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1.b) $\frac{1}{\sqrt{2\pi^d |\Sigma|}} e^{-\left(\frac{(x-\mu)\Sigma^{-1}(x-\mu)^T}{2}\right)}$

$\Rightarrow$ for (i)

$$e^{-\frac{(x-\mu)\Sigma^{-1}(x-\mu)^T}{2}} = e^{-\frac{(x-\mu)\Sigma^{-1}(x-\mu)^T}{2}}$$

taking log,

$$+(x-\mu_1)\Sigma^{-1}(x-\mu_1)^T = +(x-\mu_2)\Sigma^{-1}(x-\mu_2)^T$$

$$(x-\mu_1)(x-\mu_1)^T = (x-\mu_2)(x-\mu_2)^T$$

$\cancel{x = [3}$

$$([x_1\ x_2]-[\mu_1\ \mu_1])\begin{bmatrix} x_1-\mu_1 \\ x_2\ \mu_1 \end{bmatrix} = [x_1-\mu_2\quad x_2-\mu_2]\begin{bmatrix} x_1-\mu_2 \\ x_2-\mu_2 \end{bmatrix}$$

$$\Rightarrow [x_1-\mu_1\quad x_2-\mu_2]\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_1 \end{bmatrix} = [x_1-\mu_2,\ x_2-\mu_2]\begin{bmatrix} x_1-\mu_2 \\ x_2-\mu_2 \end{bmatrix}$$

$$\Rightarrow (x_1-\mu_1)^2+(x_2-\mu_1)^2 = (x_1-\mu_2)^2+(x_2-\mu_2)^2$$

$$\Rightarrow (x_1-3)^2+(x_2-3)^2 = (x_1-7)^2+(x_2-7)^2$$

$$\Rightarrow \cancel{x_1^2}+9-6x_1+\cancel{x_2^2}+9-6x_2 = \cancel{x_1^2}+49-14x_1+\cancel{x_2^2}+49-14x_2$$

$$\Rightarrow -\overset{3}{\cancel{6}}(x_1+x_2) = 40\cancel{\times 2} - \overset{7}{\cancel{14}}(x_1+x_2)$$

$$\Rightarrow 4(x_1+x_2) = 40$$

$$\underline{x_1+x_2=10}$$

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~~lg(1/7)~~

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b) $\Sigma_1 = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 7 & 2 \\ 1 & 7 \end{bmatrix}$.

$|\Sigma_1| = 7$ $|\Sigma_2| = 47$

$$\Rightarrow \ln \frac{1}{\sqrt{2\pi^d |\Sigma|}} e^{-\frac{(x-\mu)\Sigma^{-1}(x-\mu)}{2}}$$

$$= -\frac{\ln|\Sigma|}{2} + \ln(e)(-(x-\mu)\Sigma^{-1}(x-\mu)^T)$$

equating.

$$\Rightarrow \ln(e)(-(x-\mu_1)\Sigma^{-1}(x-\mu_1)^T) - \ln(e)(-(x-\mu_2)\Sigma^{-1}(x-\mu_2)^T)$$
$$= +\frac{\ln(7)}{2} - \frac{\ln|47|}{2}.$$

here $\frac{\ln(7)}{2} - \frac{\ln(47)}{2} \simeq 0.$

$$\Rightarrow \left([x_1-\mu_1, x_2-\mu_1]\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_1 \end{bmatrix}\right) =$$

$$\left([x_1-\mu_2, x_2-\mu_2]\begin{bmatrix} 7 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} x_1-\mu_2 \\ x_2-\mu_2 \end{bmatrix}\right)$$

$$\Rightarrow [7x_1-7\mu_2+x_2-\mu_2, 2x_1-2\mu_2+7x_2-7\mu_2]\begin{bmatrix} x_1-\mu_2 \\ x_2-\mu_2 \end{bmatrix}$$

$$= [3x_1-3\mu_1+2x_2-2\mu_2, x_1-\mu_1+3x_2-3\mu_1]\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_1 \end{bmatrix}$$

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$$\Rightarrow \left[7x_1 + x_2 - 8\mu_2 ,\ 2x_1 + 7x_2 - 9\mu_2\right]\begin{bmatrix} x_1 - \mu_2 \\ x_2 - \mu_2 \end{bmatrix} =$$

$$\left[3x_1 + 2x_2 - 5\mu_1 ,\ x_1 + 3x_2 - 4\mu_1\right]\begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_1 \end{bmatrix}$$

$$\Rightarrow \left[7x_1^2 + x_1x_2 - 8\mu_2 x_1 - 7x_1\mu_2 - x_2\mu_2 + 8\mu_2^2 + \right.$$
$$\left. 2x_1x_2 + 7x_2^2 - 9\mu_2 x_2 - 2x_1\mu_2 - 14x_2\mu_2 + 9\mu_2^2\right]$$

$$= \left[3x_1^2 + 2x_1x_2 - 5\mu_1 x_1 - 3x_1\mu_1 - 2x_2\mu_1 + 5\mu_1^2 + \right.$$
$$\left. x_1x_2 - 3x_2^2 - 4\mu_1 x_2 - x_1\mu_1 - 3x_2\mu_1 + 4\mu_1^2\right]$$

$$\Rightarrow \left\{7x_1^2 + 3x_1x_2 - 17x_1\mu_2 - 28x_2\mu_2 + 17\mu_2^2 + 7x_2^2\right\} =$$

$$3x_1^2 + 3x_1x_2 + 8\mu_1 x_1 - 9\mu_1 x_2 + 9\mu_1^2 - 3x_2^2\}$$

$$\Rightarrow \cancel{4x_1^2 \ldots - 17x_1\mu_2 - 28x_2\mu_2 + 17\mu_2^2 + 9\mu_1^2}$$

$$\Rightarrow x_1^2 + x_2^2 - 17x_1 - 17x_2 + 128 = 0$$

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Q.2 a) The ~~diment~~ rank of D should be 35, since there are only 35 kind of families sending their children hence there will be multiple students who will be sending their children who have same eating habbits and they all can be represented as Linear combination of one-another,

if D_{ij} is no. of times student order food from the restraunt then the values will just scale by the factor ~~but still~~ although the matrix might get full rank but mostly 35 values will be dominating over others.

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b) $U\Sigma V^T = \text{Svd}(D)$

$U = 2000 \times 2000$

$V = 200 \times 200$

make it $= 35 \times 200$

~~Make $U = 35 \times 35$~~

take first ~~of~~ 35 vectors of 2000 dimention.

$\Rightarrow U = \cancel{35 \times 2000}\ 2000 \times 35$

let the new Student be a vector of length 35 having 35 feature.

multiply new student $\underset{1\times35}{X} \cdot \underset{35\times200}{V}$

$=$ desired restaurants $= \underset{1\times200}{R}$

recommed the restraunt with highest value in 1×200 vector.

c) take a new restaurant with 35 features. 35×1,

for each Student ~~to~~ multiply this to feature matrix.

$\Rightarrow (35\times1) \times (1\times2000)$

So for Student who has the highest Value for that restraunt will be ~~recom~~ most likely to buy their food.

3. ~~$C = \sum \{(x-\bar{x})(x-\bar{x})^T\}$~~

$$C = \frac{1}{n} \sum_{i=1}^{n} \{(x-\bar{x})(x-\bar{x})^T\}$$

for any u,

$$u^T C u = u^T E\{(x-\bar{x})(x-\bar{x})^T\} u$$

$$= E\{u^T (x-\bar{x})(x-\bar{x})^T u\}$$

$$= \cancel{E\{} \sigma_s^2$$

Where, $s = u^T (x-\bar{x}) = (x-\bar{x})^T u$

$\because s^2 \geq 0$

$\Rightarrow \sigma_s^2 \geq 0$

$\Rightarrow u^T C u = \sigma_s^2 \geq 0$

$\Rightarrow$ it ~~is~~ PSD (Positive Semi Definite).