

## Feedback-Augmented Loss Function: Worked Example

Batch Size: 2 | Sequence Length: 3 | Vocab Size: 4

Logits:

```
[[[2. 1. 0.1 0. ]  
 [1.5 2.2 0.5 0.3]  
 [0.3 0.2 2. 1.7]]
```

```
[[0.1 2. 1. 0.5]  
 [2. 0.1 0.1 2.1]  
 [0.2 1.2 1.8 1.5]]]
```

Labels:

```
[[0 1 2]  
 [1 3 2]]
```

Feedback: solution\_score=[0.9 0.4], reasoning\_score=[0.8 0.2], is\_correct=[ True False]

### Step 1: Cross-Entropy per Sample:

Cross-entropy per token:

```
[[ -1.49753926 -1.59631951 -1.26340996]  
 [ -1.44578263 -1.32273781 -0.88710448]]
```

Cross-entropy per sample (mean):

```
[-1.45242291 -1.21854164]
```

### Step 2: Reward Loss per Sample:

Mean log-prob of correct tokens: [1.45242291 1.21854164]

Feedback score: [0.85 0.3 ]

Reward loss: [-1.23455947 -0.36556249]

### Step 3: Penalty Loss per Sample:

Mean max softmax prob per sample: [0.54352635 0.47851496]

Penalty loss: [0. 0.33496047]

### Step 4: Total Loss per Sample & Batch Mean:

Total loss per sample: [-2.68698238 -1.24914366]

Batch mean loss: -1.9681

### Mathematical Formulation:

$$L_{\text{aug}} = L_{\text{CE}} + \lambda_1 * (-\text{feedback\_score} * \text{mean\_log\_prob}) + \lambda_2 * ((1 - \text{feedback\_score}) * \text{mean\_max\_prob} * (1 - \text{is\_correct}))$$

Where  $L_{\text{CE}}$  is mean cross-entropy per sample,  $\text{mean\_log\_prob}$  is mean log-prob of correct tokens,  $\text{mean\_max\_prob}$  is average max-prob per sample,  $\text{feedback\_score}$  is in  $[0,1]$ ,  $\text{is\_correct}$  is boolean.