Poisson's Equation

Roll no: MM22B021
Name: Babar Girish Sampat
GitHub UserId: gbabar64

June 2023

1 Introduction

- Poisson's equation is an elliptic partial differential equation of broad utility in theoretical physics.

- For example, the solution to Poisson's equation is the potential field caused by a given electric charge or mass density distribution; with the potential field known, one can then calculate electrostatic or gravitational (force) field.

- It is a generalization of Laplace's equation, which is also frequently seen in physics.

- The equation is named after French mathematician and physicist Siméon Denis Poisson.

2   Statement OF Equation

- Poisson's equation is

$$\Delta\psi = f,$$

- Where Δ is a Laplace operator, and f and ψ are real or complex-valued functions on a manifold. Usually, the Laplace operator is often denoted as ∇^2, and so Poisson's equation is frequently written as

$$\nabla^2\psi = f$$

- In three-dimensional Cartesian coordinates, it takes the form

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x,y,z) = f(x,y,z)$$

3   Solutions Of Poisson's Equation

1. Poisson's equation for gravity is

$$\nabla^2 V = 4\pi G\rho$$

2. Poisson's equation in electrostatics is

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

Here,

V = Potential of respective field

G = Universal gravitational constant
ρ = Density of respective field
ϵ = Permittivity of medium

3.1   Application Of Poisson's Equations

Poisson's equation is one of the pivotal parts of Electrostatics, where we would solve the equation to find electric potential from a given charge distribution.

4   Why I Choose Poisson's Equation

- Poisson's equation is a fundamental partial differential equation that describes the behavior of a scalar potential field.

- Solving Poisson's equation allows us to understand and predict the behavior of physical phenomena like heat conduction, electomagnetism which will help me in my departmental study in future.

References

[1] Introduction To Electrodynamics, 3rd Ed. by David J.Griffith

[2] https://en.wikipedia.org/wiki/Poisson