Q.2. Consider for BG, $N(\mu_1, \sigma_1^2)$
for FG, $N(\mu_2, \sigma_2^2)$

1. $P(FG) = P(BG)$ & $\sigma_1 = \sigma_2$

for θ

$$\frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

putting $\sigma_1 = \sigma_2$

$$(x-\mu_1)^2 = (x-\mu_2)^2$$

$\Rightarrow x-\mu_1 = \pm\, x-\mu_2$

$\Rightarrow \mu_1 = \mu_2$ or $x = \frac{\mu_1+\mu_2}{2}$

$\Rightarrow$ We can use $\theta = \frac{\mu_1+\mu_2}{2}$ as threshold value

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b) $\theta = \frac{\mu_1+\mu_2}{2}$ given

$\therefore$ $P(BG) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(\theta-\mu_1)^2}{2\sigma_1^2}}$

$P(FG) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(\theta-\mu_2)^2}{2\sigma_2^2}}$

$$\Rightarrow \frac{P(BG)}{P(FG)} = \frac{\sigma_2}{\sigma_1} e^{-\left(\frac{(\mu_1-\mu_2)^2}{8\sigma_1^2} - \frac{(\mu_1-\mu_2)^2}{8\sigma_2^2}\right)}$$

$\Rightarrow$ Relation,

$$\frac{P(BG)}{P(FG)} = \frac{\sigma_2}{\sigma_1} e^{-\frac{(\mu_1-\mu_2)^2}{8} \times \frac{(\sigma_2^2-\sigma_1^2)}{(\sigma_1^2\sigma_2^2)}} \quad -(1)$$

c) $P(BG) = 4P(FG)$

$\mu_1 = 100 \quad \sigma_1 = \sigma_2$
$\mu_2 = 200$

putting in (1)

$$\cancel{\frac{1}{\sigma_1\sqrt{2\pi}}} e^{\frac{-(\theta-100)^2}{2\sigma_1^2}} = \frac{4}{\cancel{\sigma_2\sqrt{2\pi}}} e^{\frac{-(\theta-200)^2}{2\sigma_2^2}}$$

$$\Rightarrow \frac{-(\theta-100)^2}{2\sigma_1^2} = \ln 4 - \frac{(\theta-200)^2}{2\sigma_2^2}$$

$$\Rightarrow \frac{(\theta-200)^2}{2\sigma_2^2} - \frac{(\theta-100)^2}{2\sigma_1^2} = \ln 4$$

$\because \sigma_1 = \sigma_2$

$$\Rightarrow (\theta-200)^2 - (\theta-100)^2 = (\ln 4) 2\sigma_1^2$$

$\Rightarrow -100(2\theta - 300) = 2\sigma_1^2 \ln 4$

$\Rightarrow \theta = 150 - \dfrac{\sigma_1^2 \ln 4}{100}$

$\Rightarrow \boxed{\theta = 150 - \dfrac{\sigma_1^2 \ln 4}{100}}$ Ans

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