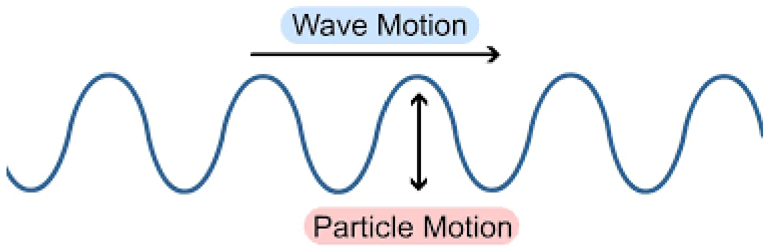
Waves

Wave motion.

It is a kind of disturbance which travels through a medium due to the repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. In a wave both information and energy propagate from one point to another but there is no motion of matter as a whole through a medium.



Three basic types of waves.

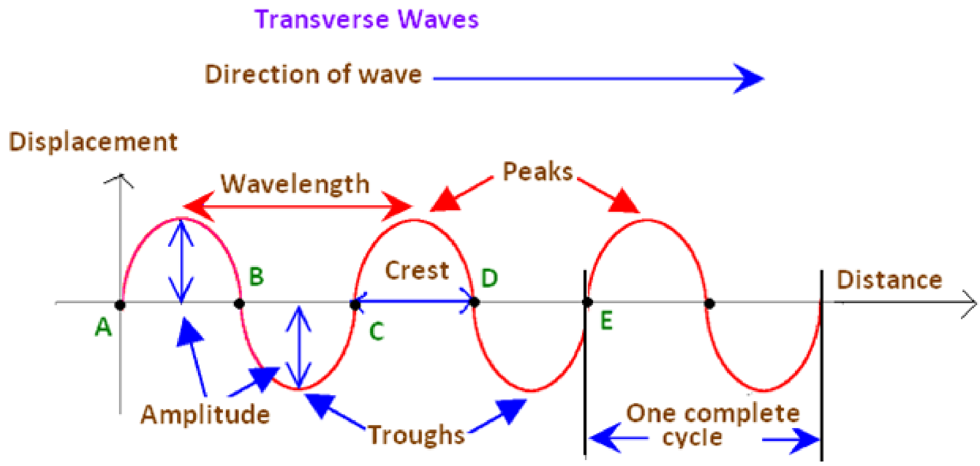
(i) Mechanical waves. The waves which require a mechanical medium for their propagation are called mechanical waves or elastic waves. For their propagation, the medium must possess the properties of inertia and elasticity. For example, water waves, sound waves, etc.

(ii) Electromagnetic waves. The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation. For example, visible light, radio waves, etc.

(iii) Matter waves. The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves. Matter waves associated with fast moving electrons are used in electron microscope.

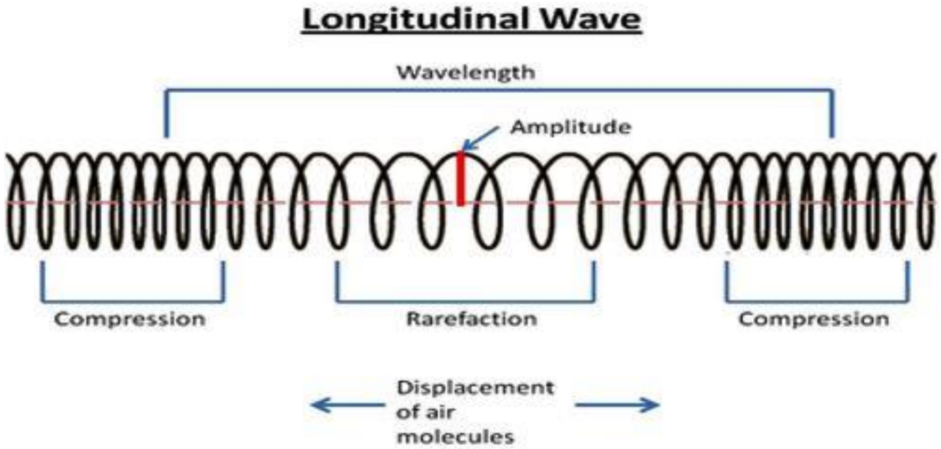
Transverse waves.

These are the waves in which particles of the medium vibrate about their mean positions in a direction perpendicular to the direction of propagation of the disturbance. These waves can propagate in those media which have a shear modulus of elasticity e.g., solids.



Longitudinal waves.

These are the waves in which particles of the medium vibrate about their mean positions along the direction of propagation of the disturbance. These waves can propagate in those media having a bulk modulus of elasticity and are therefore possible in all media: solids, liquids and gases.



Progressive wave.

A wave that moves from one point of medium to another is called a progressive wave.

Amplitude(A).

It is the maximum displacement suffered by the particle of the medium from the mean position during the propagation of the wave.

Time period (T).

It is the time in which a particle of the medium completes one vibration about its mean position.

Frequency (ϑ).

It is the number of waves produced per second in a given medium.

Wavelength (λ).

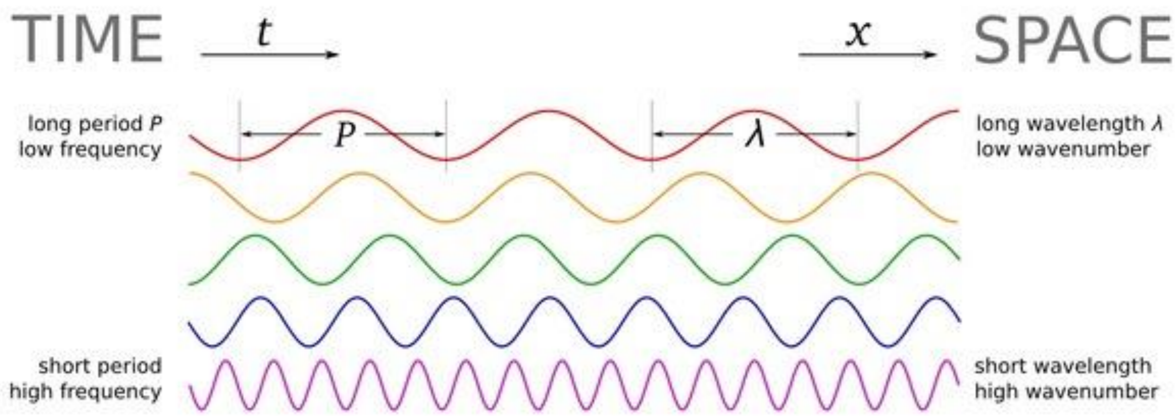
It is the distance covered by a wave during the time a particle of the medium completes one vibration about its mean position. It is the distance between two nearest particles of the medium which are vibrating in the same phase.

Angular wave number or propagation constant.

It represents the phase change per unit distance (or per unit path difference). It is equal to $2\pi/\lambda$.

$$Thus, \qquad k = \frac{2\pi}{\lambda}$$

The SI unit of k is radian per meter (rad m^{-1}).



Wave velocity (v).

It is the distance travelled by a wave in one second.

Relation between wave velocity, frequency and wavelength.

$$Wave\ velocity = Frequency\ (wavelength)$$

$$or \qquad v = \vartheta\lambda.$$

Relation between wave velocity, time period and wavelength.

$$Wave\ velocity = \frac{Wavelength}{Time\ period}$$

$$or \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad k = \frac{2\pi}{\lambda}$$

Velocity of transverse waves.

(i) Velocity of transverse waves in a solid of modulus of rigidity η *and density* ρ is given by

$$v = \sqrt{\frac{\eta}{\rho}}$$

(ii) Velocity of transverse wave in a string of mass per unit length m and stretched under tension T is given by

$$v = \sqrt{\frac{T}{m}}$$

Velocity of longitudinal waves.

(i) Velocity of longitudinal waves in an extended solid (earth's crust) of bulk modulus κ, modulus of rigidity η *and density* ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

The factor $\kappa + \frac{4}{3}\eta$ is called elongational elasticity.

(ii) Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

(iii) Velocity of longitudinal waves in a liquid of bulk modulus κ *and density* ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

(iv) Velocity of longitudinal waves in a gaseous medium of bulk modulus κ *and density* ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

According to Newton, when sound travels through gas, the changes taking place in the medium are isothermal in nature. So, Newton's formula for the speed of sound is

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}, \quad where\ P = \ pressure\ of\ the\ gas.$$

According to Laplace, when sound travels in a gas, the changes taking place in the medium the adiabatic. So, Laplace formula for the speed of sound is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

$$where, \qquad \gamma = \frac{C_p}{C_v} = specific\ heat\ ratio$$

Factors affecting velocity of sound through gases.

(i) Effect of pressure. **Pressure has no effect on the speed of sound in a gas.**

(ii) Effect of density.

$$v \propto \frac{1}{\sqrt{\rho}} \qquad or \qquad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

(iii) Effect of temperature.

$$As \ \ v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \qquad v \propto \sqrt{T} \qquad or \qquad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Temperature coefficient of velocity of sound is given by

$$\alpha = \frac{v_t - v_0}{t}$$

For air α = 0.61 ms^{-1}°C^{-1}

(iv) Effect of humidity. **Sound travels faster in moist air.**

(v) Effect of wind. **If the wind blows with velocity w in a direction making an angle θ with the direction of sound, then the resultant velocity of sound will be**

$$v' = v + w\ cos\theta$$

Wave equation.

A plane progressive harmonic wave travelling along positive X-direction may be represented as

$$(i)\ \ y = A\ sin(\omega t - kx), where\ k = \frac{2\pi}{\lambda}$$

$$(ii)\ y = A\ sin\ 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$(iii)\ \ y = A\ sin\ \frac{2\pi}{T}(vt - x)$$

Where v is the wave-velocity, λ the wavelength, A is the amplitude of the oscillating particles of the medium and y is the displacement of the particle located at position x at any instant t.

If the wave is travelling along negative X-direction, the minus sign is replaced by plus sign in the above equations. Thus

$$(i)\ \ y = A\ sin(\omega t + kx), where\ k = \frac{2\pi}{\lambda}$$

$$(ii)\ y = A\ sin\ 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$(iii)\ \ y = A\ sin\ \frac{2\pi}{T}(vt + x)$$

Phase and phase difference.

Phase is the argument of the sine or cosine function representing the wave. Thus phase,

$$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

The relation between phase difference $(\Delta\phi)$ and time interval Δt is

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

Thus, in time period T, the phase of a particle changes by 2π.

The relation between phase difference $(\Delta\phi)$ and path difference Δx is

$$\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that farther the particle is located from the origin in the positive X-direction, the more it lags behind in phase. Clearly, the phase difference between two particles located at separation $\lambda\ is\ 2\pi$.

Principle of superposition of waves.

When a number of waves travel through a medium simultaneously, the resultant displacement at any point of the medium is equal to the vector sum of the displacements of the individual waves. If $\overrightarrow{y_1}, \overrightarrow{y_2}, \overrightarrow{y_3}, \overrightarrow{y_4}, \ldots\ldots, \overrightarrow{y_n}$ are the displacements of n waves superposing each other at a point, then the resultant displacement at that point will be

$$\vec{y} = \overrightarrow{y_1} + \overrightarrow{y_2} + \overrightarrow{y_3} + \overrightarrow{y_4} + \cdots + \overrightarrow{y_n}$$

Stationary waves.

When two progressive waves of equal amplitude and frequency, travelling in opposite directions along a straight line superpose each other, the resultant wave does not travel in either direction and is called stationary or standing wave.

At some points, the particles of the medium always remains at rest. These are called nodes.

At some other points, the amplitude of oscillation is maximum. These are called antinodes.

Consider a plane progressive harmonic wave travelling along positive X-direction.

$$y_1 = A\,sin(\omega t - kx) \quad (incident\ wave)$$

If this wave is reflected from a free end, then

$$y_2 = A\,sin(\omega t + kx) \qquad (reflected\ wave)$$

The stationary wave formed by the superposition of the incident and reflected wave will be

$$y = y_1 + y_2 = 2A\,cos\,kx\,sin\,\omega t$$

In this case, the points $x = 0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots will\ be$ antinodes,

And the points $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots will\ be$ nodes.

If the wave is reflected from a rigid end, then

$$y_2 = -A\,sin(\omega t + kx)$$

The equation of the stationary wave will be

$$y = -2A\,sin\,kx\,cos\,\omega t$$

In this case, the points $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots.\ will\ be\ nodes.$

The points $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots will\ be\ antinodes.$

Separation between two successive nodes and antinodes = $\frac{\lambda}{2}$

Separation between a node and nearest antinode = $\frac{\lambda}{4}$

Modes of vibrations of strings.

On a stretched string, transverse stationary waves are formed due to superposition of direct and the reflected transverse waves.

For fundamental mode:

$$\lambda_1 = 2L, \qquad \vartheta_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{m}} = \vartheta(say)$$

For second mode:

$$\lambda_2 = L, \qquad \vartheta_2 = 2\vartheta \ \ (second\ harmonic\ or\ first\ overtone)$$

For third mode:

$$\lambda_3 = \frac{2L}{3}, \vartheta_3 = 3\vartheta \ \ (third\ harmonic\ or\ second\ overtone)$$

For p_{th} mode: When the string vibrates in p loops,

$$\vartheta_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\vartheta \;\; [pth\ harmonic\ or\ (p-1)\ overtone]$$

Fundamental frequency for a string of diameter D and density ρ,

$$\vartheta = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

Laws of transverse vibrations of a stretched string.

The fundamental frequency of a vibrating is

(i) inversely proportional to its length.

$$\vartheta \propto \frac{1}{L} \qquad (law\ of\ length)$$

(ii) Directly proportional to the square root of its tension.

$$\vartheta \propto \sqrt{T} \qquad (law\ of\ tension)$$

(iii) inversely proportional to its mass per unit length.

$$\vartheta \propto \frac{1}{\sqrt{m}} \qquad (law\ of\ mass)$$

Combining all the factors, we get

$$\vartheta = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

Where ρ is the density and D the diameter of the string.

Doppler effect in sound.

The phenomenon of the change in apparent pitch of sound due to relative motion between the source of sound and the observer is called Doppler effect. If $v, v_0, v_s\ and\ v_m$ are the velocities of sound, observer, source and medium (in the direction of sound) respectively, then the apparent frequency is given by

$$\vartheta' = \frac{v + v_m - v_0}{v + v_m - v_s} \times \vartheta$$

For the medium at rest $(v_m = 0)$,

$$\vartheta' = \frac{v - v_0}{v - v_s} \times \vartheta$$

Here the velocities are taken positive in the source to observer (S→O) direction and negative in the opposite (O→S) direction.

Special cases:

(i) when the source moves towards the stationary observer,

$$\vartheta' = \frac{v}{v - v_s} \times \vartheta \qquad (\vartheta' > \vartheta)$$

(ii) when the source moves away from the stationary observer,

$$\vartheta' = \frac{v}{v + v_s} \times \vartheta \qquad (\vartheta' < \vartheta)$$

(iii) when the observer moves towards the stationary source,

$$\vartheta' = \frac{v + v_0}{v} \times \vartheta \qquad (\vartheta' > \vartheta)$$

(iv) when the observer moves away from the stationary source,

$$\vartheta' = \frac{v - v_0}{v} \times \vartheta \qquad (\vartheta' < \vartheta)$$

(v) when both source and observer move towards each other,

$$\vartheta' = \frac{v + v_0}{v - v_s} \times \vartheta \qquad (\vartheta' > \vartheta)$$

(vi) when both source and observer move away from each other,

$$\vartheta' = \frac{v - v_0}{v + v_s} \times \vartheta \qquad (\vartheta' < \vartheta)$$

(vii) when source moves towards observer and observer away from the source,

$$\vartheta' = \frac{v - v_0}{v - v_s} \times \vartheta$$

(viii) when source moves away from observer and observer towards the source,

$$\vartheta' = \frac{v + v_0}{v + v_s} \times \vartheta$$