Department of Computer Science and Engineering

Leading University, Sylhet

Preparatory Assignment

Course Code: CSE-3213

Course Title: Digital Signal Processing

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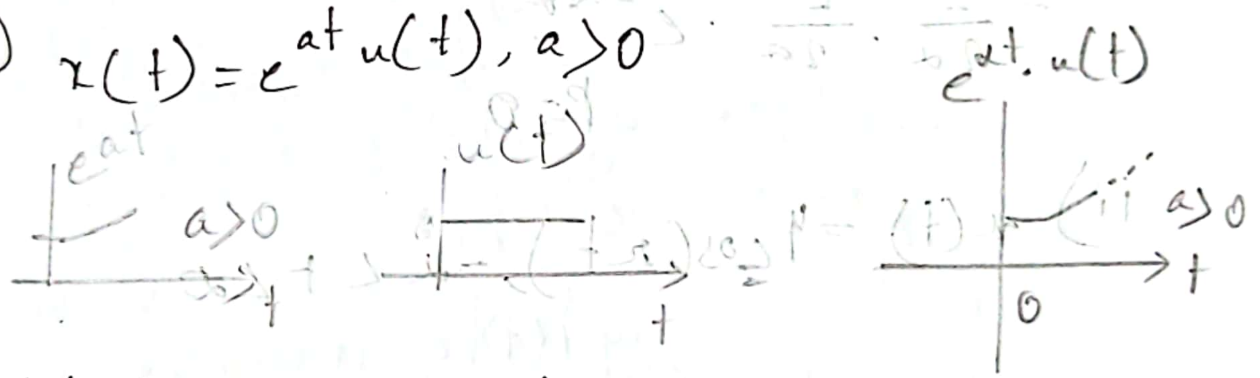
1 NO ANS

i) $x(t) = e^{at} u(t),\ a>0$



Total Energy, $E = \int_{-\alpha}^{\alpha} |x(t)|^2 dt$

$$= \int_{-\alpha}^{0} 0\,dt + \int_0^{\alpha} |e^{at}\cdot u(t)|^2 dt = \int_0^{\alpha} (e^{at}\cdot 1)^2 dt$$

$$= \int_0^{\alpha} e^{2at}\cdot dt = \frac{1}{2a} e^{2at} \Big|_0^{\alpha}$$

$$= \cancel{\frac{1}{2T}}\ \frac{1}{2a}(e^{\alpha} - e^{0})$$

$E = \alpha$

Now, $P = \lim_{T\to\alpha} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim \frac{1}{2T}$

$$\left[\int_{-T}^{0} 0\,dt + \int_0^{T} |x(t)|^2 dt\right]$$

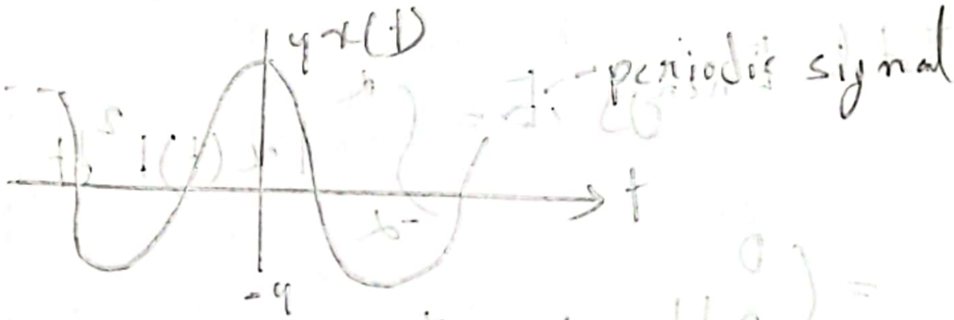
$$P = \lim_{T\to\infty} \frac{1}{2T} \cancel{\int_{-T}^{T} |x(t)|^2 dt} = \lim_{T\to\infty} \int_0^T |e^{at}\cdot u(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\cdot\frac{1}{2a}\cdot e^{2at}\Big|_0^T$$

$$= \frac{1}{2\infty}\cdot\frac{1}{2a}\cdot e^{2a\infty}$$

$$P = 0$$

ii) $x(t) = 4\cos(\pi t), -\infty < t < \infty$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} (4\cos(\pi t))^2 dt$$

$$= 16\int_{-\infty}^{\infty} \cos^2(\pi t) dt$$

$$= 8\left[t\Big|_{-\infty}^{\infty} + \frac{\sin 2\pi t}{2\pi}\Big|_{-\infty}^{\infty}\right]$$

$$= \infty$$

Now for periodic Signal

$$\text{Avg } P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)^2| dt, \quad T_0 = \text{Period}$$

3

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (4\cos(\pi t))^2 dt = \frac{16}{T_0}\int_{-T_0/2}^{T_0/2} \cos^2(\pi t)\,dt$$

$$= \frac{8}{T_0}\left[t\Big|_{-T_0/2}^{T_0/2} + \frac{\sin 2\pi t}{2\pi}\Big[_{-T/2}^{T/2}\Big]\right]$$

$$= \frac{8}{10}\left[T_0 + \frac{1}{2\pi}(\sin \pi T_0 - \sin(-\pi T_0))\right]$$

$$= \frac{8}{10}(T_0 + 0) = 8 \text{ J (Finite)}$$

1) (iii) $x(t) = rect\left(\frac{2t}{6}\right) = rect(t/3)$

Total Energy,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_{-\infty}^{-3/2} 0\,dt + \int_{-3/2}^{3/2} |x(t)|^2 dt + \int_{3/2}^{\infty} 0\,dt$$

$$= 0 + \int_{-3/2}^{3/2} (1)^2 dt + 0 = \int_{-3/2}^{3/2} dt$$

$$= t\Big|_{-3/2}^{3/2} = 3/2 - (-3/2) = \frac{6}{2} = 3 \text{ J}$$

$E = 3$ J. Finite Energy Signal

therefore $P = 0$

2 NO ANS

A signal is said to be real when it satisfies the condition,

$$x(t) = x^*(t)$$

Ex: 1. $x(t) = \cos \omega_0 t$

$$x^*(t) = \cos \omega_0 t$$

A signal is said to be imiginary when it satisfies the condition,

$$x(t) = -x^*(t)$$

Ex. $x(t) = ibt \Rightarrow x^*(t) = (ibt)^* = -ibt$

$$-x^*(t) = ibt$$

$$x(t) = -x^*(t)$$

3 NO ANS

Static System: Static system is memory less.

In this, the output is instant value, not on the past or future value.

Eg

Dynamic: It is a memory system. Output 5 will depends on the past or future input in addition to be the present input.

4 NO ANS

Dynamic system is a memory system, whose response depends on past or future inputs in addition with present
Non-causal also depends on future includes past and present.
Therefore all non-causal system are dynamic

$$y(t) = x(t) = \frac{1}{x(t+2)}$$

$$\therefore y(0) = \underbrace{x(0)}_{\text{present}} + \underbrace{\frac{1}{x(2)}}_{\text{Finite}}$$

It's a non-causal signal, also dynamic signal

5 NO ANS

$x(t) * \delta(6t-2)$

Here, $x(t) \longrightarrow$ unit parabolic signal

$\delta(6t-2) \longrightarrow$ Impulse Signal.

$= x(t) \cdot \delta(6t-2)$

$= x(t) \cdot \delta\left[6\left(t-\frac{1}{3}\right)\right]$

$= \frac{1}{6} x(t) \cdot \delta\left(t-\frac{1}{3}\right)$

$= \frac{1}{6} \cdot \frac{t^2}{2}$ at $t = \frac{1}{3}$

$= \frac{1}{6}(1/3)^2 \times \frac{1}{2}$

$= 1/108$ Ans.

GNO ANS (i) 7

i) $y(n) = x^2(n) + x(n+2)$

Now $y(0) = x^2(0) + x(2)$

$\searrow$ present $\searrow$ future

$\therefore$ Dynamic System

$\therefore$ Non-Causal System

Now, $y(n,k) = T[x(n-k)] = x^2(n-k) + x(n+2-k)$

$y(n-k) = x^2(n-k) + x(n+2-k)$

$\therefore y(n,k) = y(n-k)$ [Time invariant]

Now

$T[x_1(n)] = x_1^2(n) + x_1(n+2)$

$T[x_2(n)] = x_2^2(n) + x_2(n+2)$

Individual Response $= a_1[x_1^2(n) + x_1(n+2)]$

$+ a_2[x_2^2(n) + x_2(n+2)]$

Combined Response $= T[a_1x_1(n) + a_2x_2(n)]$

$= a_1[x_1(n) + x_2(n)]^2 + a_2[x_1(n+2) + x_2(n+2)]$

Since, Individual Response $\neq$ Combined Response

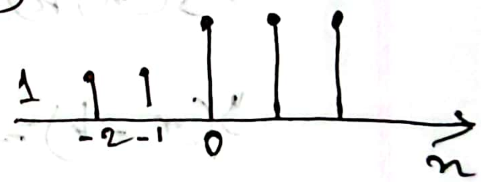
Non-linear System.

Now, $x(n) = u(n)$

$y(n) = [u(n)]^2 + u(n+2)$

$= u(n) + u(n+2)$

= Bounded



$\therefore$ Stable System

CNO ANS (ii)

$y(t) = x(t+20) + x^2(t)$

$y(0) = x(20) + x^2(0)$

Future

present

This Response depends on future and present

$\therefore$ It is a Dynamic System.

It is a Non-Causal System

Now, $y(n,k) = T[x(n-k)] = x(t+20-k)$
$+ x^2(t-k)$

$y(n-k) = y(t+20-k) + x^2(t-k)$

thus, $y(n,k) = y(n-k)$ [Time invariant]

Now, $T[x_1(t)] = x_1(t+20) + x_1^2(t)$

$T[x_2(t)] = x_2(t+20) + x_2^2(t)$

Individuals Response :- $a_1[x_1(t+20) + x_1^2(t)]$
$+ a_2[x_2(t+20) + x_2^2(t)]$

Combined Response :- $T[a_1x_1(t) + a_2x_2(t)]$
$= [a_1x_1(t+20) + a_2x_2(x+20)] +$
$[a_1x_1(t) + a_2x_2(t)]^2$

$\therefore$ Individual $\neq$ Combined Res
Res

$\therefore$ non-linear System.

Now $x(t) = u(t)$

$y(t) = u(t+20) + u(t)$ bounded

$\therefore$ Stable System.

Q NO ANS (iii)

$y(t) = y(t+5) - x(t-5)$

$y(0) = y(5) - x(-5)$

Future Past

Since, Responsive depends on past and future.

$\therefore$ It's dynamic system

It's Non-causal system

Now, $y(n,k) = T[x(n-k)] = y(t+5-k)$
$- y(t-5-k)$

$y(n-k) = y(t+5-k) - y(t-5-k)$

$\therefore y(n,k) = y(n-k)$ [$\because$ Time invariant]

Now,

$T[x_1(t)] = x_1(t+5) - x_1(t-5)$

$T[x_2(t)] = x_2(t+5) - x_2(t-5)$

Individual Response $= a_1 x_1(t+5) - a_1 x_1(t-5)$
$+ a_2 x_2(t+5) - a_2 x_2(t-5)$

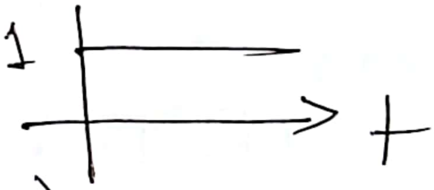
Combined Response $= T[a_1 x_1(t) + a_2 x_2(t)]$

$= a_1 x_1(t+5) - a_1 x_1(t-5) + a_2 x_2(t+5)$
$- a_2 x_2(t-5)$

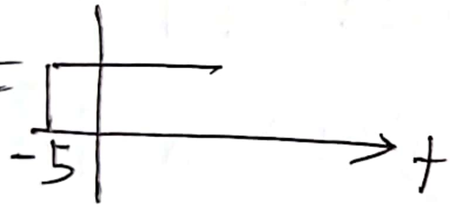
Individual Res = Combined Res

$\therefore$ Linear System

Now, $x(t) = u(t)$



$y(t) = u(t+5) - u(t-5)$

$=$ 

$=$ Bounded

$\therefore$ Stable System.

7 NO ANS

Properties of convolution:-

(i) Commutative property:- $x_1(t) * x_2(t) = x_2(t) * x_1(t)$

ii) Distributive property:- $x_1(t) * [x_2(t) + x_3(t)] = [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$

iii) Associative property:-

$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$

iv) Shift Property

if $x_1(t) * x_2(t) = z(t)$

$x_1(t-T) * x_2(t) = z(t-T)$

$x_1(t-T_1) * x_2(t-T_2) = z(t-T_1-T_2)$

8 NO ANS

Basic Signal Operations:-

Addition and Substraction:-

The sum of two signals $x(t)$ $y(t)$ is given by $z(t) = x(t) + y(t)$

the difference between two signals $x(t)$ + $y(t)$ is given by $z(t) = x(t) - y(t)$

Scaling :- Scaling is a signal $x(t)$ by a time delay T is represent by $y(t) = x(t_0 - T)$. this operation introduces a time offset in the signal.

Time Reversal :-

Reversing the time axis of the signal ~~x(t)~~ $x(t)$ is denoted as $y(t) = x(-t)$. this ~~problem~~ operation the signal across. The ~~x~~ vertical axis in the time domain.

Multiplication :- The product of two signal $x(t)$ & $y(t)$ is given by $z(t) = x(t) \cdot y(t)$

2 NO ANS

The statement " convoluted. signal. area = Multiplication of Impulse area". is generally not true.

The area of convoluted signal is not always equal to the product of the input signal area and the impulse signal area. However, there are exception. for instance if the ~~filters i~~ filter is a Dirac delta function, then the convolution of the input signal with the ~~fil~~ filter is simply the input signal itself. In this case, the area of the convoluted signal is ~~equally~~ equal to the area of the input signal.

Here is example :-

Consider the input signal $x(t) = rect(t)$ which has an area of 1. The impulse response of the filter is $h(t) = \delta(t)$ Which has an area of ∞. The convoluted signal $y(t) = x(t) * h(t) = x(t)$ which also has an area of 1.

~~NO ANS~~

Therefore in this special case., the area of convoluted signal is equal to the product of the input signal area. and the impulse signal area. However, this does not hold for all convolution operations.

10 NO ANS

LTI (Linear Time-Invariant System):- An LTI system behaves consistently over time. If we apply a time shift to the input, the output undergoes the same shift.

LTV (Linear Time-Variant System):- An LTV system's behaves can change with time. If we can apply a time shift to input. the output might respond differently. Its behavior varies with time.

i) $y(t) = \cos[x(t+4)]$ 1C

The system is not linear because the output is not a linear function of the input.

ii) $y(t) = x(t) \cdot u(t)$

This system is linear. because the output is a linear function of the input.

iii) $y(n) = x(n)/ x(n-2)$

The system is non-linear as Individu Res ≠ combined Res

11 NO ANS

Defindion of unit step function $u(t)$:-

$u(t)$. f_1 0. for $t < 0$ 1 for $t \geq 0$

·here $y(t)$:

· $u(t+4)$ is 1 for $t \geq -4$ & 0 otherwise

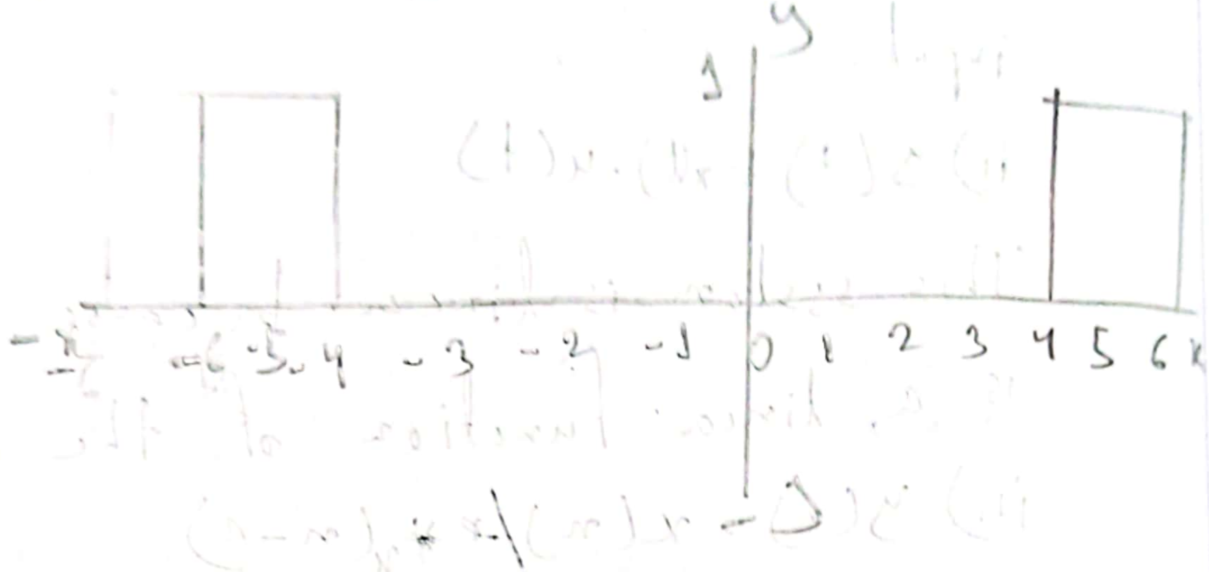
· $u(t+6)$ is 1 ~~for~~ for $t \geq -6$ & 0 otherwise

· $u(t-4)$ is 1 for $t \geq 4$ & 0 otherwise

· $u(t-6)$ is 1 for $t \geq 6$ & 0 otherwise

$$y(t) = \begin{cases} 1 & \text{for } -6 < t \le -4 \text{ or } 4 \le t < 6 \\ 0 & \text{Otherwise} \end{cases}$$ [17]

here the Sketch $y(t)$:-



12 NO ANS.

$x(n) = \{1, 2, 3, 4, 8, 5, 6, 2\}_n$
$h(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}_n$

	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2
1	1	2	3	4	8	5	6	2

$y(n) = x(n) * h(n)$
$= \{1, 3, 6, 10, 18, 23, 29, 31, 30, 28, 25, 21, 15$

$8, 2 \}$ [118

$\cancel{4} = m + n - 1 = 8 + 8 - 1 = 15$

13 NO ANS

$$x_1(t+T_1) * x_2(t) = z(t+T_1)$$

By using shift property of convolution we know,

$$x_1(t-T) * x_2(t) = z(t-T)$$

$$T = -T'$$

$$\Rightarrow x_1(t-(-T')) * x_2(t) = z(t-(-T'))$$

$$\Rightarrow x_1(t+T') * x_2(t) = z(t+T')$$

Therefore, $x_1(t+T_1) * x_2(t) = z(t-T_1)$

proved

(i) $x_1(t+8) * x_2(t) = z(t+8)$

Ex (i) $x_1(t+8) * x_2(t) = z(t+8)$

(ii) Convolution of $x_1(t+10)$ and $x_2(t)$ is $z(t+10)$