CSE-483: Mobile Robotics **Monsoon 2019**

Triangulation: Linear and Geometric Estimation

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1 Triangulation: Introduction

1. Given the relative orientation of two cameras and the images captured, we need to compute the location of corresponding points in 3D. Triangulation exactly helps in that.
2. One point to note here is that Triangulation computes the location of said 3D points with respect to the camera frame.
3. To find the 3D points in World Frame, we need to find the absolute orientation of these exploiting known control points.

2 Linear Triangulation Method

The Linear Triangulation Method is the direct analogue of the DLT Method. In each image, we have a measurement x = PX, x' = P'X, which when combined into matrix form AX = 0, result in a equation linear in X. Let's consider an example. We need to eliminate the homogeneous scale factor by cross product.

x $\times(PX) = 0$

$$x(p^{3T}X) - (p^{1T}X) = 0$$
$$y(p^{3T}X) - (p^{2T}X) = 0$$
$$x(p^{2T}X) - y(p^{1T}X) = 0$$

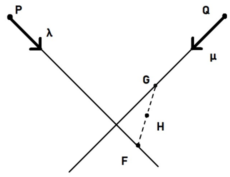
where p^{iT} are the rows of P. These equations are linear in components of X. From this, an equation of the form AX = 0 can be composed,

$$\text{A} = \begin{bmatrix} xp^{3T} - p^{1T} \\ yp^{3T} - p^{2T} \\ x^{'}p^{'3T} - p^{'1T} \\ y^{'}p^{'3T} - p^{'2T} \end{bmatrix}$$

where two equations have been included from each image, giving a total of four equations in four homogeneous unknowns. This equation can be solved using Homogeneous Method (DLT) and Inhomogeneous Method that are already discussed in previous lectures.

3 Geometric Method

The simplest way to imagine our problem is as follows:



where P, Q are the Projection Centres of our Cameras. λ, μ are the direction vectors from camera centres toward the point in the 3D world.

The problem with this simplistic approach is that these two lines may not intersect at all in 3D!

So we take the shortest distance between the two lines, FG and we take the mid-point H as the intersection of these lines, which is our interest in this case.

One more point to note here is that we're considering a purely geometric approach and completely ignoring the facts that one image might be blurry, precision in calculating both the points might not be the same etc.,

The line equations are as follows:

f(λ) $= p + \lambda.r$

$g(\mu) = q + \mu.s$

Here, we already know what p, q are. They are the rotation centres of our cameras, which can be written as:

p = $X_0^{'}$

q = $X_0^{''}$

Assuming calibrated cameras, we can write vectors r and s as follows:

r = $R^{'T}.^{k}X^{'}$

$s = R^{''T}.^{k}X^{''}$

$where$

$R^{'T}$ = rotation matrix of first cam

$K_{X'}$ = corresponding point in the first image in cam frame

Similarly

$R^{''T}$ = rotation matrix of second cam

$K_{X''}$ = corresponding point in the second image in cam frame

Again,

$K_{X'} = (x^{'}, y^{'}, c)^T$

$K_{X''} = (x'', y'', c)^T$

We need to find FG. Since it is the shortest distance, it is perpendicular to both the lines. Hence

$(f - g).r = 0$

$(f - g).s = 0$

Expanding

$(p + \lambda.r - q - \mu.s).r = 0$

$(p + \lambda.r - q - \mu.s).s = 0$

Two equations, two unknowns, λ, μ. These can be solved as follows, hence giving us the point of intersection.

$$\begin{bmatrix} r.r - s.s \\ r.s - s.s \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} (Q - P).r \\ (Q - P).s \end{bmatrix}$$

$$\begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} r.r - s.r \\ r.s - s.s \end{bmatrix} {}^{-1} \begin{bmatrix} (Q - P).r \\ (Q - P).s \end{bmatrix}$$

λ, μ are known. Hence F, G can be obtained, from which we can calculate H as the midpoint.

References

[1] Photogrammetry - II by Cyrill Strachniss

[2] Class Notes and Slides

[3] Multi-View Geometry - Hartley and Zisserman