

AI1110 Assignment 1 in L^AT_EX

Kondaparthi Anuraga Chandan*

12.13.2.7

Given that the events A and B are such that $\Pr(A) = \frac{1}{2}$, $\Pr(A + B) = \frac{3}{5}$ and $\Pr(B) = p$. Find p if they are (i) mutually exclusive (ii) independent.

Solution:

Given,

$$\Pr(A) = \frac{1}{2} \quad (1)$$

$$\Pr(A + B) = \frac{3}{5} \quad (2)$$

$$\Pr(B) = p = ? \quad (3)$$

(i) A and B are *mutually exclusive* events

$$\Pr(A + B) = \Pr(A) + \Pr(B) \quad (4)$$

$$\Rightarrow \Pr(A + B) = \Pr(A) + p \quad (5)$$

$$\Rightarrow p = \Pr(A + B) - \Pr(A) \quad (6)$$

$$\Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} \quad (7)$$

$$\Rightarrow p = \frac{1}{10} \quad (8)$$

(ii) A and B are *independent* events

$$\Pr(AB) = \Pr(A) \Pr(B) \quad (9)$$

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (10)$$

$$\Rightarrow \Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(A) \Pr(B) \quad (11)$$

$$\Rightarrow \Pr(A + B) = \Pr(A) + p - \Pr(A) p \quad (12)$$

$$\Rightarrow \Pr(A + B) - \Pr(A) = p[1 - \Pr(A)] \quad (13)$$

$$\Rightarrow p = \frac{\Pr(A + B) - \Pr(A)}{1 - \Pr(A)} \quad (14)$$

$$\Rightarrow p = \frac{\frac{3}{5} - \frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{6-5}{10}}{\frac{1}{2}} = \frac{\frac{1}{10}}{\frac{1}{2}} = \frac{2}{10} \quad (15)$$

$$\Rightarrow p = \frac{1}{5} \quad (16)$$

*The student is with the Department of Artificial Intelligence, Indian Institute of Technology, Hyderabad 502285 India e-mail: ai22btech110011@iith.ac.in.