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Theory Questions-

1. given $C(X,Y,Z) = P_1(X_1,Y_1,Z_1)\cdot\alpha_1 + \alpha_2\, P_2(X_2,Y_2,Z_2) + \alpha_3\cdot P_3(X_3,Y_3,Z_3)$

Part I

Normalized chromaticity components of P_1, P_2, P_3 are (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) respectively (say).

$$\therefore (x_1,y_1,z_1) = \left(\frac{X_1}{X_1+Y_1+Z_1}, \frac{Y_1}{X_1+Y_1+Z_1}, \frac{Z_1}{X_1+Y_1+Z_1}\right)$$

$$(x_2,y_2,z_2) = \left(\frac{X_2}{X_2+Y_2+Z_2}, \frac{Y_2}{X_2+Y_2+Z_2}, \frac{Z_2}{X_2+Y_2+Z_2}\right)$$

$$(x_3,y_3,z_3) = \left(\frac{X_3}{X_3+Y_3+Z_3}, \frac{Y_3}{X_3+Y_3+Z_3}, \frac{Z_3}{X_3+Y_3+Z_3}\right)$$

Part II

Let normalized chromaticity coordinates of C be (x,y,z)

$$(x,y,z) = \left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}, \frac{Z}{X+Y+Z}\right)$$

From given,

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$$
$$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$$
$$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$$

$$\therefore x = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3 + \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}$$

$$= \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}$$

$$y = \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_2)}$$

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$$z = \frac{\alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}$$

Part III:

Thus we need to show that chromaticity co-ordinates of any colour c can be represented as a linear combination of the chromaticity co-ordinates of their respective primaries i.e.

$\cancel{x = \alpha_1' x_1 + \alpha_2' y_2 +}$

To show: $x = \alpha_1' x_1 + \alpha_2' x_2 + \alpha_3' x_3$

$y = \alpha_1' y_1 + \alpha_2' y_2 + \alpha_3' y_3$

$z = \alpha_1' z_1 + \alpha_2' z_2 + \alpha_3' z_3$

Now: $x = \frac{X}{X+Y+Z}$ and $x_1 = \frac{X_1}{X_1+Y_1+Z_1}$

$\cancel{x =}$

$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$

$\therefore x(X+Y+Z) = \alpha_1 \cdot x_1(X_1+Y_1+Z_1) + \alpha_2 \cdot x_2(X_2+Y_2+Z_2) + \alpha_3 \cdot x_3(X_3+Y_3+Z_3)$

$$x = \frac{\alpha_1 \cdot x_1 (X_1+Y_1+Z_1)}{(X+Y+Z)} + \frac{\alpha_2 \cdot x_2 (X_2+Y_2+Z_2)}{(X+Y+Z)} + \frac{\alpha_3 \cdot x_3 (X_3+Y_3+Z_3)}{(X+Y+Z)}$$

Now let $\alpha_1' = \frac{\alpha_1 (X_1+Y_1+Z_1)}{(X+Y+Z)}$

$\alpha_2' = \frac{\alpha_2 (X_2+Y_2+Z_2)}{(X+Y+Z)}$

$\alpha_3' = \frac{\alpha_3 (X_3+Y_3+Z_3)}{(X+Y+Z)}$

By definition, we must have $\alpha_1' + \alpha_2' + \alpha_3' = 1$

$\therefore \alpha_1' + \alpha_2' + \alpha_3'$

$$= \frac{\alpha_1(X_1+Y_1+Z_1)}{(X+Y+Z)} + \frac{\alpha_2(X_2+Y_2+Z_2)}{(X+Y+Z)} + \frac{\alpha_3(X_3+Y_3+Z_3)}{(X+Y+Z)}$$

$\alpha_1' + \alpha_2' + \alpha_3' = \frac{X+Y+Z}{X+Y+Z} = 1$ $\Big(\because \alpha_1(X_1+Y_1+Z_1) = X$, $\alpha_2(X_2+Y_2+Z_2) = Y$, $\alpha_3(X_3+Y_3+Z_3) = Z \Big)$

Thus:

$$\boxed{x = \alpha_1' \cdot x_1 + \alpha_2' x_2 + \alpha_3' x_3} \quad \text{--- (1)}$$

Similarly:

$$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$$

$$\therefore y(X+Y+Z) = \alpha_1 \cdot y_1 (X_1+Y_1+Z_1) + \alpha_2 \cdot y_2 (X_2+Y_2+Z_2) + \alpha_3 \cdot y_3 (X_3+Y_3+Z_3)$$

$$y = \alpha_1 \cdot \frac{y_1 (X_1+Y_1+Z_1)}{X+Y+Z} + \alpha_2 \cdot \frac{y_2 \cdot (X_2+Y_2+Z_2)}{X+Y+Z} + \alpha_3 \cdot \frac{y_3 \cdot (X_3+Y_3+Z_3)}{X+Y+Z}$$

$$\therefore \boxed{y = \alpha_1' \cdot y_1 + \alpha_2' y_2 + \alpha_3' y_3} \quad \text{--- (2)}$$

Also

$$z = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3$$

$$z(X+Y+Z) = \alpha_1 \cdot z_1 (X_1+Y_1+Z_1) + \alpha_2 z_2 (X_2+Y_2+Z_2) + \alpha_3 \cdot z_3 (X_3+Y_3+Z_3)$$

$$z = \alpha_1 \cdot \frac{z_1 (X_1+Y_1+Z_1)}{X+Y+Z} + \alpha_2 \cdot \frac{z_2 (X_2+Y_2+Z_2)}{X+Y+Z} + \alpha_3 \cdot \frac{z_3 \cdot (X_3+Y_3+Z_3)}{X+Y+Z}$$

$$\boxed{z = \alpha_1' \cdot z_1 + \alpha_2' \cdot z_2 + \alpha_3' \cdot z_3} \quad \text{--- (3)}$$

From (1), (2), (3)

$$(x, y, z) = \alpha_1' (x_1, y_1, z_1) + \alpha_2' (x_2, y_2, z_2) + \alpha_3' (x_3, y_3, z_3)$$

Hence proved.

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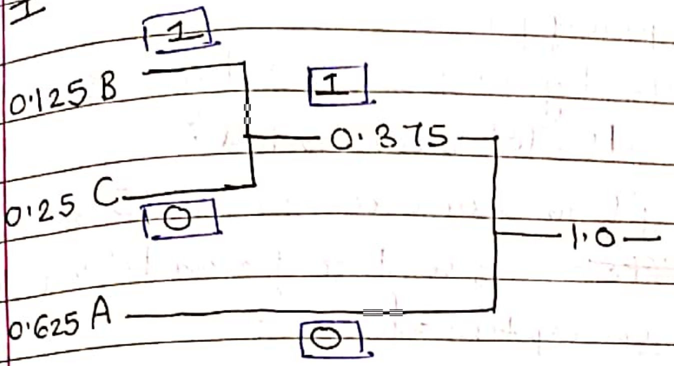
2. $S = \{A, B, C\}$, $P(A) = 0.625$, $P(B) = 0.125$, $P(C) = 0.25$

I



The Huffman code is as follows:-

A = 0 (length 1)

B = ~~10~~ 11 (length 2)

C = 10 (length 2)

~~average code length = $\frac{1+2+2}{3} = \frac{5}{3} = 1.667$~~

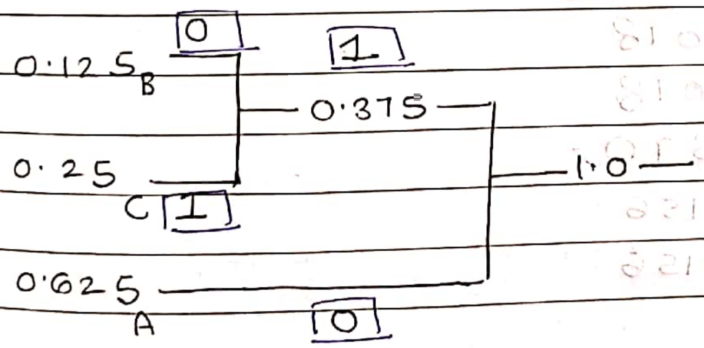
3. average code length $= \sum p_i l_i$

$= 0.625 \times 1 + 0.125 \times 2 + 0.25 \times 2$

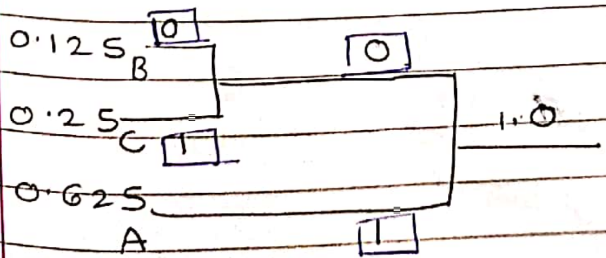
$= 1.375$ bits/symbol.

II There are four possible distinct codes. The other codes are:

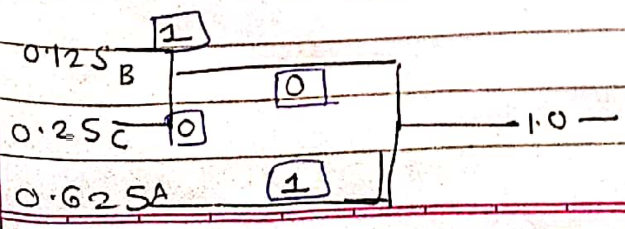
Possible Huffman nodes.



symbol	code
A	0
B	10
C	11



symbol	code
A	1
B	00
C	01



symbol	code.
A	1
B	01
C	00

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III ~~The entropy of the~~ The optimal code length is given by

$$-\sum p_i \log p_i$$

$$= -0.125 \log_2 0.125 - 0.25 \log_2 0.25 - 0.625 \log_2 0.625$$

$$= 1.298 \text{ bits/symbol.}$$

The average code length in our case is 1.375 bits/symbol which is ~~less than 1.298~~ greater than 1.298 (Entropy)

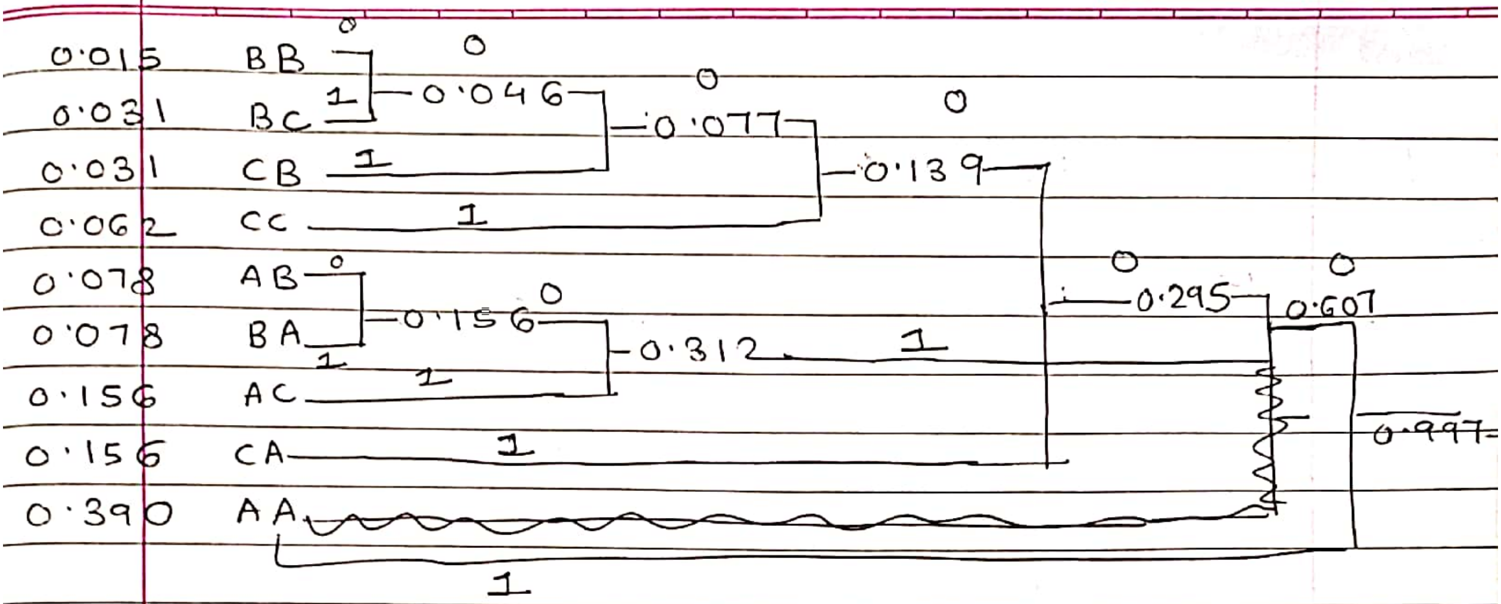
Hence this is not an optimal code.

★ Optimal techniques include arithmetic coding, block coding.

Solution: Block coding: blocks of length 2

Block	Probability
BC	0.03125
CB	0.03125
BB	0.0156
CC	0.0625
AB	0.078
BA	0.078
AA	0.390
AC	0.156
CA	0.156

Construct Huffman code.



Blocks	Encoding	Length.
BB	000000	6
BC	000001	6
CB	00001	5
CC	0001	4
AB	0100	4
BA	0101	4
AC	011	3
CA	001	3
AA	1	1

Average code length for ~~Block~~. encoding 2 symbols. $= \Sigma p_i l_i$

$= 6\times(0.015+0.031) + 5\times 0.031$
$+ 4(0.062+0.078+0.078)$
$+ 3(0.156+0.156) + 0.390$
$= 2.629$

Thus average code length per original symbol $= \frac{2.629}{2}$

~~= 1.391 bits/symbol~~
$= 1.31$ bits/symbol < 1.375

We can do even better with blocks of larger size.

Q.3.a. The entropy function is

$$H = -\sum p_i \log_2 p_i$$
$$= -P(X)\log_2 P(X) - P(Y)\log_2 P(Y)$$
$$= -x^k \log_2 x^k - (1-x^k)\log_2(1-x^k)$$

For $k = 2$

$$H = -x^2 \log_2 x^2 - (1-x^2)\log_2(1-x^2)$$

For $x = 1$, $H = 0$

The graph will be symmetric on both sides of the y-axis.

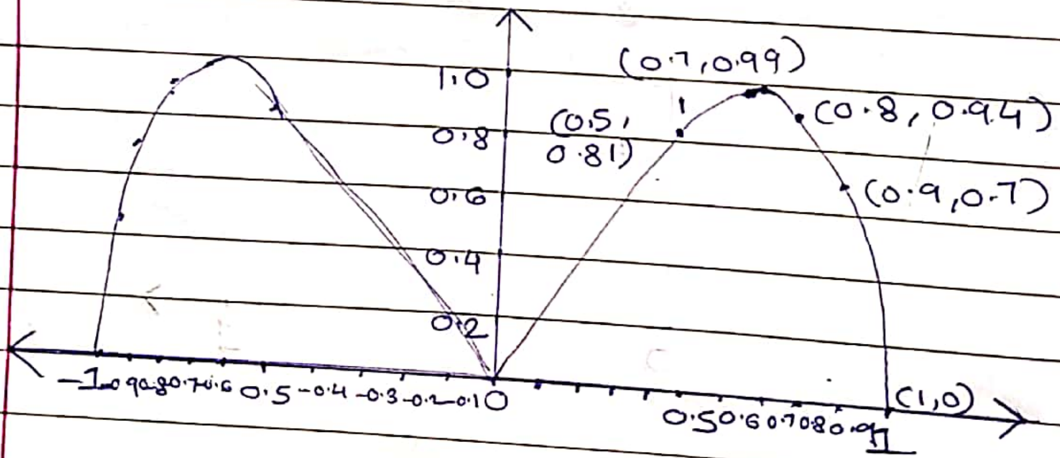
For $x = 0.5$, $H = 0.81$

For $x = 0.7$, $H = 0.99$

For $x = 0.8$, $H = 0.94$

For $x = 0.9$, $H = 0.70$

For $x = 0.95$, $H = 0.46$.



a. H becomes a minimum (i.e. 0) for $x = 0, -1, +1$.
This happens when for all p_i ($i = 1 \ldots n$), some $p_j = 1$ and all other $p_i = 0$ ($i \neq j$), i.e. the probability of occurrence of only one event is 1.

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Q. H ~~if~~ be minimum if either ~~P(X) = 1 or P(Y) = 1~~
~~(however both cannot be 1)~~

$P(X) = 1$ and $P(Y) = 0$ or. $P(X) = 0$ and $P(Y) = 1$

$x^k = 1$, $1 - x^k = 0$ $\qquad$ $x^k = 0$ and $1 - x^k = 1$

~~$\therefore H = -1\log_2 1$~~

$\boxed{x^k = 1}$ $\qquad$ $\boxed{x^k = 0}$

Thus entropy will be minimum when either
$x^k = 1$ or $x^k = 0$ (but not both at same time)

It follows that
$x = 0, 1, -1$ when k is even
$x = 0, 1$ when k is odd.

Q. From the plot we see that H is ~~maximum when~~
$H = 0.9997$ when $x = 0.7$ and $H = 0.94$ when $x = 0.8$

Thus it follows that H takes a maximum when it is very close to 0.7.

$H = 0.9997$, $x = 0.7$

H is maximum at $x = 0.7$ (approximately)

To find exact solution,
H is maximum when:

$$P(X) = P(Y)$$
$$x^k = 1 - x^k$$
$$2x^k = 1$$
$$x^k = 1/2$$
$$(k = 2) \qquad x^2 = 1/2$$
$$x = 0.7071 \text{ or } x = -0.7071$$

when $x = \pm 0.7071$ or $\pm 1/\sqrt{2}$, $H = 1$ (maximum)

Thus H is maximum when $x = \pm 0.7071 = \pm \frac{1}{\sqrt{2}}$

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Q. H is maximum when

$$P(X) = P(Y)$$

$$x^k = 1 - x^k$$

$$2x^k = 1$$

$$x^k = \frac{1}{2}$$

$$x^k = 0.5$$

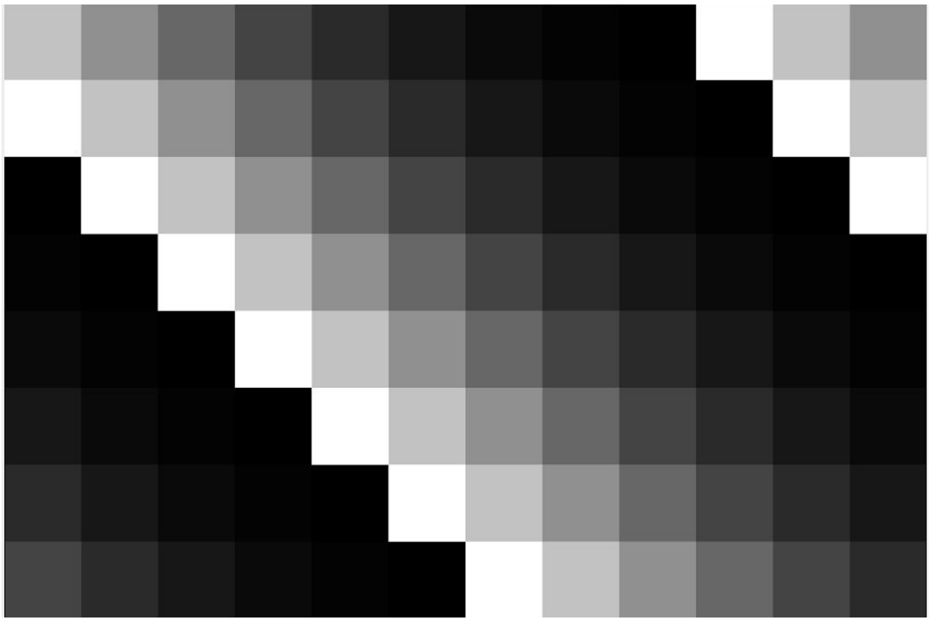
$$x = (0.5)^{1/k}$$

Thus H is maximum when $x = (0.5)^{1/k}$. $(k \neq 0)$

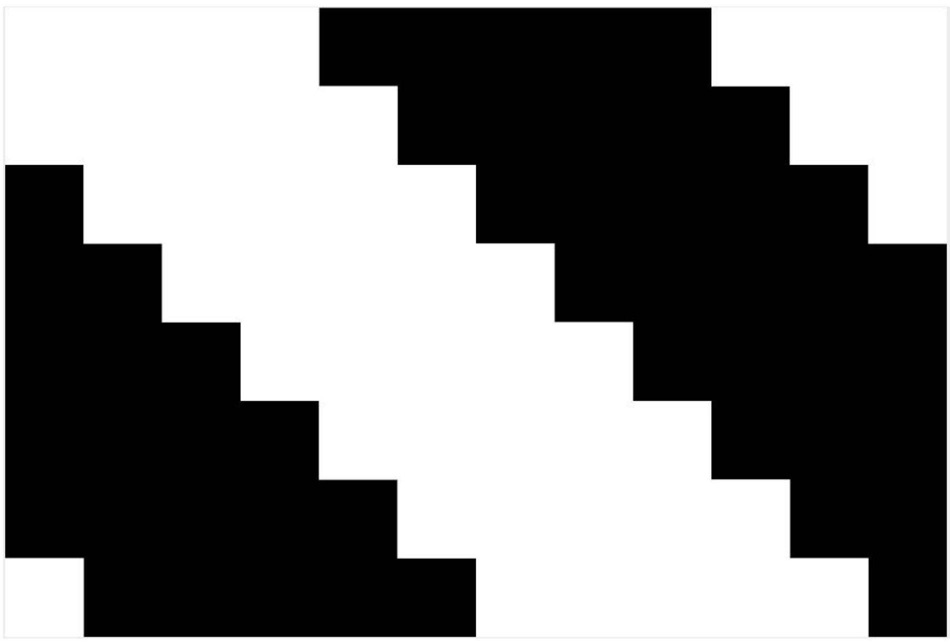
4.

4. Solution:

1. Given image with grayscale values between 0 (white) and 9 (black).

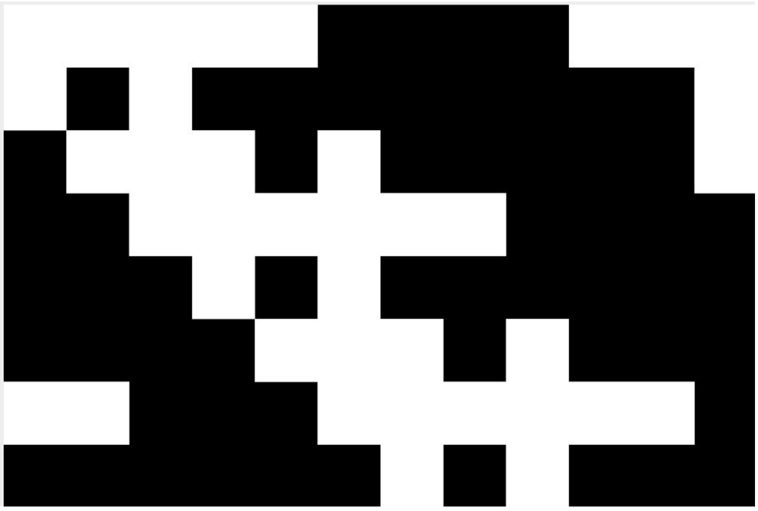


2. Image thresholded at 4.5

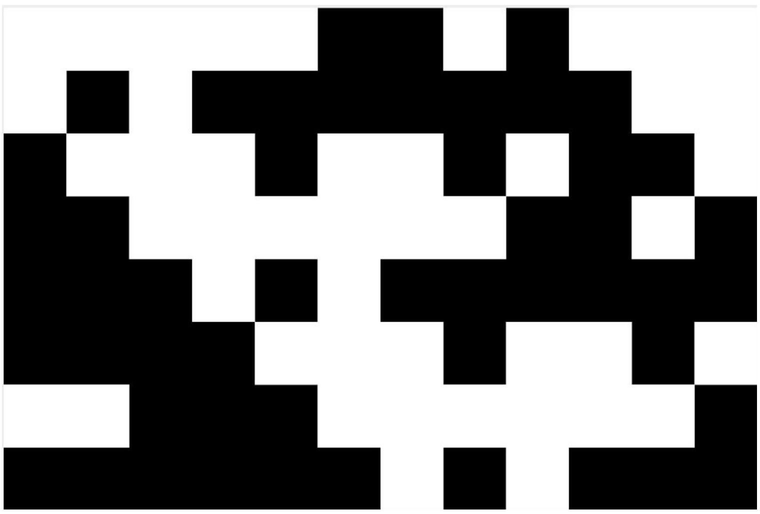


3. Output of dithering operation with top left coordinates as [0,0]

(i) If value of pixel is less than that of dither matrix , then pixel is set to 0, else it is set to 9.

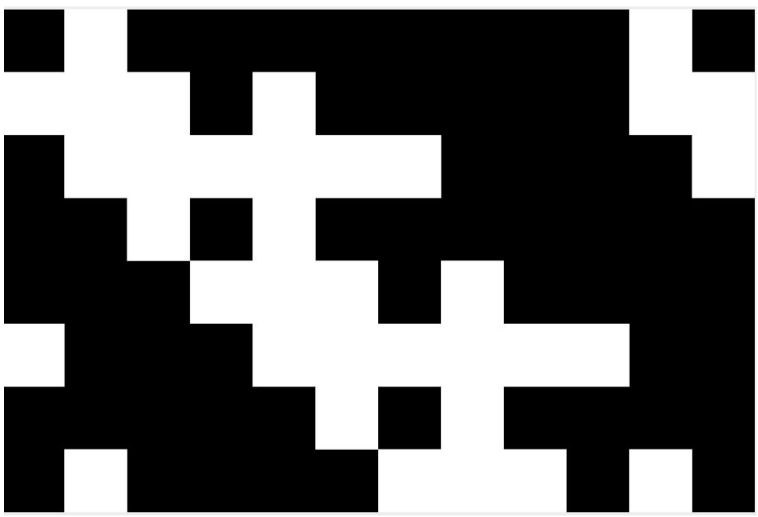


(ii) If value of pixel is less than or equal to dither matrix , then pixel is set to 0, else it is set to 9.



4. Output of dithering operation with top left coordinates as [1,1]

(i) If value of pixel is less than that of dither matrix , then pixel is set to 0, else it is set to 9.



(ii) If value of pixel is less than or equal to dither matrix , then pixel is set to 0, else it is set to 9.