Gravitation

Gravitation and gravity.

Gravitation is the force of attraction between any two bodies while gravity refers to the force of attraction between any body and the earth.

Newton's Law of Gravitation.

It states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$\text{Mathematically,} \quad F = G\frac{m_1 m_2}{r^2}$$

Where G is the universal gravitational constant. It was derived by Newton on the basis of Kepler's laws of planetary motion.

Universal Gravitational constant (G).

It is equal to the force of attraction between two bodies of unit mass each and separated by unit distance.

In SI units, $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

In CGS system, $G = 6.67 \times 10^{-8}\ dyne\ cm^2\ g^{-2}$

Dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

Properties of gravitational forces.

The gravitational force between two point masses.

. is independent of intervening medium

. obeys Newton's third law of motion

. has spherical symmetry

. is independent of the presence of the other bodies

. obeys principle of superposition

. is conservative and central

Principle of superposition of gravitational forces.

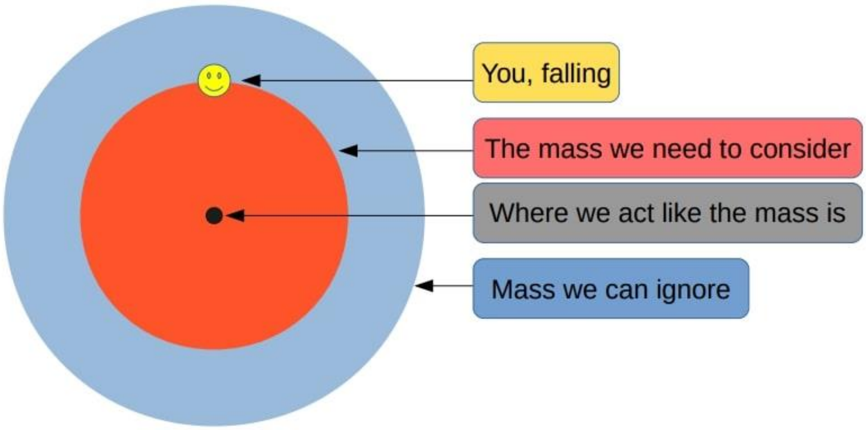
The gravitational force between two masses acts independently and uninfluenced by the presence of other bodies. Hence the resultant force on a particle due to a number of masses is the vector sum of the gravitational forces exerted by the individual masses on the given particle.

$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots + \overrightarrow{F_n} \quad = \sum_{i=0}^{n} \overrightarrow{F_\iota}$$

Shell Theorem.

. The gravitational force on a particle placed anywhere inside a spherical shell is zero.

. if a particle lies outside a spherical shell, the shell attracts the particle as though the mass of the shell were concentrated at the center of the shell



Free fall.

The motion of a body under the influence of gravity alone is called a free fall.

Acceleration due to gravity (g).

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity.

At the surface of the earth, $g = \frac{GM}{R^2}$

Where M is the mass and R is the radius of the earth.

. It is a vector quantity

. Its SI unit is ms^{-2}

Mass of the earth.

It is given by

$$M = \frac{gR^2}{G}$$

Mean density of the earth.

If the earth is a sphere of mass M and mean density ρ, then its mass would be

$$M = Volume\ x\ density = \frac{4}{3}\pi R^3 \rho$$

Also,

$$M = \frac{gR^2}{G}$$

$$\therefore \frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G}$$

$$or \quad \rho = \frac{3g}{4\pi GR}$$

Weight of a body.

It is the gravitational force with which a body is attracted towards the center of the earth.

$$\vec{W} = m\vec{g}$$

Weight of a body is a vector quantity. It is measured in newton, kg wt, etc. The weight of a body varies from place to place because of the variation in the value of g.

Variation of acceleration due to gravity.

1. Effect of altitude. At a height h,

$$g_h = g\left(1 + \frac{h}{R}\right)^{-2}$$

And, $g_h = g\left(1 - \frac{2h}{R}\right), when\ h \ll R.$

Clearly, the value of g decreases with the increase in h.

2. Effect of depth. At a depth d,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Clearly, the value of g decreases with the increase in depth d and becomes zero at the center of the earth.

3. Effect of rotation. If ω is the angular velocity of the earth about its axis of rotation, then at latitude λ,

$$g_\lambda = g - R\omega^2 \cos^2\lambda$$

As λ increases, cosλ decreases and g_λ increases. So, as we move from equator to pole, the value of g increases.

At equator, λ = 0, cosλ = 1

Hence, $$g_{equator} = g - R\omega^2$$

At poles, $\lambda = 90°,\ cos\lambda = 0$

Hence, $$g_{pole} = g - R\omega^2\ (0) = g$$

Hence, g is minimum at equator and maximum at poles.

$$g_{pole} - g_{equator} = g - (g - R\omega^2) = R\omega^2.$$

4. Effect of non-sphericity of the earth. The earth has an equatorial bulge and is flattened at the poles. As $R_e > R_p$, so the value of g is minimum at the equator and maximum at the poles.

Gravitational field.

It is the space around a material body in which its gravitational pull can be experienced by other bodies.

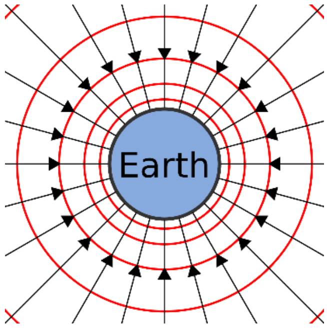
Intensity of gravitational field.

The intensity or strength of gravitational field at any point in a gravitational field is equal to the force experienced by a unit mass placed at that point. It is a vector quantity directed towards the body producing the gravitational field. It is given by

$$E = \frac{Force}{Mass} = \frac{F}{M} = \frac{GM}{r^2}$$

The intensity of gravitational field at a point is equal to the acceleration due to gravity at that point.

SI unit of E is N kg $^{-1}$ and the CGS unit is dyne g $^{-1}$.



Gravitational potential.

The gravitational potential at a point in the gravitational field of a body is the amount of work done in bringing a body of unit mass from infinity to that point. It is a scalar quantity.

Gravitational potential, $V = \frac{Work\ done}{Mass} = -\frac{GM}{r}$

SI unit of V is J kg $^{-1}$ and its CGS unit is erg g $^{-1}$.

Gravitational potential energy.

The gravitational potential energy of a body may be defined as the energy associated with it due to its position in the gravitational field of another body and is measured by the amount of work done in bringing a body from infinity to a given point in the gravitational field of the other body.

$$Gravitational\ P.E = Gravitational\ potential\ x\ mass\ of\ body$$

$$U = -\frac{GM}{r}(m)$$

Gravitational intensity (E) and gravitational potential (V) at a point are related as

$$E = -\frac{dV}{dr}$$

Escape velocity (v_c).

It is the minimum velocity with which a body must be projected vertically upward in order that it may just escape the gravitational field of the earth.

$$v_c = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

For earth, the value of escape velocity is 11.2 km s^{-1}. It is independent of the mass of body projected.

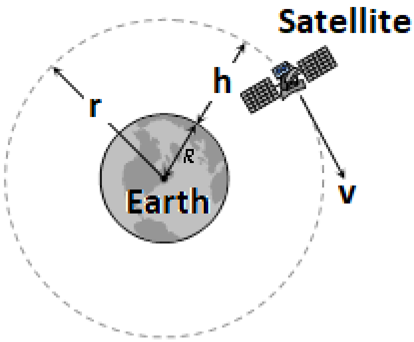
Orbital velocity of a satellite (v_0).

It is the velocity required to put a satellite in its orbit around a planet. The orbital velocity of a satellite revolving around the earth at a height h is given by

$$v_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

When a satellite revolves close to the surface of the earth (h = 0),

$$v_0 = \sqrt{gR}$$



Time period of a satellite (T).

It is the time taken by a satellite to go once around the planet.

$$T = \frac{Circumference\ of\ the\ orbit}{Orbital\ velocity}$$

$$= \frac{2\pi(R+h)}{v_0} = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$= 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

If the satellite revolves just close to the surface of the earth, h = 0, then

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

Height of a satellite above the earth's surface.

We know that,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \quad or\ (R+h)^3 = \frac{T^2\, R^2\, g}{4\pi^2}$$

$$or\ h = \left[\frac{T^2\, R^2\, g}{4\pi^2}\right]^{\frac{1}{3}} - R$$

Angular momentum of a satellite.

The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius r = (R + h) is given by

$$L = m_o v r = m\sqrt{\frac{GM}{r}}\,.r = \sqrt{GMm^2 r}$$

Total energy of a satellite.

For a satellite of mass m moving with velocity v_0 in an orbit of radius r,

Potential energy, $U = -\frac{GMm}{r}$

Kinetic energy, $K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{GM}{r}$

Total energy, $E = K + U$

Or $E = \frac{1}{2}mv_0^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$

Clearly, E = - K = U/2

Negative total energy indicates that the satellite is bound to the earth.

Conservation of quantities in motion under gravitational influence.

During the motion of an object under the gravitational influence of another object the following quantities are conserved.

. Angular momentum

. Total mechanical energy

Linear momentum is not conserved. Conservation of angular momentum leads to Kepler's second law of planetary motion.

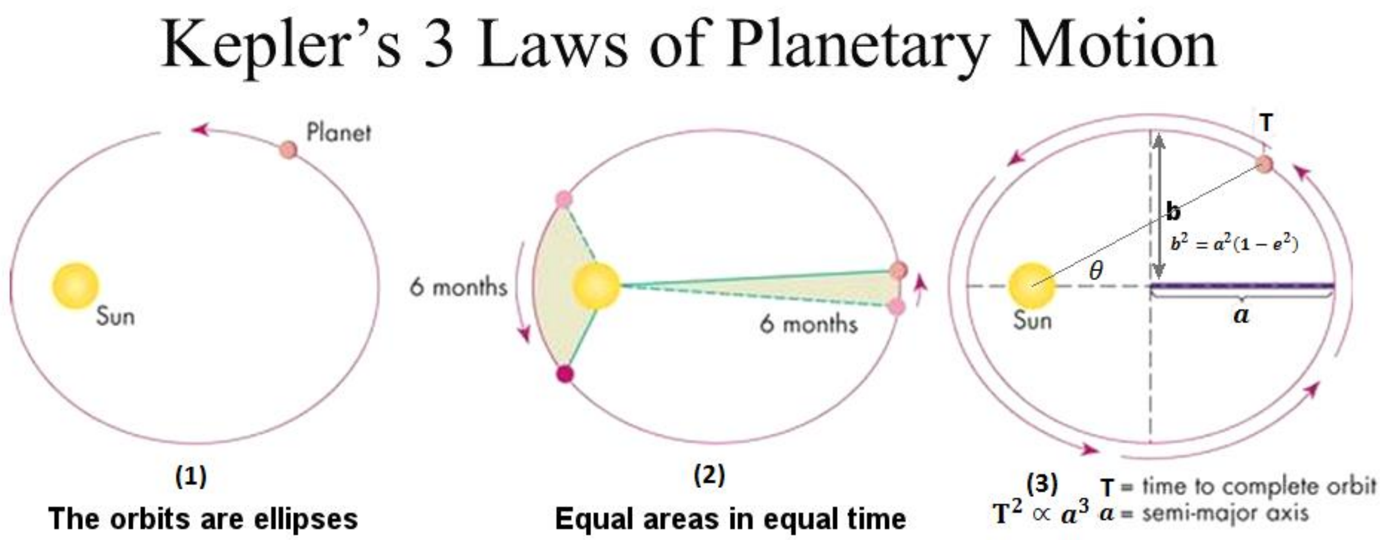
Kepler's Laws of planetary motion.

. Law of orbits. Every planet moves in an elliptical orbit around the sun, with the sun being at one of the focii.

. Law of areas. The radius vector drawn from the sun to the planet sweeps out equal areas in equal interval of time i.e., the areal velocity of a planet is constant.

. Law of periods or Harmonic law. The square of the period of revolution (T) of a planet around the sun is proportional to the cube of the semi-major axis r of the ellipse.

$$T^2 \propto r^3$$



Weightlessness.

A body is said to be in a state of weightlessness when the reaction of the supporting surface is zero or its apparent weight is zero. An astronaut experiences weightlessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in the state of free fall towards the earth.

Inertial mass.

The mass of a body which measures its inertia and which is given by the ratio of the external force applied on it to the acceleration produced in it along a smooth horizontal surface is called inertial mass.

$$Inertial\ mass = \frac{Applied\ force}{Acceleration\ produced}\ or\ m_i = \frac{F}{a}$$

Gravitational mass.

The mass of a body which determines the gravitational pull due to earth acting upon it is called gravitational mass. On the surface of the earth,

$$F = \frac{GMm_g}{R^2}$$

$$\therefore \quad gravitational\ mass, \quad m_g = \frac{FR^2}{GM}$$

Inertial mass of a body is equal to its gravitational mass.