Mechanical properties of fluids

Fluid.

A fluid is a substance that can flow. The term fluid refers to both liquids and gases.

Fluid statics.

The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics.

Fluid dynamics.

The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.

Thrust.

The total force exerted by a liquid on any surface in contact with it is called thrust. A liquid always exerts force perpendicular to the surface of the container at every point.

Pressure.

The thrust exerted by a liquid per unit area of the surface in contact with it is known as pressure.

$$Pressure = \frac{Thrust}{Area} \quad or \quad P = \frac{F}{A}$$

Pressure is a scalar quantity.

Density.

The density of any material is defined as its mass per unit volume.

$$Density = \frac{Mass}{Volume} \quad or \quad \rho = \frac{M}{V}$$

Density is a positive scalar quantity.

Units and dimension of density.

. The SI unit of density is kg m^{-3} and the CGS unit is g cm^{-3}.

. The dimensional formula of density is $[ML^{-3}]$.

Specific gravity.

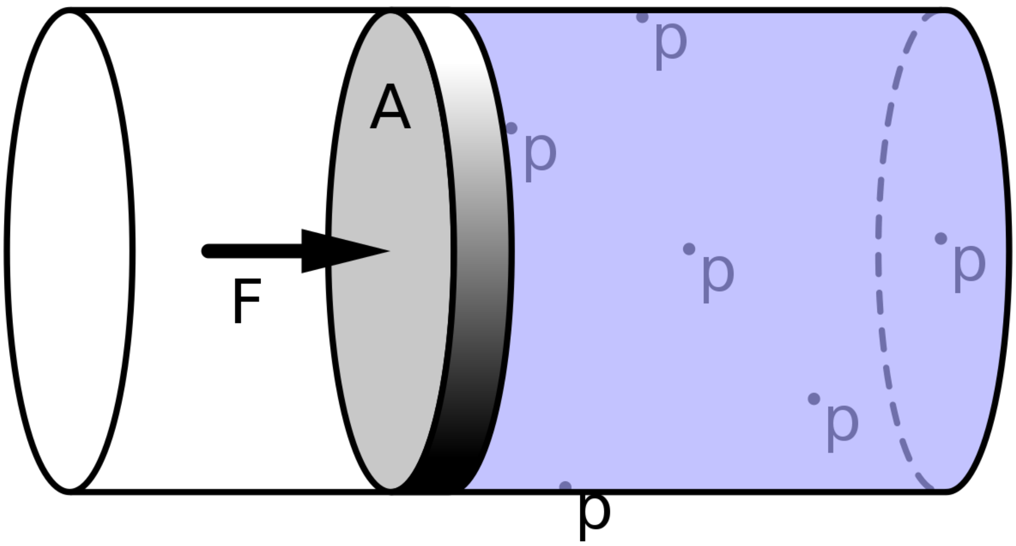
The relative density or specific gravity of a substance is defined as the ratio of the density of the substance to the density of water at 4°C.

$$Specific\ gravity = \frac{Density\ of\ substance}{Density\ of\ water\ at\ 4°C}$$

Specific gravity is a dimensionless positive scalar quantity.

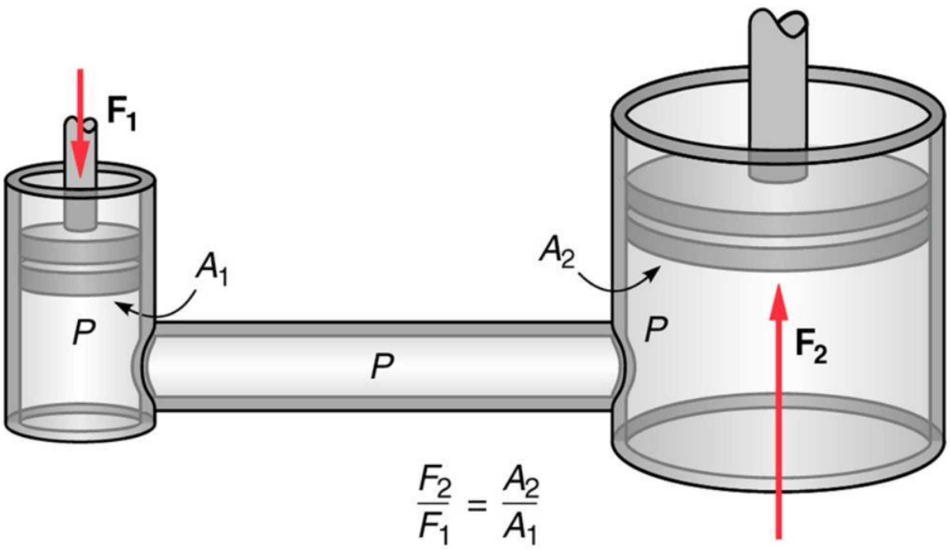
Pascal's Law.

It states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel. Or, the pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.



Hydraulic lift.

It is an application of Pascal's law. It is used to lift heavy objects.



According to Pascal's law,

Pressure applied on smaller piston = pressure transmitted to larger piston.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad or \; F_2 = P \; X \; A_2 = \frac{F_1}{A_1} \; X \; A_2$$

As, $A_2 > A_1 \quad so \quad F_2 > F_1$.

Thus, hydraulic lift acts as a force multiplier.

Pressure exerted by a liquid.

A liquid column of height h and density ρ exerts a pressure given by

$$P = h\rho g$$

Effect of gravity on fluid pressure.

The pressure in a fluid varies with depth h according to the expression

$$P = P_a + h\rho g$$

Where ρ is the density of fluid and P_a is the atmospheric pressure.

Hydrostatic paradox.

The pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.

Atmospheric pressure.

The pressure exerted by the atmosphere is called atmospheric pressure. At sea-level, we have

Atmospheric pressure = Pressure exerted by 0.76 m of Hg = 1.013 x 10^5 Nm^{-2}

Absolute pressure and gauge pressure.

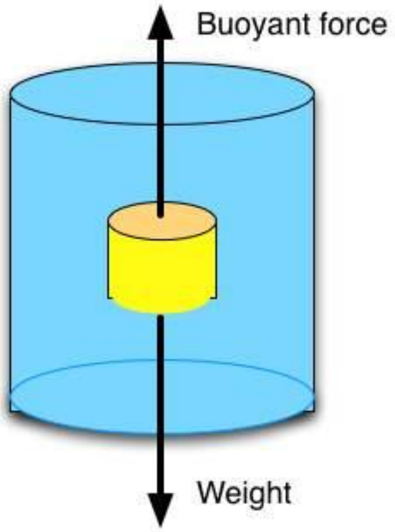
The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (absolute pressure) at a point and the atmospheric pressure. Thus,

$$P_g = P - P_a \quad or \; P = P_a + P_g$$

Absolute pressure = Atmospheric pressure + Gauge pressure.

Buoyancy and center of buoyancy.

The upward force acting on a body immersed in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy. The force of buoyancy acts through the center of gravity of the displaced fluid which is called center of buoyancy.



Archimedes' Principle.

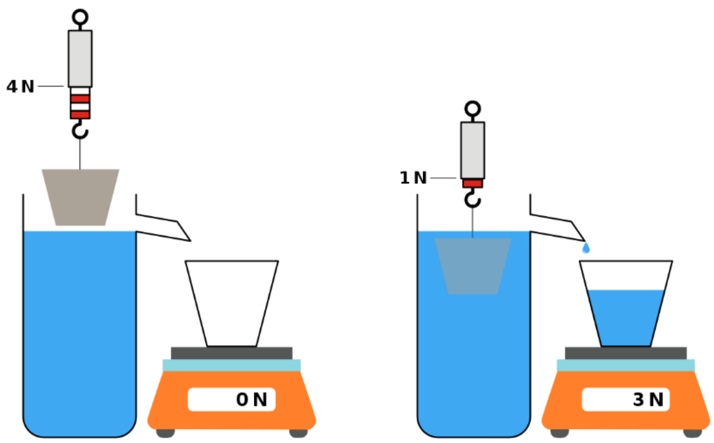
It states that when a body is immersed partly or wholly in a fluid, it loses some weight. The loss in weight is equal to the weight of the fluid displaced.

Apparent weight of a body in a fluid = True weight – Weight of fluid displaced.

$$W_{app} = W - U = V\sigma g - V\rho g$$

$$= V\sigma g \left(1 - \frac{\rho}{\sigma}\right) = W\left(1 - \frac{\rho}{\sigma}\right)$$

Where W = Vσg is the weight of the body and σ its density.



Law of floatation.

A body will float in a liquid if weight of the liquid displaced by the body is at least equal to or greater than the weight of the body.

When a body just floats,

Weight of the body = Weight of liquid displaced

$$V\sigma g = V'\rho g \quad or \quad \frac{V'}{V} = \frac{\sigma}{\rho}$$

$$or \quad \frac{Volume\ of\ the\ immersed\ part}{Total\ volume\ of\ the\ body} = \frac{Density\ of\ the\ body}{Density\ of\ liquid}$$

Viscosity.

It is the property of a fluid due to which an opposing force comes into play whenever there is relative motion between its different layers. Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction. A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion.

Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

Imagine a styrofoam cup with a hole in the bottom. If I then pour honey into the cup I will find that the cup drains very slowly. That is because honey's viscosity is large compared to other liquids' viscosities. If I fill the same cup with water, for example, the cup will drain much more quickly.

Newton's formula for viscous force.

The viscous drag between two parallel layers each of area A and having velocity gradient dv/dx is given by

$$F = -\eta A \frac{dv}{dx}$$

Where η is the coefficient of viscosity of the liquid.

Coefficient of viscosity.

It may be defined as the tangential viscous force required to maintain a unit velocity gradient between two liquid layers each of unit area. Its dimensional formula is [$ML^{-1}T^{-1}$].

Poiseuille's formula.

The volume of a liquid flowing per second through a horizontal capillary tube of length l, radius r under a pressure difference p across its two ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}.$$

Stokes' law.

It states that the backward dragging force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is

$$F = 6\pi\eta\, rv.$$

Terminal velocity.

It is the maximum constant velocity attained by a spherical body while falling through a viscous medium. The terminal velocity of a spherical body of density ρ and radius r moving through a fluid of density ρ' and viscosity η is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$$

Streamline flow and turbulent flow.

It is the flow of liquid in which each particle of the liquid passing through a point travels along the same path and with the same velocity as the preceding particle passing through the same point.

A liquid possesses streamline motion only when its velocity is less than a certain limiting value, called critical velocity. When the velocity of the liquid becomes greater than the critical velocity, the particle follow zig-zag path, such a disordered or irregular motion is called turbulent flow.

Tube of flow.

A tube of flow is a bundle of streamlines having the same velocity of fluid element over any cross-section perpendicular to the direction of flow.

Laminar flow.

The steady flow in which liquid moves in the form of layers is called laminar flow. The velocity of the layer varies from maximum at axis to zero for the layer at the wall of the tube.

Critical velocity.

The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent. It is given by

$$v_c = \frac{k\eta}{\rho r}$$

Where r is the radius of tube, ρ is the density of liquid.

Reynold's number.

It is a dimensionless number which determines the nature of the flow of the liquid. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by

$$R_e = \frac{\rho v D}{\eta}$$

. If R_e > 2000, the flow is laminar

. If R_e > 3000, the flow is turbulent

. if 2000 < R_e < 3000, the flow is unstable. It may change from laminar to turbulent and vice-versa

$$Reynold's number = \frac{Inertial\ force\ per\ unit\ area}{Viscous\ force\ per\ unit\ area}$$

Ideal fluid.

An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.

Rate of flow.

The volume of a liquid flowing per second through a pipe of cross-section a with velocity v is given by

$$Q = \frac{V}{t} = av$$

Equation of continuity.

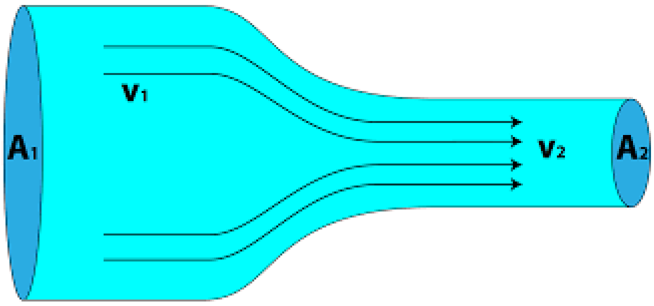
If there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant.

$$m = a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$

It is called equation of continuity. For an incompressible liquid $\rho_1 = \rho_2$, then

$$a_1 v_1 = a_2 v_2 \quad or \quad av = constant.$$

Thus, during the streamlined flow of a non-viscous and incompressible fluid through a pipe of varying cross section, the product of area of cross section and the normal fluid velocity remains constant throughout the fluid flow.



Bernoulli's principle.

It states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined, irrotational flow remains constant along a streamline. Thus,

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

$$or \qquad \frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = constant$$

The terms $\frac{P}{\rho g}$, h and $\frac{v^2}{2g}$ are called pressure head, gravitational head and velocity head respectively, For the horizontal flow of a liquid (h = constant), Bernoulli's equation takes the form

$$P + \frac{1}{2}\rho v^2 = constant$$

$$or \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

It indicates that velocity increases where pressure decreases and vice-versa.

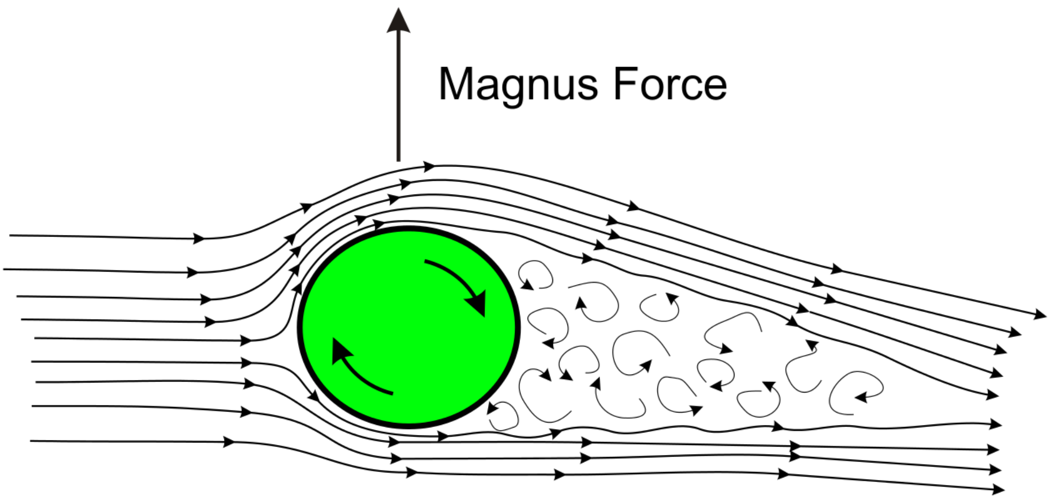
Torricelli's Law.

It states that the velocity of efflux i.e., the velocity with which the liquid flows out of an orifice (a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid. Hence the velocity of efflux of a liquid through an orifice at depth h from the liquid surface will be

$$v = \sqrt{2gh}$$

Magnus effect.

The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect.



Cohesive and adhesive forces.

The force of attraction between the molecules of the same substance is called cohesive force while the force of attraction between the molecules of two different substances is called adhesive force.

Molecular range.

It is the maximum distance up to which a molecule can exert some measurable attraction on other molecules. The order of molecular range is 10^{-9} m in solids and liquids.

Surface film.

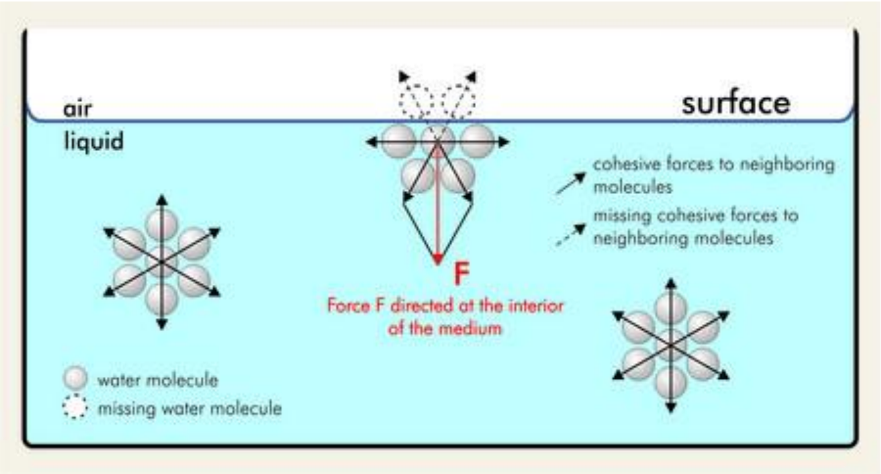
A thin film of liquid near its surface and having thickness equal to the molecular range for that liquid is called surface film. The molecules present in the surface film possess additional potential energy.

Surface tension.

It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contract so as to occupy a minimum surface area. It is measured as the force per unit length on an imaginary line drawn on the surface of liquid.

$$Surface\ tension = \frac{Force}{length}\ or\ \sigma = \frac{F}{l}$$

Its SI unit is Nm^{-1} and CGS unit is dyne cm^{-1}.



Excess pressure inside a drop and bubble.

There is excess of pressure on the concave side of a curved surface.

. Excess pressure inside a liquid drop = $\frac{2\sigma}{R}$ (one free surface)

. Excess pressure inside a liquid bubble = $\frac{4\sigma}{R}$ (two free surfaces)

. Excess pressure inside an air bubble = $\frac{2\sigma}{R}$ (one free surface)

Where R is the radius of the liquid drop, liquid bubble or air bubble.