Date
Page

1. Compute the eigen values of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1-d & 2 & 3 \\ 4 & 5-d & 6 \\ 7 & 8 & 9-d \end{bmatrix}$$

$$\Rightarrow (1-d)\left[(5-d)(9-d) - 6\times 8\right]$$

$$- 2\left[4(9-d) - 7\times 6\right]$$

$$+ 3\left[4\times 8 - 7\times(5-d)\right] = 0$$

$$\Rightarrow -d^3 + 15d^2 + 18d = 0$$

$$\Rightarrow -d(d^2 - 15d - 18) = 0$$

$$\Rightarrow d = 0 \quad -①$$

also

$$d^2 - 15d - 18 = 0$$

$$\Rightarrow d = \frac{15 \pm \sqrt{225 - 4(1)(-18)}}{2}$$

$$= \frac{15 + \sqrt{225 + 22}}{2} = \frac{15 \pm \sqrt{247}}{2}$$

$$= \frac{15 \pm 15.716}{2} = \overset{15.358}{\cancel{23.21}}, -0.35$$

$$\Rightarrow d_1 = 0, \quad d_2 = \overset{15.35}{\cancel{23.21}}, \quad d_3 = -0.35$$

for d_1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

using crammer's rule:

$$1x_1 + 2x_2 + 3x_3 = 0$$
$$4x_1 + 5x_2 + 6x_3 = 0$$

$$\Rightarrow \frac{x_1}{\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{-3} = \frac{-x_2}{-6} = \frac{x_3}{-3}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

$$\Rightarrow V_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

classmate

Date
Page

for $\lambda_2 = 15.35$

$$\begin{bmatrix} -14.358 & 2 & 3 \\ 4 & -10.35 & 6 \\ 7 & 8 & -6.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ -10.35 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -14.35 & 3 \\ 4 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -14.35 & 2 \\ 4 & -10.35 \end{vmatrix}}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 43.05 \\ 98.1 \\ 146.52 \end{bmatrix}$$

for $\lambda_3 = -0.35$

$$\begin{bmatrix} 1.35 & 2 & 3 \\ 4 & 5.35 & 6 \\ 7 & 8 & 9.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4.05 \\ 3.9 \\ 0.77 \end{bmatrix}$$

trace = 15.

Determinant = 0

rank = 2.

classmate

Date

Page

a) trace = Sum of Eigen values

trace = 15,

Sum of Eigen values =

$$\frac{15+\sqrt{247}}{2} + \frac{15-\sqrt{247}}{2}$$

$$= \frac{30+2\cancel{\sqrt{247}}+30-2\cancel{\sqrt{247}}}{4}$$

$$= \frac{60}{4} = \underline{\underline{15}}.$$

b) product of Eigen values = Detrmiant

$\Rightarrow$ product = 0

$\underline{\underline{D = 0}}$.

c) No. of non-zero elements in eigen value ~~in a Matrix~~

= rank.

No. of non-zero element in eigen values = 2

rank = $\underline{\underline{2}}$.

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2. $x \to q$ dimentional vector.
$y \to p$ dimentional Vector.

$$y_i = Ax_i$$

a) $\Rightarrow$ dimentions of A : $p \times q$

b) Condition A should have a dimention of $(q-1) \times (q)$

and x should be such that

$x_2 - x_1$, $x_3 - x_1$, $x_4 - x_1$, ... $x_q - x_1$ all should be orthogonal to each other.

Yes such A can exist.

c) (a) $q = 2$, $p = 2$.
yes it's basically no dimentional reduction therefore A can be I_2.

b) $q = 2$, $p = 1$.

$A = [1 \quad 1]$

(c) = $q = 4$. & $p = 2$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3.

3.1
There will be 1 Non Zero EigenValue in the matrix because the number of Non Zero EigenValues in a Matrix is Equal to the Rank of the Matrix, because the rows of the Matrix are linearly Dependent. Therefore there is 1 Non Zero value in Matrix

3.2
Equation of the Line perpendicular to the line w1x + w2x + w3 = 0 is **w2x - w1x + w3 = 0**.
Just like on the Previous part of the question here also the rows are linearly Dependent hence the rank of matrix is 1 Hence the Number of Non zero eigen values is one.

3.3
The Covariance matrix is having a full rank because here in the case of points around the line there is a possibility that some points go outside the line's equation hence they wont satisfy the Equation of the line and hence the Covariance matrix will have the Full rank, but here one of the eigen values wil be much greater than the other one because the points are concentrated around a particular line and most of them tend to follow the equation of the line but cannot because of the noise, and the Eigen vectors will be Orthogonal to each other with the Larger vector along the line and other smaller vector along perpendicular to the Line.

CODE for 4th Part:

```
import numpy as np
import random
import matplotlib.pyplot as plt

data = []
# Generating Random Points
for z in range(1000):
    x = random.randint(-10,10) + random.randint(-10,10)/10
    y = x+ random.randint(-10,10)/10;
    a = [x,y]
    data.append(a)

# Plotting the Points
x,y = np.array(data).T
mean_x = np.mean(x)
mean_y = np.mean(y)

print ("covariance : ")
covaar_mat = np.cov(x,y)
print(covaar_mat)
```

```
print ("Eigen Values : ")
Eigen_values, Eigen_vectors = np.linalg.eig(covaar_mat)
print(Eigen_values)
print ("Eigen Vectors : ")
print(Eigen_vectors)
plt.scatter(x,y)
plt.arrow(mean_x,mean_y,Eigen_vectors[0][0],Eigen_vectors[1][0],head_width=0.5,head_length
=0.2,color="r")
plt.arrow(mean_x,mean_y,-Eigen_vectors[0][1],-Eigen_vectors[1][1],head_width=0.5,head_lengt
h=0.2,color="g")
plt.show()
```