AI1110 Assignment 1 in LaTeX

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Given that the events A and B are such that $\Pr(A) = \frac{1}{2}$, $\Pr(A+B) = \frac{3}{5}$ and $\Pr(B) = p$. Find p if they are

(i) mutually exclusive
(ii) independent.

Solution:

Given,

$$\Pr(A) = \frac{1}{2} \tag{1}$$
$$\Pr(A+B) = \frac{3}{5} \tag{2}$$
$$\Pr(B) = p = ? \tag{3}$$

(i) A and B are *mutually exclusive* events

$$\Pr(A+B) = \Pr(A) + \Pr(B) \tag{4}$$
$$\implies \Pr(A+B) = \Pr(A) + p \tag{5}$$
$$\implies p = \Pr(A+B) - \Pr(A) \tag{6}$$
$$\implies p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} \tag{7}$$
$$\implies p = \frac{1}{10} \tag{8}$$

(ii) A and B are *independent* events

$$\Pr(AB) = \Pr(A)\Pr(B) \tag{9}$$
$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{10}$$
$$\implies \Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(A)\Pr(B) \tag{11}$$
$$\implies \Pr(A+B) = \Pr(A) + p - \Pr(A)\,p \tag{12}$$
$$\implies \Pr(A+B) - \Pr(A) = p[1 - \Pr(A)] \tag{13}$$
$$\implies p = \frac{\Pr(A+B) - \Pr(A)}{1 - \Pr(A)} \tag{14}$$
$$\implies p = \frac{\frac{3}{5} - \frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{6-5}{10}}{\frac{1}{2}} = \frac{\frac{1}{10}}{\frac{1}{2}} = \frac{2}{10} \tag{15}$$
$$\implies p = \frac{1}{5} \tag{16}$$

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