

1. if \vec{A} & \vec{B} are the vectors
then the cosine similarity between
these vectors will be

$$D = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

$$= \frac{\sum_{i=1}^n A_i B_i}{\sqrt{\sum_{i=1}^n A_i^2} \sqrt{\sum_{i=1}^n B_i^2}}$$

$$= \frac{\sum_{i=1}^n A_i B_i}{\sqrt{\sum_{i=1}^n A_i^2} \sqrt{\sum_{i=1}^n B_i^2}}$$

Demonstrating that the value can be -1 :
e.g. $[a \ b \ c \ d]^T$ & $[-a \ -b \ -c \ -d]^T$.

$$\Rightarrow D = \frac{-a^2 - b^2 - c^2 - d^2}{\sqrt{a^2 + b^2 + c^2 + d^2} \sqrt{a^2 + b^2 + c^2 + d^2}}$$

$$= \frac{-1(-a^2 - b^2 - c^2 - d^2)}{(a^2 + b^2 + c^2 + d^2)}$$

$$= -1$$

e.g. $[a \ -a \ b \ -b]^T$ & $[a \ a \ b \ b]^T$.

$$\Rightarrow D = \frac{a^2 - a^2 + b^2 - b^2}{\sqrt{a^2 + b^2} \sqrt{a^2 + b^2}} = \underline{\underline{0}}$$

$$I = [a \ b \ c \ d] [a \ b \ c \ d]^T$$

$$I = a^2 + b^2 + c^2 + d^2$$

$$\sqrt{a^2 + b^2 + c^2 + d^2} \cdot \sqrt{a^2 + b^2 + c^2 + d^2}$$

$$= \frac{a^2 + b^2 + c^2 + d^2}{a^2 + b^2 + c^2 + d^2} = \underline{\underline{1}}$$

2. a) if all the sample are multiplied by the same scalar then

$$x_i \rightarrow \alpha x_i$$

where $\alpha \in \mathbb{R}^+$

$$\text{So if } w^T x_i \geq 0$$

$$\text{then } \alpha w^T x_i \geq 0$$

$$\text{or if } w^T x_i < 0$$

$$\text{then } \alpha w^T x_i < 0.$$

hence, the accuracy of the model won't change.

b) Similar to above example it does not depend if all the sample are multiplied to same scalar or to different scalar until the scalar is changing the sign of $w^T x$ the accuracy will not change.

and since $\alpha \in \mathbb{R}^+$ then the accuracy will be same.

c) When the sample are multiplied by orthogonal matrix A , then the angle between do not change & also the length of the vectors do not change but angle of whole all the points change with x -axis (or fixed axis) but not relative to each other, hence ~~there will be no change in accuracy of the model.~~

hence the points may cross-over to the other side of line resulting in change of ~~accuracy~~ \rightarrow accuracy.

d) if the matrix is rank deficient it means that the columns are dependent this means few are independent also and this will make the accuracies change.

this is the Code for 3rd part of the Assignment

```
import random
import matplotlib.pyplot as plt
import numpy as np
```

```
def RC_Multiply(a,b):
    sum = int("0",10)
    for i in range(3):
        sum += a[i]*b[i]
    return sum;
```

```
Data = []
plt_dt = []
a = 0
for i in range(100):
    r_fra1 = random.randint(-1000,1000)/1000;
    r_fra2 = random.randint(-1000,1000)/1000;
    dat_plt = [r_fra1, r_fra2]
    plt_dt.append(dat_plt)
    a = [r_fra1, r_fra2, 1]
    if(i < 50):
        clas = "A"
    else:
        clas = "B"
    Data.append([a,clas])
```

```
w1 = [1,1,0] #plot this line
w2 = [-1,-1,0]
w3 = [0,0.5,0]
w4 = [1, -1,5] # plot this line
w5 = [1,1,0.3] # plot this line
```

```
acc_w1 = 0;
acc_w2 = 0;
acc_w3 = 0;
acc_w4 = 0;
acc_w5 = 0;
for i in range(100):
    if(RC_Multiply(w1,Data[i][0]) > 0 and Data[i][1]=="A"):
        acc_w1 += 1;
    elif(RC_Multiply(w1,Data[i][0]) <= 0 and Data[i][1]=="B"):
        acc_w1 += 1;
```

```

if(RC_Multiply(w1,Data[i][0]) > 0 and Data[i][1]=="A"):
    acc_w2 += 1;
elif(RC_Multiply(w2,Data[i][0]) <= 0 and Data[i][1]=="B"):
    acc_w2 += 1;
if(RC_Multiply(w3,Data[i][0]) > 0 and Data[i][1]=="A"):
    acc_w3 += 1;
elif(RC_Multiply(w3,Data[i][0]) <= 0 and Data[i][1]=="B"):
    acc_w3 += 1;
if(RC_Multiply(w4,Data[i][0]) > 0 and Data[i][1]=="A"):
    acc_w4 += 1;
elif(RC_Multiply(w4,Data[i][0]) <= 0 and Data[i][1]=="B"):
    acc_w4 += 1;
if(RC_Multiply(w5,Data[i][0]) > 0 and Data[i][1]=="A"):
    acc_w5 += 1;
elif(RC_Multiply(w5,Data[i][0]) <= 0 and Data[i][1]=="B"):
    acc_w5 += 1;

print("PART (i)")
print("Accuracy of a : "+str(acc_w1)+"%")
print("Accuracy of b : "+str(acc_w2)+"%")
print("Accuracy of c : "+str(acc_w3)+"%")
print("Accuracy of d : "+str(acc_w4)+"%")
print("Accuracy of e : "+str(acc_w5)+"%")

print("PART (ii)")
fig, ax = plt.subplots()
data_plot = np.array(plt_dt);
x,y = data_plot.T
ax.scatter(x,y)
x = np.array(range(-1,2))
y = (w1[0]/-w1[1])*x + (w1[2]/-w1[1])
la = ax.plot(x,y,label='line for a')
ax.set_title('With Boundry a')

fig,bx = plt.subplots()
data_plot = np.array(plt_dt);
x,y = data_plot.T
bx.scatter(x,y)
x = np.array(range(-1,2))
y = (w4[0]/-w4[1])*x + (w4[2]/-w4[1])
bx.plot(x,y,label='line for d')
bx.set_title('With Boundry b')

```

```
fig,cx = plt.subplots()
data_plot = np.array(plt_dt);
x,y = data_plot.T
cx.scatter(x,y)
x = np.array(range(-1,2))
y = (w5[0]/-w5[1])*x + (w5[2]/-w5[1])
le = cx.plot(x,y,label='line for e')
cx.set_title('With Boundry c')
plt.show()
```

PART (i)

Accuracy of a : 55%

Accuracy of b : 53%

Accuracy of c : 54%

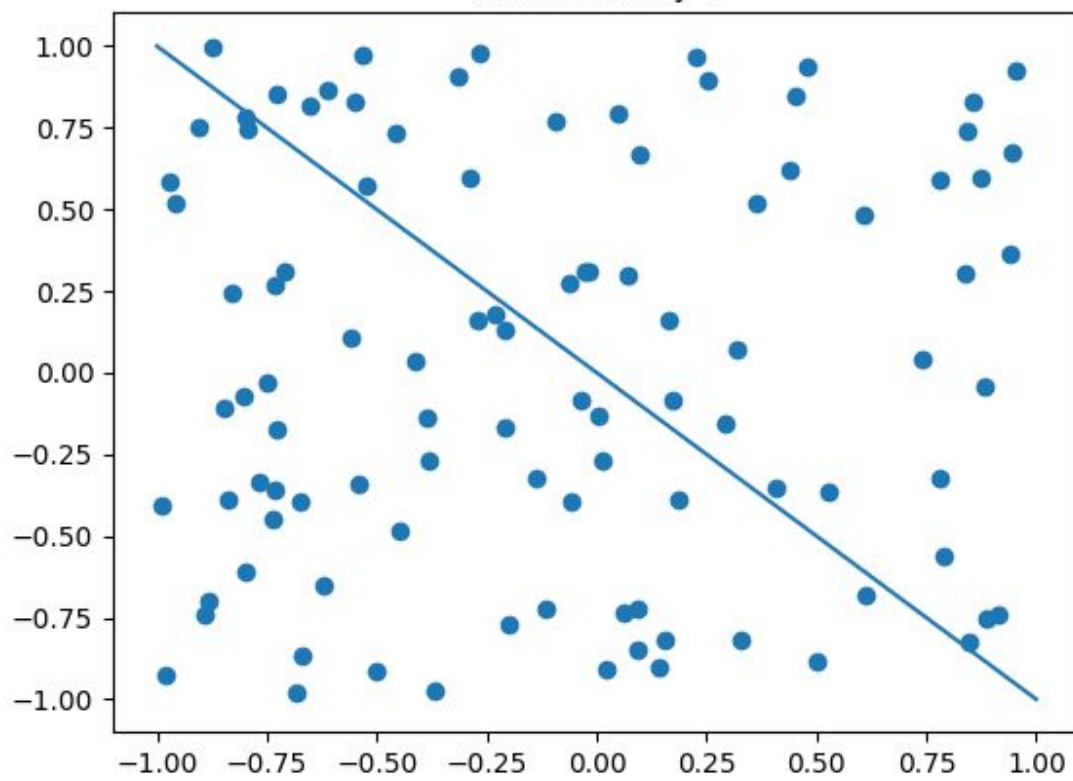
Accuracy of d : 50%

Accuracy of e : 51%

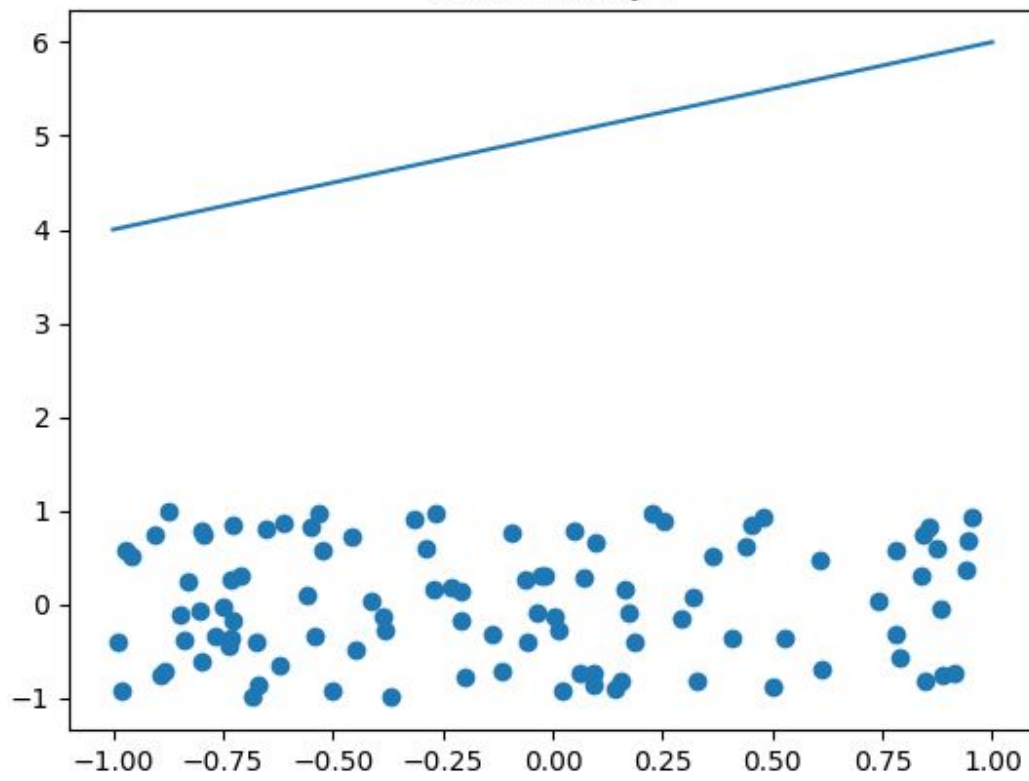
PART (ii)

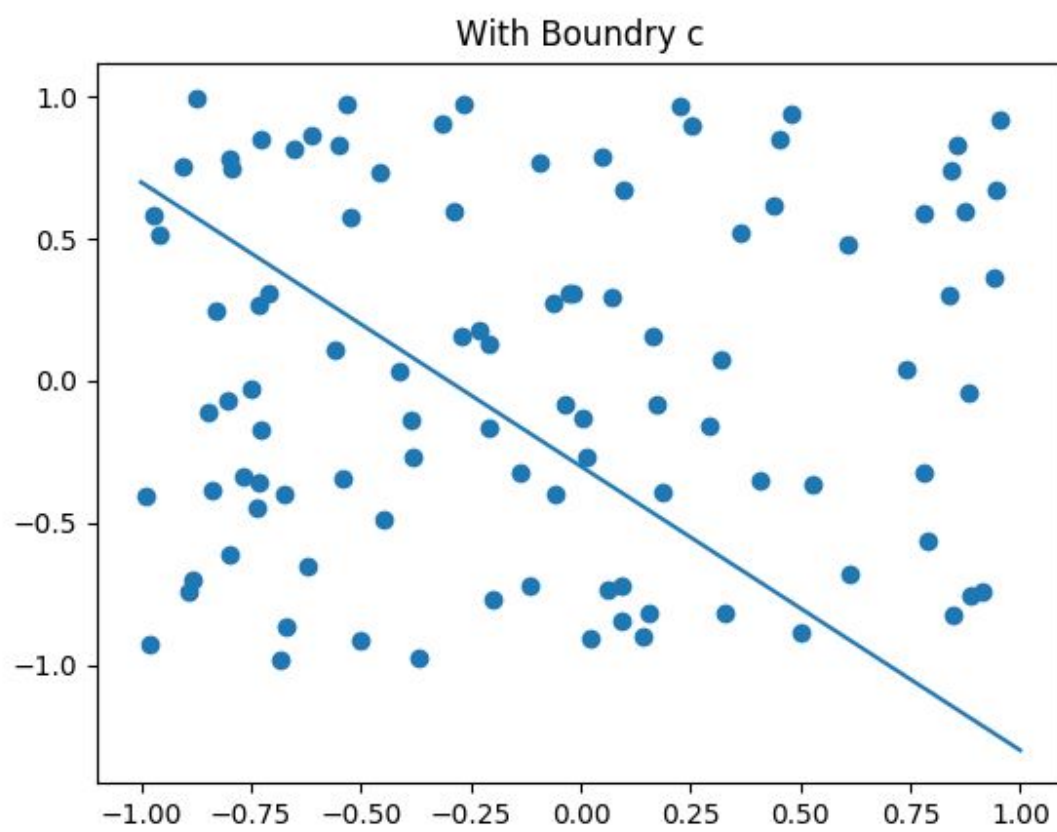
These almost show near about 50 % accuracy and since the smamples are distributed hence it is expected to show that the samples are 50 % of the time classified correctly.

With Boundry a



With Boundry b





The accuracies are random despite