3. (a) prior probability $= P(\omega_1) = 1/2$
$P(\omega_2) = 1/2$.

b) $$\mu_1 = \left[\sum_{i=1}^{N} \frac{x_{1i}}{N} \quad \sum_{i=1}^{N} \frac{x_{2i}}{N} \right]^T = \left[\frac{12}{7} \quad \frac{8}{7} \right]^T$$

$$\Sigma_1 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_1)(x_i - \mu)^T$$

$$\Sigma_1 = \begin{bmatrix} 1.57 & 0.88 \\ 0.88 & 1.47 \end{bmatrix} \quad \Sigma_1^{-1} = \begin{bmatrix} 0.95 & -0.57 \\ -0.57 & 1.01 \end{bmatrix}$$

$$|\Sigma_1| = 1.54$$

$$\mu_2 = \left[\sum_{i=1}^{N} \frac{x_{1i}}{N} \quad \sum_{i=1}^{N} \frac{x_{2i}}{N} \right]^T = \left[\frac{59}{7} \quad \frac{60}{7} \right]^T$$

$$\Sigma_2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_1)(x_i - \mu)^T$$

$$\Sigma_2 = \begin{bmatrix} 0.57 & -0.64 \\ -0.64 & 3.62 \end{bmatrix} \quad \Sigma_2^{-1} = \begin{bmatrix} 2.18 & 0.38 \\ 0.38 & 0.34 \end{bmatrix}$$

$$|\Sigma_2| = 1.65.$$

c) We know, probabilities at decision boundary are equal

let $\theta = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be decision boundary.

$\Rightarrow P(\omega_1/\theta) = P(\omega_2/\theta)$

$\Rightarrow P(\theta/\omega_1).P(\omega_1) = P(\theta/\omega_2).P(\omega_2)$

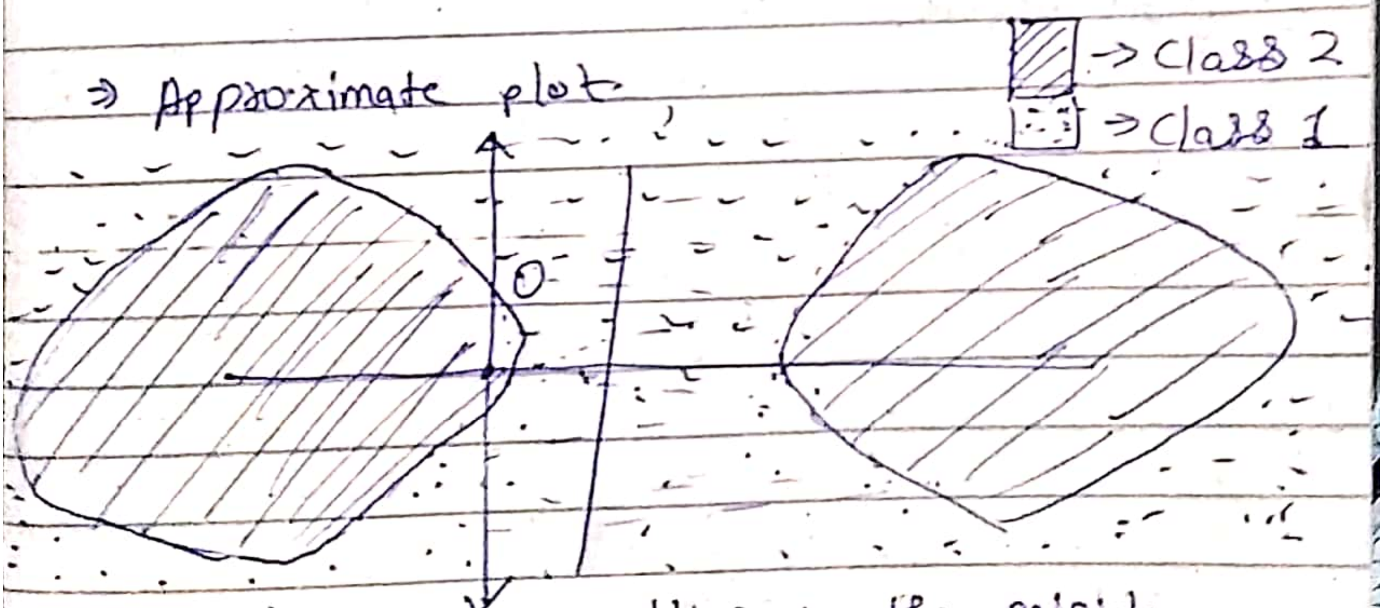
$\Rightarrow P(\theta/\omega_1) = P(\theta/\omega_2)$.

$$\Rightarrow \log|\Sigma_1| + (\theta-\mu_1)^T \Sigma_1^{-1}(\theta-\mu_1) = \log|\Sigma_2| + (\theta-\mu_2)^T \Sigma_2^{-1}(\theta-\mu_2)$$

$$\Rightarrow (\theta-\mu_2)^T \Sigma_2^{-1}(\theta-\mu_2) - (\theta-\mu_1)^T \Sigma_1^{-1}(\theta-\mu_1) = \log\frac{|\Sigma_1|}{|\Sigma_2|}.$$

$$\Rightarrow x_1^2 + \cancel{\emptyset}\,3.05x_1x_2 - 2.19x_2^2 - 9.16x_1 + 45.07x_2 - 205 = 0$$

$\Rightarrow$ Approximate plot



c) If there are penalties to the ~~misida~~ misclassification and are different to both the classes, then when we are computing decision boundary be equating probabilities we will multiply a factor $d = \dfrac{\text{penalty}(\omega_1)}{\text{penalty}(\omega_2)}$

$$\Rightarrow p(\omega_1/\theta) = p(\omega_2/\theta)\, d \Rightarrow p(\theta/\omega_1) = \frac{p(\theta/\omega_2)\, d}{\cancel{p(\theta/\omega_2)}}$$

$$\Rightarrow (\theta-\mu_2)^T \Sigma_2^{-1} (\theta-\mu_2) - (\theta-\mu_2)^T \Sigma_1^{-1} (\theta-\mu_1) = \log\left|\frac{\Sigma_1}{\Sigma_2}\right| d$$

Solving this we get a decision boundary.

if $d >> 1$, then the region with greater penalty shifts into the region with less penalty. It is going to be different from before.