1) (a)

$$Q = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix}$$

(b) For all $i = 1, 2, 3, 4$ $\sum_{n=1}^{\infty} P_{ii}^{(n)} = \infty$

Therefore all the states $\{1, 2, 3, 4\}$ are recurrent.

No state is transient.

(c) $\{1, 2\}$ is irreducible and so is $\{3, 4\}$

First stationary state:

$$\pi^{(1)} = \{\pi_1, \pi_2, 0, 0\}.$$

Solving $\pi^{(1)} Q = \pi^{(1)}$

$\Rightarrow \quad 0.5\pi_1 + 0.25\pi_2 = \pi_1 \quad , \quad 0.5\pi_1 + 0.75\pi_2 = \pi_2$

$\Rightarrow \quad 0.5\pi_1 = 0.25\pi_2 \quad , \quad 0.25\pi_2 = 0.5\pi_1$

$\Rightarrow \quad \pi_1 = \frac{1}{2}\pi_2$

Also $\pi_1 + \pi_2 = 1 \quad \Rightarrow \quad \pi_1 = \frac{1}{3}, \pi_2 = \frac{2}{3}$

$$\therefore \boxed{\pi^{(1)} = \left\{ \frac{1}{3}, \frac{2}{3}, 0, 0 \right\}} \quad \text{———①}$$

Second stationary state,

$$\Pi^{(2)} = \{0, 0, \Pi_3, \Pi_4\}.$$

$$\Pi^{(2)} q = \Pi^{(2)} \implies \Pi_3 = 0.25\Pi_3 + 0.45\Pi_4$$

$$\Pi_4 = 0.45\Pi_3 + 0.25\Pi_4$$

$$\implies \Pi_3 = \Pi_4 \quad \text{and} \quad \Pi_3 + \Pi_4 = 1$$

$$\implies \Pi_3 = \Pi_4 = \frac{1}{2}.$$

$$\therefore \boxed{\Pi^{(2)} = \left\{0, 0, \frac{1}{2}, \frac{1}{2}\right\}.} \quad — \quad \textcircled{2}$$