CURRENT ELECTRICITY

Current Electricity

The study of electric charges in motion is called current electricity.

Electric Current

The flow of electric charges through a conductor constitutes electric current. Quantitatively, electric current across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.

For a steady flow of charge,

$$I = \frac{Q}{t}$$

If the rate of flow of charge varies with time, then

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

SI unit of current is ampere(A).

1 ampere=1coulomb/1 second or 1A=1Cs^-1.

Conventional and electronic currents

The direction of motion of positive charges is taken as the direction of conventional current. Electrons being negatively charged, so the direction of electronic current is opposite to that of the conventional current.

Electric current is a scalar quantity

Although we represent current with an arrow, yet it is a scalar quantity. Electric current does not obey the laws of vector addition.

Electromotive Force(emf)

The emf of a source may be defined as the work done by the source in taking a unit positive charge from its lower potential terminal to the higher potential

terminal. OR, it is the energy supplied by the source in taking a unit positive charge once round the complete circuit. It is equal to the terminal potential difference measured in open circuit.

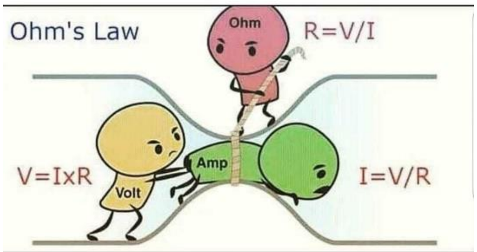
$$EMF = \frac{Work\ done}{Charge} \quad or \quad \mathcal{E} = \frac{W}{q}$$

Ohm's Law

The current flowing through a conductor is directly proportional to the potential difference across its ends, provided the temperature and other physical conditions remain unchanged.

$$V \propto I \qquad or \qquad V = RI \qquad or \qquad R = V/I$$

Here R is called the resistance of the conductor.



Resistance

It is the property by virtue of which a conductor opposes the flow of charges through it. It is equal to the ratio of potential difference applied across the conductor to the current flowing through it. It depends on the length l and area of cross-section A of the conductor through the relation:

$$R = \rho\frac{l}{A}, \quad \rho = resistivity\ of\ the\ material.$$

SI unit of resistance is ohm

The resistance of a conductor is 1 ohm if a current of 1 ampere flows through it on applying a potential difference of 1 volt across its ends.

$$1\ ohm = \frac{1\ volt}{1\ ampere} \quad or\ \ 1\ \Omega = 1\ VA^{-1}$$

Resistivity or Specific resistance

It is the resistance offered by a unit cube of the material of a conductor.

$$\rho = \frac{RA}{l}$$

SI unit of $\rho = \Omega\, m$

It depends on the nature of the material of the conductor and the physical conditions like temperature, pressure, etc.

Current density

It is the amount of charge flowing per second per unit area normal to the flow of charge. It is a vector quantity having the same direction as that of the motion of the positive charge.

For normal flow of charge,

$$j = \frac{q/t}{A} = \frac{I}{A}$$

In general,

$$I = jA\,cos\theta = \vec{j}\,.\vec{A}$$

SI unit of current density=Am^-2.

Conductance

It is the reciprocal of resistance.

Conductance=1/Resistance

Or G=1/R

SI unit of conductance =ohm^-1=mho

Conductivity or specific conductance

It is the reciprocal of resistivity.

Conductivity=1/Resistivity

$$or \qquad \sigma = \frac{1}{\rho}$$

SI unit of conductivity=ohm^-1m^-1==mho m^-1

Resistivities of different substances

Metals have low resistivities in the range of 10^-8 ohm m to 10^-6 ohm m. Insulators have resistivities more than 10^4 ohm m. Semiconductors have intermediate resistivities lying between 10^-6 ohm m to 10^4 ohm m.

Color	Value	Multiplier	Tolerance
Black	0	$x\ 10^{0}$	±20%
Brown	1	$x\ 10^{1}$	± 1%
Red	2	$x\ 10^{2}$	± 2%
Orange	3	$x\ 10^{3}$	± 3%
Yellow	4	$x\ 10^{4}$	-0,+100%
Green	5	$x\ 10^{5}$	±0.5%
Blue	6	$x\ 10^{6}$	±0.25%
Violet	7	$x\ 10^{7}$	±0.10%
Gray	8	$x\ 10^{8}$	±0.05%
White	9	$x\ 10^{9}$	± 10%
Gold	-	$x\ 10^{-1}$	± 5%
Silver	-	$x\ 10^{-2}$	±10%
None	-	-	±20%

Color Code of Carbon Resistor

Tolerance:

Gold	Silver	No color
+-5%	+-10%	+-20%

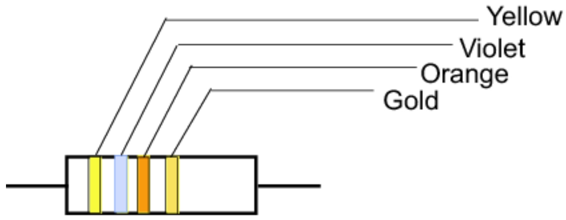
A set of colored co-axial bands is printed on the resistor which reveals the following facts:

(1) The first band indicates the significant figure.

(2) The second band indicates the second significant figure.

(3) The third band indicates the power of ten with which the above two significant figures must be multiplied to get the resistance value in ohms.

(4) The last band indicates the tolerance in percent of the indicated value.



Resistance = 47 x 10^3 ohms ± 2 x 10^3 ohms

Carries of current

In metals, free electrons are charge carriers. In ionized gases, electrons and positively charged ions are the charge carriers. In an electrolyte, both positive and negative ions are the charge carriers. In semiconductors like Ge and Si, conduction is due to electrons and holes. A hole is a vacant state from which an electron has been removed and acts as a positive charge carrier.

Drift velocity and relaxation time

The average velocity acquired by the free electrons of a conductor in the opposite direction of the externally applied electric field is called drift velocity. The average time that elapses between the two successive collisions of an electron is called relaxation time (τ).

$$v_d = \frac{eE\tau}{m} \quad ; \quad R = \frac{ml}{ne^2\tau A} \quad ; \quad \rho = \frac{m}{ne^2\tau}$$

$$I = en\,Av_d \quad ; \quad j = en\,v_d$$

Here n = number of free electrons per unit volume or free electron density and m = mass of an electron.

Other forms of ohm's law.

In terms of vector quantities like current density $\vec{j}$ and electric field $\vec{E}$, ohm's law may be expressed as

$$\vec{j} = \sigma\vec{E} \qquad or \qquad \vec{E} = \rho\,\vec{j}$$

The equation $\vec{E} = \rho \vec{j}$ leads to another statement of ohm's law i.e., a conducting material obeys ohm's law when the resistivity of the material does not depend on the magnitude and direction of the applied electric field.

Temperature coefficient of resistance (α).

It is defined as the change in resistance per unit original resistance per degree rise in temperature. It is given by

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

$$if \ \ t_1 = 0°\text{C} \qquad and \quad t_2 = t°\text{C}, \ \ then$$

$$\alpha = \frac{R_t - R_0}{R_0 \times t} \quad or \quad R_t = R_0(1 + \alpha t)$$

The unit of α is $°\text{C}^{-1}$ or K^{-1}.

Effect of temperature on resistance.

For metals α is positive i.e., resistance of metals increases with the increase in temperature.

For semiconductors and insulators, α is negative i.e., their resistance decreases with the increase in temperature.

For alloys like constantan and manganin, the temperature coefficient of resistance α is very small. So, they are used for making standard resistors.

Mobility of a charge carrier.

The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field. It is given by

$$\mu = \frac{v_d}{E} = \frac{q\tau}{m}$$

For an electron,

$$\mu_e = \frac{e\tau_e}{m_e}$$

For a hole,

$$\mu_h = \frac{e\tau_h}{m_h}$$

SI unit of mobility = $m^2\ V^{-1}\ s^{-1}$

Practical unit of mobility = $cm^2\ V^{-1}\ s^{-1}$.

Relation between electric current and mobility.

For a conductor,

$$I = en\,A\mu_e E$$

For a semiconductor,

$$I = eAE\,(n\mu_e + p\mu_h)$$

$$and \qquad \sigma = e(n\mu_e + p\mu_h)$$

$$where\ n\ and\ p\ are\ the\ electron\ and\ hole\ densities.$$

Non-ohmic conductors.

The conductors which do not obey Ohm's law are called non-ohmic conductors. The non-ohmic situations may be of the following types:

(i) The straight-line V-I graph does not pass through the origin.

(ii) V-I relationship is not linear.

(iii) V-I relationship depends on the sign of V for the same absolute value of V.

(iv) V-I relationship is not unique.

Examples of non-ohmic conductors are water voltameter, thyristor, a p-n junction, etc.

Resistances in series.

When a number of resistances are connected in series, their equivalent resistance (R_s) is equal to the sum of the individual resistances.

$$R_s = R_1 + R_2 + R_3 + \cdots$$

Resistances in parallel.

When a number of resistances are connected in parallel, the reciprocal of their equivalent resistance (R_p) is equal to the sum of the reciprocals of the individual resistances.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Internal resistance.

The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal resistance of the cell. It depends on

(i) nature of the electrolyte

(ii) concentration of the electrolyte

(iii) distance between the electrodes

(iv) common area of the electrodes dipped in the electrolyte and

(v) temperature of the electrolyte.

Relations between emf ($\mathcal{E}$), terminal potential difference (V) and internal resistance (r).

The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference. It is less than the emf of the cell in a closed circuit.

$$\mathcal{E} = V + Ir\,; \qquad V = \mathcal{E} - Ir; \qquad V = IR = \frac{\mathcal{E}R}{R + r}$$

$$r = \frac{\mathcal{E} - V}{I} = R\left[\frac{\mathcal{E} - V}{V}\right]; \qquad I = \frac{\mathcal{E}}{R + r}; \qquad I_{max} = \frac{\mathcal{E}}{r}$$

Terminal p.d. of a cell when it is being charged is

$$V = \mathcal{E} + Ir, \qquad V > \mathcal{E}$$

Cells in series.

If n cells of emf $\mathcal{E}$ and internal resistance r each are connected in series, then current drawn through external resistance R is

$$I = \frac{n\mathcal{E}}{R + nr}$$

Cells in parallel.

If m cells are connected in parallel, then current drawn through external resistance R is

$$I = \frac{m\mathcal{E}}{mR + r}$$

Cells in mixed grouping.

If n cells are connected in series in each row and m such rows are connected in parallel, then current drawn through an external resistance R is

$$I = \frac{mn\mathcal{E}}{mR + nr}$$

For maximum current, the external resistance must be equal to the total internal resistance, i.e.,

$$R = \frac{nr}{m}$$

$$or \quad mR = nr$$

Joule's law of heating.

It states that the amount of heat H produced in a resistor is

- **directly proportional to the square of current for a given R,**
- **directly proportional to the resistance R for a given I,**
- **inversely proportional to the resistance R for a given V, and**
- **directly proportional to the time t for which the current flows through the resistor.**

Mathematically,

This law can be expressed as

$$H = VIt\ joule$$

$$= I^2Rt\ \ joule = \frac{V^2t}{R}\ joule$$

$$or \qquad H = \frac{VIt}{4.18}\ cal$$

$$= \frac{I^2Rt}{4.18}\ cal\ = \frac{V^2t}{4.18R}\ cal.$$

Electric power.

It is the rate at which an electric appliance converts electric energy into other forms of energy. Or, it is the rate at which work is done by a source of emf in maintaining an electric current through a circuit.

Electric power,

$$P = \frac{W}{t} = VI = \ I^2R = \frac{V^2}{R}$$

SI unit of power is watt.

Electric energy.

It is the total work done in maintaining an electric current in an electric circuit for a given time.

$$electric\ energy = electric\ power\ \times time$$

$$W = Pt$$

$$= VI\ t\ \ joule = \ I^2Rt\ joule$$

Efficiency of a source of emf.

It is the ratio of the output power to the input power. If a source of emf $\mathcal{E}$ and internal resistance r is connected to an external resistance, then its efficiency will be

$$\eta = \frac{output\ power}{input\ power} = \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{R}{R+r}$$

Maximum power theorem.

It states that the output power of a source of emf is maximum when the external resistance in the circuit is equal to internal resistance of the circuit i.e., when R = r.

$$P_{max} = \frac{\mathcal{E}^2}{4r}$$

The efficiency of a source of emf is 50% when it delivers maximum power.

Kirchhoff's law.

These laws enable us to determine the currents and voltages in different parts of the electrical circuits.

First law or junction law. In an electric circuit, the algebraic sum of currents at any junction is zero. Or, the sum of currents entering a junction is equal to the sum of the currents leaving that junction.

Mathematically,

$$\sum I = 0$$

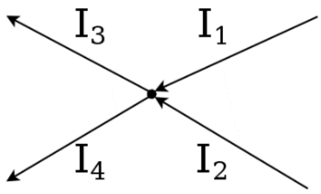
Justification. This law is based on the law of conservation of charge. When the currents in a circuit are steady, charges cannot accumulate or originate at any point of the circuit.

Second law or loop rule. Around any loop of a network, the sum of changes in potential must be zero. Or, the algebraic of the emfs in any loop of a circuit is equal to the sum of the products of currents and resistance in it.

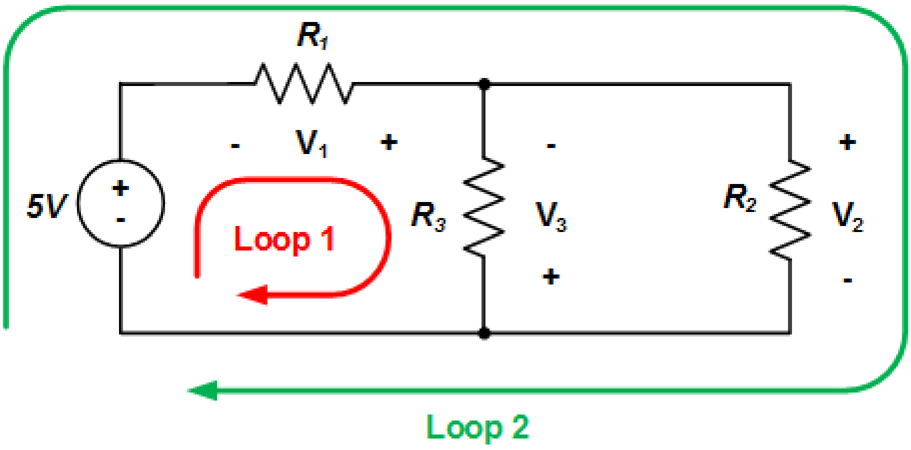
$$\sum \Delta V = 0$$

$$\text{or} \quad \sum \mathcal{E} = \sum IR$$

Justification. This law is based on the law of conservation of energy. As the electrostatic force is a conservative force, the total work done by it along any closed path must be zero.



$$I_1 + I_2 = I_3 + I_4$$



Potentiometer.

It is a device used to compare emfs of two sources. Its working is based on the principle that when a constant current flows through a wire of uniform cross-sectional area and composition, the p.d. across any length of the wire is directly proportional to that length.

$$V \propto l$$

$$or \quad V = kl$$

Where k is the potential drop per unit length which is called potential gradient. Potentiometer has two main uses.

(i) To compare the emfs of two cells. If l_1 and l_2 are the balancing lengths of the potentiometer wire for the cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively,

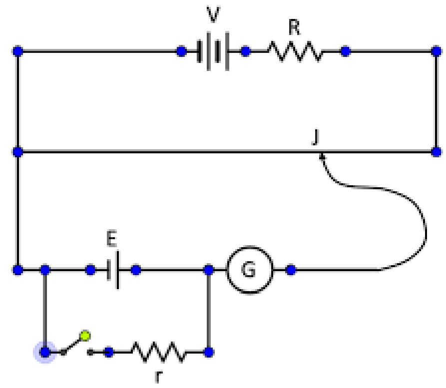
Then

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$$

(ii) To find internal resistance r of a cell.

If l_1 is the balancing length of the potentiometer wire without shunt and l_2 the balancing length with shunt R across the cell, then internal resistance of the cell will be

$$r = \frac{\mathcal{E} - V}{V} \times R = \frac{l_1 - l_2}{l_2} \times R$$

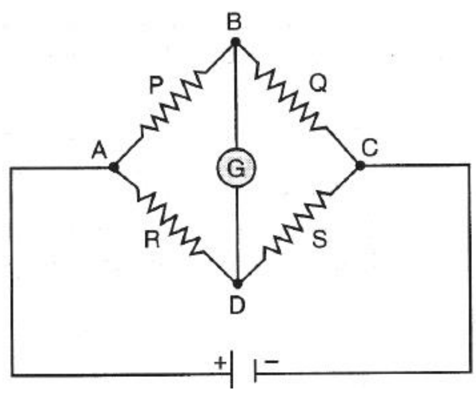


Wheatstone bridge.

It is an arrangement of four resistances P, Q, R and S joined to form a quadrilateral ABCD with a battery between A and C and a sensitive galvanometer between B and D. The resistances are so adjusted that no current flows through the galvanometer. The bridge is then said to be balanced. In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

Knowing any three resistances, the fourth resistance can be computed. A Wheatstone bridge is most sensitive when the resistances in its four arms are of the same order.



Slide wire bridge or meter bridge.

It is an application of Wheatstone bridge in which R is fixed and a balance point is obtained by varying P and Q i.e., by adjusting the position of a jockey on a 100 cm long resistance wire stretched between two terminals. If the balance point is obtained at length l_1, then

$$\frac{P}{Q} = \frac{R}{S} = \frac{l_1}{100 - l_1}$$

$$or \qquad S = \left(\frac{100 - l_1}{l_1}\right) R$$

Resistivity,

$$\rho = \frac{SA}{l_1} = \frac{S \times \pi r^2}{l_1}$$