Aeroelastic Analysis of Hypersonic Double-wedge Lifting Surface

Koorosh Gobal[1], (Add your name)[1], and (Add your name)[1]

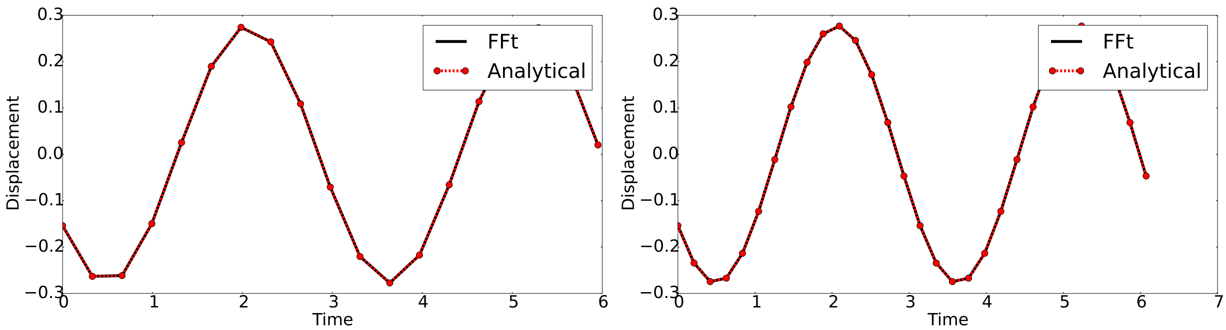
[1]Department of Mechanical and Materials Engineering, Wright State University

Abstract

1 Demonstration Results

1.1 Linear MCK System

$$\ddot{x} + \dot{x} + x = \sin 2t \tag{1}$$



(a) n = 19

(b) n = 30

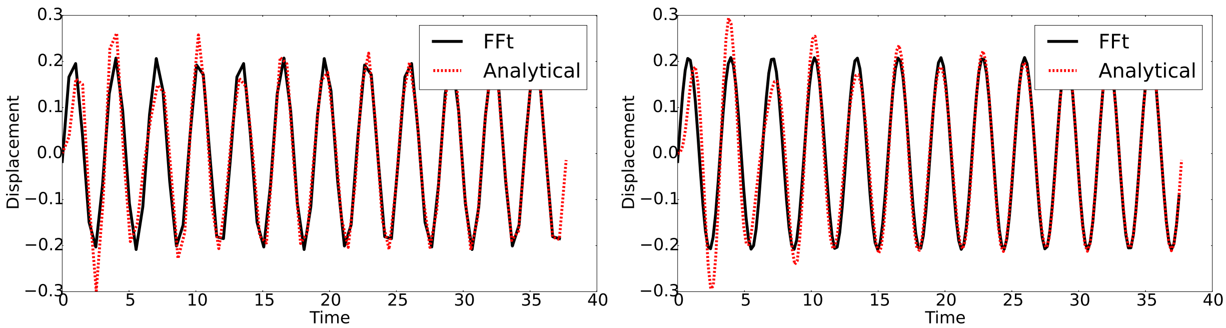
Figure 1: Comparison between HB and analytical result.

1.2 Nonlinear Oscilator

(Problem 2.45 on page 140 of Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods (Nayfeh))

$$\ddot{x} + 2\mu\dot{x} + \frac{g}{R}\sin x - \alpha^2 \sin x \cos x = \sin 2t \tag{2}$$

mu = 0.1 g = 9.81 R = 1.0 alpha = 1



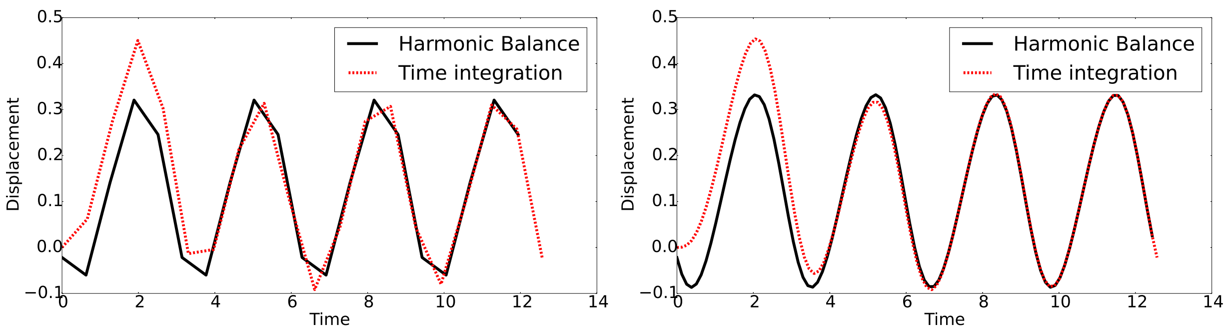
(a) n = 75 (b) n = 199

Figure 2: Comparison between HB and analytical result.

1.3 Parametrically excited Duffing Oscillator

$$\ddot{x} + x + \epsilon\left[2\mu\dot{x} + \alpha x^3 + 2kx\cos\omega t\right] = \sin 2t \tag{3}$$

epsilon = 1.0 mu = 1.0 alpha = 1.0 k = 1.0 omega = 2.0



(a) n = 20 (b) n = 99

Figure 3: Comparison between HB and analytical result.

1.4 Hypersonic Flutter

The governing equation can be written as

$$m\ddot{\theta} + k\theta = f(\theta, t) \tag{4}$$

where m is the mass of airfoil, k is the stiffness, θ is the angle of attack and f is the aerodynamic load. We use the Piston Theory to calculate the aerodynamic load acting on the airfoil. The pressure distribution on the airfoil can be written as follows

$$\frac{P(x,t)}{P_\infty} = \left(1 + \frac{\gamma - 1}{2}\frac{v_n}{a_\infty}\right)^{\frac{2\gamma}{\gamma-1}} \tag{5}$$

where $P(x,t)$ is the pressure at point x on the airfoil, P_∞ is the free stream pressure, γ is the ratio of specific heats and a_∞ is the speed of sound. v_n is calculated using the following equation.

$$v_n = \frac{\partial Z(x,t)}{\partial t} + V\frac{\partial Z(x,t)}{\partial x} \tag{6}$$

where V is the free stream velocity and $Z(x,t)$ is the position of airfoil surface. The position of airfoil surface can be related to the angle of attack using the following equation:

$$Z(x,t) = M_r\left(\theta(t)\right)Z_0(x) \tag{7}$$

where M_r is the rotation matrix, and $Z_0(x)$ is the initial shape of the airfoil. As can be seen here, the location of surface at time t only depends on the angle of attach at that time, $\theta(t)$. The rotation matrix is defined as

$$M_r = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

By expanding Equation (5), and substituting for $Z(x,t)$ from Equation (7), Equation (4) can be rewritten as

$$m\ddot{\theta} + k\theta = \oint_{airfoil} P_\infty \left[\gamma \frac{v_n}{a_\infty} + \frac{\gamma(\gamma+1)}{4} \left(\frac{v_n}{a_\infty} \right)^2 + 1 \right] ds \tag{8a}$$

$$v_n = \frac{\partial M_r(\theta)}{\partial t} Z_0(x) + V M_r(\theta) \frac{\partial Z_0(x)}{\partial x} \tag{8b}$$

Equation (8) needs to be solved for θ using Harmonic Balance method.