Problem 1.2

$$h(x) = \text{sign}(\vec{w}^T \vec{x})$$

$$= \text{sgn}(g(x))$$

Here $x_2 = a x_1 + b$

$a =$ slope of line

when $a = 0$ then the above line is horizontal as slope $= 0$.

For $a > 0$ it is increasing & b is the offset (y-intercept). Hence we intersect x_2 axis at $x_2 = b$ & line will pass through origin if and only if $b = 0$.

$\therefore$ $x_2 = a x_1 + b$.

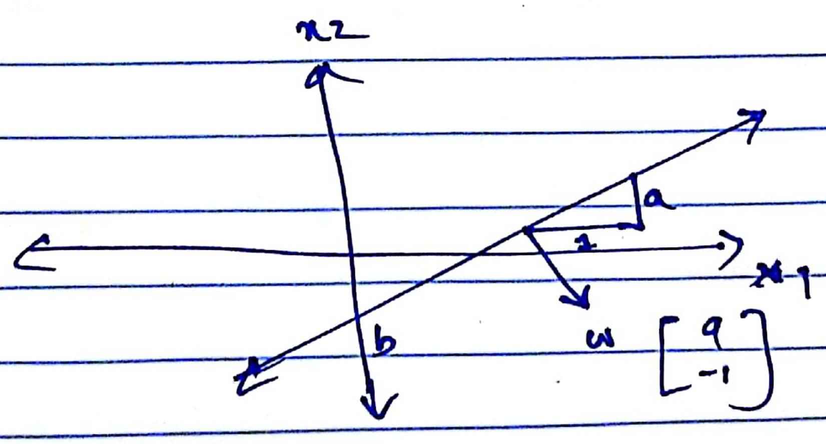
$\Rightarrow$ $-a x_1 + x_2 + b = 0$

converting in vector form

$$[a, -1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b = 0$$

Considering $x_0 = 1$

$$[b \;\; -a \;\; 1] \cdot \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$$



linear function is defined by.

$$y(x) = 0 \quad \& \quad y(x) = w^T x$$

$$\therefore (w_0, w_1, w_2) \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0.$$

$$\therefore w_0 + w_1 x_1 + w_2 x_2 = 0.$$

This is the hyperplane with normal vector w

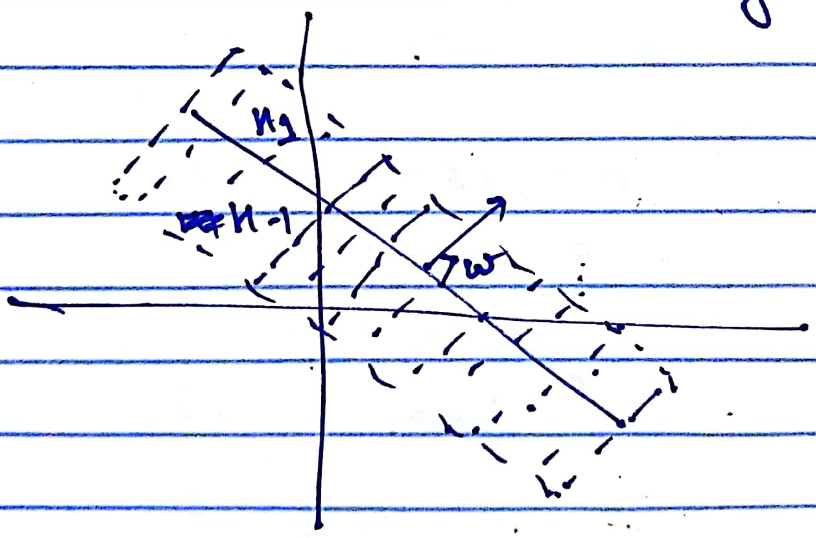
This hyperplane seperates space into two halfs $h(x) = 1$ & $h(x) = -1$

i.e

$$\mathcal{H}_{+1} = \{ x \mid y(x) > 0 \} \quad \& \quad \mathcal{H}_{-1} \{ x \mid y(x) < 0 \}$$

Hence $h(x) = \mathrm{sgn}(y(x)) = 1$ for $x \in \mathcal{H}_1$

& $h(x) = \mathrm{sgn}(y_n) = -1$ for $x \in \mathcal{H}_{-1}$



b) Draw the picture for cases $\vec{w} = [1, 2, 3]^T$ & $\bar{w} = -[1, 2, 3]^T$

Case 1:

$y(x) = 0.$

$\Rightarrow w^T x = 0.$

$$(1, 2, 3)\begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0.$$

$(2, 0)$

$(0, 3) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3}{2}$ $m = y/x$

$y = -1/3$

$x = -1/2$

$m = y/x =$

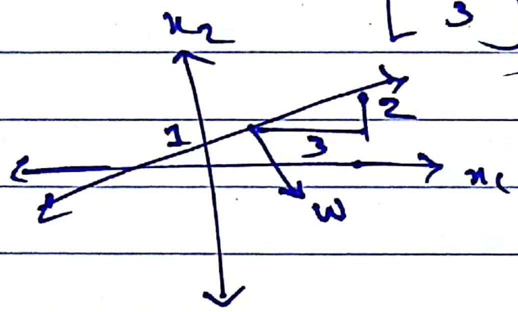
$1 + 2x_1 + 3x_2 = 0$

$2x_1 + 3x_2 = -1$

$\therefore w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$

$x_1 = -1/2$ $x_2 = 0$

$x_2 = -1/3$ $x_1 = 0$



Case 2:

$y(x) = 0$

$\therefore w^T(x) = 0.$

$$[-1, -2, -3]\begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$$

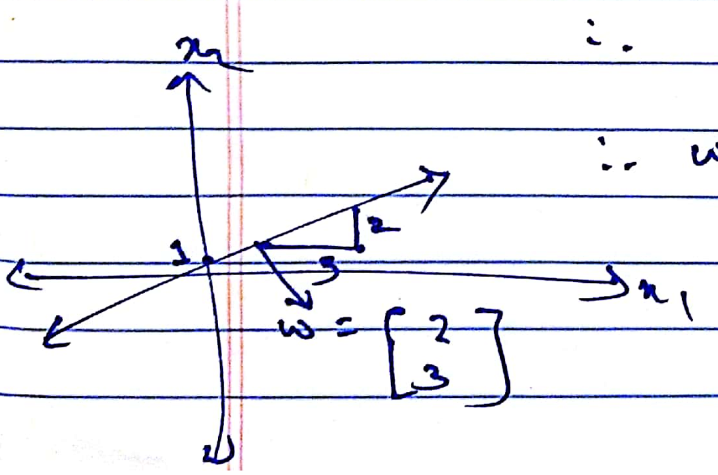
$-1 - 2x_1 - 3x_2 = 0$

$2x_1 + 3x_2 + 1 = 0$

$2x_1 + 3x_2 = -1$

$$\therefore [2, 3]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1 = 0$$

$\therefore w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$



Exercise 1.8

ν = Fraction of red balls

$\therefore P(\nu \leq 0.1) = P\left(\frac{red}{N} \leq 0.1\right)$

$\Rightarrow P(red \leq 0.1N)$

$N=10$

$P(red \leq 0.1N) = P(red \leq 1) = P(red=0) + P(red=1)$ ↳ ①

$\mu = 0.9 = P(red)$ $\therefore P(green) = 1-0.9 = 0.1$

$P(red=0) = \cancel{10 \times 0.1 \times 10^{-10}}\ 10 \times (0.1)^{10} = 10^{-10}$

$P(red=1) = 10 \times 0.1^{9} \times 0.9 = 9 \times 10^{-9}$

$\therefore$ From Eq^n ①

$P(red \leq 1) = 10^{-10} + 9 \times 10^{-9}$

$= 0.1 \times 10^{-9} + 9 \times 10^{-9}$

$= 9.1 \times 10^{-9}$

$\therefore P(\nu \leq 0.1) = 9.1 \times 10^{-9}$

Problem ~~Ex~~ 1.11

For supermarket we have to penelize more ~~for~~ if a true customer is reject. (we are not considering ~~po~~ correct case as error = 0 in this case).
General form ~~is~~ of Ein is given by

$$E_{in}(h) = \frac{1}{N} \sum_{n=1}^{N} e\left(h(x_n), f(x_n)\right).$$

Penelizing more $\Rightarrow$

$$= \frac{1}{N}\left[\sum_{y_n=1} e\left(h(x_n), 1\right) + \sum_{y_n=-1} e\left(h(x_n), -1\right)\right]$$

$$= \frac{1}{N}\left[\sum_{y_n=1} 10\,[h(x) \neq 1] + \sum_{y_n=-1} [h(x) \neq -1]\right]$$

For ~~super market~~ CIA we have to penalize more if algorithm accepts an ~~#~~ intruder (we are not considering correct case as error = 0)

$\therefore$ General form of Ein is given by

$$E_{in}(h) = \frac{1}{N} \sum_{n=1}^{N} e\left(h(x_n), f(x_n)\right)$$

$$= \frac{1}{N}\left[\sum_{y_n=1} e\left(h(x_n), 1\right) + \sum_{y_n=-1} e\left(h(x_n), -1\right)\right]$$

$$= \frac{1}{N}\left[\sum_{y_n=1} [h(x_n) \neq 1] + \sum_{y_n=-1} 1000\,[h(x) \neq -1]\right]$$

Exercise 1.9

General formula

$$P(|\nu - \mu| > \epsilon) \le 2e^{-2\epsilon^2 N} \rightarrow (1)$$

Here $\mu = 0.9$

$\nu = 0.1$

$N = 10$

$\Rightarrow$ ~~$|\nu - \mu| \le -0.8$~~ $\quad \nu - \mu \le -0.8$

$\therefore \quad |\nu - \mu| \geq 0.8$

$\therefore$ we get $\epsilon = 0.8$ by comparing with above form (1)

$$\therefore \quad P(|\nu - \mu| > 0.8) \le 2e^{-2 \times 0.8^2 \times 10}$$

$$\approx 5.52 \times 10^{-6}$$

From solution of ~~Exer~~ exercise 1.8 we get $P(\nu \le 0.1) \le P(|\nu - \mu| > 0.8)$

$$\Rightarrow \quad 9.1 \times 10^{-9} \le 5.52 \times 10^{-6}$$

Hence we prove that