_1 g + m_2 g}{m_1+m_2} = \boxed{g}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2} \rightarrow \int \vec{a}_{cm}\,dt = \frac{\int m_1\vec{a}_1\,dt + m_2\int \vec{a}_2\,dt}{m_1+m_2}$$

$$\boxed{\vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1+m_2}}$$

$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}}$$

Position of COM

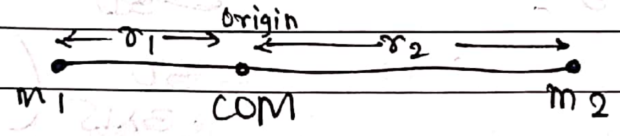
COM lies on the line joining the 2 particle.

$$\vec{r}_{cm} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1+m_2}$$ → weighted average of position

(x, y) → co-ordinates

$$\boxed{x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2}} \qquad \boxed{y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1+m_2}}$$

Q.



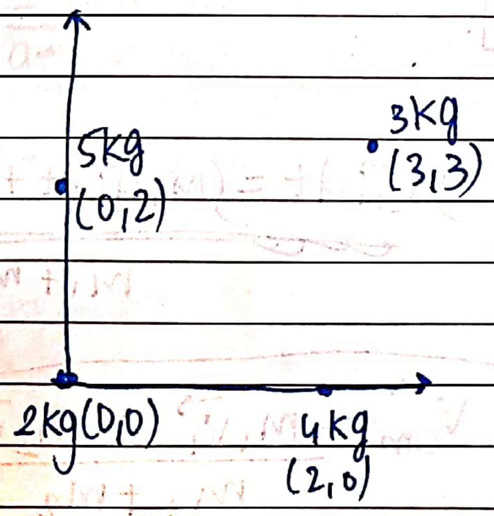
$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2}$$

$$\Rightarrow 0 = \frac{m_1(-r_1) + m_2r_2}{m_1+m_2} \Rightarrow m_1r_1 = m_2r_2$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

COM divides the line joining in inverse ratio of their mass.

Q> Find coordinates of COM.

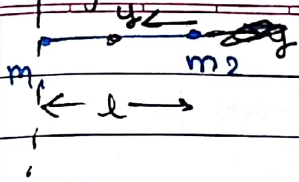


$$x_{cm} = \frac{0+9+0+8}{14} = \frac{17}{14}$$

$$y_{cm} = \frac{10+9+0+0}{14} = \frac{19}{14}$$

Scanned by CamScanner

origin.



Q> If m_1 is shifted 'r' meters towards m_2, find distance moved by m_2 so that COM does not shift.

→ $$x_{com} = \frac{m_2 l}{m_1+m_2} = \frac{m_1 r + m_2(l - y)}{m_1+m_2}$$

$$\Rightarrow m_2 l = m_1 r + m_2 l - m_2 y$$

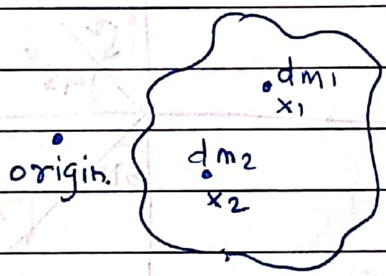
$$\Rightarrow m_1 r = m_2 y$$

$$\Rightarrow \boxed{y = \frac{m_1 r}{m_2}}$$

COM of continous bodies :-

$$X_{cm} = \frac{dm_1 x_1 + dm_2 x_2 + \dots}{M}$$

$$= \frac{\int dm \times x}{\int dm}$$



$$\boxed{X_{cm} = \frac{\int x\, dm}{\int dm}, \quad Y_{cm} = \frac{\int y\, dm}{\int dm}}$$

* continued ahead.

Mass densities

linear mass density (λ)

$$\lambda = \frac{mass}{length}$$

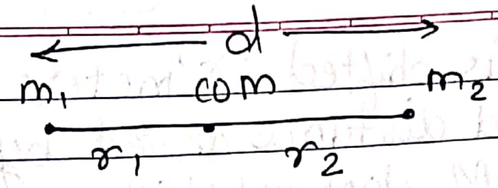
Areal (surface) mass density) (σ)

$$\sigma = \frac{mass}{Area}.$$

Volume mass density (ρ)

$$\rho = \frac{mass}{volume}$$

#

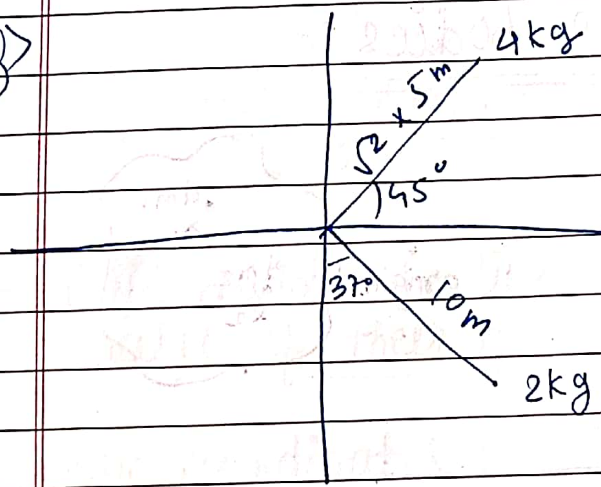


$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$\Rightarrow \frac{r_1}{r_1 + r_2} = \frac{m_2}{m_1 + m_2}$$

$$\Rightarrow \boxed{r_1 = \frac{m_2 d}{m_1 + m_2}} \qquad \boxed{r_2 = \frac{m_1 d}{m_1 + m_2}}$$

Q)

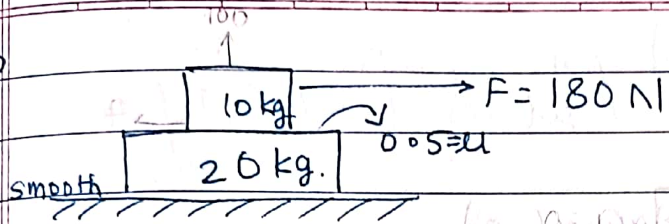


Dist. of COM from origin.

$$\rightarrow \frac{(5\hat{i}+5\hat{j})4 + 2(6\hat{i}-8\hat{j})}{6} = \frac{32\hat{i}+4\hat{j}}{6} = \frac{16\hat{i}+2\hat{j}}{3}$$

$$\text{dist.} = \sqrt{\frac{256}{9} + \frac{4}{9}} = \frac{2}{3}\sqrt{64+1} = \boxed{\frac{2}{3}\sqrt{65}\,m}$$

Find a_{cm}

Q) 

→ friction is internal

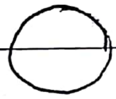
So, $a_{cm} = \frac{180}{30} = \boxed{6 \, m/s^2}$

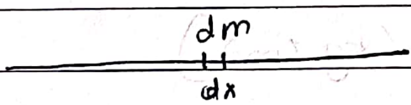
Mass Densities

Linear Mass density (λ)

$$\lambda = \frac{\text{Total mass}}{\text{Total length}}$$

Ex) Ring(M,R)

 → $\boxed{\lambda = \frac{M}{2\pi R}}$

Ex)  diff. material of each point

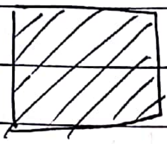
$$\lambda = \frac{dm}{dx}$$

$$\boxed{dm = \lambda dx}$$

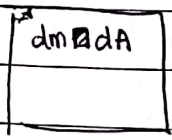
Areal (Surface) Mass Density

$$\boxed{\sigma = \frac{\text{mass}}{\text{area}}} \quad (\text{kg/m}^2)$$

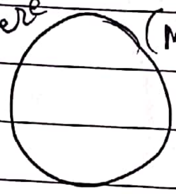
Ex)



$$\boxed{\sigma = \frac{M}{L^2}}$$



$$\sigma = \frac{dm}{dA} \qquad \boxed{dm = \sigma\, dA}$$



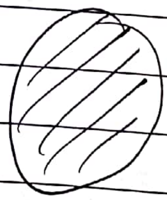
$$\boxed{\sigma = \frac{M}{4\pi R^2}}$$

Volume Mass Density

$$\boxed{\rho = \frac{\text{mass}}{\text{volume}}} \quad (\text{kg/m}^3)$$

$$(dV, dm) \longrightarrow \frac{dm}{dV} = \rho \Rightarrow \boxed{dm = \rho\, dV}$$

Ex)

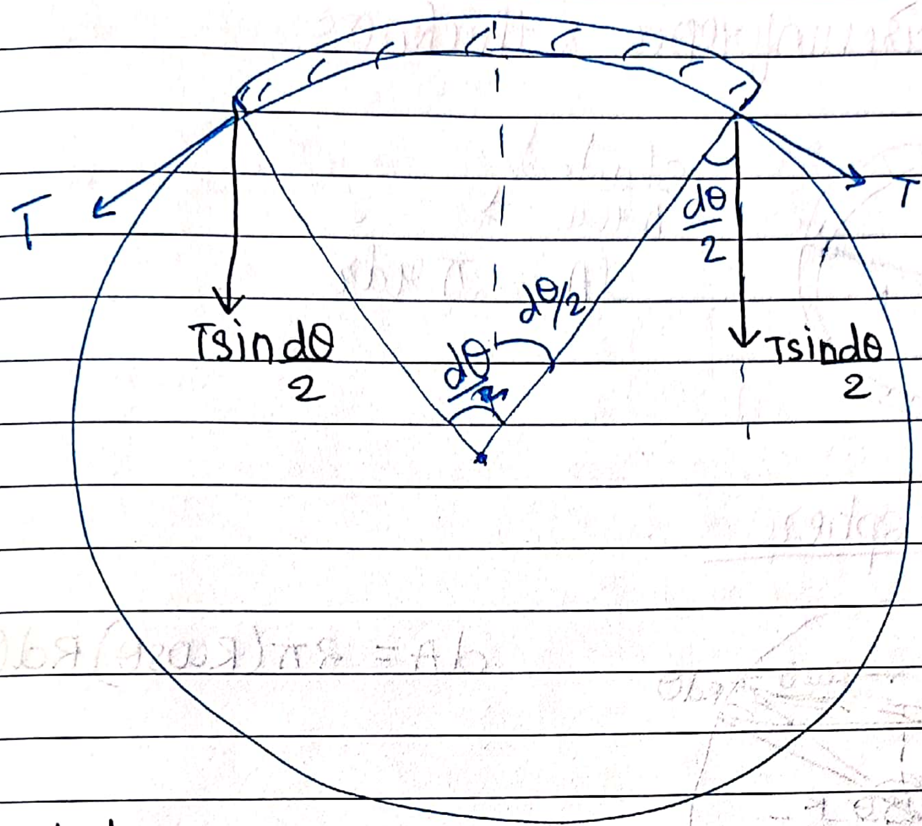


$$\boxed{\rho = \frac{M}{\frac{4}{3}\pi R^3}}$$

$\boxed{dm = \lambda\, dx = \sigma\, dA = \rho\, dV}$

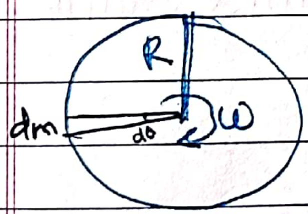
Page No.
Date

12/4/22



Total force in y-drxn $= 2T\sin\frac{d\theta}{2} = 2T\frac{d\theta}{2} = \boxed{Td\theta}$

Q> Tension developed in wire = ?



$$dm\, R\omega^2 = Td\theta$$

$$\cancel{\lambda dx R\omega^2} = Td\theta$$

$$\lambda dx R\omega^2 =$$

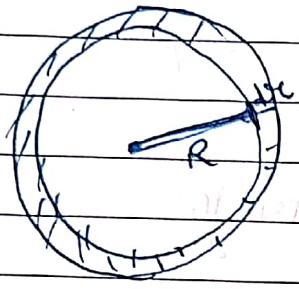
$$\lambda (Rd\theta) R\omega^2 = Td\theta$$

$$T = \lambda R^2\omega^2$$

$$\boxed{T = \frac{mR\omega^2}{2\pi}}$$

Area = circumference x thickness

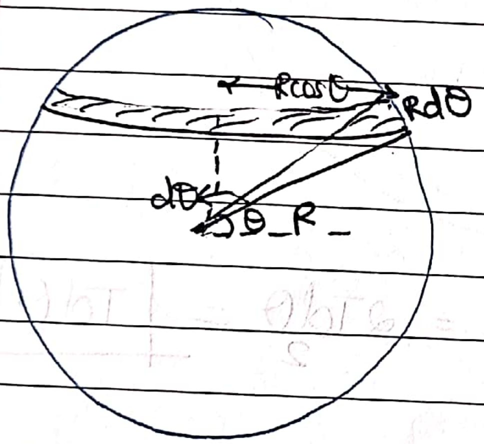
Ex)



shaded Area ⇒

$dA = 2\pi r dr$

Ex) Hollow sphere

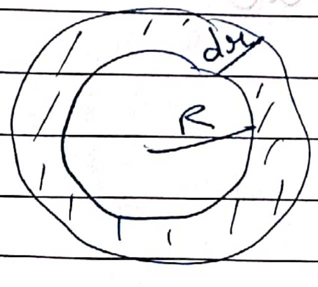


$dA = 2\pi(R\cos\theta)Rd\theta$

Note: For any hollow figure, always take slant thickness as actual thickness.

volume = Area x thickness

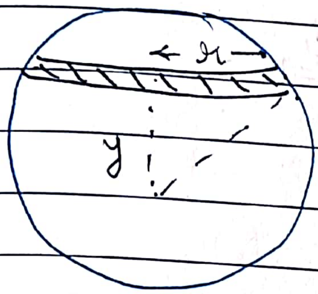
Ex) sphere



shaded vol^n

$dv = 4\pi r^2 dr$

Ex) Solid sphere



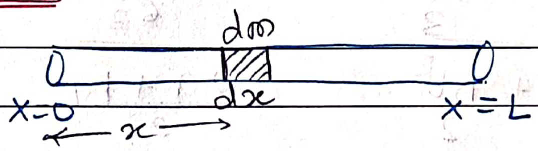
vol^m = $\pi r^2 \times dy$

Note: for any solid figure, actual thickness should be taken.

COM of continuous bodies.

$$X_{com} = \frac{\int x\,dm}{\int dm} \qquad Y_{com} = \frac{\int y\,dm}{\int dm}$$

① Thin Rod



(mass → M)

$\lambda = \frac{M}{L}$

$$X_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int x \lambda\,dx}{M}$$

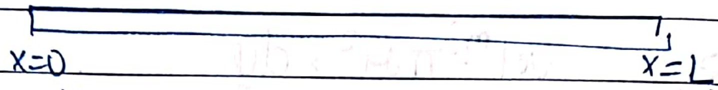
$$= \frac{\lambda}{M}\int_0^L x\,dx$$

$$= \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L$$

$$\boxed{X_{cm} = \frac{L}{2}}$$

Page No.
Date

Q) λ varies with distance $\lambda = a + bx$



Position of COM.

$\rightarrow$ $$X_{cm} = \frac{\int x(a+bx)dx}{\int (a+bx)dx}$$

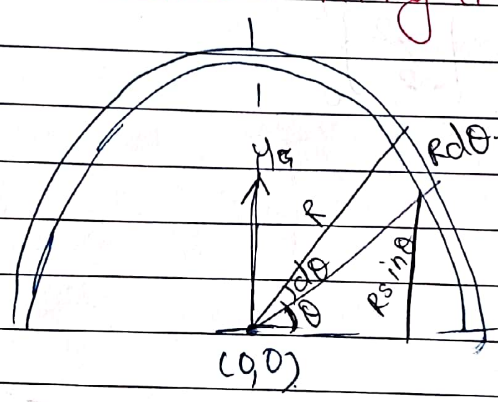
$$= \frac{a\int x dx + b\int x^2 dx}{a\int dx + b\int x dx}$$

$$= \frac{a\frac{x^2}{2}\Big]_0^L + b\frac{x^3}{3}\Big]_0^L}{ax\Big]_0^L + b\frac{x^2}{2}\Big]_0^L}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^3}{3}}{aL + \frac{bL^2}{2}} = \frac{\frac{3aL + 2bL^2}{\cancel{6}\,3}}{\frac{2a+bL}{\cancel{2}}}$$

$$= \frac{3aL + 2bL^2}{3(2a+bL)}$$

② semicircular Ring (m, R)



$$\lambda = \frac{M}{\pi R}$$

$$dm = \frac{M}{\pi R} Rd\theta$$

$$= \frac{Md\theta}{\pi}$$

$$Y_{cm} = \frac{\int dm\, Y}{\int dm} = \frac{1}{M}\int_0^{\pi} \frac{M\, d\theta}{\pi}(R\sin\theta)$$

$$= \cancel{\frac{R}{\pi}\left(\sin\theta\right)_0^{\pi}}$$

$$= \cancel{\frac{R}{\pi}\Big(}$$

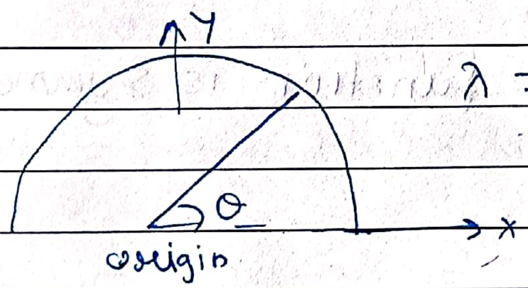
$$Y_{cm} = \frac{R}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{R}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$= \frac{R}{\pi}\left(1-(-1)\right)$$

$$Y_{cm} = \frac{2R}{\pi}$$

$$\boxed{C.O.M = \left(0, \frac{2R}{\pi}\right)}$$

Q) 

$\lambda = \lambda_0 \sin\theta$ Radius $\rightarrow R$.

$$\cancel{X_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int x(\lambda_0 \sin\theta) R\, d\theta}{\int \lambda_0 \sin\theta\, R\, d\theta}}$$

$$\cancel{= \lambda_0 R^2 \int \cos\theta \sin\theta\, d\theta}$$

$$\cancel{X_{cm} = \frac{\int R^2\cos\theta\, \lambda_0 \sin\theta\, d\theta}{\int R\lambda_0 \sin\theta\, d\theta}}$$

$$\cancel{= \frac{R}{2}\left(\cos\theta(-\cos\theta) + \int \sin\theta(-\cos\theta)\right)}$$

$$\rightarrow \quad Y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int R\sin\theta\, \lambda_0 \sin\theta\, R\,d\theta}{\int \lambda_0 R\,d\theta \sin\theta}$$

$$= \frac{R\int \sin^2\theta\, d\theta}{\int_0^{\pi} \sin\theta\, d\theta}$$

$$= \frac{R\int \frac{1-\cos 2\theta}{2}\, d\theta}{[-\cos\theta]_0^{\pi}}$$

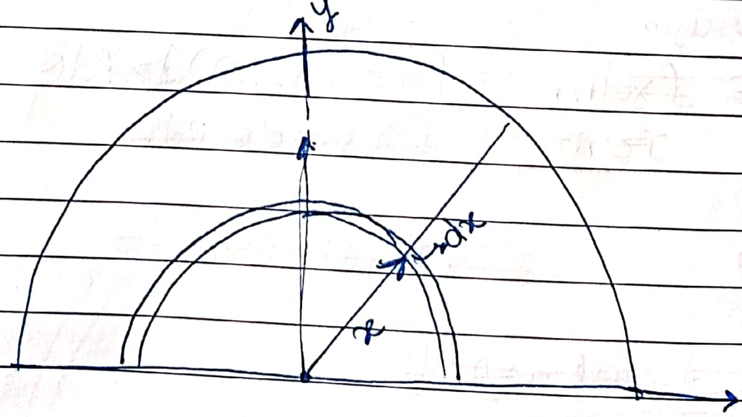
$$= \frac{\frac{R}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]}{2}$$

$$= \frac{R}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$\boxed{Y_{cm} = \frac{\pi R}{4}}$$

X_{cm} will be ~~..~~ 0 as function is symmetric.

③ Semicircular Disc.



$$Y_{cm} = \frac{\int y\,dm}{m} = \frac{1}{m}\int \frac{2x}{\pi}\,dm$$

$$dm = \frac{M}{\pi r^2/2} \times \frac{2\pi x\,dx}{2} = \frac{2M}{R^2}\,x\,dx$$

$$Y_{cm} = \frac{4}{\pi R^2}\int_0^R x^2\,dx$$

$$= \frac{4}{\pi R^2}\left[\frac{R^3}{3}\right]_0^R$$

$$= \boxed{\frac{4R}{3\pi}}$$

Q) σ varies with radius as $\sigma = \frac{\sigma_0}{r}$, find position of COM of semicircular disc.

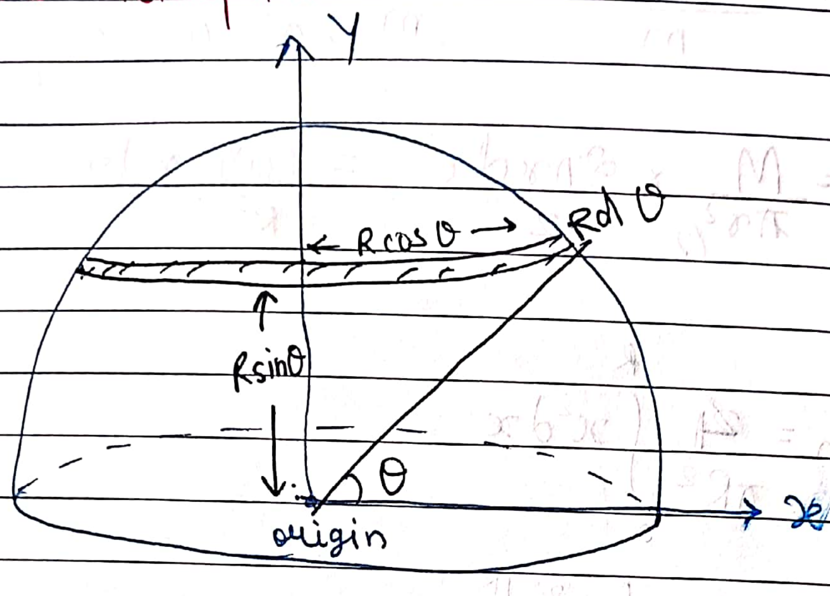
→ $dm = \sigma \pi r\,dr = \frac{\sigma_0}{r}\pi r\,dr = \sigma_0 \pi\,dr$.

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^R \frac{2r}{\pi}\sigma_0 \pi\,dr}{\int_0^R \sigma_0 \pi\,dr}$$

$$= \boxed{\frac{R}{\pi}}$$

Page No.
Date

④ Hollow Hemisphere



$$dm = \sigma\, dA$$
$$= \sigma\, 2\pi R\cos\theta\, R\, d\theta$$

$$Y_{cm} = \frac{\int y\, dm}{\int dm} = \frac{\int R\sin\theta \times \sigma \times 2\pi R\cos\theta \times R\, d\theta}{M}$$

$$= \frac{R^3 \sigma 2\pi}{M} \int \sin\theta\cos\theta\, d\theta$$

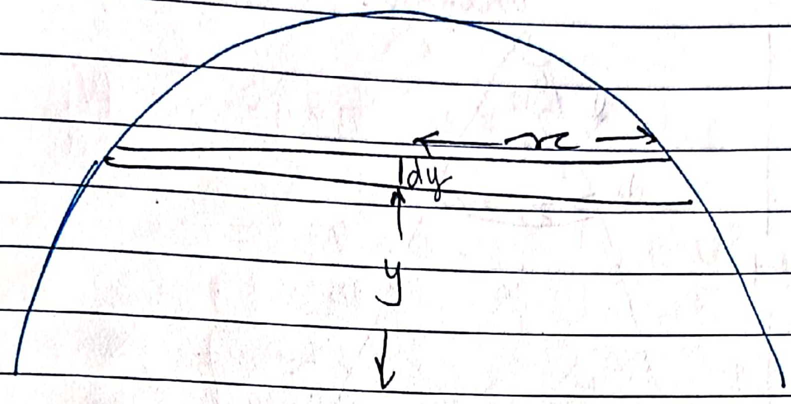
$$= \frac{\cancel{2}\pi R^3 M}{2\pi R^2 M} \int \sin 2\theta\, d\theta$$

$$= \frac{R}{2}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{R}{4}(1-(-1))$$

$$= \boxed{\frac{R}{2}}$$

⑤ Solid Hemisphere.



$x^2+y^2=R^2$

$$\rho = \frac{M}{\frac{2}{3}\pi R^3} = \frac{3M}{2\pi R^3}$$

$$dm = \rho \times dV$$
$$= \frac{3M}{2\pi R^3} \times \pi x^2 \times dy$$

$$dm = \frac{3M\,x^2 dy}{2R^3}$$

$$Y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int y \times \frac{3m}{2R^3} x^2 dy}{M} = \frac{3}{2R^3}\int x^2 y\,dy$$

$$= \frac{3}{2R^3}\left(\int R^2 y\,dy - \int y^3 dy\right)$$

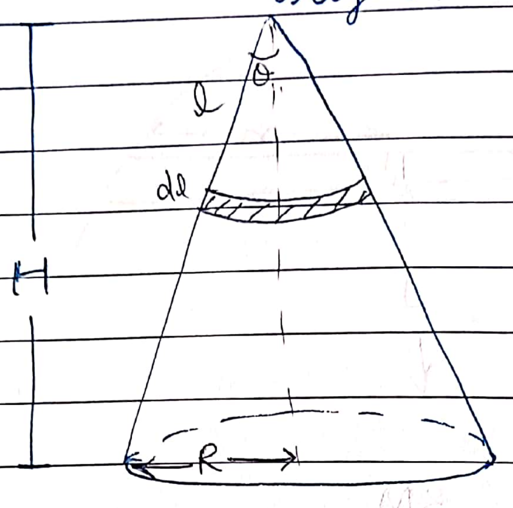
$$= \frac{3}{2R^3}\left[\left[\frac{R^2y^2}{2}\right]_0^R - \left[\frac{y^4}{4}\right]_0^R\right]$$

$$= \frac{3}{2R^3}\left(\frac{R^4}{2} - \frac{R^4}{4}\right)$$

$$= \boxed{\frac{3R}{8}}$$

Page No.
Date

⑥ Hollow Cone



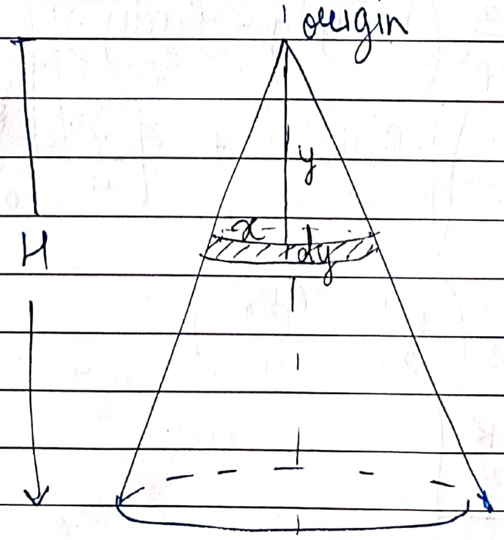
$$y_{cm} = \frac{\int y\,dM}{\int dM} = \frac{\int l\cos\theta \times \sigma \times 2\pi l \sin\theta\, dl}{M}$$

$$= \frac{\cos\theta\ \sigma\ 2\pi \sin\theta}{M}\left[\frac{l^3}{3}\right]_0^L$$

$$= \frac{H}{\cancel{L}} \times \frac{\cancel{M}}{\cancel{\pi} R \cancel{L}} \times \frac{2\cancel{\pi}}{\cancel{M}} \times \frac{\cancel{R}}{\cancel{L}} \times \frac{\cancel{L^3}}{3}$$

$$= \boxed{\frac{2H}{3}}$$

⑦ Solid Cone



$$\frac{x}{y} = \frac{R}{H}$$

$$x = \frac{Ry}{H}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int y \times \rho \times \pi x^2 dy}{M}$$

$$= \frac{\rho\pi \int y x^2 dy}{M}$$

$$= \frac{\cancel{M}}{\frac{1}{3}\cancel{\pi} R^2 H} \times \cancel{\pi} \times \frac{1}{\cancel{M}} \int y \frac{R^2 y^2}{H^2}\, dy$$

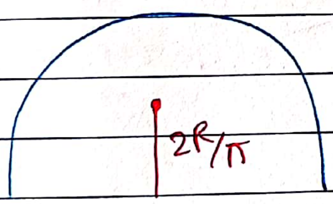
$$= \frac{R^2}{H^2} \times 3 \times \frac{1}{\cancel{R^2} H} \left[\frac{y^4}{4}\right]_0^H$$

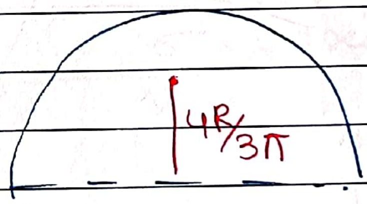
$$= \frac{3H^4}{4H^3}$$

$$= \boxed{\frac{3H}{4}}$$

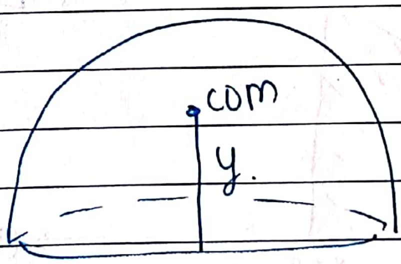
① S.C. Ring

② S.C. Disc





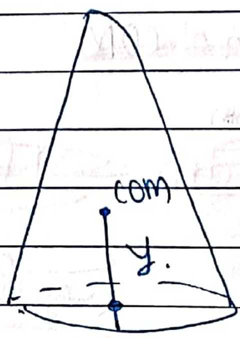
③ Hemisphere



$y = \frac{R}{2}$ (Hollow)

$y = \frac{3R}{8}$ (solid)

④ Cone



$y = \frac{H}{3}$ (Hollow)

$y = \frac{H}{4}$ (solid)

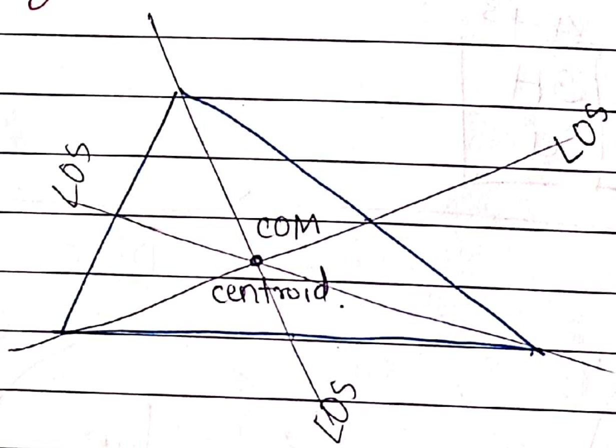
Note:

① COM can be outside the body.

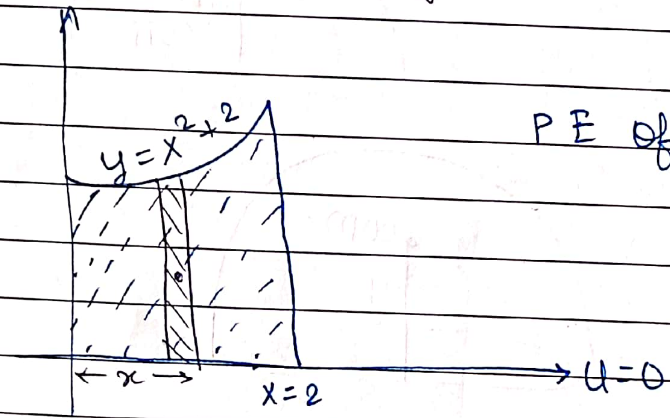
② i) For continuous bodies,
C.O.M. lies on the axis (line) of symmetry.

ii) com lies on the intersection of axis of symmetry.

⑧ Triangular Plate



Q)



PE of plate of mass M.

$\cancel{PE = mg \times h \text{ of } COM}$

$\cancel{= mg\, Y_{com}}$

$\cancel{= mg \frac{\int y\,dm}{m}}$ $\cancel{= g\int \frac{x^2+2}{2}\,\sigma\, dy\,dx}$

$\cancel{= \frac{g}{2}\int (x^2+2)^2\,dx}$

~~$PE = \frac{g}{2}\int x^4 + 4x^2 + 2\,dx$~~

~~$= \frac{g}{2}\left[\frac{x^5}{5} + \frac{4x^3}{3} + 2x\right]$~~

~~$= 0.5\int$~~

$PE = mgh$

$Area = \int y\,dx =$

$M = A \times \sigma$

$$h = y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int \frac{y}{2}\,\sigma \times \underbrace{y\,dx}_{\text{area of element}}}{M}$$

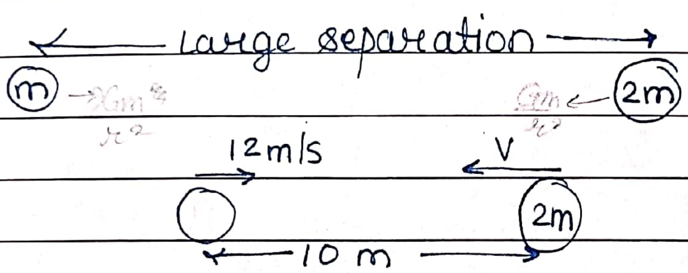
Page No.
Date

19/4/22

$$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M_{total}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{M}$$

Note :- $F_{ext} = 0$
$a_{cm} = 0$
$V_{cm} =$ constant

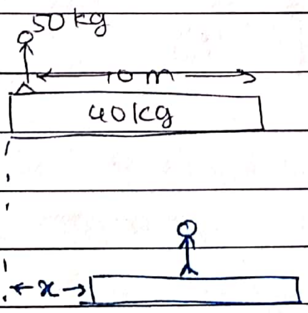
If $V_{cm} = 0$
it will rem. 0

Q)


$V_{cm} = ?$ (when they are 10m apart)

→ no external force
$V_{icm} = 0$
⇒ $\boxed{V_{cmf} = 0}$

① Case-1 $F_{ext} = 0$ $V_{cm} = 0$
⇒ COM does not shift.

Q)


when man reaches middle of plank, plank shifts by.

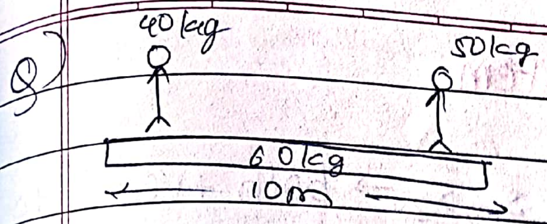
$F_{ext} = 0$ $V_{cm} = 0$

$X_{cm} = X_{cm}$

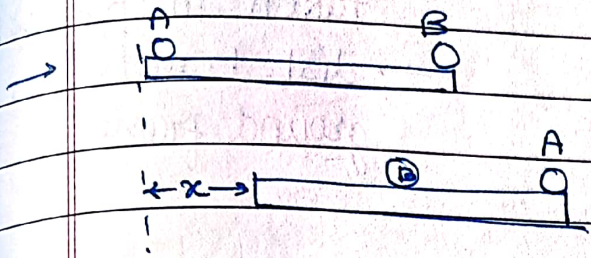
$50 \times 0 + 40 \times 5 = 50(x+5) + 40(x+5)$

$20 = 9x + 45$ ⇒ $\boxed{x = \frac{-25}{9}}$

Scanned by CamScanner

Q) 

If A reaches to right end & B reaches centre of plank, shift = ?

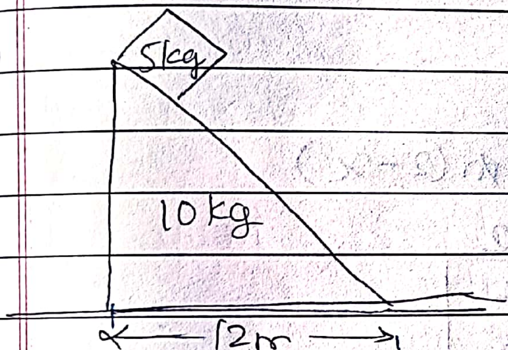


$X_{cm} = X_{cm}$

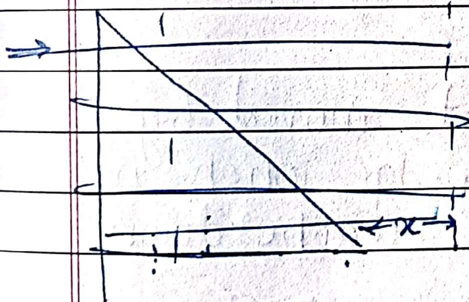
$40 \times 0 + 5\cancel{0} \times 10 + 6\cancel{0} \times 5 = 4\cancel{0} \times (x+10) + 5\cancel{0}(x+5) + 6\cancel{0}(x+5)$

$80 = 15x + 95$

$\Rightarrow \boxed{x = -1} \Rightarrow$ 1m shift backward.

Q) 

when 5kg reaches ground 10kg shifts by.



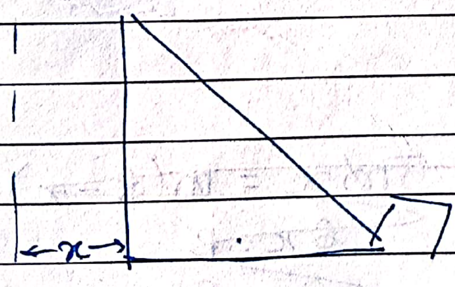
$5 \times 0 + 10 \times 6 = 5(12+x) + 10(x+6)$

$12 = 12 + x + 2x + 6$

$\Rightarrow -12 = 3x$

$\Rightarrow x = -4$

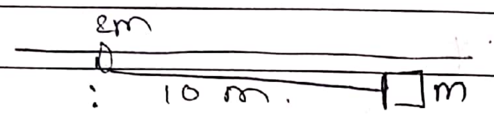
$\Rightarrow$ 4 m backward.



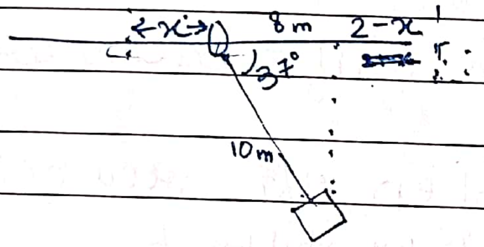
Note: for a two particle system com does not shift

$$\boxed{m_1 s_1 = m_2 s_2}$$

$s_1, s_2 \rightarrow$ both +ve & to be taken in opp. dirxn
$\rightarrow$ dist. travelled in ground frame.

Q) 

find displ. of ring when string rotates by 37°



$\cancel{2mx = m(2+x)}$

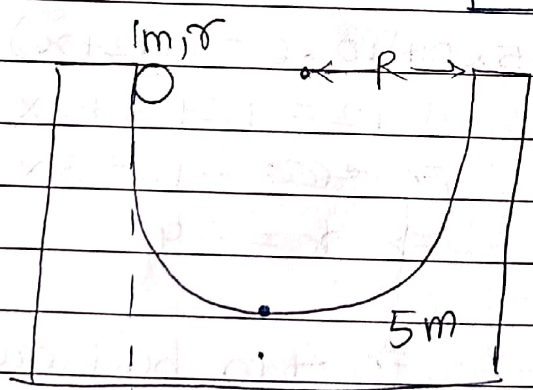
$\cancel{2x = 2+x}$

$\cancel{x = 2/m}$

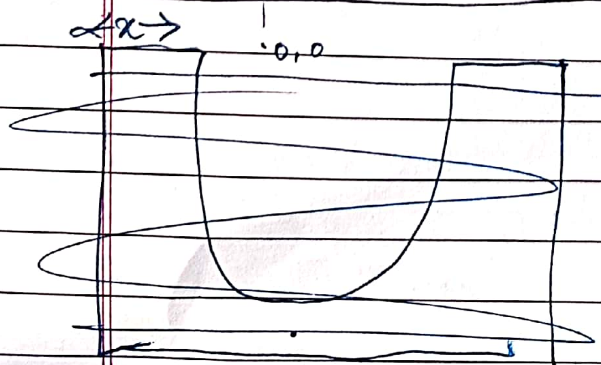
$2\cancel{m}x = \cancel{m}(2-x)$

$3x = 2$

$$\boxed{x = \frac{2}{3}}$$

Q) 

Dist moved by wedge when ball reaches down



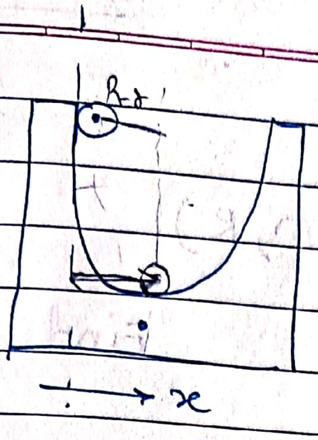
$\cancel{5mx = m(R-x)}$

$\cancel{6x = R}$

$\cancel{x = \frac{R}{6}}$

$\cancel{5mR = 5m(R+x)} + m(R+x)$

Page No.
Date



$$m\frac{r}{\cancel{2}} + 5mR = m\left(R - \frac{r}{\cancel{2}} + x\right) + 5m(x + R)$$

$$\frac{r}{\cancel{4}} + \cancel{5R} = R - \frac{r}{\cancel{2}} + x + 5x + \cancel{R5}$$

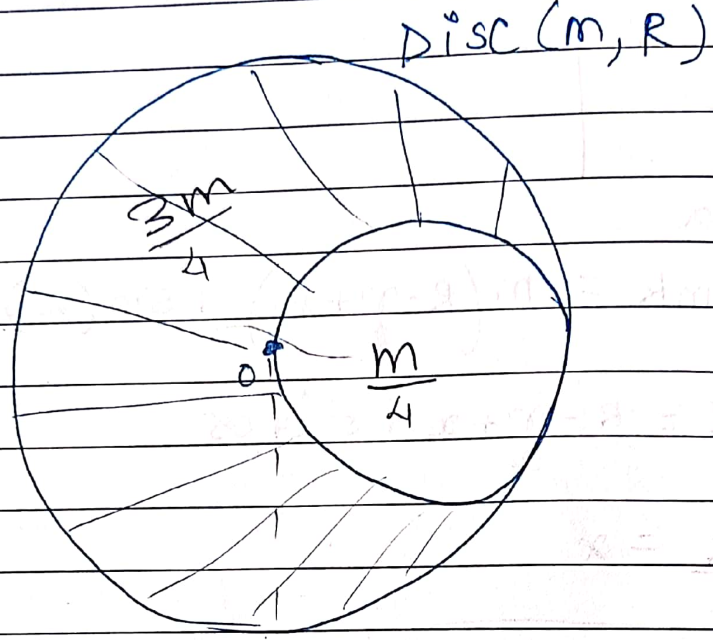
$$\frac{r - R}{6} = x$$

$$\Rightarrow \boxed{x = \frac{R - r}{6} \text{ backward}}$$

M-II
$$5m \times s_1 = m(R - r - s_1)$$
$$6s_1 = R - r$$
$$\boxed{s_1 = \frac{R - r}{6}}$$

× ×

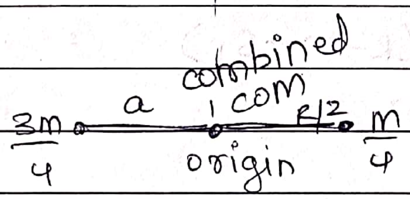
Bodies with cavity



Find COM

$\pi r^2 \rightarrow m$

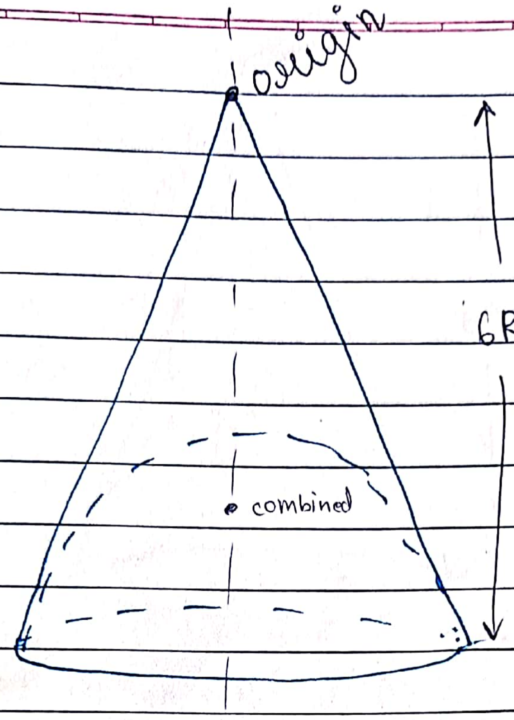
$\pi (r/2)^2 \rightarrow m/4$



$$\frac{a}{R/2} = \frac{m/4}{3m/4}$$

$$\boxed{a = \frac{R}{6}}$$

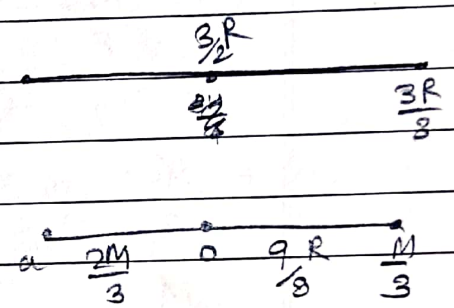
Page No.
Date



Hemisphere of R is rem.
COM = ?

$\frac{4}{3}\pi r^3 2 = M$

$\frac{2}{3}\pi r^3 = \frac{M}{3}$

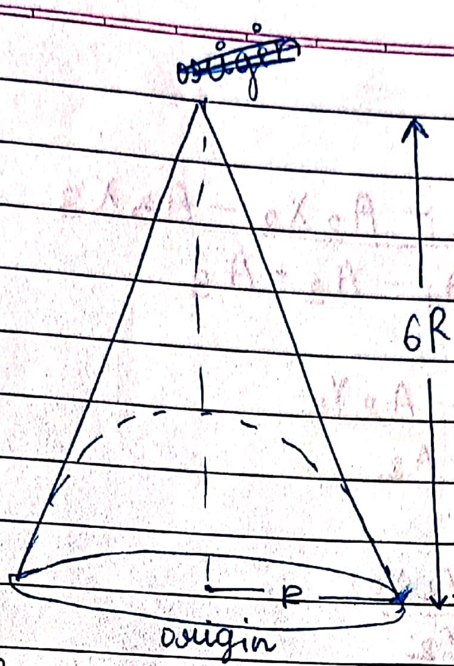


$a \frac{2M}{3} \times \frac{3}{M} = \frac{9R}{8}$

$a = \frac{9}{8}R$

$COM = \frac{3R}{2} + \frac{9R}{8} = \frac{21R}{8}$

Page No.
Date

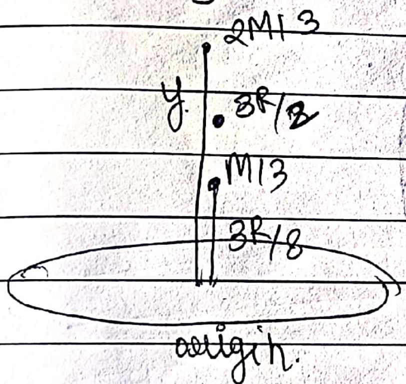


COM = ?

$V_{cone} = \frac{1}{3}\pi R^2 H = 2\pi R^3 = M$

$V_{hemisp} = \frac{M}{3}$

Rem. $= \frac{2M}{3}$



$Y_{cm} = \dfrac{\frac{M}{3} \times \frac{3R}{8} + \frac{2M}{3} \times y}{M}$

$\frac{3R}{2} = \frac{R}{8} + \frac{2y}{3}$

$\frac{12R}{8} - \frac{R}{8} = \frac{2y}{3}$

$\boxed{\frac{33}{16}R = y}$

COM from vertex $= \boxed{\frac{63R}{16}}$

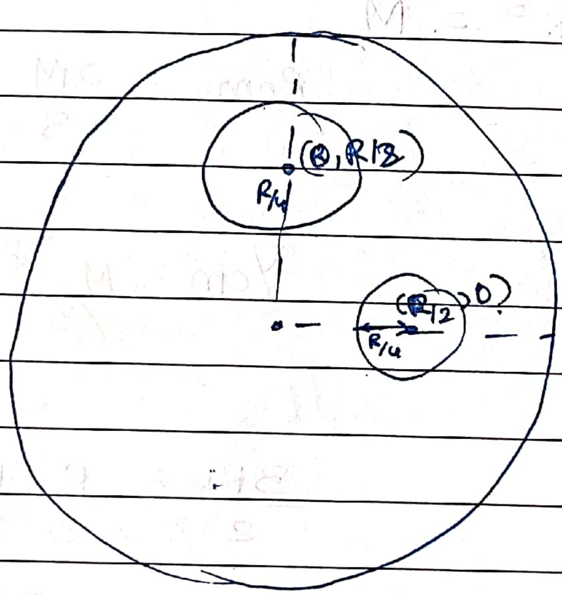
for cavities

Area
$$X_{cm} = \frac{A_1X_1 - A_2X_2 - A_3X_3}{A_1 - A_2 - A_3}$$

$$Y_{cm} = \frac{A_1Y_1 - A_2Y_2}{A_1 - A_2}$$

$$X_{cm} = \frac{V_1X_1 - V_2X_2}{V_1 - V_2}$$

Q)



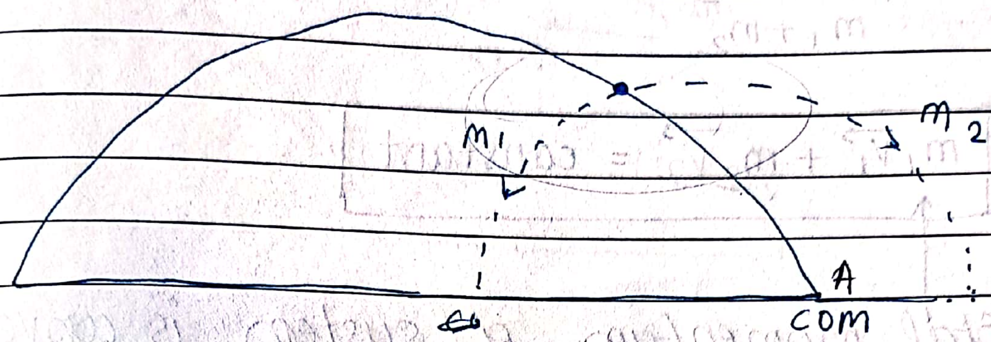
$$X_{cm} = \frac{\pi R^2 \times 0 - \frac{\pi R^2}{16} \cdot \frac{R}{2} - 0}{\pi R^2 - \frac{\pi R^2}{8}}$$

$$= \boxed{\frac{-R}{28}}$$

$$Y_{cm} = \boxed{\frac{-R}{28}}$$

Page No.
Date

Internal forces do not affect motion of COM



Note - For COM to fall on A, all particles should reach ground simultaneously.
$\Rightarrow$ V_y should be same.
$\Rightarrow$ If ~~one~~ m_1 falls before m_2 then COM will fall right ~~of A~~.

Q) Ball explodes at H.P. in mass ratio 2:3 lighter masses retraces the path, where will heavier mass fall.

$\rightarrow$ $R = \frac{2m \times 0 + 3m \times y}{5m}$

$\Rightarrow$ $\boxed{y = \frac{5R}{3}}$

Page No.
Date

If $F_{ext} = 0$ $\quad \vec{V_{cm}} = \text{constant}$

$$\frac{m_1\vec{V_1} + m_2\vec{V_2}}{m_1 + m_2} = \text{constant}$$

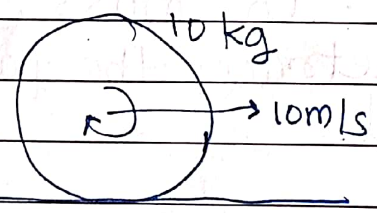
$$\boxed{m_1\vec{V_1} + m_2\vec{V_2} = \text{constant}}$$

Total momentum of system is converted when $F_{ext} = 0$

Total momentum of the system = Total mass x $\vec{V_{cm}}$

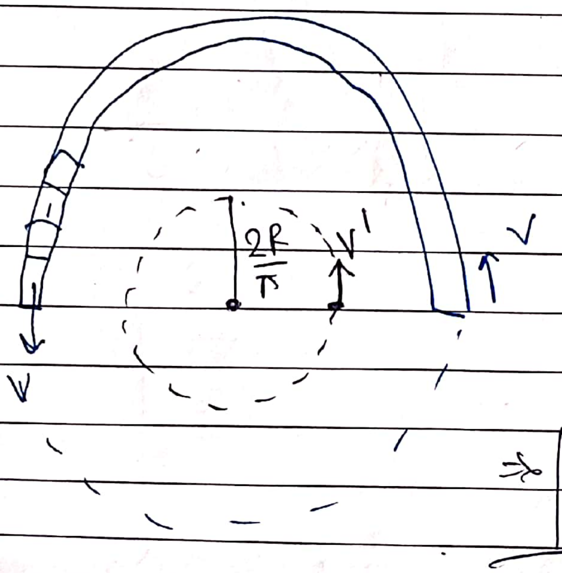
$$\boxed{\vec{P} = M\vec{V_{cm}}}$$

Q)



momentum = MV = $\boxed{100 \text{ kgm/s}}$

Q)



Train → mass M

$\omega = \omega$

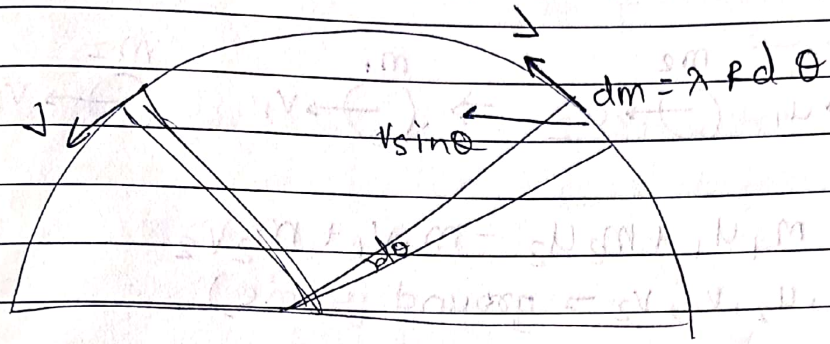
$$\frac{V}{R} = \frac{V'}{2R/\pi}$$

$$\Rightarrow \frac{2V}{\pi} = V'$$

$$\Rightarrow \boxed{P = \frac{M2V}{\pi}}$$

Page No.
Date

(M-2)



$$P = \int dm\, V\sin\theta$$
$$= \int \lambda R d\theta \sin\theta\, V$$
$$= \frac{M}{\pi R} \times RV(-\cos\theta)_0^{\pi}$$
$$= \boxed{\frac{2MV}{\pi}}$$

Notes:-

① Total momentum of a system in COM frame = zero

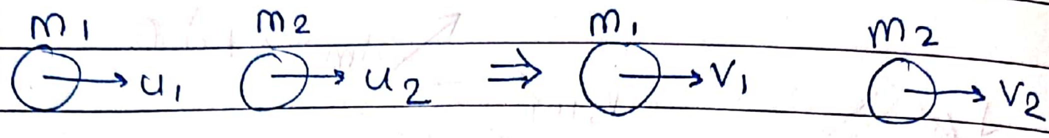
② Total external force in COM frame = zero

③ Kinetic energy of 2 particle system in COM frame
$= \frac{1}{2}\mu V_{rel}^2$ → rel. vel. of two particles
$\mu \to \frac{m_1 m_2}{m_1+m_2}$

$m_1 \to v_1$
COM($\vec{V}_{cm}$)
$m_2 \to v_2$

$$KE_{com} = \frac{1}{2}m_1(\vec{V_1}-\vec{COM})^2 + \frac{1}{2}m_2(\vec{V_2}-\vec{V}_{cm})^2$$
$$= \frac{1}{2}\frac{m_1 m_2^2(\vec{V_1}-\vec{V_2})^2}{m_1+m_2} + \frac{1}{2}\frac{m_1^2 m_2}{m_1+m_2}(\vec{V_2}-\vec{V_1})^2$$
$$= \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)^2}\left(m_2(v_1-v_2)^2 + m_1(v_1-v_2)^2\right)$$
$$= \boxed{\frac{1}{2}\mu(v_1-v_2)^2}$$

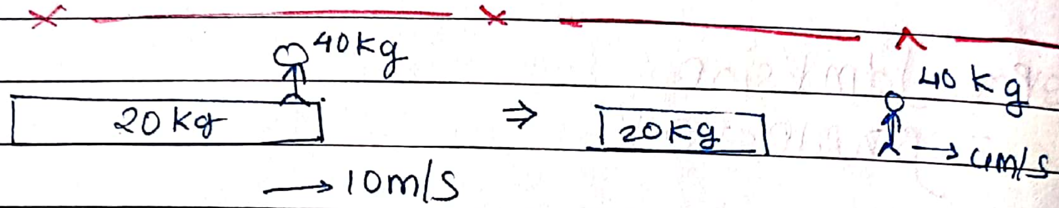
Momentum Conservation



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

($u_1, u_2, v_1, v_2 \rightarrow$ ground frame).

Q)

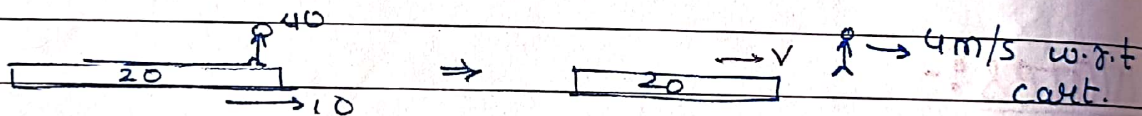


vel. of 20kg = ?

$\rightarrow 60\cancel{0} = 2\cancel{0}V + 16\cancel{0}$

~~22~~ $\boxed{V = 22\ m/s}$

Q)



final velocity of cart.

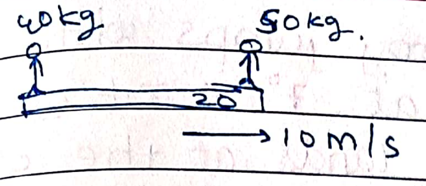
$$600 = 20(v) + 40(v+4)$$

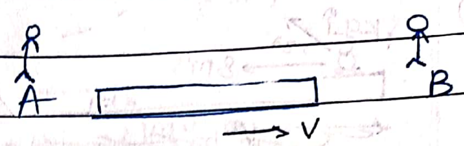
$$600 = 60v + 160$$

$$\frac{440}{60} = v$$

$$\Rightarrow \boxed{v = \frac{22}{3}\ m/s}$$

Note: w.r.t cart → always take vel. of cart after jumping.

Q) 



A → jumps 20 m/s w.r.t. cart after jump
B ⇒ jumps 10 m/s w.r.t. cart before jump.

vel. of cart = ?

$A \to -(V+20)$

$B \to 20$

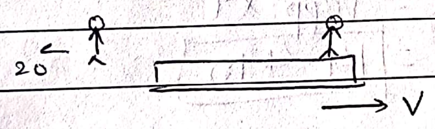
$1100 = 40V \cancel{+} 800 + 1000 + 20V$

$\cancel{-700 = 60V}$ $\quad \frac{90V}{6}$

$\cancel{\Rightarrow V = \frac{-70}{6} m/s}$

$\Rightarrow \boxed{V = 15 m/s}$

Q) same as above
first A jumps then B.

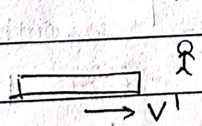


$A = V - 20$

$1100 = 40V - 800 + 70V$

$1900 = 110V \Rightarrow V = \frac{190}{11} m/s.$

$B \to 10 + \frac{190}{11} = \frac{\cancel{410}\ 300}{11}$



$\frac{2100}{11} = 20V_1 + 50 \times \frac{300}{11} \to \frac{\cancel{7000}\ \cancel{1500}}{11}$

$\cancel{\frac{600}{11} = V_1 20 \Rightarrow V_1 = \frac{30}{11} m/s}$

$20V_1' = -\frac{4900}{11}$

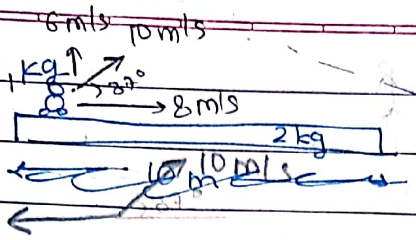
$V' = -\frac{245}{11} m/s$

~~0 = 6 - 10t~~

~~t = 6x8/5 = 4.8m~~

Page No.

Date

Q) 

frog jumps with 10 m/s at 37° w.r.t cart & lands at the other end. find length of plank

~~4.8 m~~

∵ 10 m/s at 37° is already w.r.t cart.

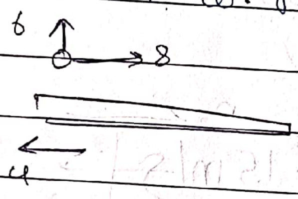
~~0 = 8 - 2V ⇒ V = 4 m/s~~ $d = R$

~~2.4 m~~

$\Rightarrow$ l of planck = 9.6 m

~~length of planck = 7.2 m~~

b) 10 m/s at 37° w.r.t ground.

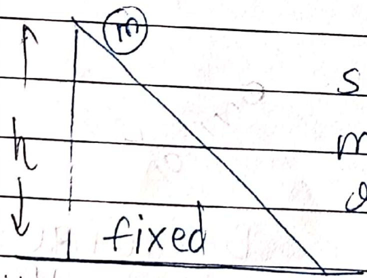


$V_{app} = 12$ m/s

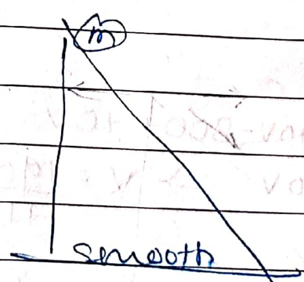
time of flight = 1.2 s

length of planck = $V_{app} \times t$

$= \boxed{14.4\ m}$

Q) 

speed of m more on reaching ground?



Here entire P.E of m gets converted to K.E. of m.

so V ↑

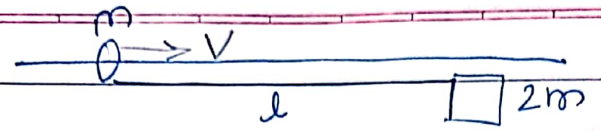
Here PE of m gets converted to K.E of wedge & m

so V ↓

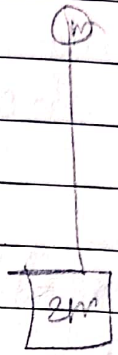
$mv = 2mv'$

$v' = \frac{v}{2}$

Page No.
Date

Q)


Find speed of 2m when it reach bottom.



$$2mgl = \frac{mv^2}{2} + \frac{2mv^2}{2\times 4}$$

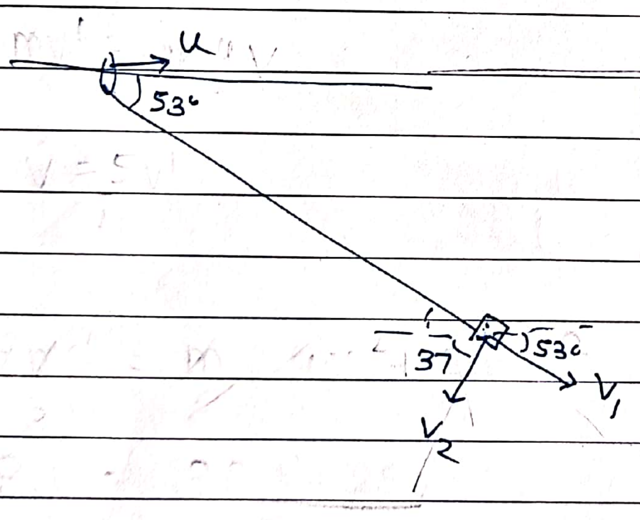
$$8gl = 3v^2$$

$$V = \sqrt{\frac{8gl}{3}}$$

$$V_2 = \sqrt{\frac{2gl}{3}}$$

Q) Find speed of 2m when turn by 53°

Ans.



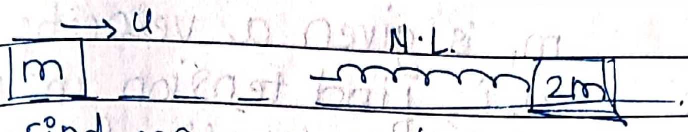
$$mu = 2m(v_2 \cos 37 - v_1 \cos 53^\circ)$$

$$u \cos 53^\circ = v_1$$

$$2mg(l \sin 53^\circ)$$

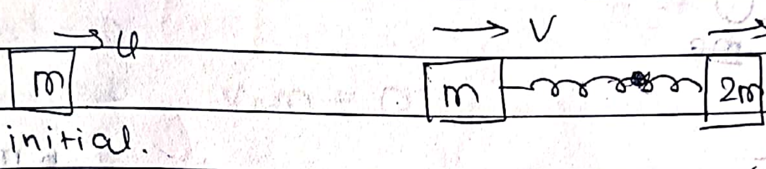
$$= \frac{1}{2} 2m(v_1^2 + v_2^2) + \frac{1}{2} mu^2$$

Q)



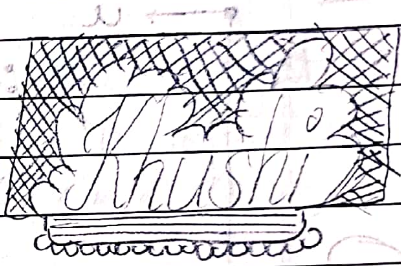
Find max ~~comprehension~~ compression in spring

→ At Max compression / elongation velocity of both blocks is same.



$$mu = 3mV$$

$$\frac{u}{3} = V$$



$$W_{spring} = \Delta KE$$

$$\frac{1}{2}k(0^2 - x^2) = \frac{1}{2} \times 3mV^2 - \frac{1}{2}mu^2$$

$$kx^2 = mu^2 - \frac{3mu^2}{9}$$

$$kx^2 = \frac{2mu^2}{3}$$

$$\boxed{x = \sqrt{\frac{2mu^2}{3k}}}$$

M-II COM frame $\frac{1}{2}k(0^2 - x^2) = \frac{1}{2}\mu 0^2 - \frac{1}{2}\frac{2m^2}{3m}u^2$

$$\boxed{x = \sqrt{\frac{2u^2}{3k}}}$$

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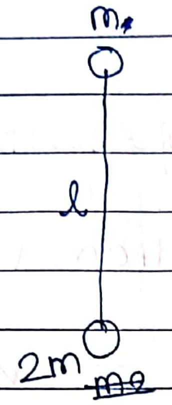
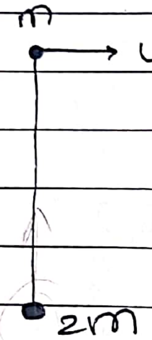
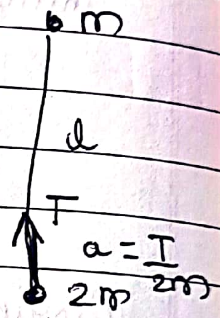
Q) m_1 is given a velocity $u\hat{i}$, find tension in the string of length l.



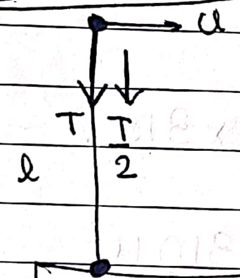


(M-I)

$\because$ 2m is accelerated pseudo force should be applied.

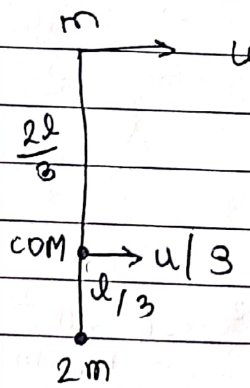


w.r.t. 2m



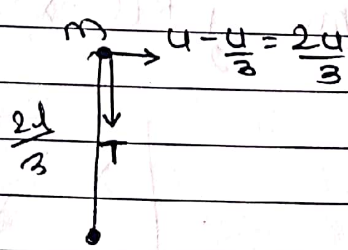
$$\frac{mu^2}{l} = \frac{3T}{2} \Rightarrow \boxed{T = \frac{2mu^2}{3L}}$$

(M-II)



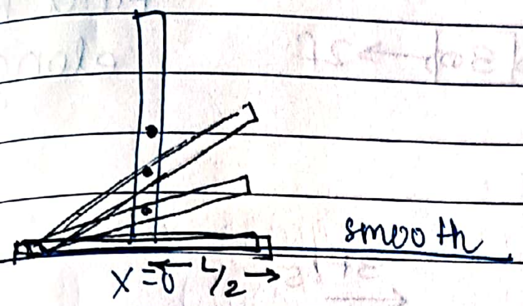
$$\frac{mu + 2m \times 0}{3m} = \frac{u}{3}$$

w.r.t COM



$$T = \frac{m(2u/3)^2}{2l/3}$$

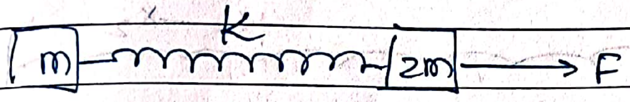
$$= \boxed{\frac{2mu^2}{3l}}$$

Q) 

Rod (length - l)
Rod is ~~clearly~~ slightly disturbed & released. Find x-coordinate where top end will strike the ground.

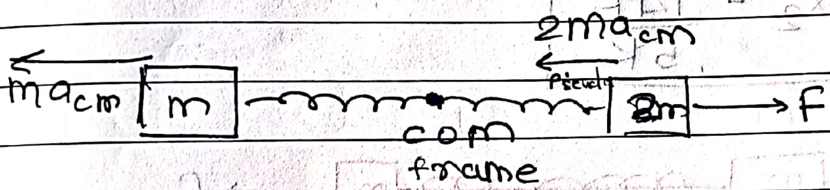
$F_{ext} = 0$

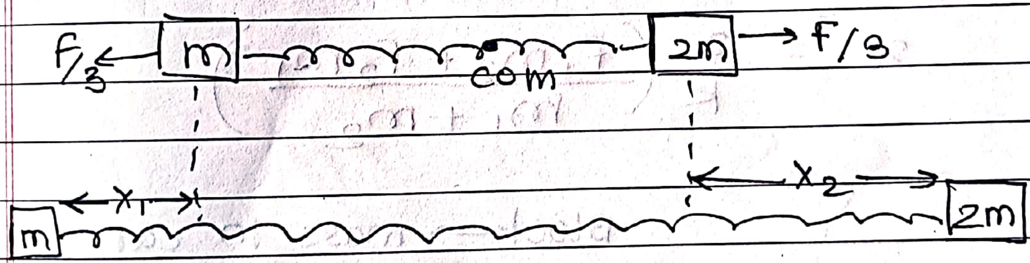
$a_{cm} = 0 \quad v_{cm} = 0$

Q) 

Max elongation in spring.

$\rightarrow a_{cm} = \frac{F}{m+2m} = \frac{F}{3m}$





W·E· Therom

$$\frac{F}{3}x_1 + \frac{F}{3}x_2 + \frac{1}{2}k\left(0^2 - (x_1+x_2)^2\right) = 0 - 0$$

$$\frac{F}{3}(x_1+x_2) = \frac{k}{2}(x_1+x_2)^2$$

$$\boxed{x_1 + x_2 = \frac{2F}{3k}}$$

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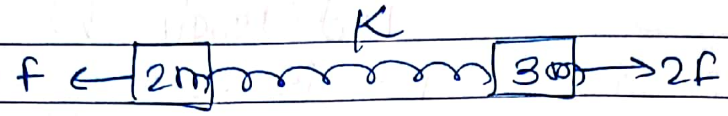
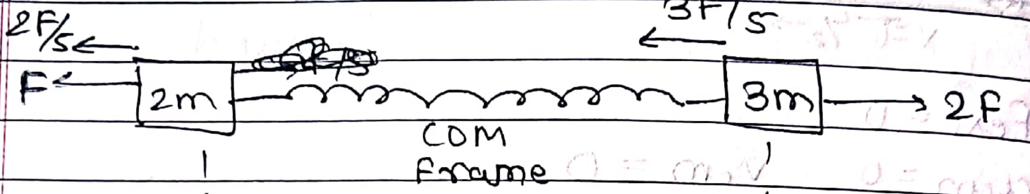
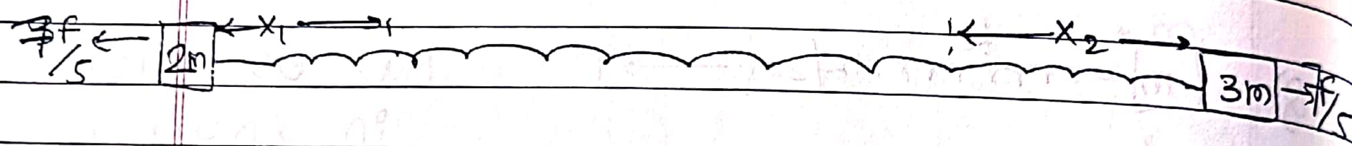
Q)



Find max elongation

$a_{cm} = \frac{F}{5m}$

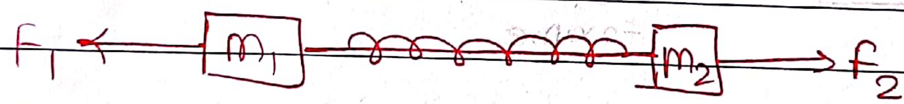




$$\frac{7F}{5}(x_1+x_2) + \frac{1}{2}k\left(0-(x_1+x_2)^2\right) = 0$$

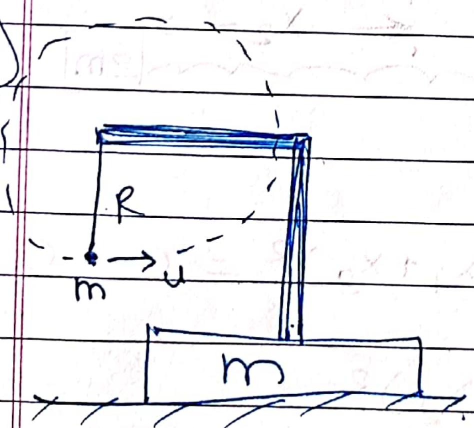
$$\boxed{x_1 + x_2 = \frac{14F}{5K}}$$

#



$$x = \frac{2}{K}\left(\frac{f_1 m_2 + f_2 m_1}{m_1 + m_2}\right)$$

Q)



Block → mass m can slide only on smooth surface.

L-shaped rigid system is massless.

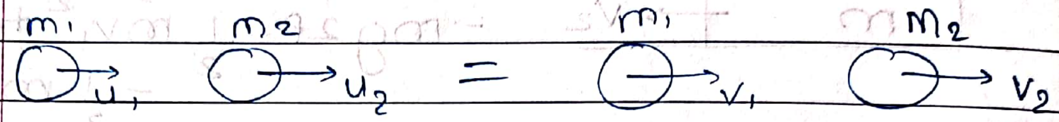
Ball is given a velocity u as shown.

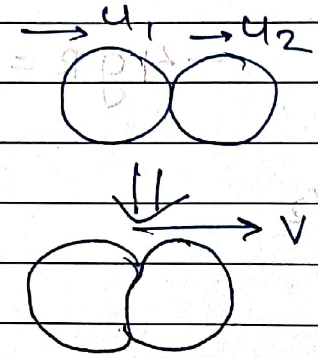
Find tension in string when ball is at highest position.

25/5/22

Collision:

$u_1 > u_2$ $\qquad\qquad$ $V_2 > V_1$





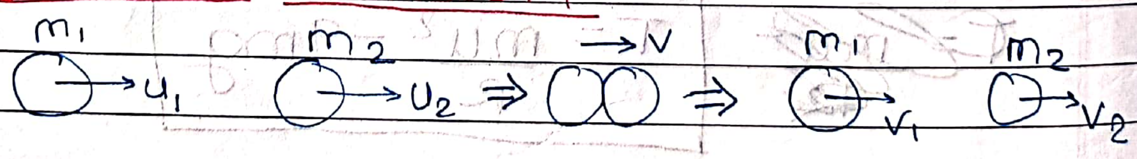
max. deformation

(i) If object moves in max deformed state, collision is perfectly inelastic.

(ii) If objects completely reform their shape, collision is elastic (perfectly elastic)

(iii) If objects separate, but shapes are not completely reformed, collision is inelastic.

(1) ELASTIC COLLISION



(i) $m_1u_1 + m_2u_2 = (m_1+m_2)V = m_1V_1 + m_2V_2$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

(ii) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_1v_1^2$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$$

$$\frac{(ii)}{(i)}$$

$$\Rightarrow \quad v_1 + u_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1$$

$$\Rightarrow \boxed{\frac{v_2 - v_1}{u_1 - u_2} = 1}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

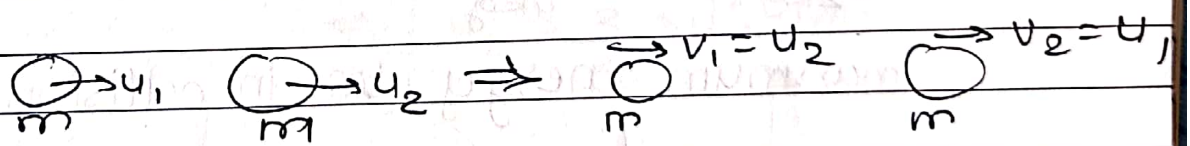
$$u_1 - u_2 = v_2 - v_1$$

On solving $\longrightarrow$

$$v_1 = \frac{2m_2 u_2 + (m_1 - m_2)u_1}{m_1 + m_2}$$

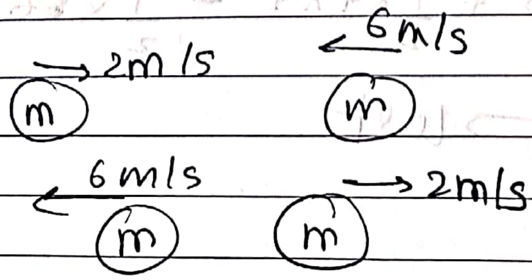
$$v_2 = \frac{2m_1 u_1 + (m_2 - m_1)u_2}{m_1 + m_2}$$

Note :



When two bodies of same mass collides elastically, velocities gets interchanged.

eg



Q)

$\rightarrow 2\,m/s$			$\leftarrow 5\,m/s$
m	m	m	m
0	2 →	← 5	0
0	← 5	2 →	0
← 5	0	0	2 →

Coefficient of restitution

$$e = \frac{\text{velocity of separation}}{\text{velocity of aproach}}$$

$$= \frac{V_2 - V_1}{u_1 - u_2}$$

$= 1 \longrightarrow$ elastic

$0 <$ inelastic < 1

$0 \longrightarrow$ perfectly inelastic

Q)

$\rightarrow 4\,m/s$ (2kg) $\rightarrow 2\,m/s$ (4kg)

$e = 0$

maximum energy lost in collision.

M-I final $v = \frac{8}{3}\,m/s$

$$\Delta E = \frac{1}{2} \times 2 \times 16 + \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 6 \times \frac{8}{3} \times \frac{8}{3}$$

$$= \boxed{\frac{8}{3}\,J} \longrightarrow \text{lost.}$$

M-II from COM frame,

$$\frac{1}{2} \times \frac{2 \times 4}{2+4} \times (4-2)^2$$

$KE_{COM} = 0$

$$\text{lost} = \frac{2}{3} \times 4 = \boxed{\frac{8}{3} \text{ J}}$$

Q) $\xrightarrow{} 4\text{ m/s}$ $\xleftarrow{} 1\text{ m/s}$ $e = \frac{1}{2}$

[2kg] [4kg] $\Longrightarrow$ (2) $\xrightarrow{} V_1$ (4) $\xrightarrow{} V_2$

vel. after collision = ?

$$8 - 4 = 2V_1 + 4V_2 = 4$$

$$\frac{V_2 - V_1}{5} = \frac{1}{2}$$

$$\Rightarrow V_2 - V_1 = \frac{5}{2} \Rightarrow 2V_2 - 2V_1 = 5$$

$$2V_1 + 4V_2 = 4$$

$$\Rightarrow 6V_2 = 9$$

$$\Rightarrow \boxed{V_2 = \frac{3}{2} \text{ m/s}}$$

$$\Rightarrow \boxed{V_1 = -1 \text{ m/s}}$$

Q) Equal mass (elastic collision) find no. of collisions

1 2 3 - - - - 2n-1 2n

odd no. blocks → $V\hat{i}$
even no. blocks → $-V\hat{i}$

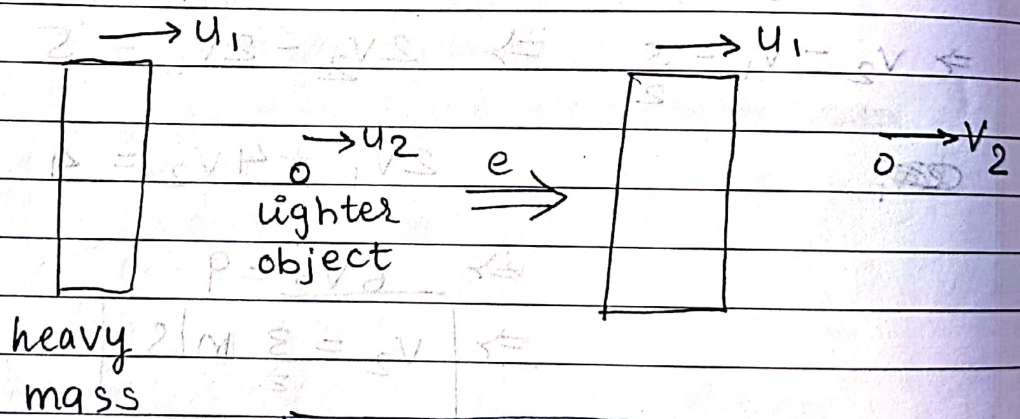
1 2 3 4 5 - - - 2n-1 2n

→v v← →v ← →v →v v←

$\to n+(n-1)+(n-2)+ \dots 1$

$$= \boxed{\frac{n(n+1)}{2}}$$

collision of light & heavy mass



$$\boxed{e = \frac{V_2 - u_1}{u_1 - u_2}}$$

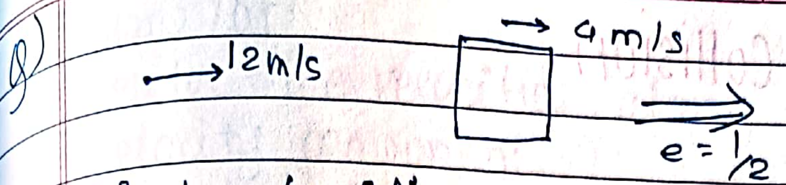
Special Case

$u_1 = 0$

$e = 1$ (elastic) $e = e$

$\boxed{V_2 = -u_2}$ $\boxed{V_2 = -eu_2}$

Q)



find velocities of both after collision.

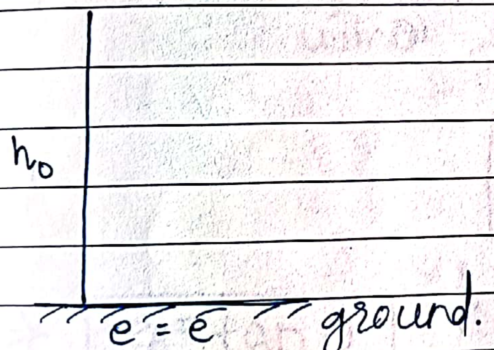
$\rightarrow \frac{1}{2} = \frac{V_2 - 4}{4 - 12} = \frac{V_2 - 4}{-8}$

$\Rightarrow -4 = V_2 - 4$

$\boxed{V_2 = 0 \text{ m/s}}$

$\boxed{V_{sep} = e\, V_{app}}$

Q)



height to which ball will rebound.

$\rightarrow v^2 = 0 + 2h_0 g$

$\Rightarrow V_{rebound} = e\sqrt{2h_0 g}$

$\Rightarrow 0^2 = V_R^2 - 2hg$

$\Rightarrow \cancel{2}hg = e^2 \cancel{2}h_0 g$

$\Rightarrow \boxed{h' = e^2 h_0}$

displacement total $= h_0$

distance $= \frac{h_0(1+e^2)}{1-e^2}$

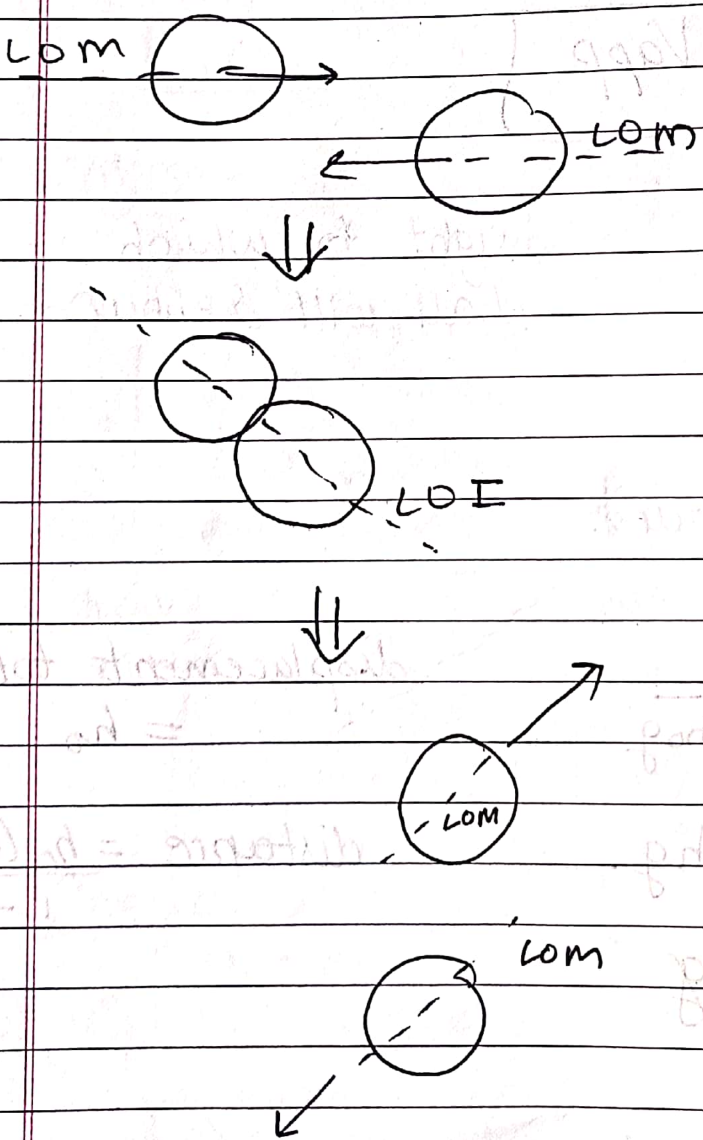
Page No.
Date

26/5/22

Types of Collision

(i) Head on collision
Line of motion before and after collision is same. line of motion is same as line of impact.

(ii) Oblique collision
Line of motion before and after collision is different. ~~Line of motion is different~~.



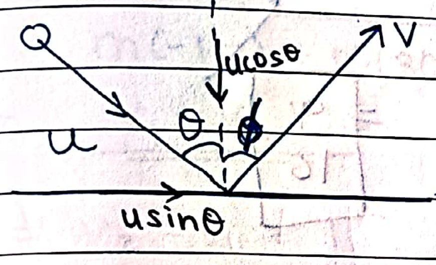
(i) Draw LOI

(ii) Resolve the velocities along & $\perp$ to the LOI

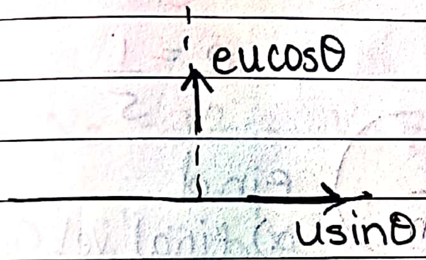
(iii) velocity component $\perp$ to LOI does not change.

(iv) $e = \dfrac{\text{vel. of sep.}}{\text{vel. of app.}}$ along the LOI.

Q) find V & ϕ ($e \to$ given)



→



$$V = \sqrt{e^2u^2\cos^2\theta + u^2\sin^2\theta}$$

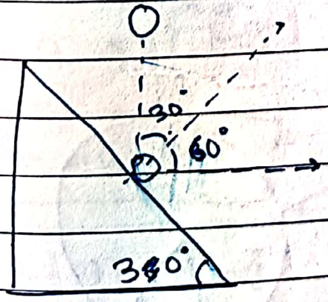
$$\boxed{V = u\sqrt{e^2\cos^2\theta + \sin^2\theta}}$$

$$\tan\phi = \frac{u\sin\theta}{eu\cos\theta} = \frac{\tan\theta}{e}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\tan\theta}{e}\right)}$$

$$\Rightarrow \boxed{e = \frac{\tan\theta}{\tan\phi}}$$

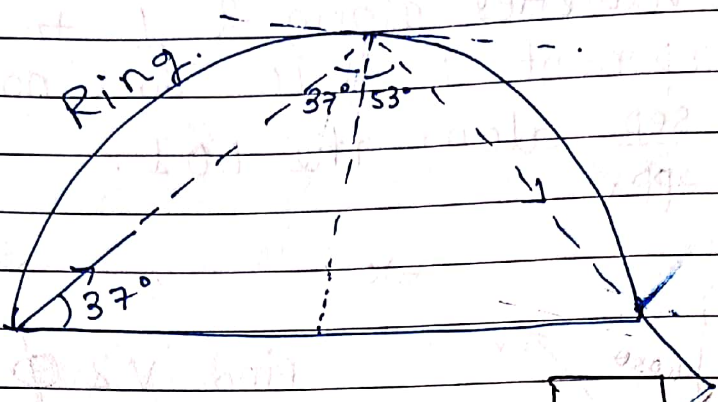
Q) Ball falls vertically goes horizontally. Find e.



$$e = \frac{\tan 30}{\tan 60} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \boxed{\frac{1}{3}}$$

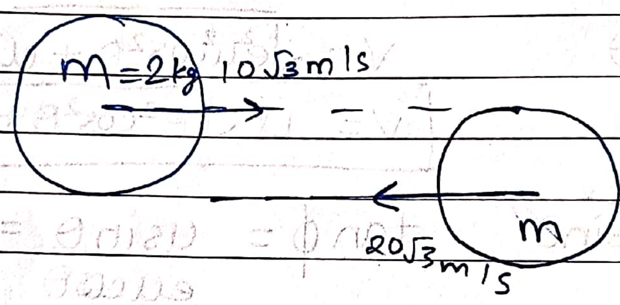
Page No.
Date

Q)



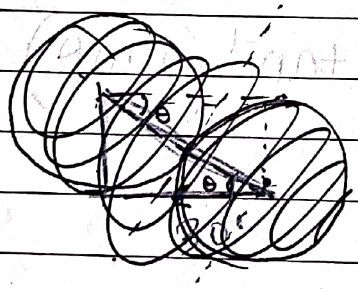
$$e = \frac{\tan 37}{\tan 53} = \frac{3}{4} \times \frac{3}{4} = \boxed{\frac{9}{16}}$$

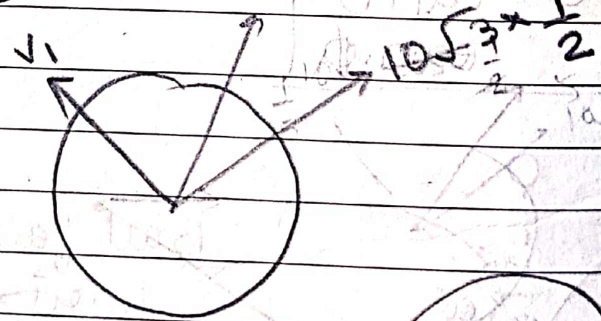
Q)

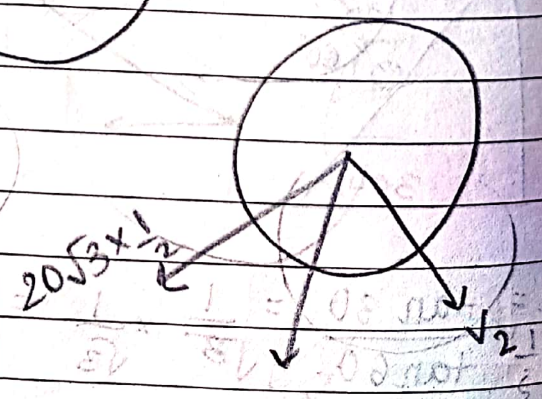


$e = \frac{1}{5}$

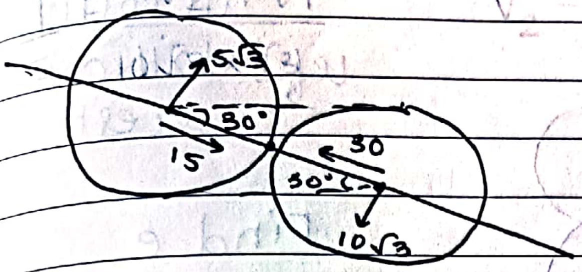
find
a) final vel after collistion
b) Loss in KE

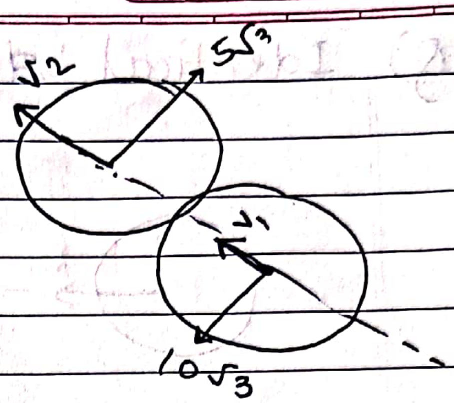






Page No.
Date





LOCM →

$$m(-15) + m(30) = mv_1 + mv_2$$

$$\Rightarrow \underline{v_1 + v_2 = 15}$$

$$e = \frac{v_2 - v_1}{45} = \frac{1}{5} \Rightarrow v_2 - v_1 = 9$$

$$\Rightarrow v_2 = 12, \quad v_1 = 3$$

(i) $v_1' = \sqrt{v_2^2 + 5\sqrt{3}^2} = \sqrt{144 + 75} = \boxed{\sqrt{219} \text{ m/s}}$

$v_2' = \sqrt{9 + 300} = \boxed{\sqrt{309} \text{ m/s}}$

(ii) $-\frac{1}{2} m \left(300 + 1200 - 219 - 309\right)$

$= -\frac{1}{2} m \left(81 + 891\right)$

$= -\frac{m}{2} \times \overset{486}{\cancel{972}}$

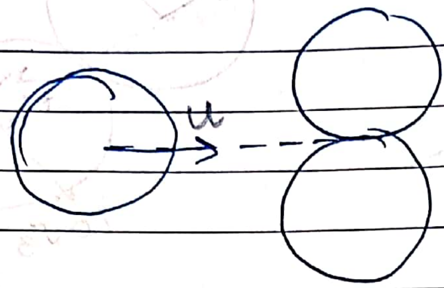
$= \boxed{-486 m}$

$= \boxed{-972 \text{ J}}$

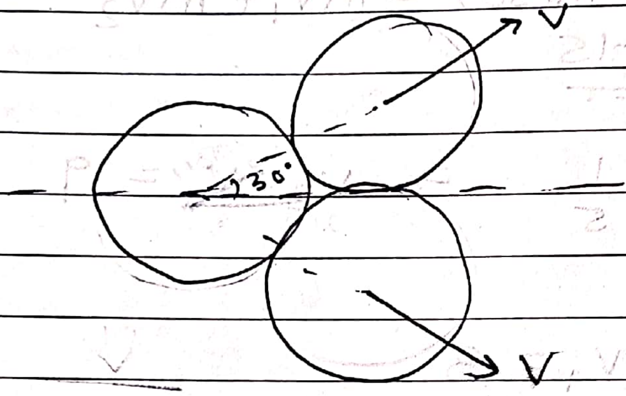
Page No.
Date

Q) Identical balls

All collision, left ball comes to rest



Find e.



L.O.C.M

$mu = 2mv\cos 30$

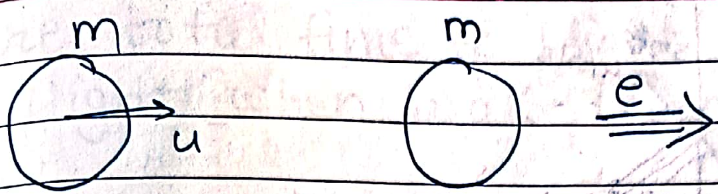
$\boxed{u = v\sqrt{3}}$

$$e = \frac{v}{u\cos 30} = \frac{1}{\sqrt{3} \times \frac{\sqrt{3}}{2}} = \boxed{\frac{2}{3}}$$

Q) Prove that
when two objects of which one is at rest collide elastically & obliquely, they move ⊥ to each other.

Page No.			
Date			

8)



Find e if $\frac{1}{4}$ of KE is lost.

M-I

$$\frac{1}{2}mu^2 \times \frac{3}{4} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$3u^2 = 4v_1^2 + 4v_2^2$$

$$mu = mv_1 + mv_2$$

$$\Rightarrow v_1^2 + v_2^2 = 6v_1v_2$$

$$\Rightarrow (v_1 - v_2)^2 = 4v_1v_2$$

$$(v_1 + v_2)^2 = 8v_1v_2$$

$$\Rightarrow \boxed{u^2 = 8v_1v_2}$$

$$\Rightarrow e = \frac{v_1 - v_2}{u}$$

$$e = \frac{1}{\sqrt{2}}\sqrt{\frac{4v_1v_2}{8v_1v_2}} \Rightarrow \boxed{e = \frac{1}{\sqrt{2}}}$$

M-II

$$u = v_1 + v_2$$

$$eu = v_2 - v_1$$

$$v_1 = \frac{(1-e)u}{2}$$

$$v_2 = \frac{(1+e)u}{2}$$

$$\frac{1}{2}m(v_1^2 + v_2^2) = \frac{3}{4} \times \frac{1}{2}mu^2$$

$$\left(\frac{1+e^2}{2}\right)\cancel{u^2} = \frac{3}{4}\cancel{u^2}$$

$$e^2 = \frac{1}{2}$$

$$\boxed{e = \frac{1}{\sqrt{2}}}$$

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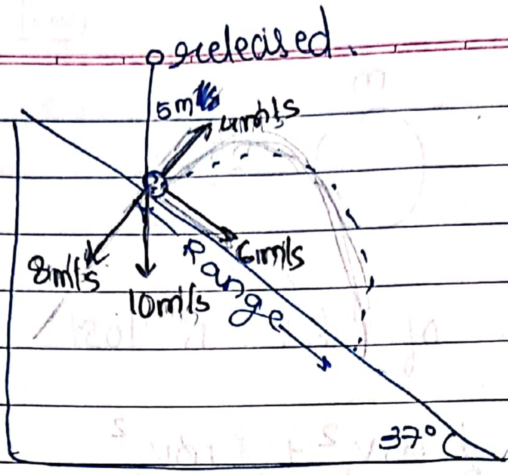
27/5/22

Q)



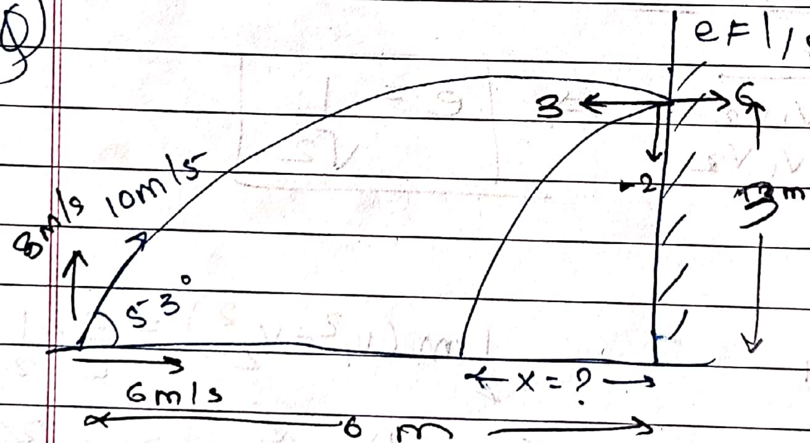
$$T = \frac{2u_y}{g\cos\theta} = \frac{8}{8} = 1s$$

$$S_x = u_x t + \frac{g\sin\theta}{2} t^2$$

$$= 6 + 3\times 1$$

$$= \boxed{9m}$$

Q)



M-1

$$S_y = 8\times 1 - \frac{5}{5}\times 1 = 3m$$

$$V_y = 8 - 10\times 1 = -2m/s$$

$$3 = 8t - 5t^2$$

$$5t^2 - 8t + 3 = 0$$

$$(5t - 3)(t-1)$$

$$-3 = -2t - 5t^2$$

$$5t^2 + 2t - 3 = 0$$

$$(5t - 3)(t+1) = 0$$

$$t = 3/5$$

$$x = \frac{3}{5}\times 3 = \boxed{\frac{9}{5}\ m}$$

Page No.
Date

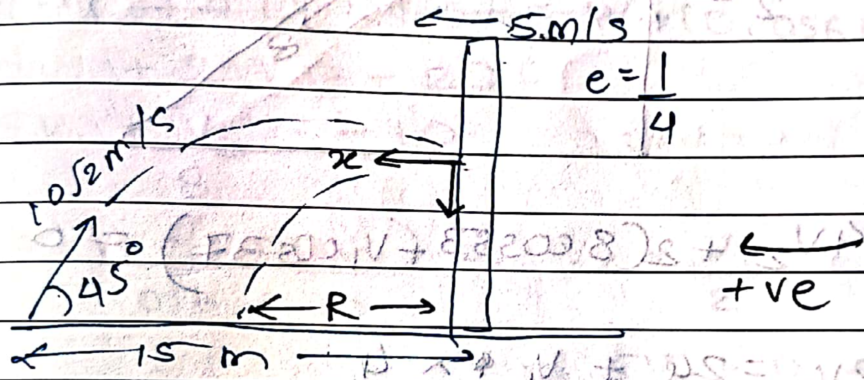
Here, total time of flight is actual time of flight (when wall is not there)

$t_1 + t_2 = \frac{2u\sin\theta}{g}$

$1 + t_2 = 1.6$

$t_2 = 0.6$

$x = 0.6 \times 3 = \boxed{1.8\ m}$



$t_1 + t_2 = \frac{2 \times 10}{10} = 2s$

$t_2 = 2 - 1.5 = 0.5s$

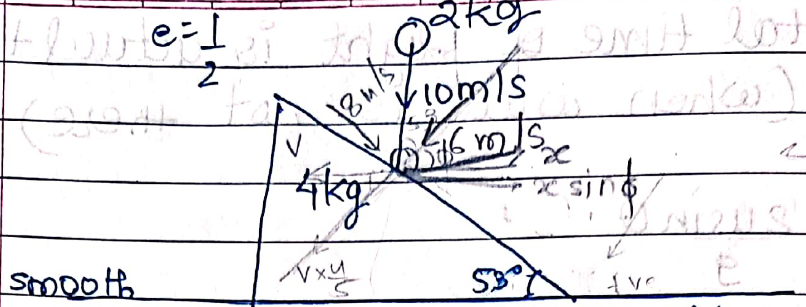
~~$\frac{1}{4} = \frac{x-5}{15}$ $x = \frac{5}{4} - 15 = \frac{45}{4}$ m/s~~

~~$R = \frac{45}{4} \times \frac{1}{2} = \boxed{\frac{15}{8}\ m}$~~

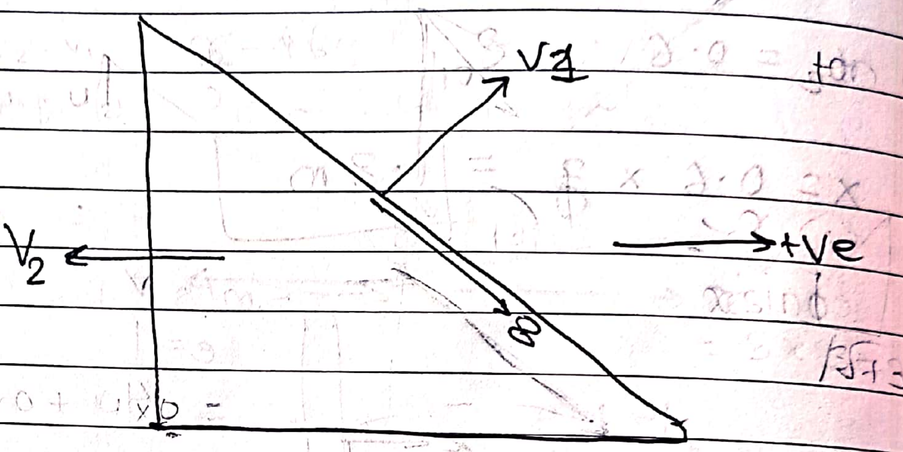
$e = \frac{V_2 - V_1}{u_1 - u_2} = \frac{x-5}{5+10} = \frac{1}{4}$ $\quad x = \frac{15}{4} + 5 = \frac{35}{4}$

$\boxed{R = \frac{35}{8}\ m}$

Q) $e = \frac{1}{2}$



Final velocity of both after collision.



$$-4V_2 + 2(8\cos 53 + V_1 \cos 37) = 0$$

$$2V_2 = \frac{24}{5} + V_1 \times \frac{4}{5}$$

$$5V_2 = 12 + 2V_1 \quad \text{—— (i)}$$

$$\frac{1}{2} = \frac{V_1 + \frac{4V_2}{5}}{0-(-6)} \Rightarrow 3 = V_1 + \frac{4V_2}{5}$$

$$\Rightarrow 2V_2 - 6 = -\frac{8V_2}{5}$$

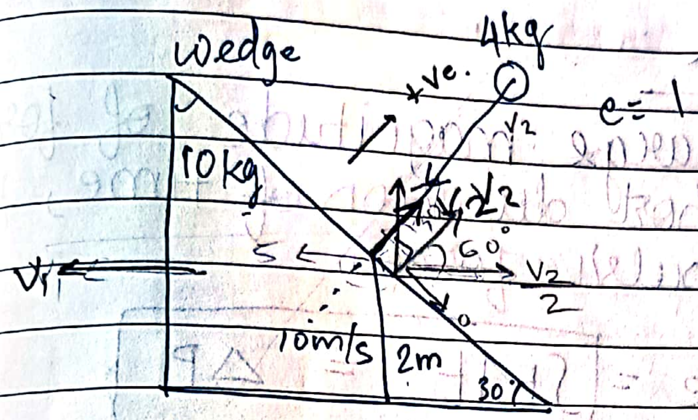
$$18 = \frac{33V_2}{5} \Rightarrow \boxed{V_2 = \frac{30}{11} \text{ m/s}} \text{ wedge vel.}$$

$$V_1 = \frac{\cancel{150}\,75}{11} - \cancel{12}\,6 = \boxed{\frac{9}{11} \text{ m/s}}$$

$$\text{Ball vel.} = \sqrt{\frac{81}{121} + 64} = \boxed{8.44 \text{ m/s}}$$

Page No.			
Date			

8)



Find range of projectile assuming it falls on ground.

$10(-V_1) + 4(V_2 \cos 60) = 4(-10\cos 60)$

$-10V_1 + 2V_2 = -20$

$-5V_1 + V_2 = -10$

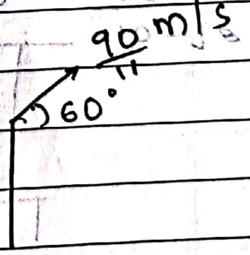
$e = \frac{V_2 - (-V_1 \cos 60)}{0 - (-10)} \Rightarrow V_2 + \frac{V_1}{2} = 10$

$5V_1 - 10 + \frac{V_1}{2} = 10$

$11V_1 = 40$

$V_1 = \frac{40}{11}$

$\boxed{V_2 = \frac{90}{11} \text{ m/s}}$



~~$-2 = \frac{90t}{11} - 5t^2$~~

~~$-22 = 90t - 55t^2$~~

~~$55t^2 - 90t - 22 = 0$~~

$-2 = x \times \sqrt{3} - \frac{10 \times x^2}{2 \times \frac{8100}{121} \times \frac{1}{4}}$

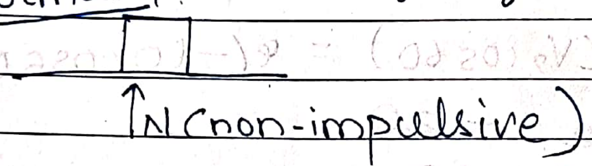
Impulse

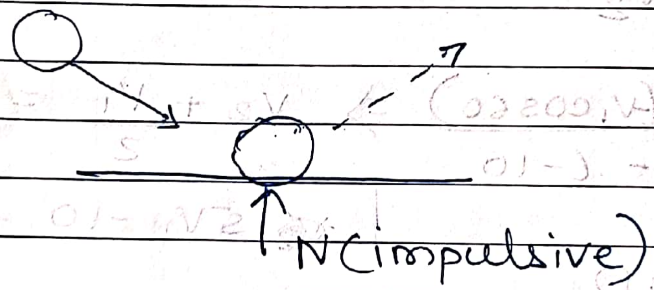
When a large magnitude of force acts for a short duration of time, force is called impulsive force.

$$\text{Impulse} = \boxed{\int F\,dt = \Delta P}$$

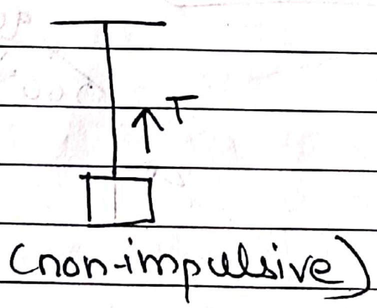
Impulse = change in momentum

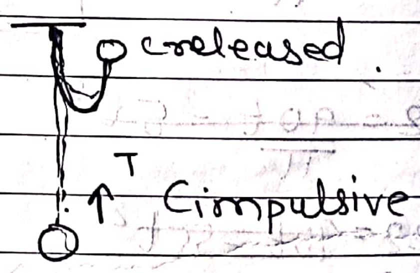
eg. Normal / friction (if rough surface)





Tension

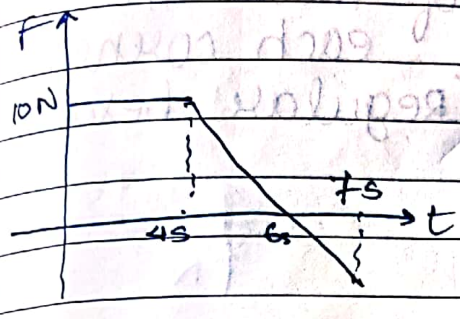




Note → (i) mg, kx, electrostatic force are always non-impulsive.

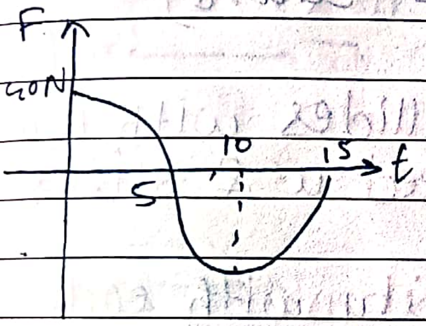
(ii) Non-impulsive forces can be neglected in presence of impulsive force.

(iii) Area under F-t curve gives impulse



change in momentum in 7 sec.

$40 + 10 - \frac{5}{2} = 47.5$ kgm/s



$F = 40\cos\frac{\pi t}{10}$

If initial velocity is 4 m/s $(-\hat{i})$ find velocity at t = 15 s. (mass = 15 kg)

$\rightarrow \int F\,dt = \int_0^{15} 40\cos\frac{\pi t}{10}\,dt$

$= 40\left(\frac{\sin\frac{\pi t}{10}}{\pi/10}\right)\Big]_0^{15}$

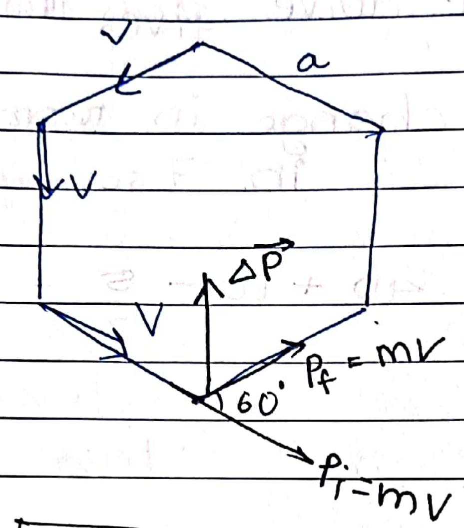
$= \frac{400}{\pi}(-1-0) = \frac{-400}{\pi}$

$\frac{-400}{\pi} = P_f - (-60)$

$15V = \frac{-400 - 60\pi}{\pi}$

$$\boxed{V_f = \frac{-80 - 12\pi}{3\pi}}$$

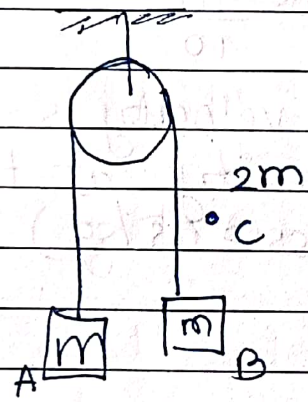
Q) Impulse on particle of mass m at each corner. (Regular hexagon)

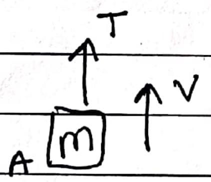


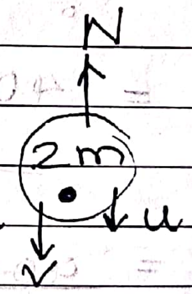
$\boxed{\Delta\vec{P} = m\vec{V}}$ (towards centre)

Q) C collides with B with speed u & sticks to B.

Velocity with each system will move.

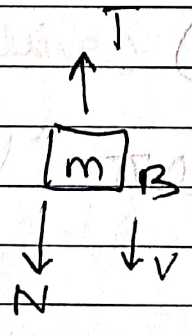






$\int T\,dt = mV - 0$

$\int N\,dt = 2m(u - V)$

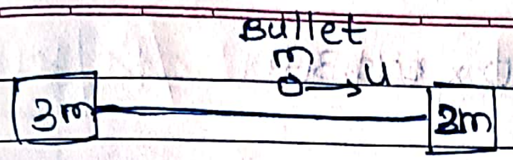


$\int N\,dt - \int T\,dt = mV$

$2mu - 2mV - mV = mV \Rightarrow 2mu = 4mV$

$\Rightarrow \boxed{V = u/2}$

Q)



Bullet gets embedded in 2m. Find velocity with which system moves.

→ No ext force

M-I

$\Rightarrow \quad mu = 6mv$

$$\boxed{v = \frac{u}{6}}$$

M-II

$3m \longrightarrow T \qquad \int T\,dt = 3mv$

$N \longleftarrow m \longrightarrow v \qquad -\int N\,dt = mv - mu$

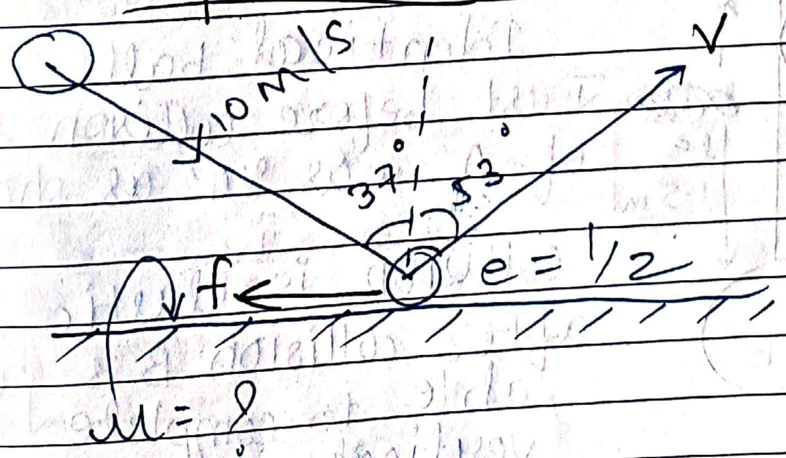
$T \longleftarrow 2m \longrightarrow N \qquad \int N\,dt - \int T\,dt = 2mv$

$$mu - mv - 3mv = 2mv$$

$$mu = 6mv$$

$$\boxed{\frac{u}{6} = v}$$

Q)



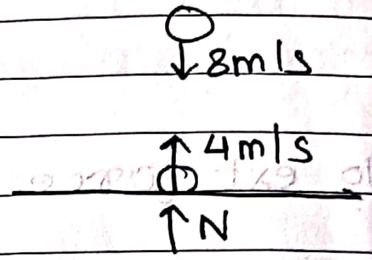
Khushi Kucheria

Page No.
Date

$V\cos 53 = \frac{1}{2} \times 10 \times \cos 37$

$V \times \frac{3}{5} = 5 \times \frac{4}{5}$

$V = \frac{20}{3}\,m/s$



$\int N\,dt = 4m - (-8m)$

$\Rightarrow \int N\,dt = 12m$

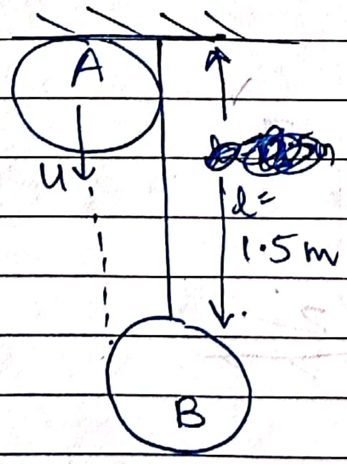
$-\int f\,dt = m \times \frac{20}{3} \times \frac{4}{5} - m \times 10 \times \frac{3}{5}$

$-\mu \int N\,dt = \frac{16m}{3} - 6m$

$\Rightarrow -\mu \times 12m = \frac{16m - 18m}{3}$

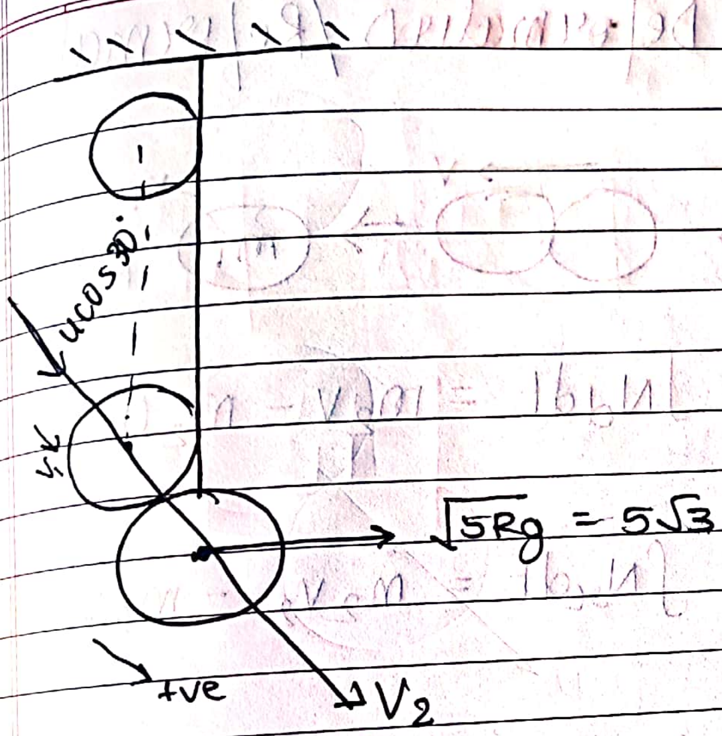
$\Rightarrow \boxed{\mu = \frac{1}{18}}$

8)



Identical balls.
Just before collision speed of A was 'u' as shown.
collision is elastic & after collision B is just able to complete the vertical circle.

Find u.



$$e = \frac{v_2 - v_1}{u\cos 30} = 1$$

$$V_2 = 5\sqrt{3}\cos 60$$

$$V_2 - V_1 = \frac{u\sqrt{3}}{2}$$

$$V_2 = \frac{5\sqrt{3}}{2}$$

$$\Rightarrow V_1 = (5-u)\frac{\sqrt{3}}{2}$$

(A) $\rightarrow -\int N dt = mV_1 - mu\cos 30°$

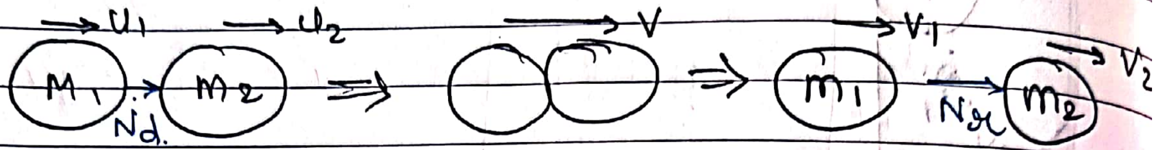
(B) $\rightarrow \int N\cos 60\, dt = m\sqrt{75} - 0$

$$\frac{1}{2}\left(mu\frac{\sqrt{3}}{2} - mV_1\right) = m\sqrt{75}$$

$$\frac{u\cancel{\sqrt{3}}}{2} - \frac{5\cancel{\sqrt{3}}}{2} + \frac{u\cancel{\sqrt{3}}}{2} = 10\cancel{\sqrt{3}}$$

$$\boxed{u = \frac{25}{2}\, m/s}$$

Impulse of Deformation / Reformation



Impulse of deformation $= \int N_d\,dt = m_2V - m_2u_2$

Impulse of Reformation $= \int N_r\,dt = m_2V_2 - m_2V$

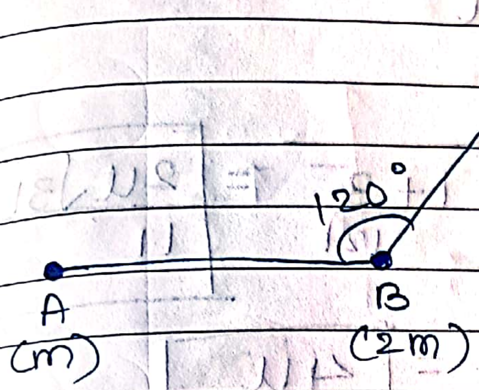
$$\frac{\text{Impulse of Reform}}{\text{Impulse of deform}} = \frac{m_2V_2 - m_2V}{m_2V - m_2u_2}$$

$$= \frac{V_2 - V}{V - u_2}$$

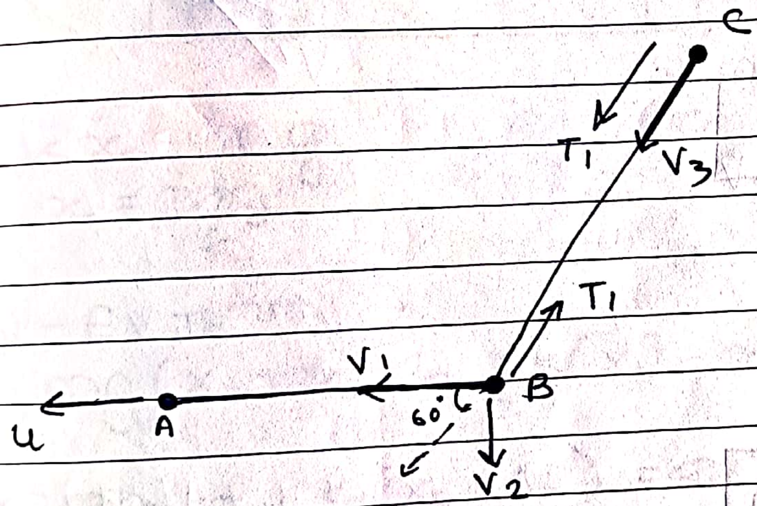
$$= \frac{V_2 - \dfrac{m_1V_1 + m_2V_2}{m_1+m_2}}{\dfrac{m_1u_1 + m_2u_2}{m_1+m_2} - u_2} \quad \begin{matrix} \leftarrow A \\ \leftarrow B \end{matrix}$$

$$\boxed{e = \frac{V_2 - V_1}{u_1 - u_2}}$$

Page No.	
Date	

Q) 

Particle lie on smooth table & connected by light strings. Impulse applied to particle A along BA so that it acquires velocity u. find initial speed of B & C.



$u = V_1$

$V_1 \cos 60 + V_2 \cos 30 = V_3$

$\int T_1 dt = m\left(\frac{V_1}{2} + \frac{V_2\sqrt{3}}{2}\right)$

$\int T_1 \cos 30 dt = -2mV_2$

$\left(\frac{mV_1}{2} + \frac{mV_2\sqrt{3}}{2}\right)\frac{\sqrt{3}}{2} = -2mV_2$

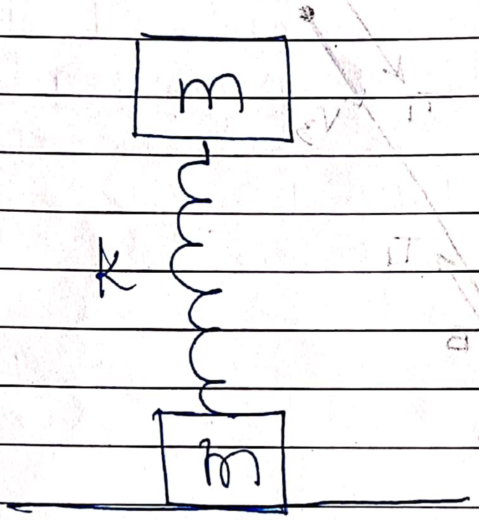
$\sqrt{3}V_1 + 3V_2 = -8V_2 \quad \Rightarrow \sqrt{3}\,V_1 = -11V_2$

$V_1 = u$, $V_2 = \frac{-\sqrt{3}\,u}{11}$

$v_B = \sqrt{V_1^2 + V_2^2} = u\sqrt{1 + \frac{3}{121}} = \boxed{\frac{2u\sqrt{31}}{11}}$

$v_c = \frac{u}{2} + \left(\frac{-\sqrt{3}u}{11}\right) \times \frac{\sqrt{3}}{2} = \boxed{\frac{4u}{11}}$

Q)



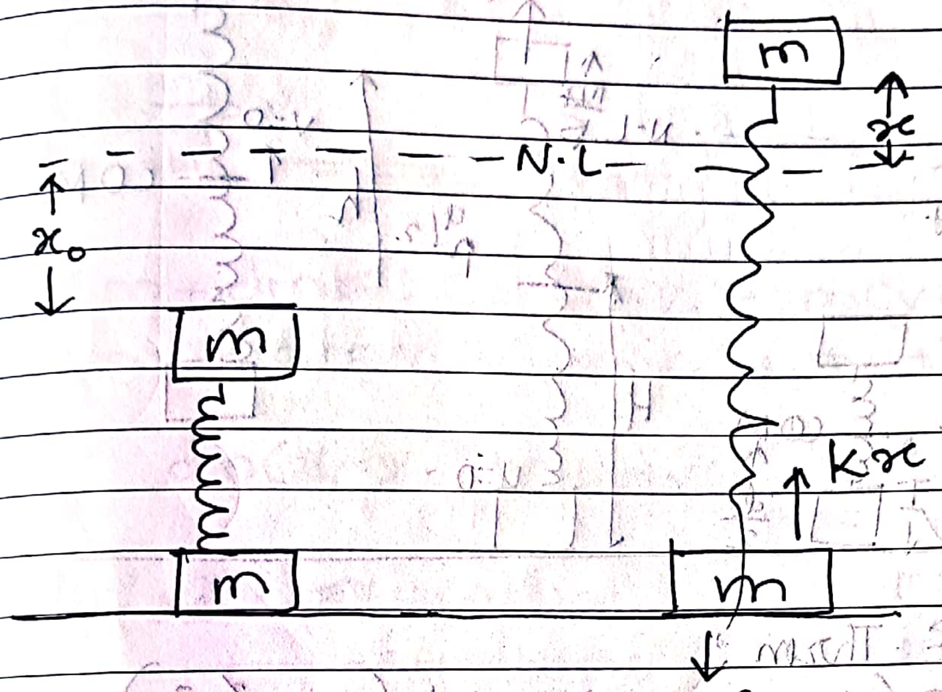
Blocks are connected by a thread which is burnt.

(a) At what value of initial compression, the lower cube will bounce up after thread is burnt.

(b) To what height COM of this system will rise if initial compression $= \frac{7mg}{K}$

Page No.
Date

(a)



$kx = mg$

$x = \frac{mg}{k}$

W - E Th^{rm}

$-mg(x_0 + x) + \frac{1}{2}k(x_0^2 - x^2) = 0$

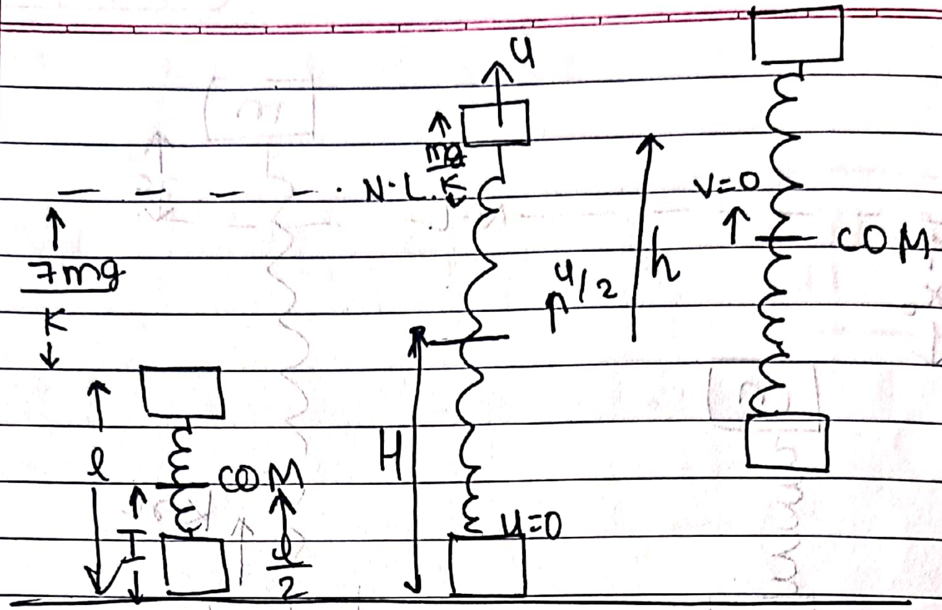
$\cancel{mg} + \frac{1}{2}k(x_0 - x) = mg$

$x_0 - \frac{mg}{k} = \frac{2mg}{k}$

$$\boxed{x_0 = \frac{3mg}{k}}$$

Page No.
Date

(b)



W.E. Thrm,

$$-mg\left(\frac{7mg}{k}+\frac{mg}{k}\right)+\frac{1}{2}k\left(\frac{48m^2g^2}{k^2}\right)=\frac{1}{2}mu^2$$

$$-\frac{8mg^2}{k}+\frac{24mg^2}{k}=\frac{u^2}{2}$$

$$\frac{16mg^2}{k}=\frac{u^2}{2} \Rightarrow \boxed{u^2=\frac{32mg^2}{k}}$$

$$0^2=\frac{u^2}{4}-2gh$$

$$h=\frac{u^2}{8g}=\boxed{\frac{4mg}{k}}$$

$$H=\frac{l}{2}+8mg/k$$

Rise = H + h − I

$$I=l/2$$

$$\text{Rise}=\frac{l}{2}+\frac{4mg}{k}+\frac{8mg}{k}-\frac{l}{2}$$

$$=\boxed{\frac{8mg}{k}}$$

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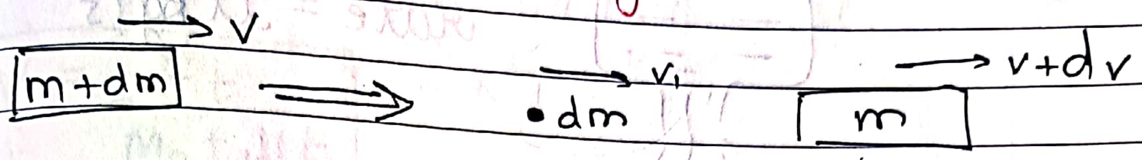
Variable Mass System



$(m+dm)v = dm\,v_1 + m(v+dv)$

$mv + dm\,v = dm\,v_1 + mv + m\,dv$

$dm(v - v_1) = m\,dv$

v_{rel} = vel. of ejected mass w.r.t original mass $= v_1 - v$

$-v_{rel}\,dm = m\,dv$

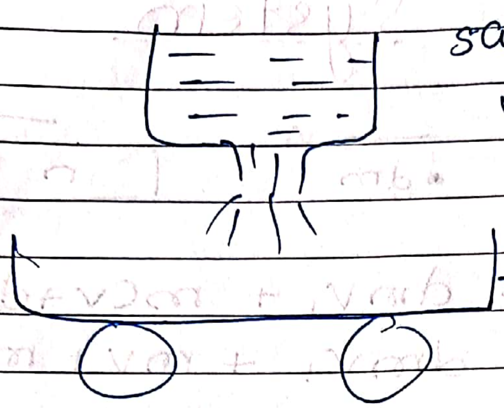
$$\boxed{-v_{rel}\frac{dm}{dt} = m\frac{dv}{dt}}$$

Thrust force $= -v_{rel}\dfrac{dm}{dt} = -\mu v_{rel}$ $\left(\mu = \dfrac{dm}{dt}\right)$

(i) when mass is increasing $(dm/dt = +ve)$
$(\text{Thrust force})_{dirxn}$ = same as v_{rel}.

(ii) when mass is decreasing, dirxn of thrust force is opp. to v_{rel}.

8)



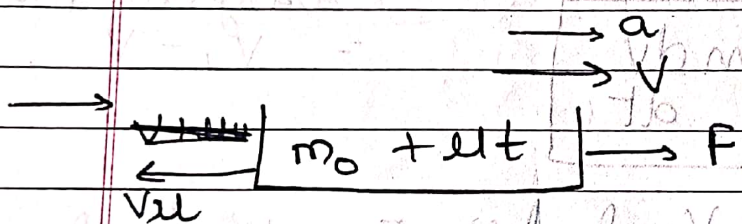
sand falling rate = μ kg/s

$\rightarrow F$ (F starts at $t=0$)

$u = 0$

(mass of cart $= m_0$)

Find velocity & acceleration at any time 't'



$$(m_0 + \mu t) \times a = F - \mu V$$

$$(m_0 + \mu t) \times \frac{dv}{dt} = F - \mu V$$

$$\int_0^V \frac{dV}{F - \mu V} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\left. \frac{\ln(F - \mu V)}{-\mu} \right]_0^V = \left[\frac{\ln(m_0 + \mu t)}{\mu} \right]_0^t$$

$$\ln(F - \mu V) - \ln F = \ln m_0 - \ln(m_0 + \mu t)$$

$$\frac{F - \mu V}{F} = \frac{m_0}{m_0 + \mu t}$$

$$\cancel{Fm_0 + F\mu t \quad \mu V m_0 \quad \mu^2 V t}$$

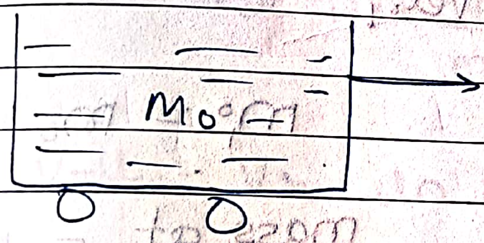
$$\mu V = \frac{F m_0 + \mu F t - F m_0}{m_0 + \mu t}$$

$$\boxed{V = \frac{Ft}{m_0 + \mu t}}$$

$$a = \frac{dV}{dt} = \frac{(m_0 + \mu t)F - Ft(\mu)}{(m_0 + \mu t)^2}$$

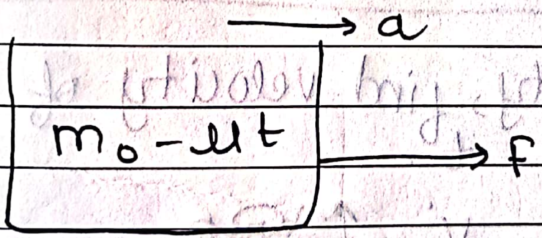
$$\boxed{a = \frac{F m_0}{(m_0 + \mu t)^2}}$$

Q) $u = 0$



F starts ~~stactings~~ acting at $t = 0$

Hole is made at bottom & sand starts spilling at μ kg/s. Find vel. at t.



$V_{rel} = 0$

$F_{thrust} = 0$

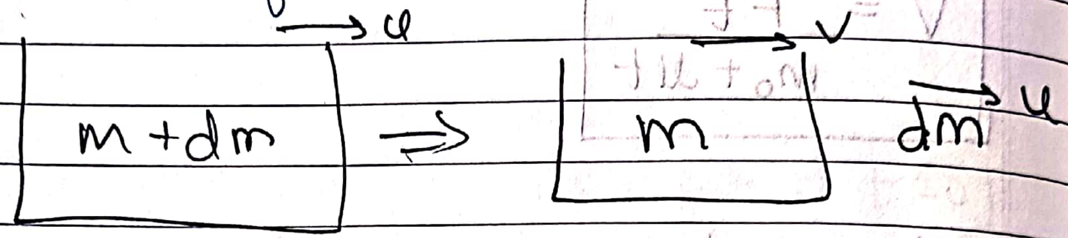
$$(m_0 - \mu t)a = F$$

$$\frac{dV}{dt} = \frac{F}{m_0 - \mu t}$$

$$\int_0^v dV = \int \frac{F}{m_0 - \mu t} dt$$

$$V = \frac{F \ln(m_0 - \mu t)}{-\mu}\Bigg|_0^t = \boxed{\frac{F}{\mu} \ln \frac{m_0}{m_0 - \mu t}}$$

Q) If a cart carrying sand is given vel. u & after sometime hole is made. Vel. at time 't' after hole is made.

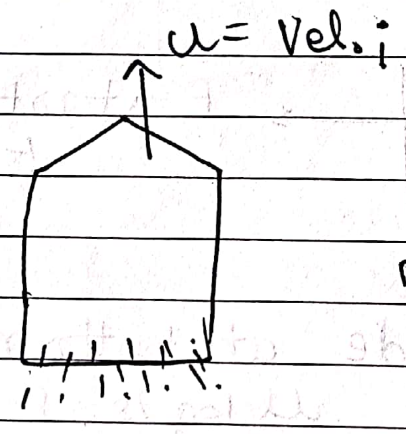


$$(m+dm)u = mV + dm\,u$$

$$\boxed{V = u}$$

Rocket Propulsion

$\frac{dm}{dt} = -\mu$



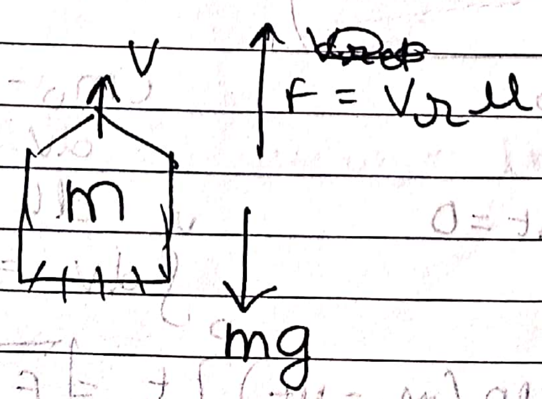
$m_i = m_0$

mass at time t $= m$

constant $V_{rel} = V_r$

Q) Consider gravity, find velocity of rocket at any time t.

→ At time t



Page No.
Date

$$ma = \mu V_r - mg$$

$$(m_0 - \mu t)\frac{dv}{dt} = \mu V_r - (m_0 - \mu t) g$$

$$(m_0 - \mu t)\, dv = \mu V_r\, dt - (m_0 - \mu t) g\, dt$$

$$\int_u^v dv = \mu V_r \int_0^t \frac{dt}{m_0 - \mu t} - \int_0^t g\, dt$$

$$v - u = \mu V_r \left. \frac{\ln(m_0 - \mu t)}{-\mu} \right]_0^t - gt$$

$$v - u = V_r \ln \frac{m_0}{m_0 - \mu t} - gt$$

$$\Rightarrow v - u = V_r \ln \frac{m_0}{m} - gt$$

$$\boxed{V = u - gt + V_r \ln \frac{m_0}{m}}$$