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2.a) let the first elements be a $\Rightarrow$ mean $= \mu = a$ & $N = 1$.
as we get more elements we get a change in mean,

$$\mu_N = \frac{1}{N}\sum_{i=1}^{N} x_i \quad , \quad \mu_{N+1} = \frac{1}{N+1}\sum_{i=1}^{N+1} x_i$$

$$\Rightarrow \mu_d = \mu_{N+1} - \mu_N$$

$$= \frac{1}{N+1}\sum_{i=1}^{N+1} x_i - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)$$

$$= \frac{N\sum_{i=1}^{N+1} x_i - (N+1)\sum_{i=1}^{N} x_i}{N(N+1)}$$

$$= \frac{\cancel{N\sum_{i=1}^{N} x_i} + N x_{N+1} - \left(\cancel{N\sum_{i=1}^{N} x_i} + \cancel{N}\sum_{i=1}^{N} x_i\right)}{(N+1)N}$$

$$= \frac{\cancel{N}(x_{N+1} - \mu_N)}{(N+1)\cancel{N}}$$

$$\Rightarrow \mu_d = \frac{x_{N+1} - \mu_N}{N+1}$$

$$\Rightarrow \mu_{N+1} = \mu_N + \mu_d = \mu_N + \frac{x_{N+1} - \mu_N}{N+1}$$

derivation finding covariance

$$\sigma_N^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_N)^2.$$

$$\sigma_N = x_N - \mu_N$$

$$\Rightarrow \sigma_N^2 = \frac{\sigma_N^T \sigma_N}{N}$$

$$\Rightarrow \sigma_{N+1}^2 = \frac{\tilde{x}_{N+1}^T \sigma_{N+1}}{N+1} \qquad \sigma_{N+1} = \tilde{x}_{N+1}$$

$$\cancel{= \frac{1}{N+1}(\tilde{x}_{N+1} - \mu_{N+1})^T(\tilde{x}_{N+1} - \mu_{N+1})}$$

$$\cancel{= \frac{1}{N+1}(\tilde{x}}$$

$$= \frac{1}{N+1}(x_{N+1}^T - \mu_{N+1})^T(x_{N+1} - \mu_{N+1})$$

$$= \frac{1}{N+1}(x_{N+1}^T x_{N+1} - (N+1)\mu_{N+1}^2)$$

$$= \frac{1}{N+1}(x_N^T x_N + x_{N+1}^2 - (N+1)\mu_{N+1}^2)$$

$$= \frac{N\sigma_N^2 + x_{N+1}^2}{N+1} - \mu_{N+1}^2.$$

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⇒ Algorithms.

for mean.

if a is the first element.

$Sum = a$

$N = 1$

$\mu = a$

else

$$\mu_{N+1} += \frac{a - \mu_N}{N+1}$$

for covariance

if a is the first element

$\sigma_N^2 = 0$

$N = 1.$

$\mu = a$

else

$$\sigma_{N+1}^2 = \frac{N\sigma_N^2 + a^2}{N+1} - \mu_{N+1}^2.$$

b) for mean, we don't have to recalculate the mean & covariance again & again for the window.

$$\mu_N = \mu_{N-1} + \frac{x_N - x_{N-M}}{M}$$

for covariance.

$$Sum\_2 = \sum_{i=0}^{N-1} x_i^2$$

$$Sum\_1 = \sum_{i=0}^{N-1} x_i$$

σ_N = Covariance of present window

σ_N' = Covarionace of next window.

$$\sigma_N' = \left(Sum\_2 - x_{i-m}^2 + x_i^2\right) + \cancel{N} \frac{(Sum\_1 - x_{i-m} + x_i)^2}{N^2}$$

$$- 2\frac{(Sum\_1 - x_{i-m} + x_i)}{N} (Sum\_1 - x_{i-m} + x_i)$$

$$= \left(Sum\_2 - x_{(i-m)}^2 + x_i^2\right) - \frac{(Sum\_1 - x_{(i-m)} + x_i)^2}{N}$$

Derivation:

Covariance of M elements.

$$\sigma_M^2 = \sum_{i=0}^{M-1} (x_i - \mu_M)^2$$

$$= \sum_{i=0}^{M-1} (x_i^2 + \mu_M^2 - 2x_i\mu_M)$$

$\Rightarrow \sigma_{M'}^2$ (of Next window)

$$= \sum_{i=1}^{M} (x_i^2 + \mu_M'^2 - 2x_i\mu_M')$$

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$\Rightarrow \sigma_{M'}^2$ from σ_M^2

$$\sigma_{M'}^2 = \sum_{i=0}^{M-1} (x_i)^2 - x_0^2 + x_M^2 +$$

$$N\mu_M^2 - N\mu_M^2 + N\mu_{M'}^2 +$$

$$-2\mu_M \sum_{i=0}^{M-1} x_i + 2\mu_M \sum_{i=0}^{M-1} x_i - 2\mu_{M'} \sum_{i=1}^{M} x_i$$

$$\Rightarrow \sigma_{M'}^2 = \left(Sum\_2 - x_{(i-M)}^2 - x_i^2\right) + \left(\frac{Sum\_1 - x_{i-m} + x_i}{N}\right)^2$$

$$-2\left(Sum\_1 - x_{im} + x_i\right)\left(Sum\_1 - x_{i-m} + x_i\right)$$