Laws of motion

Force.

It may be defined as an agency (a push or pull) which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body. Force is a vector quantity.

Inertia.

It is the inherent property of a material body by virtue of which it cannot change, by itself, its state of rest or of uniform motion in a straight line. Inertia is of three types:

(i) Inertia of rest

(ii) Inertia of motion and

(iii) Inertia of direction

Mass as the measure of inertia.

If a body has more mass, it has more inertia i.e., it is more difficult to change its state of rest or of uniform motion.

Momentum.

It is the quantity of motion in a body. It is equal to the product of mass m and velocity v of the body.

$$momentum, \quad p = mv \quad or\, \vec{p} = m\vec{v}$$

Momentum is a vector quantity having the direction of velocity $\vec{v}$.

Its SI unit is kg ms^{-1}.

Newton's first law of motion.

It states that every body continues in its state of rest or of uniform motion along a straight line, unless an external force is applied to change that state. This law defines force.

Newton's second law of motion.

It states that the rate of change of momentum of a body is directly proportional to the applied force and the change in momentum takes place in the direction of the applied force. This law gives a measure of the force.

Mathematically,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma = m\left(\frac{v-u}{t}\right)$$

(i) When m is constant and v changes,

$$F = m\frac{dv}{dt}$$

(ii) When v is constant and m changes,

$$F = v\frac{dm}{dt}$$

In component form, Newton's second law may be expressed as

$$F_x = \frac{dp_x}{dt} \quad ; \quad F_y = \frac{dp_y}{dt} \quad ; \quad F_z = \frac{dp_z}{dt}.$$

Newton's third law of motion.

It states that to every action, there is an equal and opposite reaction.
Mathematically,

$$\overrightarrow{F_{BA}} = -\overrightarrow{F_{AB}}$$

Forces of action and reaction never cancel out because they act on different bodies.

Impulse of a force.

Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum. It is defined as the product of the force and the time for which it acts and equal to the total change in momentum.

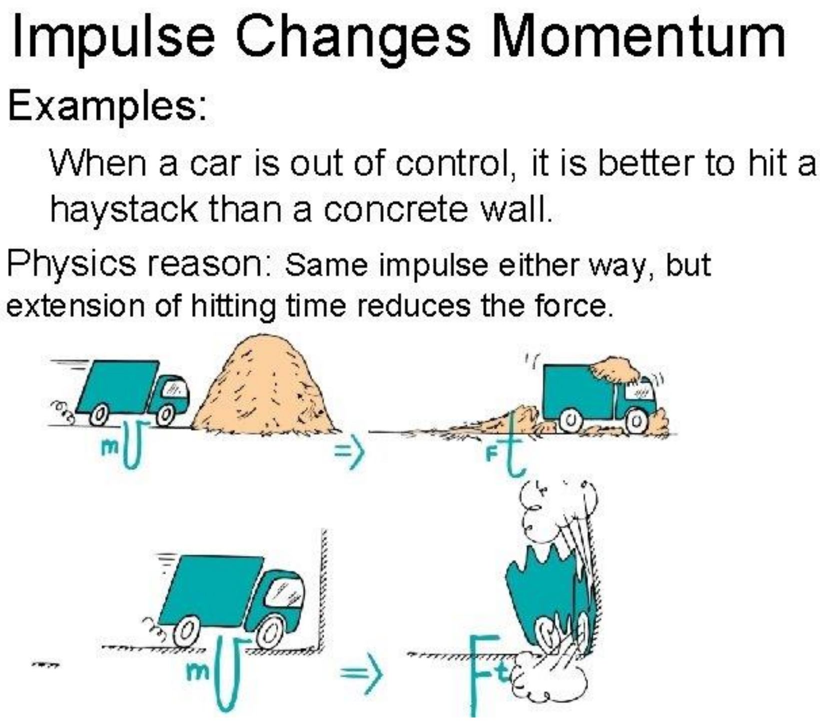
$$Impulse = Force \times time\ duration = Total\ change\ in\ momentum.$$

Impulse is a vector quantity, denoted by $\vec{J}$.

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}.dt = Area\ under\ the\ force - time(F-t)\ graph.$$

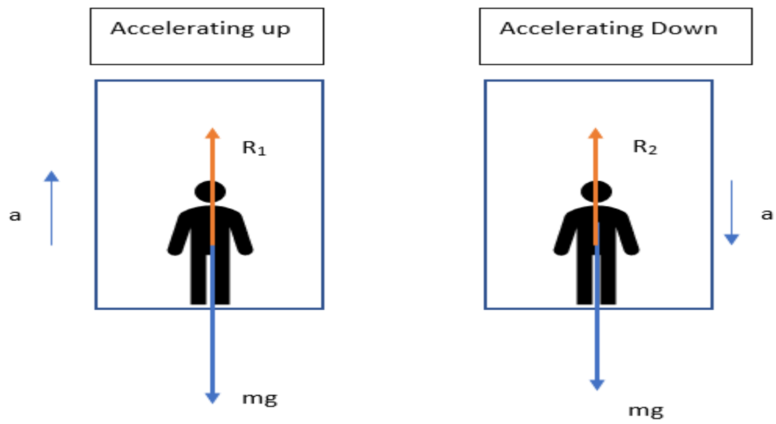
$$or \qquad \vec{J} = \overrightarrow{F_{av}} \times t = \overrightarrow{p_2} - \overrightarrow{p_1} = m(\vec{v} - \vec{u})$$

The SI unit of impulse is Ns or kg ms^{-1}.



Apparent weight of a body in a lift.

(i) When a lift moves upward with uniform acceleration a, apparent weight of a body in the lift increases.



$$R - mg = ma \quad or \quad R = m(g+a) \quad (m\ is\ the\ mass\ of\ object\ in\ the\ lift)$$

(ii) When a lift moves downwards with acceleration a, the apparent weight of a body in the lift decreases.

$$mg - R = ma \qquad or \qquad R = m(g-a)$$

(iii) When a lift is at rest or moves with uniform velocity, a = 0, the apparent weight of the body is equal to its true weight.

$$R = mg$$

(iv) When a lift falls freely, (a = g) the apparent weight of a body in the lift becomes zero.

$$R = m(g-g) = 0.$$

Law of conservation of linear momentum.

(i) In the absence of any external force, vector sum of the linear momenta of a system of particles remains constant.

$$\vec{p} = \overrightarrow{p_1} + \overrightarrow{p_2} + \overrightarrow{p_3} + \cdots + \overrightarrow{p_n} = constant$$

$$or \qquad m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + m_3\overrightarrow{v_3} + \cdots + m_n\overrightarrow{v_n} = constant$$

(ii) When two bodies collide,

Total momentum before collision = Total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

(iii) When a bullet of mass m is fired with velocity v from a gun of mass M, the gun recoils with velocity V.

Momentum of gun = - Momentum of bullet

$$MV = -mv$$

Recoil velocity of gun,

$$V = -\frac{mv}{M}.$$

Rocket propulsion.

It is an example of momentum conservation in which the large backward momentum of the ejected gases imparts an equal forward momentum of the rocket. Due to the decrease in mass of the rocket-fuel system, the acceleration of the rocket keeps on increasing. Let

$$u = velocity\ of\ exhaust\ gases$$

$$v_o, v = initial\ velocity\ and\ velocity\ of\ the\ rocket\ at\ any\ instant\ t$$

$$m_0, m, m_e = initial\ mass, mass\ of\ rocket\ at$$

$$any\ instant\ t\ and\ mass\ of\ empty\ rocket.$$

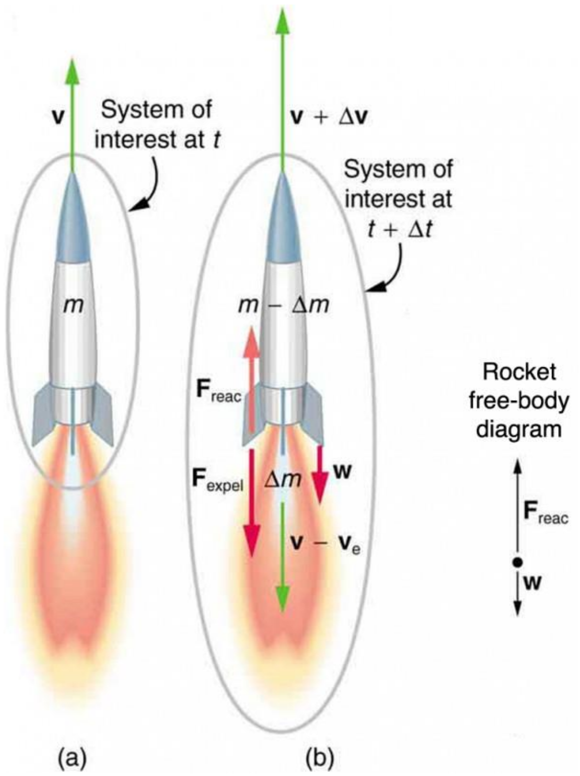
$$\frac{dm}{dt} = rate\ of\ ejection\ of\ fuel.$$

$$Thrust\ on\ rocket: \qquad F = -u\frac{dm}{dt}$$

$$Acceleration\ of\ rocket: \qquad a = \left[\frac{u}{m_0 - t\frac{dm}{dt}}\right]\frac{dm}{dt}$$

$$Velocity\ of\ rocket: \qquad v = v_0 + u\log_c\frac{m_0}{m}$$

$$Burnt - out\ speed\ of\ rocket: \qquad v_b = v_0 + u\log_e\frac{m_0}{m_e}$$



Concurrent forces.

The forces acting at the same point of a body are called concurrent forces.

Equilibrium of concurrent forces.

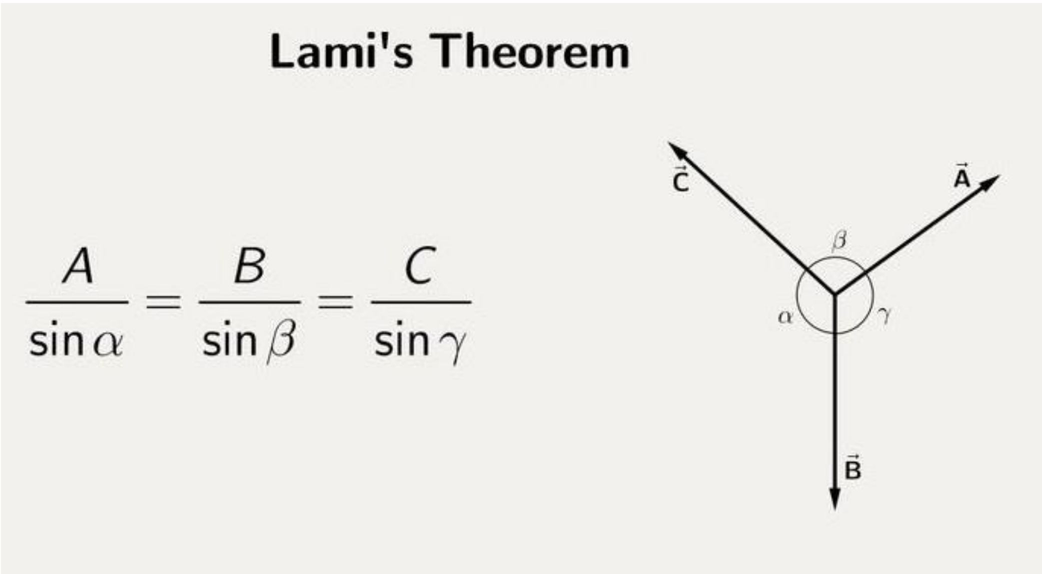
A number of concurrent forces acting on a body are said to be in equilibrium if their vector sum is zero or if these forces can be completely represented by the sides of closed polygon taken in the same order.

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots + \overrightarrow{F_n} = 0$$

Lami's theorem

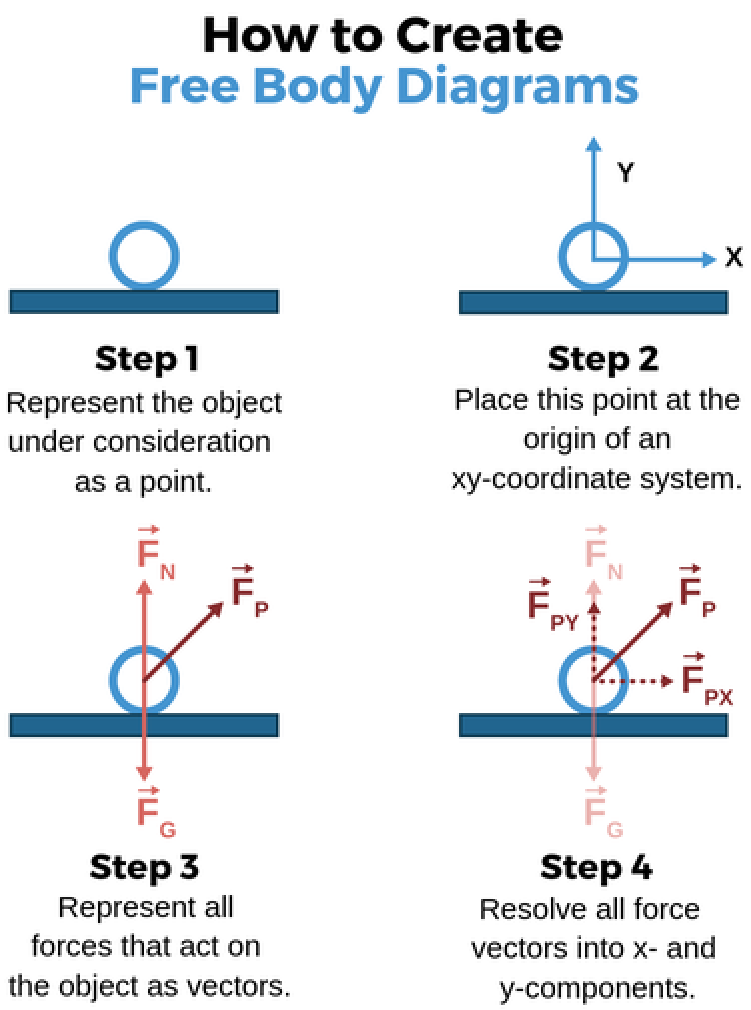
It states that if three forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between other two forces. If α, β and γ are the angles between $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$; $\overrightarrow{F_3}$ and $\overrightarrow{F_1}$; $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ respectively, then according to Lami's theorem:

$$\frac{F_1}{sin\ \alpha} = \frac{F_2}{sin\ \beta} = \frac{F_3}{sin\ \gamma}.$$



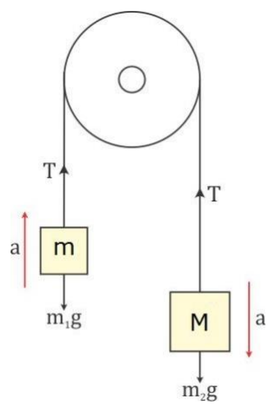
Free-body diagram.

A diagram for each body of the system showing all the forces exerted on the body by the remaining parts of the system is called free-body diagram.



Motion of connected bodies.

Suppose two bodies of masses M and m (M > m) are tied at the ends of an inextensible string passing over a frictionless pulley. Then



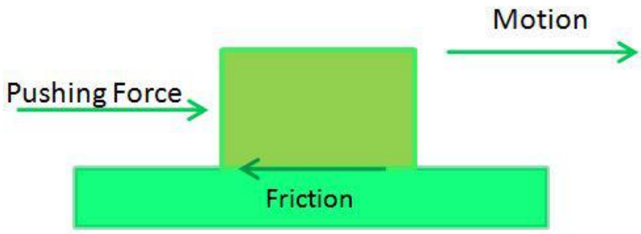
$$\textit{Acceleration of the masses,}\quad a = \frac{M-m}{M+m}.g$$

$$\textit{Tension in the string,}\quad T = \frac{2Mm}{M+m}.g$$

Clearly, a < g.

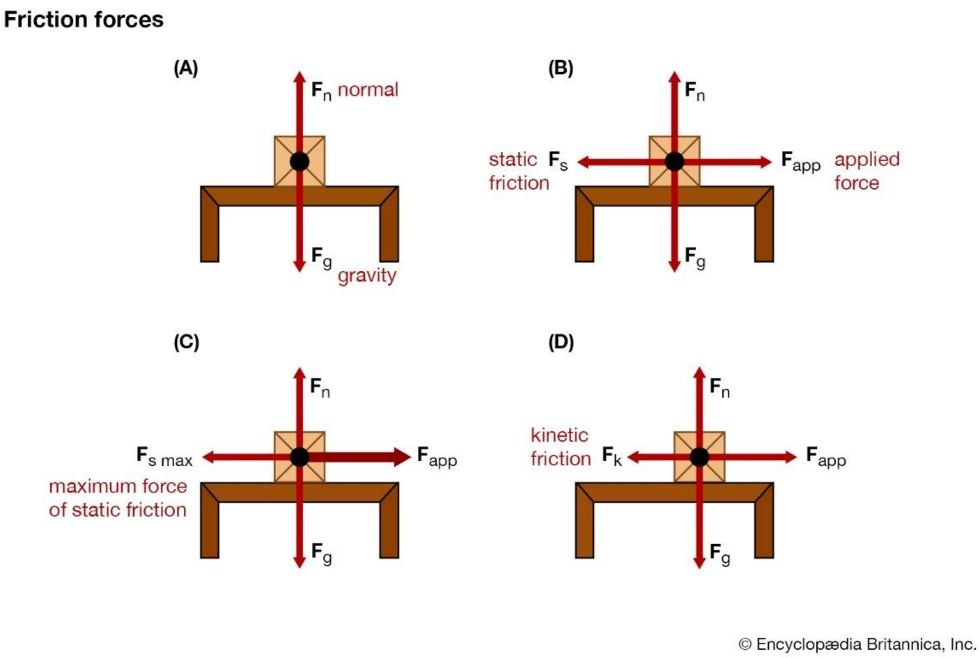
Friction.

Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called friction.



Static friction.

The force of friction which comes into play between two bodies before one body actually starts moving over the surface of another body is called static friction (f_s). Static friction is self-adjusting force.

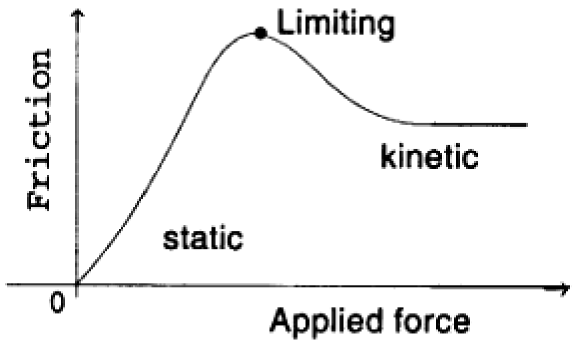


Limiting friction.

The maximum force of static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction (f_s^{max})

Kinetic friction.

The force of friction which comes into play when a body is in a steady motion over the surface of another body is called kinetic or dynamic friction (f_k), **kinetic friction is less than limiting friction.**



Laws of limiting friction.

(i) The force of limiting friction depends upon the nature of the two surfaces in contact and their state of roughness.

(ii) The force of limiting friction acts tangential to the two surfaces in contact and in a direction opposite to that of the applied force.

(iii) The force of limiting friction between any two surfaces is independent of the shape or area of the surfaces in contact so as long as the normal reaction remains the same.

(iv) The force of limiting friction between the two given surfaces is directly proportional to the normal reaction between the two surfaces.

$$f \propto R \qquad or \qquad f = \mu_s R$$

Where the constant of proportionality μ_s is called the coefficient of limiting friction.

Coefficient of limiting friction.

It is the ratio of limiting friction to the normal reaction.

$$\mu_s = \frac{f_s^{max}}{R} = \frac{limiting\ friction}{normal\ reaction}$$

Coefficient of kinetic friction.

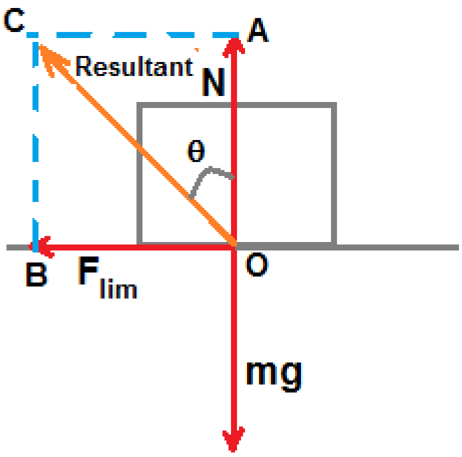
It is the ratio of kinetic friction to the normal reaction.

$$\mu_k = \frac{f_k}{R} = \frac{kinetic\ friction}{normal\ reaction}$$

As $\quad f_k < f_s^{max} \quad or \quad \mu_k R < \mu_s R \quad \therefore\ \mu_k < \mu_s$

Angle of friction.

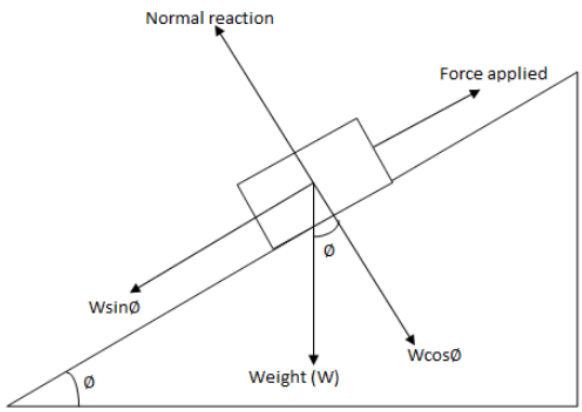
It is the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction. If θ is the angle of friction, then



$$\tan\theta = \mu_s$$

Angle of repose.

It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down. If $\emptyset$ is the angle of repose, then



$$\tan\emptyset = \mu_s$$

Motion along a rough horizontal surface.

If a body of mass m is moved over a rough horizontal surface through distance s, then

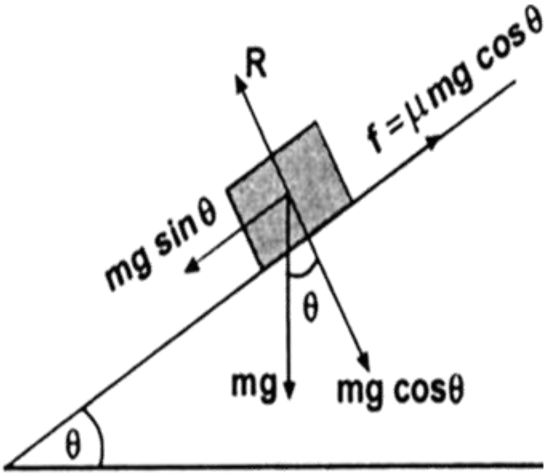
$$Force\ of\ friction, \quad f = \mu R = \mu mg$$

$$Retardation\ produced, \quad a = \frac{f}{m} = \mu g$$

$$Work\ done\ against\ friction, \quad W = f \times s = \mu mgs$$

Motion along a rough inclined plane.

(i) When a body moves down an inclined plane with uniform velocity (a = 0), net downward force needed is



$$F = mg\ sin\theta - f = mg(sin\theta - \ \mu cos\theta)$$

$$Work\ done,\ \ W = mg(sin\theta - \ \mu cos\theta)s$$

(ii) When a body moves up an inclined plane with uniform velocity (a=0), net upward force needed is

$$F = mg\ sin\theta + f = mg\ (sin\theta + \ \mu cos\theta)$$

$$W = Fs = mg\ (sin\theta + \mu cos\theta)s$$

(iii) When a body moves up an inclined plane with acceleration a, net upward force needed is

$$F = ma + mg\ sin\theta + f$$

$$= m(a + g\ sin\theta + \mu g\ cos\theta)$$

$$W = m(a + g\ sin\theta + \ \mu g\ cos\theta)s.$$

Acceleration of a body sliding down a rough inclined plane.

When the angle of inclination is greater than the angle of repose, the acceleration produced is

$$a = g(sin\theta - \mu_k\ cos\theta)$$

Centripetal force.

It is the force required to make a body move along a circular path with a uniform speed. It always acts along the radius and towards the center of the

circular path. The centripetal force required to move a body of mass m along a circular path of radius r with speed v is given by

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi\nu)^2 = mr\left(\frac{2\pi}{T}\right)^2$$

Centrifugal force.

It is a fictional force acting radially outwards on a particle moving in a circle and is equal in magnitude to the centripetal force.

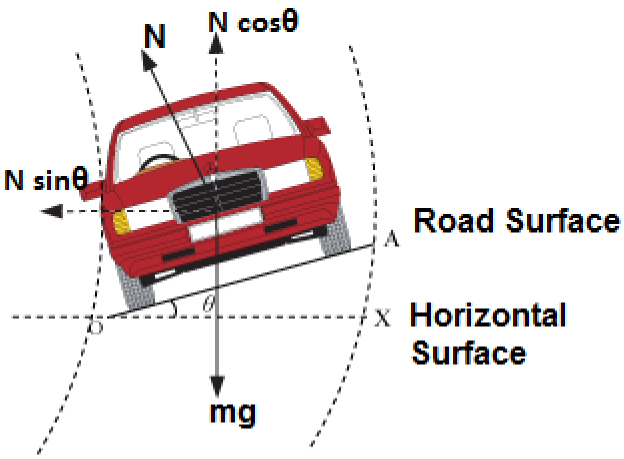
A vehicle taking circular turn on a level road.

If μ is the coefficient of friction between tyres and road, then the maximum velocity with which the vehicle can safely take a circular turn of radius r is given by

$$v = \sqrt{\mu r g}$$

Banking of roads.

The maximum angle with which a vehicle (in the absence of friction) can negotiate a circular turn of radius r and banked at an angle θ is given by



$$v = \sqrt{rg\,tan\theta}$$

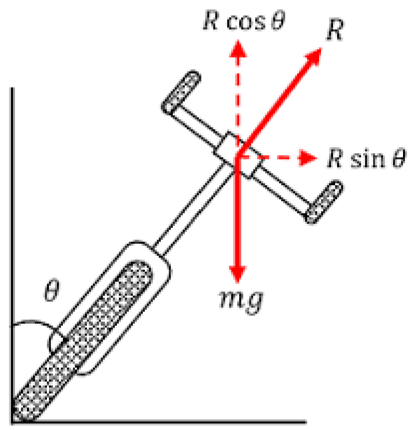
When the frictional forces are also taken into account, the maximum safe velocity is given by

$$v = \sqrt{rg\left(\frac{\mu + tan\theta}{1 - \mu\,tan\theta}\right)}$$

Bending of a cyclist.

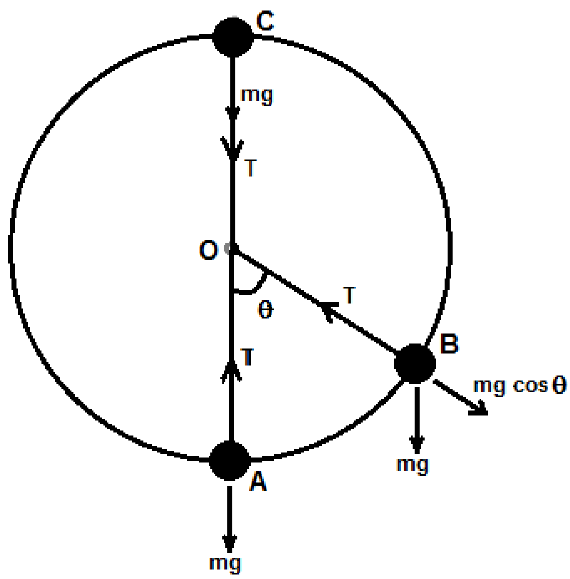
In order to take a circular turn of radius r with speed v, the cyclist should bend himself through an angle θ from the vertical such that

$$\tan\theta = \frac{v^2}{rg}$$



Motion in a vertical circle.

Consider a body of mass m tied at the end of a string and rotating in a vertical circle of radius r.



Then,

(i) Velocity at any point B at a height h from the lowest point A,

$$v = \sqrt{u^2 - 2gh}$$

(ii) Tension in the string at any point B,

$$T = \frac{m}{r}(u^2 - 3gh + gr)$$

(iii) Tension at the lowest point (h = 0),

$$T_L = \frac{m}{r}(u^2 + gr)$$

(iv) Tension at the highest point H (h = 2r),

$$T_H = \frac{m}{r}(u^2 - 5gr)$$

(v) Difference in tensions at highest and lowest points,

$$T_L - T_H = 6mg$$

(vi) Minimum velocity at the lowest point A for looping the loop,

$$v_L = \sqrt{5gr}$$

(vii) Velocity at the highest point for looping the loop,

$$v_H = \sqrt{gr}$$

Inertial frame of reference.

An inertial frame of reference is one in which Newton's first law of motion holds good. All frames moving with uniform motion relative to an inertial frame are also inertial.

Non-inertial frame of reference.

A frame of reference which is accelerating with respect to an inertial frame of reference is called non-inertial frame of reference. In a non-inertial frame, the second law of motion $\vec{F} = m\vec{a}$ is not valid. Instead, it takes the form

$$\vec{F} - m\vec{\alpha} = m\vec{a} \quad or \quad \vec{F} + \vec{F_P} = m\vec{a}$$

Where $\vec{\alpha}$ is the acceleration of the non-inertial frame relative to any inertial frame and $\vec{a}$ is the acceleration of the body relative to the non-inertial frame. The force $\vec{F_P} = -m\vec{\alpha}$ is an example of pseudo force.