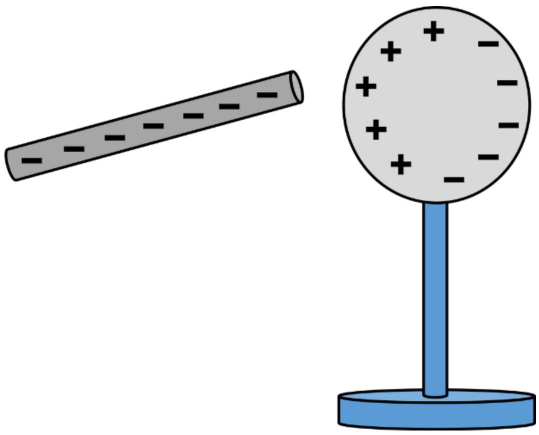
Electric Charges and Field

Electrostatics: It is the study of electric charges at rest.

Electric charge: It is an intrinsic property of elementary particles of matter which gives rise to electric force between various object. it is a scalar quantity and its SI unit is coulomb.

Fundamentals laws of electrostatics: Like charges repel and unlike charges attract each other.

Electrostatic Induction: It is the phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its father end in the presence of a nearby charged body. An insulated conductor can be positively or negatively charged by induction.



Three basic properties of electricity charges:

(i) quantization,

(ii) additivity, and

(iii) conservation.

Additivity of electric charge: This means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

Quantization of electric charge: This means that the total charge (*q)* of a body is always an integral multiple of a basic quantum of charge (*e) i.e.*

$q = ne$, where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

Faraday's laws of electrolysis and Millikan's oil drop experiment established the quantum nature of electric charge.

For macroscopically large charges, the quantization of charges can be ignored.

Basic Quantum of charge: The smallest amount of charge or the basic quantum of charge is the charge on an electron or proton. Its exact magnitude is

$e = 1.602182 \times 10^{-19}$ C.

Law of conservation of charge: It states that the total charge of a system remains unchanged with time. This means that when bodies are charged through friction, there is only transfer of charge from one body to another but no net creation or destruction of charge takes place.

Coulomb's laws: The force of attraction or repulsion between two stationary point charges q_1 and q_2 is directly proportional to the product q_1q_2 and inversely proportional to the square of the distance r between them. Mathematically,

$$F = k\frac{q_1q_2}{r^2}$$

The proportional constant k depends on the nature of the medium between the two charges and the system of units chosen to measure F, q_1, q_2 and r. For free space and in SI unit,

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2C^{-2}$$

ε_0 is called permittivity of free space and its value is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m$

Hence Coulomb's law in SI units maybe expressed as

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r^2}$$

SI unit of charge is coulomb (C): it is that amount of charge that repels an equal and similar charge with a force of 9×10^9 N when placed in vacuum at a distance of one metre from it.

Permittivity (ε): it is the property of a medium which determines the electric force between two charges situated in that medium.

Dielectric constant or relative permittivity: The ratio ($\varepsilon/\varepsilon_0$) of the permittivity of the given medium to that of free space is known as relative permittivity (ε_r) or dielectric constant (k) of the given medium,

$$\varepsilon_r \quad or \quad k = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{vac}}{F_{med}}$$

The dielectric constant of a medium may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium

Coulomb's law for any medium other than vacuum can be written as

$$F_{med} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 k} \cdot \frac{q_1 q_2}{r^2} = \frac{F_{vac}}{k}.$$

Electric force *vs* gravitational force: Electrostatic forces are much stronger than gravitational forces. The ratio of the electric force and gravitational force between a proton and an electron is

$$\frac{F_e}{F_G} = \frac{ke^2}{Gm_p m_e} = 2.27 \times 10^{39}$$

Principle of superposition of electrostatic forces: When a number of charges are interacting, the total force on a given charge is the vector sum of the forces exerted on it due to all other charges. The force between two charges is not affected by the presence of other charges. The total force on charge q_1 due to the charges q_2, q_3, …. Will be

$$\overrightarrow{F_1} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \cdots + \overrightarrow{F_{1N}}$$

$$= \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{N} \frac{q_i}{r_{1i}^2} \widehat{r_{1\imath}}$$

$$= \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{N} \frac{q_i(\overrightarrow{r_1} - \overrightarrow{r_\imath})}{|\overrightarrow{r_1} - \overrightarrow{r_\imath}|^3}$$

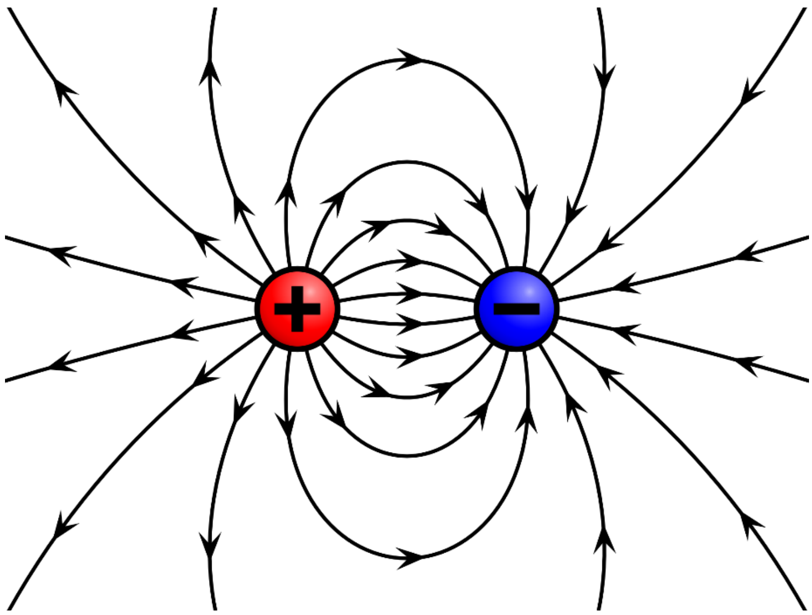
where,

$$\widehat{r_{1\imath}} = \frac{\overrightarrow{r_1} - \overrightarrow{r_\imath}}{|\overrightarrow{r_1} - \overrightarrow{r_\imath}|} = a\ unit\ vector\ pointing\ from\ q_i\ to\ q_1$$

Electric field: An electric field is said to exist at a point, if a force of electrical origin is exerted on a stationary charge placed at that point. Quantitatively, it is defined as the electrostatic force per unit test charge acting on a vanishingly small positive test charged place at the given point. Mathematically,

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

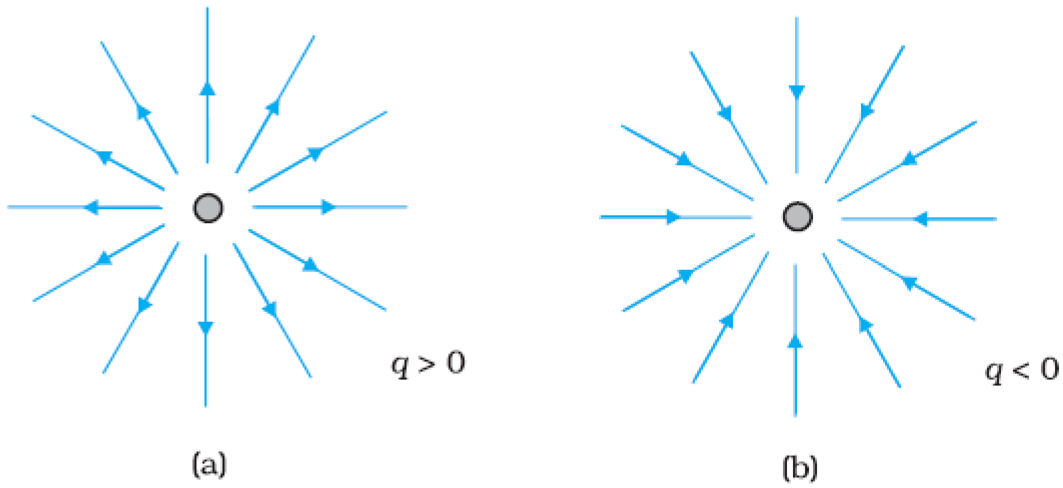
Electric field is a vector quantity whose direction is same as that of the force exerted on a positive test charge.



Electric field due to a point charge: The electric field of a point charge *q* at distance *r* from it is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

If *q* is positive, *E* points radially outwards and if *q* is negative, *E* points radially inwards. This field is spherically symmetric.



Electric field due to a system of point charges: superposition principal for electric fields: The principal states that the electric field at any point due to a group of point charges is equals to the vectors sum of the electric fields produced by each charge individually at the point, when all other charges are assumed to be absent.

$$\vec{E} = \vec{E_1} + \vec{E_2} + \cdots + \vec{E_N}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{N} \frac{q_i}{r_{iP}^2} \widehat{r_{\iota P}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r_\iota}|^3} (\vec{r} - \vec{r_\iota})$$

Continuous charge distribution: When the charge involved is much greater than the charge on an electron, we can ignore its quantum nature and assume that the charge is distributed in a continuous manner. This is known as a continuous charge distribution.

$$Volume\ charge\ density, \quad \rho = \frac{dq}{dV} \quad Cm^{-3}$$

$$Surface\ charge\ density, \quad \sigma = \frac{dq}{dS} \quad Cm^{-2}$$

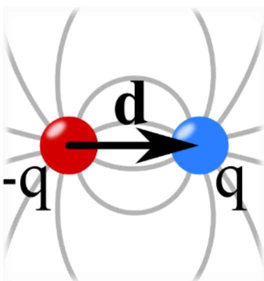
$$Linear\ charge\ density, \quad \lambda = \frac{dq}{dL} \quad Cm^{-1}$$

Electric dipole and dipole moment: An electric dipole is a pair of equal and opposite charges +*q* and -*q* separated by some distance 2*a*. Its dipole moment is given by

$$\vec{p} = Either\ charge\ \times vector\ drawn\ from - q\ to + q = q \times 2\vec{a}$$

$$Magnitude\ of\ dipole\ moment, \quad p = q\ \times 2a$$

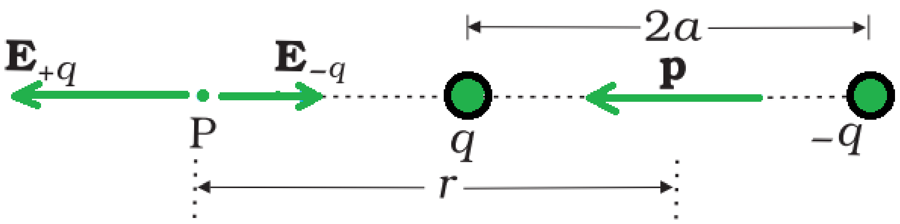
Dipole moment is a vector quantity having direction along the dipole axis from -*q* to +*q*. Its SI unit is coulomb metre (Cm).



Electric field at an axial point of a dipole: The dipole field on the axis as distance *r* from the centre is

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2} \approx \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \quad for\ r \gg a.$$

At any axial point, the direction of dipole field is along the direction of dipole moment $\vec{p}$.

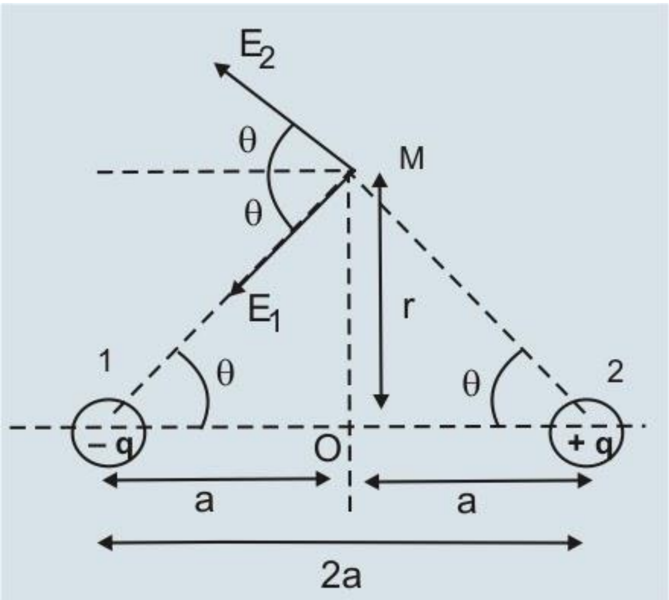


Electric field at an equatorial point of dipole: The electric field at point on the perpendicular bisector of the dipole at distance *r* from its centre is

$$E_{equa} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \quad for\, r \gg a.$$

At any equation equatorial point, the direction of dipole field is anti- parallel to the direction of dipole.

In contrast to $1/r^2$ dependence of the electric field of a point charge, the dipole field has $1/r^3$ dependence. Moreover, the electric field due to a short dipole at a certain distance along the axis is twice the electric field at the same distance along the equatorial line.



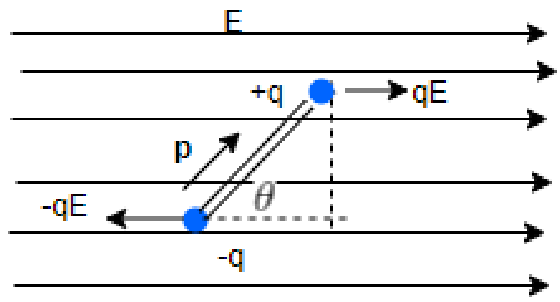
Torque on a dipole in a uniform electric field: The torque on a dipole of moment *p* when placed in a uniform electric field at an angle θ with it is given by

$$\tau = pE\sin\theta$$

$$In\ vector\ form, \quad \vec{\tau} = \vec{p} \times \vec{E}$$

When the dipole is released, the torque tends to align dipole along the field $\vec{E}$.

If E = 1 unit and θ = $90°$, then τ = p. So, dipole moment may also be defined as the torque acting on electric dipole placed perpendicular to a uniform electric field of unit strength.



Important properties of electric lines of force: There are:

(i) Lines of force are continuous curves without any breaks.

(ii) No two lines of force can cross each other.

(iii) They start at positive charges and end at negative charges – they cannot form closed loops.

(iv) The relative closeness of the lines of force indicates the strength of electric field at different points.

(v) They are always normal to the surface of a conductor.

(vi) They have a tendency to contract length-wise and expand laterally.

Electric flux: The electric flux through a given area represents the total number of electric lines of force passing normally through that area. If the electric field *E* makes an angle θ with the normal to the area element ΔS, then the electric flux is

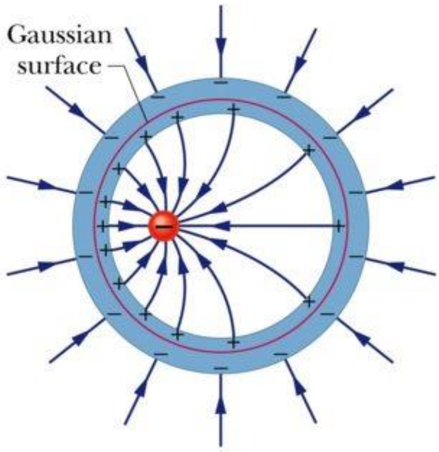
$$\Delta\phi_E = E\Delta S\, cos\theta = \vec{E}\,.\overrightarrow{\Delta S}$$

The electric flux through any surface any surface *S,* open or closed, is equal to the surface integral of $\vec{E}$ over the surface *S.*

$$\phi_E = \int_{Surface} \vec{E}.\overrightarrow{dS}$$

Electric flux is a scalar quantity.
SI unit of electric flux = $Nm^2\, C^{-1}$

Gaussian Surface: Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.



Gauss's theorem: The total flux of electric field $\vec{E}$ through a closed surface $\vec{S}$ is equal to $1/\varepsilon_0$ times the charge *q* enclosed by the surface $\vec{S}$*.*

$$\phi_E = \oint \vec{E}.\overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

Electric field of a line charge: The electric field of a long straight wire of uniform linear charge density λ,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad i.e., \quad E \propto \frac{1}{r}$$

Where *r* is the perpendicular distance of the wire from the observation point.

Electric field of an infinite plane sheet of charge: It does not depend on the distance of the observation point from the plane sheet.

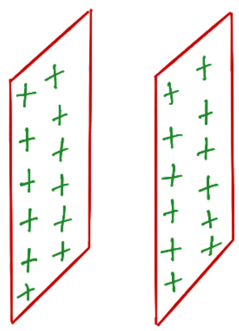
$$E = \frac{\sigma}{2\varepsilon_0}$$

Where $\sigma = uniform\ surface\ charge\ density$

Electric field of two positively charged parallel plates: If the two plates have surface charge densities σ_1 and σ_2 such that $\sigma_1 > \sigma_2 > 0$, then

$$E = \pm\frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2) \quad (outside\ the\ plates)$$

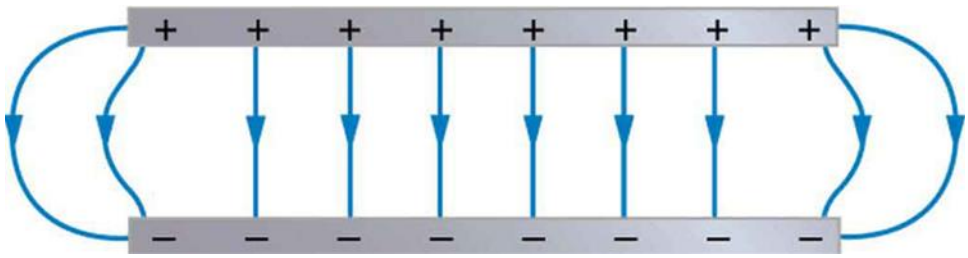
$$E = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2) \quad (inside\ the\ plates)$$



Electric field of two equally and oppositely charged parallel plates: If the two plates have surface charge densities $\pm\sigma$ then

$$E = 0 \qquad (for\ outside\ points)$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad (for\ inside\ points)$$



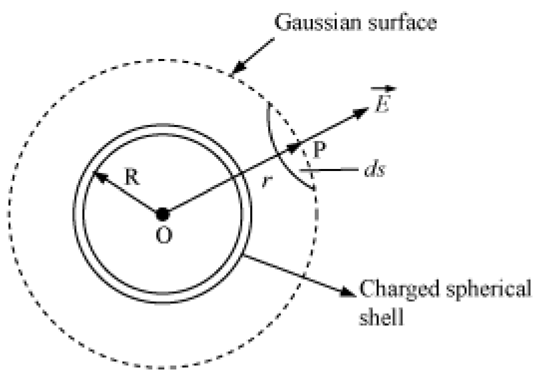
Electric field of a thin spherical shell: If *R* is the radius and σ, the surface charge density of the shell, then

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \qquad for\, r > R\ \ (outside\ points)$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \qquad for\, r = R\ \ (At\ the\ surface)$$

$$E = 0 \qquad for\, r < R\ \ (inside\ points)$$

$$where, \qquad q = 4\pi R^2\sigma$$



Electric field of a uniformly charged solid sphere: If ρ is the uniform volume charge density and *R* radius of the sphere, then

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \qquad for\, r > R\ \ (outside\ points)$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qr}{R^3} \qquad for\, r < R\ \ (inside\ points)$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \qquad for\, r = R\ \ (At\ the\ surface)\ where\ q = \frac{4}{3}\pi R^3\rho$$