CSE-483: Mobile Robotics	**Monsoon 2019**

Epipolar Geometry - II

Prepared by: Pranathi Bhuvanagiri , Shreya Terupally

1 Fundamental Matrix

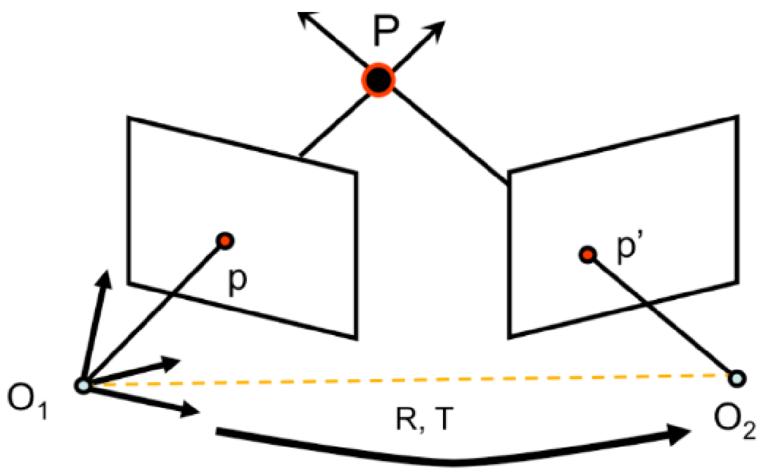

Figure 1: Epipolar Geometry

1. From the figure, we know that O1, O2 are the centers of 2 cameras with intrinsic matrix K. Let P be a 3D point in the world. p and p' are the projections of the point P in O1 and O2 respectively. If the relative rotation and translation of O2 with respect to O1 is R,t.Then the Fundamental matrix

$$F = K^{-T}[t_X]RK^{-1}$$

where tx (3x3) is the skew symmetric cross product matrix of t

2. Fundamental matrix has 7 dof and rank 2.

1.1 Epipolar Constraint

1. Since O1P, O2P and O1O2 lie on a single plane, we write the coplanarity equation to get the epipolar constraint.

$$p^T F p' = 0 \tag{1}$$

   where p and p' are the corresponding image coordinates on O1 and O2 respectively

2. If we rip F of camera intrinsics, K, we get Essential matrix

$$E = K^T F K$$

   The new epipolar constraint is =

$$x^T E x' = 0$$

   Where, x and x' are camera canonical image co-ordinates.

2 Estimation of Fundamental Matrix(F)

2.1 Eight Point Algorithm

1. Eight-Point Algorithm is used to estimate the Fundamental matrix if we are given two images of the same scene without knowing the extrinsic or intrinsic parameters of the camera. The Eight-Point Algorithm assumes that a set of at least 8 pairs of corresponding points between two images is known.

2. These correspondences gives us the epipolar constraint(discussed above).

$$p^T F p' = 0$$

   where p, p' are homogenised image coordinates of the form [u v 1] and [u' v' 1] respectively.
   Note : p and p' are column vectors of dimensions 3x1.

3. This constraint can be reformulated as:

$$A * f = 0$$

where A =

$$Kroneckerproductofp^T andp'^T = [u_i u_i^{'}, v_i v_i^{'}, u_i, v_i u_i^{'}, v_i v_i^{'}, v_i, u_i^{'}, v_i^{'}, 1]$$

and f = reshaped F matrix (9x1)

$$f = [F11, F12, F13, F21, F22, F23, F31, F32, F33]^T$$

4. Since this constraint is a scalar equation, it only constrains one degree of freedom. Since we can only know the Fundamental matrix up to scale, we require eight of these constraints to determine the Fundamental matrix:

Estimating F

$$\underbrace{\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' & 1 \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' & 1 \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' & 1 \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' & 1 \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' & 1 \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' & 1 \end{pmatrix}}_{\mathbf{W}} \underbrace{\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix}}_{\mathbf{f}} = 0$$

- Homogeneous system $\mathbf{W}\,\mathbf{f} = 0$

   (a) where W is an Nx9 matrix derived from N>=8 correspondences and f is the values of the Fundamental matrix.

5. It is better to use more than eight correspondences and create a larger W matrix because it reduces the effects of noisy measurements.

6. The solution to this system of homogeneous equations can be found in the least squares sense by Singular Value Decomposition.

7. The estimate of the Fundamental matrix F may have full rank. However, we know that the true Fundamental matrix has rank 2.So, we have to enforce the rank 2 constraint and recreate the Fundamental matrix.

$$F = U \sum V^T$$

8. Rank 2 enforcement : We need

$$det|F| = 0$$

The New Fundamental matrix F =

$$U \begin{vmatrix} \sum_1 & 0 & 0 \\ 0 & \sum_2 & 0 \\ 0 & 0 & 0 \end{vmatrix} V^T$$

2.2 The Normalized Eight Point Algorithm

1. A typical homogeneous representation of the 2D image coordinate (u1,v1)

$$\mathbf{p} = \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$

where both u1,v2 lie in the range 0 to 1000-2000 for a modern digital camera. This means that the first two coordinates in y vary over a much larger range than the third coordinate.

2. To solve this problem, the coordinate system of each of the two images should be transformed, independently, into a new coordinate system by translating it to the centroid of image points and uniformly scaling so that the mean distance from the origin to a point equals 2.

3. This principle results in a distinct coordinate transformation for each of the two images. As a result, new homogeneous image coordinates y, y' are given by

$$y = Tp$$
$$y' = T'p'$$

where T , T' are the transformations (translation and scaling) from the old to the new normalized image coordinates

4. Let the fundamental matrix from these normalised coordinates be Fn. The final Fundamental matrix F

$$F = T^T F_n T'$$

3 RANSAC in F matrix Estimation

1. We discussed earlier that we need at least 8 correspondences to estimate F matrix. These correspondences are detected and matched using a feature detector(Example: SIFT). There can be some correspondences that don't fit the model(Outliers). So, we don't want to use these points in our estimation.

2. The least square estimation discussed above is sensitive to outliers. So, we need a method that is robust to these outliers.

3.1 RANdom SAmple Consensus(RANSAC) Algorithm

(a) RANSAC is a general parameter estimation approach which is robust to outliers in a data.

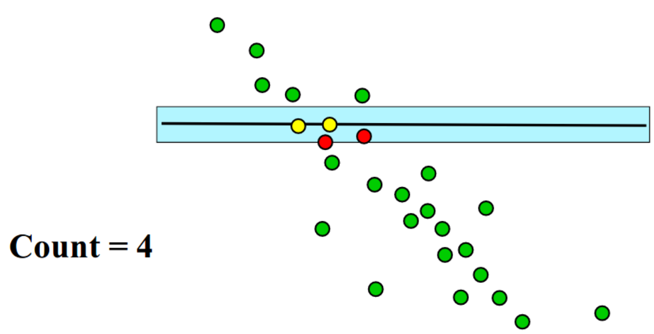

Figure 2: In this image, for the model fitted by the 2 yellow points, the 2 red points lie in the threshold distance, therefore the count of inliers is 4.

(b) In this model, a sample of s points is selected randomly from the given data. If the distance from a point to the line is less than the threshold(d) then the point fits the model(inlier). We pick the model which has maximum number of inliers or has the no of inliers greater than the required.

(c) The RANSAC Algorithm for fitting a model

```
Determine:
    s — the smallest number of points required
    N — the number of iterations required
    d — the threshold used to identify a point that fits well
    T — the number of nearby points required
      to assert a model fits well
Until N iterations have occurred
    Draw a sample of s points from the data
      uniformly and at random
    Fit to that set of s points
    For each data point outside the sample
      Test the distance from the point to the line
        against d if the distance from the point to the line
        is less than d the point is close
    end
    If there are T or more points close to the line
      then there is a good fit. Refit the line using all
      these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion
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(d) To use the RANSAC algorithm in F matrix estimation, we take the s as 8 since we need atleast 8 points to determine F.

(e) While fitting the model to that set of s points, we use 8-point algorithm to determine F and enforce the rank 2 constraint.

(f) The number of inliers is the number of elements in W*f vector that are less than or equal to threshold value, where W is (Nx9) matrix with N = the total number of correspondences.

3.2 N: Number of iterations and Termination

(a) Let
e = probability that a point is outlier
s = number of samples in a subset
p = The probability of getting a subset of all inliers
N = The number of iterations required

(b) The probability that we get a subset of all inliers =

$$(1-e)^s$$

The probability that we do not get this subset in an iteration =

$$1-(1-e)^s$$

The probability that we do not get this subset in N iterations =

$$(1-(1-e)^s)^N$$

We need this subset of all inliers at least once in N iterations with probability = p
Therefore the probability that we do not get this subset even once in N iterations = 1-p

(c) The final equation =

$$1-p=(1-(1-e)^s)^N$$

Therefore, required N =

$$\frac{log(1-p)}{log(1-(1-e)^s)}$$

3.3 T : Minimum number of required inliers for a model

(a) Let e be the probability that a point is outlier. Then required inliers to total points(P) ratio for a model = 1-e

$$\frac{T}{P}=1-e$$

References

[1] https://web.stanford.edu/class/cs231a/coursenotes/03epipolargeometry.pdf

[2] Lecture Slides