

Assignment 2 in L^AT_EX

AI1110: Probability and Random Variables

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12.13.6.10 Question: How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Solution:

Let us say the man has tossed the fair coin 'n' times. The event of having no heads is same as the event of having tails in all tosses.

Parameter	Value	Description
n	?	number of times the coin is tossed
Pr(no heads)	$\frac{1}{2^n}$	probability of having no heads in tossing the fair coin 'n' times
Pr(all tails)	$\frac{1}{2^n}$	probability of getting all tails in tossing the fair coin 'n' times
Pr(atleast one head)	$1 - \frac{1}{2^n}$	probability of having at least one head in tossing the fair coin 'n' times

$$\therefore \text{Pr(no heads)} = \text{Pr(all tails)} = \frac{1}{2^n}. \quad (1)$$

$$\therefore \text{Pr(atleast one head)} = 1 - \text{Pr(no heads)}. \quad (2)$$

$$\text{Pr(atleast one head)} = 1 - \frac{1}{2^n} \quad (3)$$

$$1 - \frac{1}{2^n} > \frac{90}{100} \quad (4)$$

$$\frac{1}{2^n} < \frac{10}{100} \quad (5)$$

$$\frac{1}{2^n} < \frac{1}{10} \quad (6)$$

$$2^n > 10 \quad (7)$$

$$n > \log_2 10 \quad (8)$$

$$n > 3.3219 \quad (9)$$

$$\therefore n \geq 4 \quad (10)$$

\therefore the man should toss the fair coin atleast 4 times so that probability of having at least one head is more than 90%.