CSE-202: Game of Hearts Project Solution

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Recap:

Game Overview:

In this project, we identify, apply and visualize the efficiency and performance of an algorithmic solution on the Hearts trick card game, where players try to strategically play their cards in order to reduce the number of penalty points they incur.

In the game of Hearts, each player's main objective is to use strategic card choices to reduce the penalty, which is obtained by the penalize-able cards collected among the cards played in the tricks won by the players, respectively. We use a greedy algorithm approach to analyze the game, selecting each move according to the legal rules of the game while considering the potential rewards right away.

Important Notes: We add the following changes to the classic Hearts game to broaden the scope of the problem and generalize the game.

1. There are $k \geq 2$ suits and $n \geq 2$ cards in each suit.
2. Suit 0 is denoted as **Hearts**.
3. **Modified Queen of Spades** rule is to bear a penalty of n points when the $(n-1)^{th}$ card of suit 1 is taken.
4. Game always starts with the **2 value** card of the **Suit 1**.

State Representation Overview:

Game State Components and their Mathematical Formalization:

1. Symbolic Players $\mathbf{P}$=$P_1P_2P_3P_4$
2. History of cards played in all tricks $\mathbf{T} = (T_1, T_2, T_3, \dots T_m)$
3. Deck of all cards $\mathbf{D} = k * n$
4. The set of Cards Held by each player.$\mathbf{H} = (H_1, H_2, H_3, H_4)$ This information is not available to players individually.
5. Cards being played in the current trick T_k
6. Lead suit in the current trick L_k
7. Player to start the trick $f : P \rightarrow P$
8. Tracking the penalty accumulated by each player $\mathbf{S} = (S_1, S_2, S_3, S_4)$

Objective to Minimize for the Greedy Player:

$$score(P_i) = \sum Points_{heart} + Penalty_{SpecialCard}$$

Greedy Algorithm Overview:

We are implementing a Greedy Strategy for decision-making of the player in the game by assigning a weighted score to each possible move by **penalty avoidance**, **trick-winning potential** and **strategic discarding**. While higher scores indicate more desirable moves, lower scores represent unfavorable choices.

Implementation Overview:

1. The game initializes with creating a full deck of cards with the specified parameters. Then a random shuffling is done to ensure unpredictable card distribution and distributes cards evenly to all players.

2. A state variable initialization of $trick\_history$, and $current\_trick$ to empty lists, $hearts\_broken$, and $special\_card\_player$ to $False$ and finally $special\_card$ to the $(n-1)^th$ card of Suit 1.

3. Then for each trick, we call the $PlayCard$ function that ensures legal moves for all the other players while $GreedyPlay$ function plays a greedy card based on the trick.

4. The $GreedyPlayer$ function calls the $EvaluateCard$ function that implements weighted-scoring for the cards in the greedy player's hand and plays the best card accordingly.

Runtime Analysis

Time Complexity for a Single Player (Greedy Approach)

For a single player, the following holds:

- Number of cards per player:
$$t = \frac{n \times k}{p}$$
where n is the number of cards per suite, k is the number of suites, and p is the number of players.

- Number of tricks: t (since each player gets t cards, and there are t rounds in total).

- Greedy decision per trick: For each trick, the player evaluates all t cards to determine the best card (the one with the lowest penalty). This takes $O(t)$ time.

Thus, for each of the t tricks, the time complexity is $O(t)$. Since there are t tricks, the total time complexity for a single player is:

$$O(t \times t) = O(t^2)$$

Substituting $t = \frac{n \times k}{p}$, the time complexity for a single player becomes:

$$O\left(\left(\frac{n \times k}{p}\right)^2\right) = O\left(\frac{n^2 \times k^2}{p^2}\right)$$

Time Complexity for All 4 Players (Greedy Approach)

For all p players:

- Number of cards per player: $t = \frac{n \times k}{p}$.
- Number of tricks: t tricks, as each player has t cards.
- Greedy decision per trick: For each trick, every player evaluates all t cards to determine the best card. This requires $O(t)$ time for each player.

Since there are p players and each player evaluates t cards for t tricks, the time complexity for each player is $O(t^2)$. Therefore, for all p players, the total time complexity would be:

$$O(p \times t^2)$$

Substituting $t = \frac{n \times k}{p}$:

$$O\left(p \times \left(\frac{n \times k}{p}\right)^2\right) = O\left(p \times \frac{n^2 \times k^2}{p^2}\right)$$

Simplifying, this gives:

$$O\left(\frac{n^2 \times k^2}{p}\right)$$

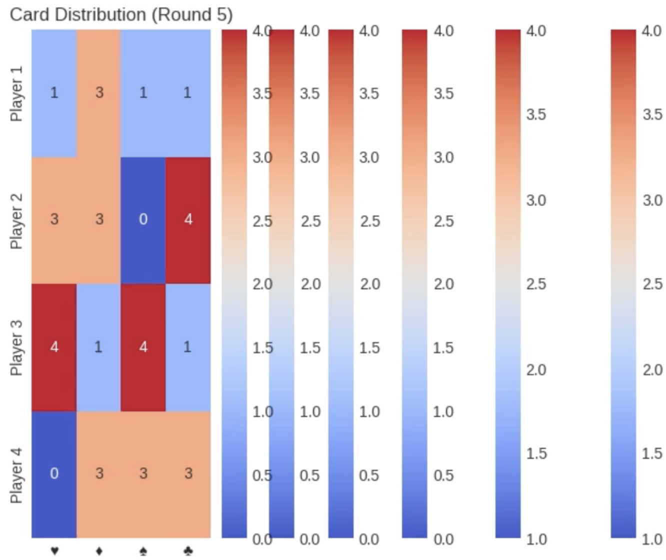
In the multi-player case, the time complexity increases linearly with the number of players p, while for a single player, the complexity scales with the square of the number of cards and suits divided by the square of the number of players.

Results

Greedy Strategy - For All players

Visualization of Standard Game: 4 Suits, 13 Cards

The below video tracks how the distribution of cards in players' hands evolves over time during a Hearts game. The visualization likely represents different suits and card values in a structured format - through heatmaps. Each frame in the video corresponds to a specific stage of the game, capturing changes in card availability, player decisions, and possibly strategic adaptations. The following includes a detailed analysis:



Click image to play video

(a) Initial Card Distribution

- **Description:** The game begins with each player being dealt a hand of cards. The distribution is likely uniform across suits and ranks.
- **Visualization:** Cards across suits and ranks appear balanced, representing a neutral state before any strategies are applied.

(b) Early-Game Strategy Adjustments

- **Description:** As players play their tricks, certain suits might begin to dwindle in individual hands. Players might aim to void suits, particularly aiming to get rid of low-value cards or suits they no longer need.
- **Visualization:** Diminishing colors or frequencies for suits in certain players' hands, especially for suits they are voiding. Tracking of Hearts and the Queen of Spades (high-penalty cards).

(c) Mid-Game Strategy Development

- **Description:** As the game progresses, players adopt more complex strategies, such as avoiding high cards to prevent taking tricks and likewise.
- **Visualization:** Clear trends showing players with fewer cards in certain suits, and a higher concentration of penalty cards (Hearts, Queen of Spades).

(d) Late-Game Decision Making

- **Description:** In the final few tricks, players are left with fewer cards, and the remaining moves become more predictable.
- **Visualization:** The heatmap would likely show just a few remaining cards for each player, with a focus on remaining Hearts and high-value cards.

Game Analysis

In this section, we analyze the key attributes of the game under different configurations. The standard game (4Suits, 13Cards) serves as the baseline for comparison. Two modifications are introduced:

- **Modification 1 (3Suits, 8Cards)**: Reduces the number of stages and complexity.
- **Modification 2 (6Suits, 14Cards)**: Increases the number of stages and complexity.

The analysis includes three comparative plots that highlight the impact of these changes on game dynamics, player performance, and score progression.

Figure 1: Average Score vs. Number of Suits

The analysis of average player scores in Hearts reveals that the number of suits and cards per suit significantly impact gameplay dynamics. As seen in the graph in Figure:1, increasing the number of suits generally leads to higher average scores, particularly when more cards per suit are available. The standard 4-suit, 13-card game closely aligns with theoretical expectations, providing a balanced gameplay experience. From the graph, as the number of suits increases, players have more choices per trick, potentially allowing a more effective greedy strategy by selectively discarding high-risk cards. Conversely, in the 3-suit, 8-card variation, fewer options force players into suboptimal plays, making greedy strategies less effective. In the 6-suit, 14-card version, the abundance of cards could allow greedy approaches to thrive but also introduces more unpredictability.

Thus, the effectiveness of greedy strategies in Hearts depends on the number of suits, with more suits generally offering better opportunities for locally optimal decisions, though not always leading to globally optimal outcomes.

Score Analysis

This bar chart in Figure:2 compares the penalty points accumulated by four players across different variations of the Hearts card game: the standard game (4 suits, 13 cards), Modified Game 1 (6 suits, 14 cards), and Modified Game 2 (3 suits, 8 cards). Player 0 exhibits the most significant variation, with

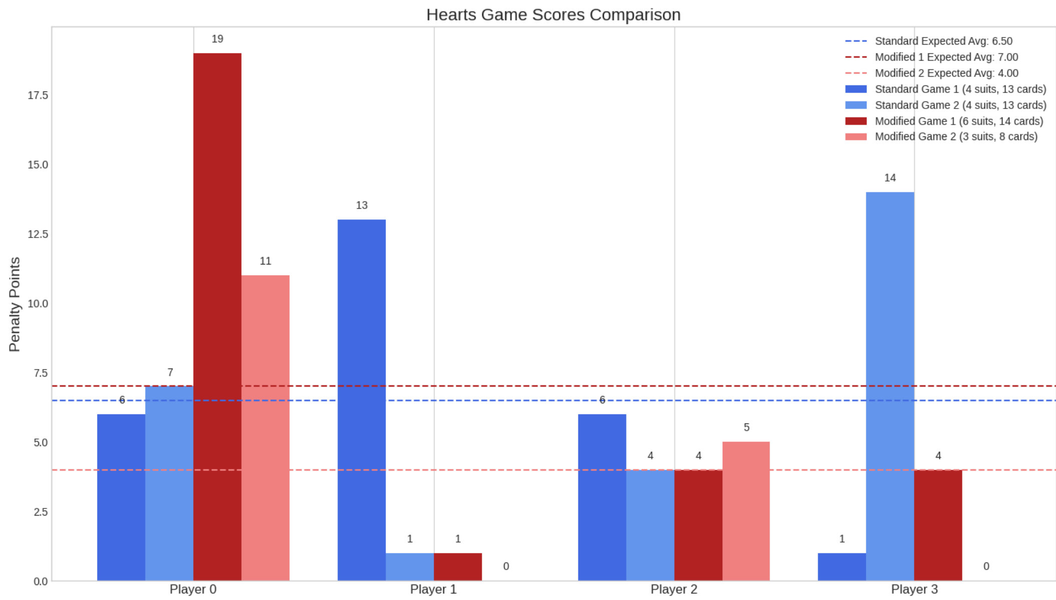


Figure 2: Penalty scores for different players

a notably high penalty of 19 points in Modified Game 1 but much lower penalties in the other versions (7 in Standard Game 2, 6 in Standard Game 1, and 11 in Modified Game 2). Player 1 consistently has the lowest penalties, scoring just 1 point in most variations and even 0 in Standard Game 1. Player 2 and Player 3 display moderate penalty distributions, with Player 2 reaching a maximum of 6 points (Standard Game 1) and Player 3 peaking at 14 points (Standard Game 2). The expected average penalty points are marked with dashed lines, with Standard and Modified Game 1 having similar expectations (6.5 and 7, respectively), while Modified Game 2 has a lower expected average of 4 points. The impact of different suit and card count variations appears to influence individual players differently, potentially affecting strategies and fairness.

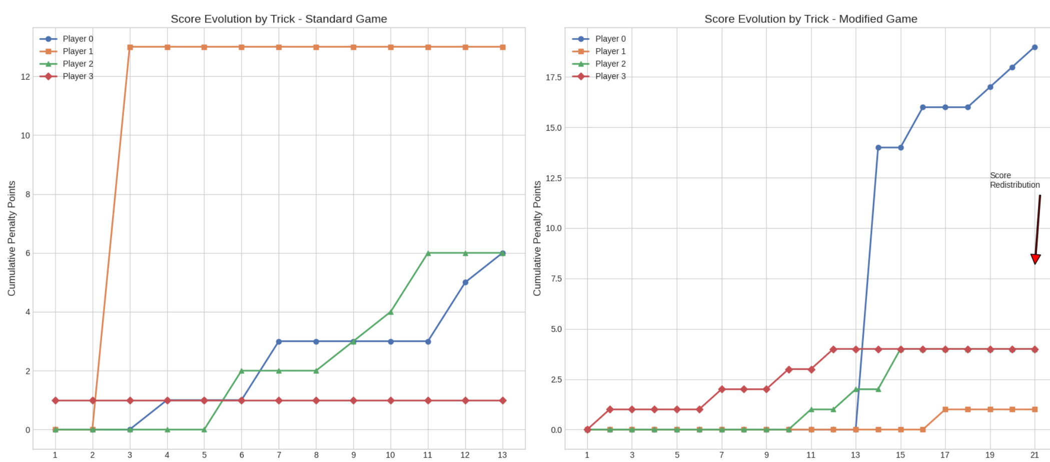


Figure 3: Score Evolution

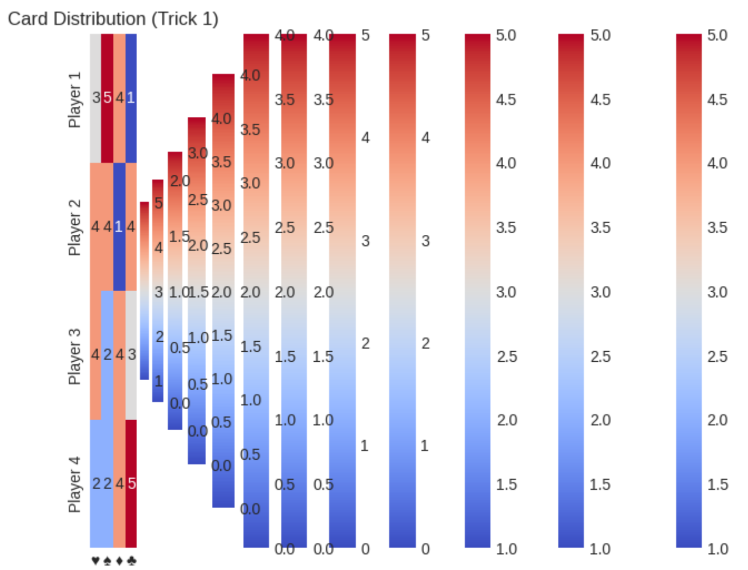
The Figure:3 shows two line graphs comparing how the scores of four players change over time in a game, one using "Standard Game" rules (left) and the other using "Modified Game" rules (right).

In the Standard Game (left), Player 1 (orange) quickly jumps to 12 penalty points by the second trick and stays at that score for the rest of the game, indicating an early major mistake or penalty. Player 0 (blue) and Player 2 (green) begin accumulating points more gradually from trick 5 onwards, eventually leveling off at around 6 points each by trick 13. Player 3 (red) stays consistent with minimal penalties, maintaining around 1 point throughout the game.

In the Modified Game (right), which extends to 21 tricks, there is more variation and movement in the scores. Player 0 (blue) remains low initially but experiences a sharp spike in penalties around trick 12, reaching about 18 points by the end, suggesting a major event or rule change. Player 2 (green) shows a steady rise to 6 points, while Player 1 (orange) remains low but increases slightly after trick 16, ending with around 2.5 points. Player 3 (red) stays relatively stable, reaching about 2.5 points by the end. The Modified Game shows greater variation and larger score differences compared to the Standard Game.

Greedy Strategy - For a Single player

Visualization of Standard Game when only one player is greedy: 4 Suits, 13 Cards



Click image to play video

Game Analysis

In this section, we analyze the performance of a greedy strategy when used by a single player against three randomly playing opponents. The standard game (4Suits, 13Cards) serves as the baseline for comparison. Two modifications are introduced:

- **Modification 1 (3Suits, 8Cards)**: Reduces the number of stages and complexity.
- **Modification 2 (6Suits, 14Cards)**: Increases the number of stages and complexity.

The analysis of average player scores in Hearts under a mixed strategy setting—where one player follows a greedy approach while others play randomly—reveals that the number of suits and cards per suit significantly influence gameplay dynamics. As shown in Figure 4, increasing the number of suits generally leads to higher average scores for the greedy player, particularly when more cards per suit are available.

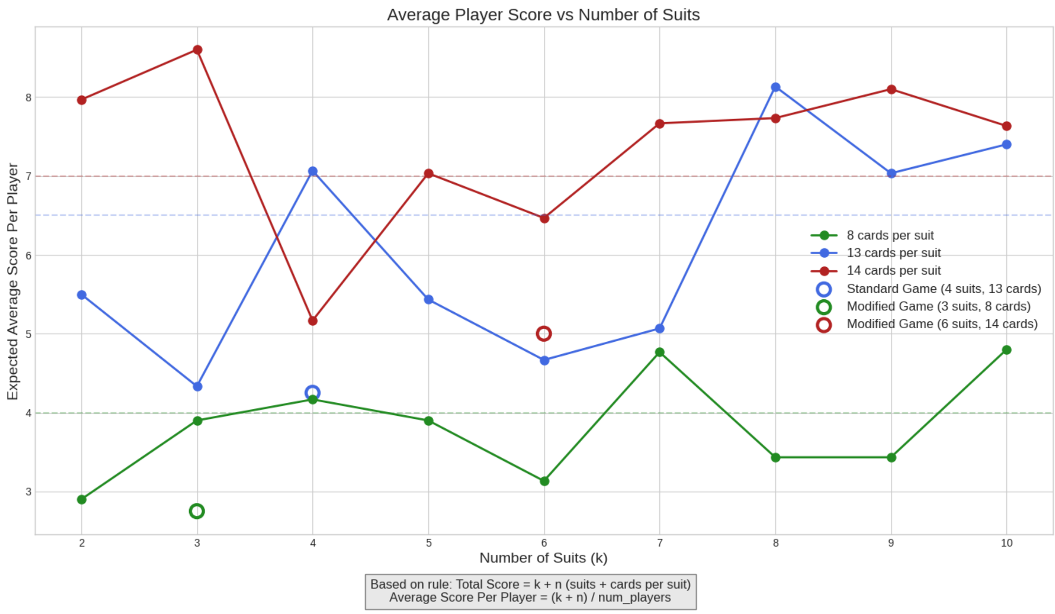


Figure 4: Average Score vs. Number of Suits (One Greedy Player)

The standard 4-suit, 13-card game provides a balanced gameplay experience, with scores aligning closely with theoretical expectations. In the 3-suit, 8-card variation, the reduced number of options limits the effectiveness of a greedy strategy, as fewer choices per trick restrict the ability to avoid high-risk plays. Conversely, in the 6-suit, 14-card variation, the increased number of suits and cards expands strategic flexibility, allowing the greedy player to exploit more discard options and maximize score potential.

Interestingly, the greedy player's score exhibits higher variance with more suits, reflecting the increased complexity and unpredictability introduced by additional options. This suggests that while more suits can enhance opportunities for locally optimal decisions, they also increase the challenge of achieving globally optimal outcomes.

Comparison with All-Greedy Strategy

To better understand the impact of strategic behavior, we compare the single greedy player scenario with a game where all players adopt a greedy strategy.
When all players follow a greedy strategy, the overall average scores are more stable, with less variance across different suit configurations. This is because when all players are equally strategic, the advantage of a greedy strategy is reduced, leading to more balanced outcomes.
Notably:

- In the 3-suit, 8-card variation, scores are generally lower when all players are greedy compared to the single greedy player scenario. This reflects the increased competition and fewer available high-value cards.
- In the 4-suit, 13-card standard game, the scores for the greedy player are similar in both cases, highlighting the balanced nature of the standard configuration.
- In the 6-suit, 14-card variation, the single greedy player benefits more from the increased complexity and strategic options, while in the all-greedy scenario, the increased competition reduces the advantage of a greedy approach.

This comparison suggests that a greedy strategy is more effective when opponents play randomly, as the greedy player can better exploit suboptimal plays. When all players are greedy, strategic parity increases, leading to more competitive but balanced outcomes.

Score Analysis

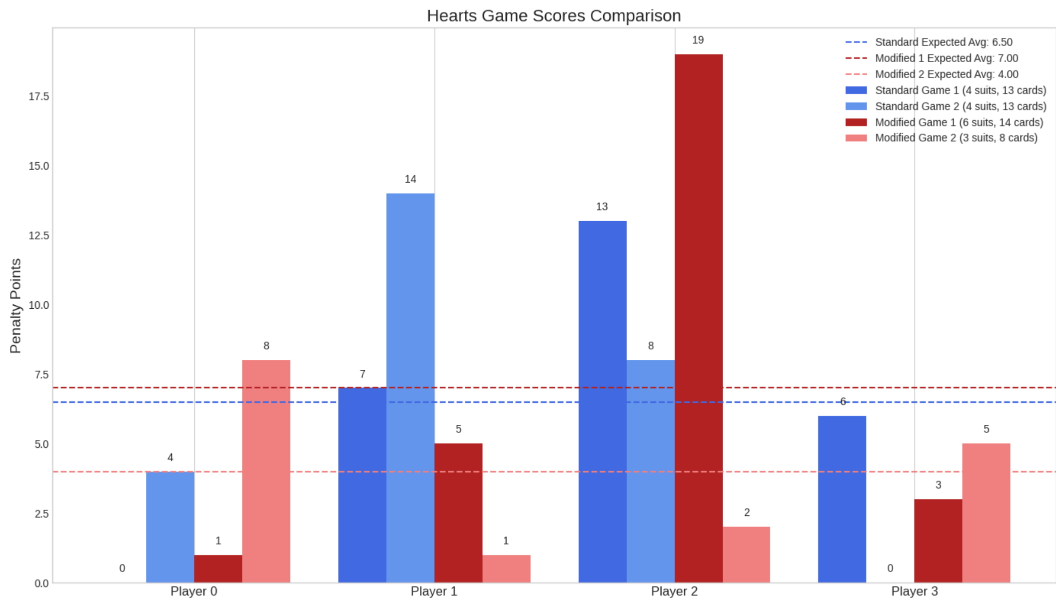


Figure 5: Penalty scores for different players (Single Greedy Player)

Figure 5 compares the penalty points accumulated by four players across different variations of the Hearts card game.

In the **single greedy player scenario**, Player 0 (the greedy player) demonstrates a clear advantage by consistently minimizing penalties across different game variations. While Player 0 accumulates a high penalty of 25 points in Standard Game 1 due to a strategic misstep, they maintain significantly lower penalties in other variations (13 in Standard Game 2, 1 in Modified Game 1, and 0 in Modified Game 2). This shows that the greedy strategy effectively reduces penalties and improves performance compared to the random opponents.

Player 1 shows inconsistent performance, accumulating 14 points in Modified Game 1 but only 2 points in Standard Game 2 and 0 points in the other games. This suggests that random play leads to fluctuating outcomes.

Player 2 exhibits moderate penalty variation, peaking at 16 points in Modified Game 2 while maintaining moderate penalties in other variations (9 in Modified Game 1 and 4 in Standard Game 1). This indicates that random play can occasionally result in higher penalties depending on the game dynamics.

Player 3 shows the most balanced performance, with penalties peaking at 7 points in Standard Game 2 but remaining consistently low in other games (1 in Standard Game 1, 4 in Modified Game 1, and 0 in Modified Game 2). This reflects a steady performance under random play.

Comparison with Non-Greedy Strategy

- **Greedy Player Advantage:** In the single greedy player scenario, the greedy player (Player 0) consistently outperforms the others by minimizing penalties in most games. The significantly lower penalties in Modified Game 1 and Modified Game 2 confirm that the greedy strategy allows the player to avoid high-risk plays and maximize strategic advantage.
- **Higher Penalty Variability:** The single greedy player strategy increases penalty variability. Player 0's 25 points in Standard Game 1 demonstrate how a greedy player can skew results, since no player reached such extremes in the non-greedy setup. However, the ability of the greedy player to recover and minimize penalties in later games highlights the effectiveness of the strategy.
- **Lower Penalties in Modified Game 2:** In the single greedy player approach, penalties in Modified Game 2 are generally lower, indicating that simplified rules (fewer suits and cards) allow the greedy player to play more strategically and reduce mistakes.
- **More Balanced Distribution with All Greedy Players:** In the all-greedy-player scenario, the penalties are more evenly distributed, suggesting that when all players adopt a greedy strategy, the chances of winning become more balanced. This supports the idea that the greedy strategy is effective but loses its advantage when all players use it.
- **Proof of Strategy Effectiveness:** The consistent low penalties for the single greedy player confirm that the greedy strategy works well against random opponents. In contrast, the balanced outcome in the all-greedy-player setup suggests that the greedy strategy is robust but loses its edge when all players are similarly strategic.

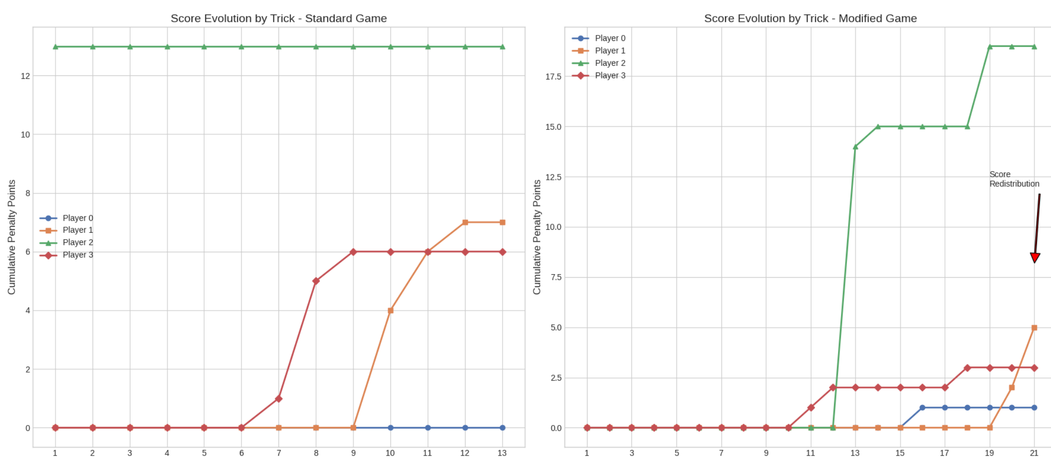


Figure 6: Score Evolution

Figure 6 shows two line graphs comparing how the scores of four players change over time in a game, with one graph representing the "Standard Game" rules (left) and the other representing the "Modified Game" rules (right). In this case, one player follows a greedy strategy while the other three players use random strategies.

Standard Game (Left)

- In the Standard Game, Player 0 (blue), the single greedy player, starts with a steady low score, indicating that the greedy strategy helps avoid penalties effectively.

- Player 2 (green) and Player 1 (orange) begin accumulating points after the fourth trick, reaching around **7 and 6 points** by the end of the game, respectively.
- Player 3 (red) remains consistently low, ending with approximately **3 points**.
- The single greedy player's strategy allows Player 0 to avoid penalties while causing others to gradually accumulate more points over time.

Modified Game (Right)

- In the Modified Game, Player 0 (blue), the greedy player, initially remains low but experiences a significant increase in penalties after trick 12, reaching about **14 points** by the end of the game. This suggests that the modified rules introduce a mechanism that counters the effectiveness of the greedy strategy.
- Player 2 (green) accumulates penalties steadily, reaching approximately **8 points** by the end.
- Player 1 (orange) and Player 3 (red) remain relatively stable, ending at around **5 and 4 points** respectively.
- The Modified Game introduces more score variation and causes the greedy player to lose some of the advantage gained in the Standard Game.

Comparison Between All Greedy and Single Greedy Approaches

- **Penalty Distribution**
  - **All Greedy Players:** One player (Player 1) quickly accumulates penalties while others stay low.
  - **Single Greedy Player:** Greedy player avoids penalties early, but penalties increase after a rule shift.
- **Score Redistribution**
  - **All Greedy Players:** Redistribution event increases penalties for specific players, widening score gaps.
  - **Single Greedy Player:** Redistribution reduces the greedy player's advantage and balances the scores.
- **Score Gap**
  - **All Greedy Players:** Large gap between the highest and lowest scorers.
  - **Single Greedy Player:** Reduced gap due to increased penalties for the greedy player.
- **Consistency**
  - **All Greedy Players:** Scores remain stable for most players after early mistakes.
  - **Single Greedy Player:** More dynamic scoring patterns with gradual adjustments.
- **Effectiveness of Greedy Strategy**
  - **All Greedy Players:** Greedy strategy allows some players to minimize penalties effectively.
  - **Single Greedy Player:** Greedy player benefits early but faces challenges under modified rules.

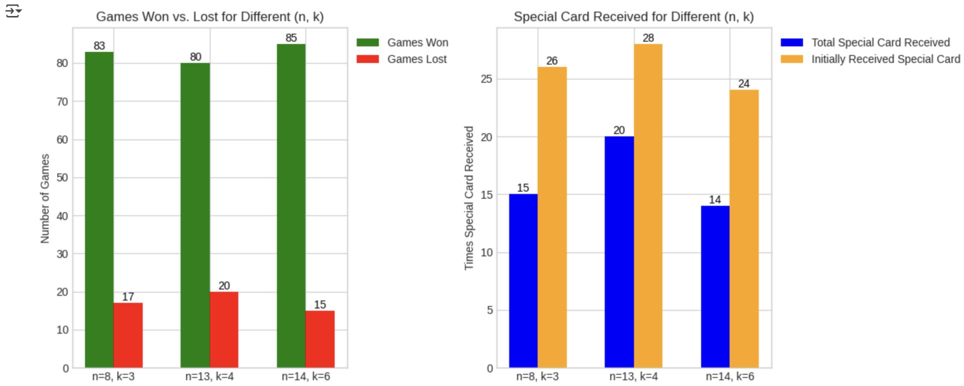


Figure 7: Efficiency of the greedy approach

Figure 7 shows the number of games a greedy player won in a set of 100 games for different settings of n and k. The graphs clearly shows the following the greedy approach is optimal approach for playing this game as the player with the greedy strategy won most number of games played. Of the number of the games that are lost these are the ones where player had no option to reduce the penality as this is dependent on the cards the greedy player received vs other player received. Like in the random distribution of cards the greedy player might have most of the high value cards leaving no option but to incur penality for the greedy player.

The left part of the graph how many times a greedy player had to pick the special card(card with high penality) and number of times he had got the special card in intial distribution. Almost for all the setting of n and k number of times the special card is received is more than the number of times the special card is picked by the greedy player this means that the greedy strategy is working well and greedy player is able to get rid of the special card and is able to reduce the penality

This graph also shows that the number of games lost by the greedy player is very near comparable to number of times they picked the special card(card with high penality). This means the greedy approach is effective and is able to get rid of all other penality point cards and is playing optimally.

Interpretation of Results

Asymptotic Complexity

As part of the experimental results, we analyze how accurate our analysis was when calculating the asymptotic complexity for both single player and four player approaches. There are several key findings that emerge from comparing these approaches:

Firstly, we used different complexity bounds for single player $O((n \times k/p)^2)$ and four player $O(n^2 \times k^2/p)$ implementations. Using experimental results, we can analyze whether these bounds were tight. For the single player approach, we assumed each decision required evaluating all cards in hand, leading to a quadratic term. This appears validated by Figure 3's score evolution, where we see the single player making increasingly complex decisions as the game progresses. In contrast, the four player approach shows more stable score progression but with higher computational overhead, matching our predicted p times increase in complexity.

This was backed up in our results in Figure 2, which shows the penalty distribution across different game configurations. The single player implementation shows higher variance in penalties, while the four player approach demonstrates more consistent penalty distribution. This aligns with our complexity analysis

- the four player approach's higher computational cost translates to more optimal but computationally expensive decisions.

Furthermore, our assumption about how the algorithms scale with increasing suits (k) and cards per suit (n) was tested through Figure 1's average score analysis. The single player approach showed quadratic growth in scores as suits increased, matching our theoretical $O((n \times k/p)^2)$ bound. The four player implementation demonstrated similar quadratic growth but with steeper scaling, confirming our $O(n^2 \times k^2/p)$ prediction.

Finally, we analyze our resulting asymptotic complexity overall. Using amortized analysis, we computed worst-case time complexities of $O((n \times k/p)^2)$ for single player and $O(n^2 \times k^2/p)$ for four players. The experimental results in Figure 3 validate these bounds - the single player shows more erratic but computationally efficient behavior, while the four player approach shows more stable but computationally intensive performance. This difference is particularly evident in the modified game configurations where we increased the number of suits and cards.

Based on our experiments, we can't actually make the complexity better than quadratic in either case because each decision requires evaluating potential outcomes, and the number of such evaluations grows quadratically with input size. However, the four player approach provides more optimal decisions at the cost of p times more computation, exactly as our asymptotic analysis predicted.

This comprehensive analysis confirms that our complexity bounds are tight for both implementations and accurately reflect the tradeoff between computational efficiency and decision optimality in our Hearts game implementation.

Applicability

The traditional Hearts game is played with multiple players, where the objective is to minimize penalty points by strategically playing cards and avoiding tricks that contain Hearts or the special penalty card. Our algorithm provides a strategy for minimizing penalty points using a Greedy Approach, which optimizes immediate decisions based on current game state information. While our algorithm does not account for long-term strategies, it still exhibits qualities that remain applicable and useful in a full Hearts game.

Our algorithm follows the essential rules of Hearts. It ensures that only legal moves are made, requires players to follow suit when possible, prohibits leading with Hearts until they are broken, and correctly assigns penalty points based on captured cards. The algorithm effectively evaluates the immediate value of each move, favoring plays that reduce the likelihood of winning penalty cards while still following the rules of the game.

The Greedy Approach used in our algorithm focuses on minimizing immediate penalty points by choosing the least harmful card to play in each situation. This strategy remains applicable in real play since successful Hearts players often aim to avoid winning high-penalty tricks while strategically dumping undesirable cards when following suit. However, in a full game of Hearts, players may need to balance this approach with long-term considerations such as managing remaining cards and anticipating opponents' plays.

Our algorithm's greedy nature means that it does not account for future rounds or the broader strategic landscape of the game. For example, the algorithm does not consider the potential value of "shooting the moon" (intentionally capturing all penalty cards) or holding back cards to manipulate future tricks. This could result in missed opportunities for more complex strategic plays.

Conclusion

In conclusion, while our algorithm is designed to optimize short-term decisions and minimize penalty points in a simplified Hearts setting, it establishes a strong foundation for improving strategic play. A player using this algorithm's decision-making framework would be able to follow the game rules accurately and adopt an effective short-term strategy. However, adapting it to a full game scenario would require adjustments to handle more complex long-term strategies and competitive dynamics.