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$$\vec{\nabla}(h^T x) = \vec{\nabla}\left(\sum_{j=1}^{n} h_j \; x_j\right) = h$$

$$\tfrac{1}{2}x^T R x = \tfrac{1}{2}x^T \sum_{j=1}^{n} \begin{bmatrix} R_{1j} \; x_j \\ R_{2j} \; x_j \\ \vdots \\ R_{nj} \; x_j \end{bmatrix} = \tfrac{1}{2}\left(x_1 \sum_{j=1}^{n} R_{1j} \; x_j + x_2 \sum_{j=1}^{n} R_{2j} \; x_j + \cdots + x_n \sum_{j=1}^{n} \; x_j \right) \tag{1}$$

$$\frac{1}{2}\vec{\nabla}(x^T R x) = \frac{1}{2}\begin{bmatrix} x_1 R_{11} + \sum_{j=1}^{n} R_{1j} \; x_j + \sum_{j=2}^{n} R_{j1} \; x_j \\ x_1 R_{12} + x_2 R_{22} + \sum_{j=1}^{n} R_{2j} \; x_j + \sum_{j=3}^{n} R_{j2} \; x_j \\ \vdots \\ \sum_{j=1}^{n} R_{nj} \; x_j + \sum_{j=1}^{n} R_{jn} \; x_j \end{bmatrix} = \frac{1}{2}\left(\sum_{j=1}^{n} \begin{bmatrix} R_{1j} \; x_j \\ R_{2j} \; x_j \\ \vdots \\ R_{nj} \; x_j \end{bmatrix} + \sum_{j=1}^{n} \begin{bmatrix} R_{j1} \; x_j \\ R_{j2} \; x_j \\ \vdots \\ R_{jn} \; x_j \end{bmatrix} \right)$$

$$= \frac{1}{2}\left(Rx + R^T x\right) = \frac{1}{2}(R + R^T)x = Rx.$$

So, $\vec{\nabla}F(x) = h + Rx$.

$\mathbf{H}_h = 0 \because h(x_i^1), \text{and } \frac{\partial^2 h}{\partial x_i \partial x_j} = 0.$

From (1):

$$\mathbf{H}_{\frac{1}{2}x^T R x} = \frac{1}{2}\begin{bmatrix} \dfrac{\partial^2(x_1 R_{11} x_1)}{\partial x_1^2} & \dfrac{\partial^2(x_1 R_{12} x_2 + x_2 R_{21} x_1)}{\partial x_1 \, \partial x_2} & \cdots & \dfrac{\partial^2(x_1 R_{1n} x_n + x_n R_{n1} x_1)}{\partial x_1 \, \partial x_n} \\ \dfrac{\partial^2(x_1 R_{12} x_2 + x_2 R_{21} x_1)}{\partial x_2 \, \partial x_1} & \dfrac{\partial^2(x_2 R_{22} x_2)}{\partial x_2^2} & \cdots & \dfrac{\partial^2(x_2 R_{2n} x_n + x_n R_{n2} x_2)}{\partial x_2 \, \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2(x_1 R_{1n} x_n + x_n R_{n1} x_1)}{\partial x_n \, \partial x_1} & \dfrac{\partial^2(x_2 R_{2n} x_n + x_n R_{n2} x_2)}{\partial x_n \, \partial x_2} & \cdots & \dfrac{\partial^2(x_n R_{nn} x_n)}{\partial x_n^2} \end{bmatrix} = R$$

Hence, $\mathbf{H}_{F(x)} = R$.