CBSE Class 12 Maths Question Paper Solution 2015

QUESTION PAPER CODE 65/1/D

EXPECTED ANSWERS/VALUE POINTS SECTION - A

Marks

1.
$$p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$$

2.
$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \implies \lambda = 7$$

$$\frac{1}{2} + \frac{1}{2} \text{ m}$$

3.
$$\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \implies \theta = \frac{\pi}{6}$$
 $\frac{1}{2} + \frac{1}{2} m$

4.
$$a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$$

5.
$$\frac{dv}{dr} = -\frac{A}{r^2}, \implies r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$$
 $\frac{1}{2} + \frac{1}{2}m$

6. I.F =
$$e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$$

SECTION - B

7. Getting
$$A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$
 1½ m

$$A^{2} - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$1 \text{ m}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}$$
 1 m

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$
¹/₂ m

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$
 1 m

$$|A'| = 1(-9)-2(-5) = -9+10 = 1 \neq 0$$
 ½ m

$$Adj A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$
 2 m

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$

8.
$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

 $R_2 \rightarrow R_2 - x R_1$ and $R_3 \rightarrow R_3 - x^2 R_1$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix}$$
 (For bringing 2 zeroes in any row/column 1+1 m

$$f(x) = a (a^2 + 2ax + x^2) = a (x + a)^2$$

$$f(2x) - f(x) = a [2x + a]^{2} - a (x + a)^{2}$$

$$= a x (3x + 2a)$$
1 m

9.
$$\int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x (1 + 2\cos x)} = \int \frac{\sin x \cdot dx}{(1 - \cos x) (1 + \cos x) (1 + 2\cos x)} = 1 \text{ m}$$

$$= -\int \frac{dt}{(1-t)(1+t)(1+2t)} \text{ where } \cos x = t$$
 \(\frac{1}{2} m\)

$$= \int \left(\frac{-\frac{1}{6}}{1-t} + \frac{\frac{1}{2}}{1+t} - \frac{\frac{4}{3}}{1+2t} \right) dt$$

$$= + \frac{1}{6} \log \left| 1 - t \right| + \frac{1}{2} \log \left| 1 + t \right| - \frac{2}{3} \log \left| 1 + 2t \right| + c$$

$$= \frac{1}{6} \log \left| 1 - \cos x \right| + \frac{1}{2} \log \left| 1 + \cos x \right| - \frac{2}{3} \log \left| 1 + 2 \cos x \right| + c$$

$$\int \frac{x^2 - 3x + 1}{\sqrt{1 - x^2}} dx = \int \frac{2 - 3x - (1 - x^2)}{\sqrt{1 - x^2}} dx$$

$$= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx$$

$$= 2\sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + c \qquad (\frac{1}{2}+1+1)m$$

or =
$$\frac{3}{2} \sin^{-1}x + \frac{1}{2} (6-x) \sqrt{1-x^2} + c$$

10.
$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^{2} dx = \int_{-\pi}^{\pi} (\cos^{2}ax + \sin^{2}bx) dx - \int_{-\pi}^{\pi} 2\cos ax \sin bx dx$$
$$= I_{1} - I_{2}$$
¹/₂ m

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx$$
 (being an even fun.)

$$I_2 = 0$$
 (being an odd fun.)

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx$$
¹/₂ m

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi}$$
¹/₂ m

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b}\right] \text{ or } 2\pi$$

11. Let E_1 : selecting bag A, and E_2 : selecting bag B.

:.
$$P(E_1) = \frac{1}{3}$$
, $P(E_2) = \frac{2}{3}$

Let A: Getting one Red and one balck ball

$$\therefore P(A|E_1) = \frac{{}^{4}C_1 \cdot {}^{6}C_1}{{}^{10}C_2} = \frac{8}{15}, \ P(A|E_2) = \frac{{}^{7}C_1 \cdot {}^{3}C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45}$$

OR
$$x : 0 1 2 3 4 \frac{1}{2} m$$

$$P\left(x\right) \quad : \quad {}^{4}C_{0}\left(\frac{1}{2}\right)^{4} \quad {}^{4}C_{1}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right) \quad {}^{4}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2} \quad {}^{4}C_{3}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{3} \quad {}^{4}C_{4}\left(\frac{1}{2}\right)^{4} \quad 1\frac{1}{2}m$$

$$= \frac{1}{16} \qquad = \frac{4}{16} \qquad = \frac{6}{16} \qquad = \frac{4}{16} \qquad = \frac{1}{16} \qquad \frac{1}{2} \text{ m}$$

$$x P (x) : 0 \frac{4}{16} \frac{12}{16} \frac{12}{16} \frac{4}{16}$$

$$x^{2}P(x)$$
: 0 $\frac{4}{16}$ $\frac{24}{16}$ $\frac{36}{16}$ $\frac{16}{16}$ $\frac{16}{16}$ $\frac{1}{2}m$

Mean =
$$\sum x P(x) = \frac{32}{16} = 2$$

Variance =
$$\sum x^2 P(x) - (\sum x P(x))^2 = \frac{80}{16} - (2)^2 = 1$$

12.
$$\vec{r} \times \vec{i} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)x\hat{i} = -y\hat{k} + z\hat{j}$$

$$\vec{r} \times \vec{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\hat{j} = x\hat{k} - z\hat{i}$$
1½ m

$$(\overrightarrow{r} \times \widehat{i}), (\overrightarrow{r} \times \overrightarrow{j}) = (\widehat{o} + \widehat{z} - \widehat{j} - \widehat{y} + \widehat{k}) \cdot (-\widehat{z} + \widehat{i} + \widehat{j} + \widehat{x} + \widehat{k}) = -xy$$
¹/₂ m

$$(\overrightarrow{r} \times \overrightarrow{i}) \cdot (\overrightarrow{r} \times \overrightarrow{j}) + xy = -xy + xy = 0$$
¹/₂ m

13. Any point on the line
$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$
 is $(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$

If this is the point of intersection with plane x - y + z = 5

then
$$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \implies \lambda = 0$$
 1 m

$$\therefore$$
 Point of intersection is $(2, -1, 2)$ 1 m

Required distance =
$$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$

14. Writing
$$\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$$

and
$$tan^{-1}x = cos^{-1} \frac{1}{\sqrt{1+x^2}}$$
 1½ m

$$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1 + (x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1 + x^2}} \right)$$

$$1 + x^2 + 2x + 1 = 1 + x^2 \implies x = -\frac{1}{2}$$

$$\left(\tan^{-1}x\right)^{2} + \left(\cot^{-1}x\right)^{2} = \frac{5\pi^{2}}{8} \implies \left(\tan^{-1}x\right)^{2} + \left(\frac{\pi}{2} - \tan^{-1}x\right)^{2} = \frac{5\pi^{2}}{8}$$

$$\therefore 2 \left(\tan^{-1} x \right)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} = 0$$

$$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\frac{\pi}{4}, -\frac{\pi}{4}$$

$$\Rightarrow x = -1$$

15. Putting
$$x^2 = \cos \theta$$
, we get

$$\frac{1}{2}$$
 m

$$y = tan^{-1} \left(\frac{\sqrt{1 + cos\theta} + \sqrt{1 - cos\theta}}{\sqrt{1 + cos\theta} - \sqrt{1 - cos\theta}} \right)$$
¹/₂ m

$$= \tan^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left(\frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right)$$

$$1 + \frac{1}{2} \text{ m}$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}}$$

16.
$$\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$$
 ½ m

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{a \sin \theta + b \cos \theta} = -\frac{x}{y}$$

or
$$y \frac{dy}{dx} + x = 0$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0$$

Using (i) we get
$$y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0$$
 1/2 m

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

17. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.}$$

Area (A) =
$$\frac{\sqrt{3}x^2}{4}$$
 1 m

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$$
 1 m

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2 | \text{s}$$

18. Writing
$$x + 3 = -\frac{1}{2}(-4 - 2x) + 1$$

$$\therefore \int (x+3)\sqrt{3-4x-x^2} \ dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} \ dx + \int \sqrt{7-(x+2)^2} \ dx \qquad \frac{1}{2} + \frac{1}{2} m$$

$$= -\frac{1}{3} \left(3 - 4x - x^2 \right)^{3/2} + \frac{x+2}{2} \sqrt{3 - 4x - x^2} + \frac{7}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{7}} \right) + c$$
 1+1 m

19. HF. M P

A
$$\begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ C & 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix}$$

1½ m

Funds collected by school A: Rs. 7000,

SECTION - C

20. $\forall a, b \in N, (a, b) R (a, b) as ab (b + a) = ba (a + b)$

: R is reflexive(i)

 $2 \, \mathrm{m}$

Let (a, b) R (c, d) for $(a, b), (c, d) \in N \times N$

: ad(b+c) = bc(a+d)(ii)

Also (c, d) R (a, b) : cb(d+a) = da(c+b) (using ii)

: R is symmetric(iii)

 $2 \, \text{m}$

Let (a, b) R (c, d) and (c, d) R (e, f), for $a, b, c, d, e, f \in N$

 \therefore ad (b+c) = bc (a+d) and cf (d+e) = de (c+f)

1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

i.e $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

adding we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

 \Rightarrow af (b+e) = be (a+f)

Hence (a, b) R (e, f) \therefore R is transitive(iv)

 $\frac{1}{2}$ m

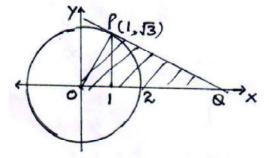
Form (i), (iii) and (iv) R is an equivalence relation

 $\frac{1}{2}$ m

 $\frac{1}{2}$ m

21.

Correct Fig. 1 m



Eqn. of normal (OP):
$$y = \sqrt{3}x$$
 $\frac{1}{2} + \frac{1}{2}m$

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}} (x-1) \text{ i.e. } y = \frac{1}{\sqrt{3}} (4-x)$$
 1 m

Coordinates of
$$Q(4, 0)$$

$$\therefore \text{ Req. area } = \int_{0}^{1} \sqrt{3}x \, dx + \int_{1}^{4} \frac{1}{\sqrt{3}} (4-x) dx$$

$$= \sqrt{3} \frac{x^2}{2} \bigg]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units}$$

$$\int_{1}^{3} (e^{2-3x} + x^{2} + 1) dx \qquad \text{here } h = \frac{2}{n}$$

$$= \lim_{h \to 0} h \left[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h) \right]$$

$$= \lim_{h \to 0} \, h \left[\! \left(\! e^{-1} + 2 \right) \! + \left(\! e^{-1-3h} + 2 + 2h + h^2 \right) \! + \left(\! e^{-1-6h} + 2 + 4h + 4h^2 \right) \! + \ldots \right]$$

+
$$\left(e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^2h^2\right)$$
 1 m

$$= \lim_{h \to 0} h \left[e^{-l} \left(1 + e^{-3h} + e^{-6h} + \ldots + e^{-3(n-1)h} \right) + 2n + 2h \left(1 + 2 + \ldots + (n-1) \right) + h^2 \left(1^2 + 2^2 + \ldots + \left(n - 1 \right)^2 \right) \right] \ \, 1 \frac{1}{2} \ \, m + 2 h \left(1 + 2 + \ldots + (n-1) + 2 h \left($$

$$= \lim_{h \to 0} h \left(e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2 \frac{nh (nh - h)}{2} + \frac{nh (nh - h)(2nh - h)}{6} \right)$$
 1 m

$$= e^{-1} \cdot \frac{\left(e^{-6} - 1\right)}{-3} + 4 + 4 + \frac{8}{3} = - e^{-1} \cdot \frac{\left(e^{-6} - 1\right)}{3} + \frac{32}{3}$$

22. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2}$$

$$\therefore$$
 Integrating factor is $e^{\tan^{-1} y}$ 1 m

:. Solution is:
$$x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy$$
 1½ m

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^{t} dt \text{ where } \tan^{-1}y = t$$

$$= t e^{t} - e^{t} + c = e^{tan^{-1}y} (tan^{-1}y - 1) + c$$

or
$$x = tan^{-1}y - 1 + c e^{-tan^{-1}y}$$

Given differential equation is
$$\frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

Putting
$$\frac{y}{x} = v$$
 to get $v + x \frac{dv}{dx} = \frac{v}{1 + v^2}$

$$\therefore x \frac{dv}{dx} = \frac{v}{1 + v^2} - v = \frac{-v^3}{1 + v^2}$$
 1½ m

$$\Rightarrow \int \frac{v^2 + 1}{v^3} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log |v| - \frac{1}{2v^2} = -\log |x| + c$$

$$\therefore \log y - \frac{x^2}{2y^2} = c$$

$$x = 0, y = 1 \implies c = 0 : log y - \frac{x^2}{2y^2} = 0$$
 1/2 m

23. Any point on line
$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$
 is $(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$

$$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \implies \lambda = -\frac{3}{2}, \text{ hence } k = \frac{9}{2}$$

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$
1 m

$$\Rightarrow -5(x-1)+2(y+1)+1(z-1) = 0$$
 1 m

i.e.
$$5x-2y-z-6=0$$
 ½ m

24.
$$P(\overline{A} \cap B) = \frac{2}{15} \Rightarrow P(\overline{A}) \cdot P(B) = \frac{2}{15}$$

$$P(A \cap \overline{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\overline{B}) = \frac{1}{6}$$

:
$$(1-P(A))P(B) = \frac{2}{15} \text{ or } P(B)-P(A)\cdot P(B) = \frac{2}{15} \dots (i)$$
 1 m

$$P(A)(1-P(B)) = \frac{1}{6} \text{ or } P(A)-P(A)\cdot P(B) = \frac{1}{6} \dots (ii)$$
 1 m

From (i) and (ii)
$$P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$$
 1/2 m

Let P (A) = x, P (B) = y :
$$x = \left(\frac{1}{30} + y\right)$$

(i)
$$\Rightarrow y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0$$
 \(\frac{1}{30} + y\)

Solving to get $y = \frac{1}{6}$ or $y = \frac{4}{5}$

$$x = \frac{1}{5}$$
 or $x = \frac{5}{6}$

Hence
$$P(A) = \frac{1}{5}$$
, $P(B) = \frac{1}{6}$ OR $P(A) = \frac{5}{6}$, $P(B) = \frac{4}{5}$

25. $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

$$f'(x) = 0 \implies \cos x + \sin x = 0 \text{ or } \tan x = -1,$$
 1 m

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$f''(x) = \cos x - \sin x$$

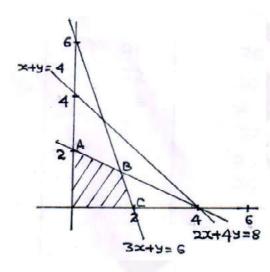
$$f''(3\pi/4) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$
 i.e - ve so, $x = 3\pi/4$ is Local Maxima

and
$$f''(7\pi/4) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$
 i.e + ve so, $x = 7\pi/4$ is Local Minima

Local Maximum value
$$=\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Local Minimum value
$$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2}$$

26.



Correct graphs of three lines 1x3 = 3 mCorrectly shading

feasible region 1 m

Vertices are

Z = 2x + 5y is maximum

at
$$A(0, 2)$$
 and maximum value = 10 1 m