Assignment 2 in LaTeX

AI1110: Probability and Random Variables

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12.13.4.4 Question:
Find the probability distribution of

(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

Solution:

(i) Let X be the random variable representing the number of heads in two tosses of a fair coin. X can take values {0, 1, 2}.

$$P(X=r) = {}^{n}C_r p^r q^{n-r}, p = \frac{1}{2}, q = \frac{1}{2}, n = 2. \tag{1}$$

$$\therefore P(X=r) = \frac{{}^{2}C_r}{2^2} \tag{2}$$

$$P(X=0) = \frac{{}^{2}C_0}{2^2} = \frac{1}{4} \tag{3}$$

$$P(X=1) = \frac{{}^{2}C_1}{2^2} = \frac{2}{4} = \frac{1}{2} \tag{4}$$

$$P(X=2) = \frac{{}^{2}C_2}{2^2} = \frac{1}{4} \tag{5}$$

(ii) Let Y be the random variable representing the number of tails in simultaneous tosses of 3 coins. Y can take values {0, 1, 2, 3}.

$$P(Y=r) = {}^{n}C_r p^r q^{n-r}, p = \frac{1}{2}, q = \frac{1}{2}, n = 3. \tag{6}$$

$$\therefore P(Y=r) = \frac{{}^{3}C_r}{2^3} \tag{7}$$

$$P(Y=0) = \frac{{}^{3}C_0}{2^3} = \frac{1}{8} \tag{8}$$

$$P(Y=1) = \frac{{}^{3}C_1}{2^3} = \frac{3}{8} \tag{9}$$

$$P(Y=2) = \frac{{}^{3}C_2}{2^3} = \frac{3}{8} \tag{10}$$

$$P(Y=3) = \frac{{}^{3}C_3}{2^3} = \frac{1}{8} \tag{11}$$

(iii) Let Z be the random variable representing the number of heads in four tosses of a coin. Z can take values {0, 1, 2, 3, 4}.

$$P(Z=r) = {}^{n}C_r p^r q^{n-r}, p = \frac{1}{2}, q = \frac{1}{2}, n = 4. \tag{12}$$

$$\therefore P(Z=r) = \frac{{}^{4}C_r}{2^4} \tag{13}$$

$$P(Z=0) = \frac{{}^{4}C_0}{2^4} = \frac{1}{16} \tag{14}$$

$$P(Z=1) = \frac{{}^{4}C_1}{2^4} = \frac{4}{16} = \frac{1}{4} \tag{15}$$

$$P(Z=2) = \frac{{}^{4}C_2}{2^4} = \frac{6}{16} = \frac{3}{8} \tag{16}$$

$$P(Z=3) = \frac{{}^{4}C_3}{2^4} = \frac{4}{16} = \frac{1}{4} \tag{17}$$

$$P(Z=4) = \frac{{}^{4}C_4}{2^4} = \frac{1}{16} \tag{18}$$

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