ASSIGNMENT.-4

A) Let X be the no. of increasing moves

$\therefore \; S_t = S_0 + X - (10-X)$

$\Rightarrow S_0 + 2X - 10$

for $S_t > K \Rightarrow S_0 + 2X - 10 > K$

$\Rightarrow 100 + 2X - 10 > 105 \Rightarrow 2X > 15 \Rightarrow X > 7.5$

$\therefore \; \boxed{X \geq 8}$

$\therefore$ Required probability $= P(X \geq 8)$

$$= \sum_{x=8}^{10} \binom{10}{x} \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10} \left[\frac{10 \times 9}{2} + 10 + 1\right]$$

$$= \frac{1}{2^{10}} \times 56 = \frac{56}{1024} = \boxed{\frac{7}{128}} \text{ (Ans (a))}$$

Payoff $= \max(S_t - K, 0)$

for $x < 8 \Rightarrow$ Payoff $= 0$.

$\therefore$ Payoff $= \max(S_t - K, 0) \quad x \geq 8 = 2x - 15$

$$E[\text{Payoff}] = \sum_{x=8}^{10} \binom{10}{x} \left(\frac{1}{2}\right)^{10} (2x - 15)$$

$$= \frac{1}{2^{10}} \left[45 \times 1 + 10 \times 3 + 5\right]$$

$$= \boxed{\frac{5}{64}} \text{ (Ans (b))}$$

Fair value = Payoff (No discounting)
$= \boxed{\frac{5}{64}}$ (Ans (c))

Part B :

A) $\sigma = \sqrt{\frac{\pi}{2}}\, E\left[|x|\right] = \boxed{\sqrt{\frac{\pi}{2}}}$ (Ans).

For 10 days, $\sigma_{10} = \sqrt{10}\,\sigma = \boxed{\sqrt{5\pi}}$ (Ans)

b) $S_T = S_0 e^{R} \Rightarrow \log\left(\frac{S_T}{S_0}\right) = R \sim N(0, \sigma^2)$.

$\Rightarrow \log S_T \sim N(\log S_0, 10\sigma^2)$

$\therefore$ $S_T \sim \text{lognormal}(\mu = \log S_0, \sigma^2 = 10\sigma^2)$

$\therefore f_{S_T}(s) = \frac{1}{s \cdot \sigma\sqrt{2\pi}} e^{-\frac{(\log s - \mu)^2}{2\sigma^2}}$

$$\therefore \boxed{E\left[\max(S_T - K, 0)\right] = \int_K^{\infty} (s-K) \cdot \frac{1}{s \cdot \sigma\sqrt{2\pi}} e^{-\left(\frac{\log s - \mu}{2\sigma^2}\right)^2} ds}$$

(Ans).

c) The expectation : 18.0218

Teacher's Signature.....................

Part C:

(a) Assuming it is symmetrical. i.e $U(-a, a)$

$\frac{1}{2a}\int_{-a}^{a} x \, dx = 1 \Rightarrow \frac{a}{2} = 1 \Rightarrow \boxed{a = 2}$.

$\therefore$ Support = $[-2, 2]$

(b) Binomial distribution allows only for ± 1 changes.

Normal distribution is unbounded.

Uniform distribution can provide a range of values in $[a, b]$.

(c) Estimated fair price = $\boxed{0.1397}$.