

AI1110 Assignment 2 in L^AT_EX

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12.13.4.4 Question:

Find the probability distribution of

- (i) number of heads in two tosses of a coin.
- (ii) number of tails in the simultaneous tosses of three coins.
- (iii) number of heads in four tosses of a coin.

Solution:

- (i) Let X be the random variable representing the number of heads in two tosses of a fair coin. X can take values {0, 1, 2}.

$$P(X=r) = \binom{n}{r} (p)^r (1-p)^{n-r}, \text{ here } p = \frac{1}{2}, n = 2.$$

$$\therefore P(X=r) = \binom{2}{r} \left(\frac{1}{2}\right)^2$$

$$P(X=0) = \binom{2}{0} \frac{1}{2^2} = \frac{1}{4} \quad (1)$$

$$P(X=1) = \binom{2}{1} \frac{1}{2^2} = \frac{2}{4} = \frac{1}{2} \quad (2)$$

$$P(X=2) = \binom{2}{2} \frac{1}{2^2} = \frac{1}{4} \quad (3)$$

- (ii) Let Y be the random variable representing the number of tails in simultaneous tosses of 3 coins. Y can take values {0, 1, 2, 3}.

$$P(Y=r) = \binom{n}{r} (p)^r (1-p)^{n-r}, \text{ here } p = \frac{1}{2}, n = 3.$$

$$\therefore P(Y=r) = \binom{3}{r} \left(\frac{1}{2}\right)^3$$

$$P(Y=0) = \binom{3}{0} \frac{1}{2^3} = \frac{1}{8} \quad (4)$$

$$P(Y=1) = \binom{3}{1} \frac{1}{2^3} = \frac{3}{8} \quad (5)$$

$$P(Y=2) = \binom{3}{2} \frac{1}{2^3} = \frac{3}{8} \quad (6)$$

$$P(Y=3) = \binom{3}{3} \frac{1}{2^3} = \frac{1}{8} \quad (7)$$

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- (iii) Let Z be the random variable representing the number of heads in four tosses of a coin. Z can take values $\{0, 1, 2, 3, 4\}$.

$$P(Z=r) = \binom{n}{r} (p)^r (1-p)^{n-r}, \text{ here } p = \frac{1}{2}, n = 4.$$

$$\therefore P(Z=r) = \binom{4}{r} \left(\frac{1}{2}\right)^4$$

$$P(Z = 0) = \binom{4}{0} \frac{1}{2^4} = \frac{1}{16} \quad (8)$$

$$P(Z = 1) = \binom{4}{1} \frac{1}{2^4} = \frac{4}{16} = \frac{1}{4} \quad (9)$$

$$P(Z = 2) = \binom{4}{2} \frac{1}{2^4} = \frac{6}{16} = \frac{3}{8} \quad (10)$$

$$P(Z = 3) = \binom{4}{3} \frac{1}{2^4} = \frac{4}{16} = \frac{1}{4} \quad (11)$$

$$P(Z = 4) = \binom{4}{4} \frac{1}{2^4} = \frac{1}{16} \quad (12)$$