d. Compute the eigen values of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1-d & 2 & 3 \\ 4 & 5-d & 6 \\ 7 & 8 & 9-d \end{bmatrix}$$

$$\Rightarrow (1-d)\left[(5-d)(9-d) - 6\times 8\right]$$
$$- 2\left[4(9-d) - 7\times 6\right]$$
$$+ 3\left[4\times 8 - 7\times(5-d)\right] = 0$$

$$\Rightarrow -d^3 + 15d^2 + 18d = 0$$
$$\Rightarrow -d(d^2 - 15d - 18) = 0$$
$$\Rightarrow d = 0 \quad -①$$

also

$$d^2 - 15d - 18 = 0$$

$$\Rightarrow d = \frac{15 \pm \sqrt{225 - 4(1)(-18)}}{2}$$

$$= \frac{15 \pm \sqrt{225 + 22}}{2} = \frac{15 \pm \sqrt{247}}{2}$$

$$= \frac{15 \pm 15.716}{2} = \overset{15.358}{\cancel{23.21}}, -0.35.$$

$$\Rightarrow d_1 = 0, \quad d_2 = \overset{15.35}{\cancel{23.21}}, \quad d_3 = -0.35.$$

for d_1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

using crammer's rule:

$$1x_1 + 2x_2 + 3x_3 = 0$$
$$4x_1 + 5x_2 + 6x_3 = 0$$

$$\Rightarrow \frac{x_1}{\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{-3} = \frac{-x_2}{-6} = \frac{x_3}{-3}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ \cancel{3} \end{bmatrix}.$$

$$\Rightarrow V_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

for $d_2 = 15.35$

$$\begin{bmatrix} -14.358 & 2 & 3 \\ 4 & -10.35 & 6 \\ 7 & 8 & -6.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ -10.35 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -14.35 & 3 \\ 4 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -14.35 & 2 \\ 4 & -10.35 \end{vmatrix}}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 43.05 \\ 98.6 \\ 146.52 \end{bmatrix}$$

for $d_3 = -0.35$

$$\begin{bmatrix} 1.35 & 2 & 3 \\ 4 & 5.35 & 6 \\ 7 & 8 & 9.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4.05 \\ 3.9 \\ 0.77 \end{bmatrix}$$

trace = 15.
Determinant = 0
rank = 2.

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a) trace = Sum of Eigen values

trace = 15,

Sum of Eigen values =

$$\frac{15+\sqrt{247}}{2} + \frac{15-\sqrt{247}}{2}$$

$$= \frac{30 + 2\cancel{\sqrt{247}} + 30 - 2\cancel{\sqrt{247}}}{4}$$

$$= \frac{60}{4} = \underline{\underline{15}}.$$

b) product of Eigen values = Detrmiant

⇒ product = 0

$\underline{\underline{D = 0}}$.

c) No. of non-zero elements in eigen value ~~in a Matrix~~ = rank.

No. of non-zero element in eigen values = 2

rank = $\underline{\underline{2}}$.

2. $x \rightarrow q$ dimentional vector
$y \rightarrow p$ dimentional Vector.

$$y_i = A x_i$$

a) => dimentions of A : $p \times q$

b) Condition A should have a dimention of $(q-1) \times (q)$

and x should be such that

$x_2 - x_1$, $x_3 - x_1$, $x_4 - x_1$, ... $x_q - x_1$
all should be orthogonal to each other.

Yes such A can exist.

c) (a) $q = 2$, $p = 2$.
yes it's basically no dimentional reduction therefore A can be I_2.

b) $q = 2$, $p = 1$.

$A = [1 \quad 1]$

(c) = $q = 4$, & $p = 2$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3.