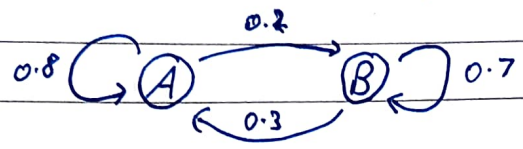
Q-2.) Let A = win

B = lose

Markov Chain Probability Graph



a.) $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.7 \end{bmatrix}$

we want $\bar{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ s.t. $v = P \cdot v$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad -①$$

$x + y = 1 \quad -②$

① $\Rightarrow$ $x = 0.8x + 0.2y \Rightarrow 0.2x = 0.3y \Rightarrow x = \frac{3}{2}y \quad -③$

$y = 0.2x + 0.7y \Rightarrow$ ↗

②, ③,

$\frac{3}{2}y + y = 1 \Rightarrow y = 2/5 = 0.4$

$y = 0.4$

$x = 0.6$

P(win) in long run = 0.6.

b.) P(dinner) = P(win) × P(dinner/win) + P(lose) × P(dinner/lose)

= 0.6 × 0.7 + 0.4 × 0.2

= 0.42 + 0.08

P(dinner) = 0.50

c.) $E(X) = \sum_{i=1}^{\infty} i \times P(\text{dinner at } i^{th} \text{ match}) = \sum i \underbrace{(1/2)^{i-1}}_{\text{no dinner for } i-1 \text{ matches}} \underbrace{(1/2)}_{\text{dinner after } i^{th} \text{ match}}$

$= \sum i (1/2)^i = 2 \quad (= 1/p)$