Aeroelastic Analysis of Hypersonic Double-wedge Lifting Surface

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Abstract

1 Introduction

Nonlinear oscillations problem are important issues in physical science, mechanical structures and other engineering researches. Nonlinear systems display different behaviors than linear systems. They can exhibit

1. multiply steady state solutions, stable and unstable, in response to the same inputs (bifurcation),
2. response at frequencies other than forcing frequency
3. hysterisis with jump phenomenon, that includes discontinuous variations in the response of the system when system parameters are varied,
4. irregular motions that are extremely sensitive to the initial conditions (chaos).

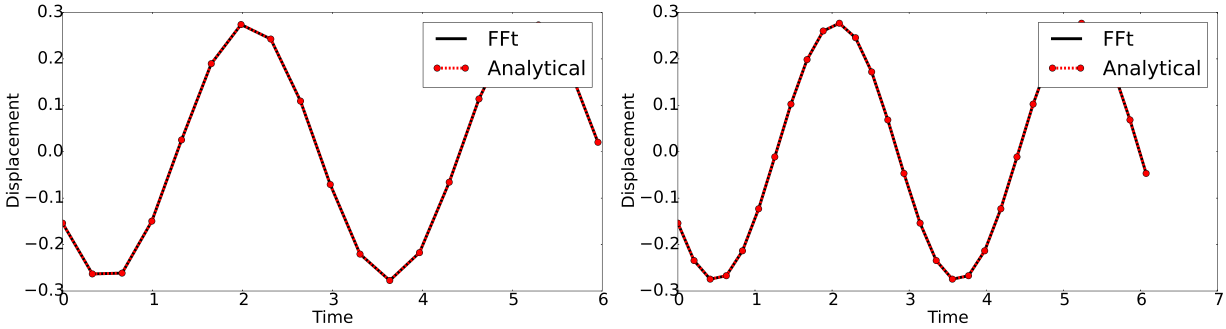
Hence studying nonlinearieties is of extreme importance. As most of the real systems are modeled with non-linear differential equations, solving them is a critical task. There are many approximation methods to determine steady state solution in time domain. The drawback of such time-domain methods is the excessive need for computational time and resources. This can become a major bottle neck in design space exploration efforts of such systems where mutiple solutions for different confgurations is needed. However, simulation in frequency domain eases the problem by transforming differential equations into algebraic complex equations and reduces the simulation cost by orders of magnitude. One such frequency domain approach to find steady state solution is the Harmonic Balance (HB) method.

In this report, Harmonic balance methodology is first build on closed form equation for structural response of linear equations and then extended to basic non-linear equations and non-linear equations with parametric excitations. The results are compared with time integrated solutions of the corresponding systems.

2 Demonstration Results

2.1 Linear MCK System

$$\ddot{x} + \dot{x} + x = \sin 2t \tag{1}$$



(a) n = 19 (b) n = 30

Figure 1: Comparison between HB and analytical result.

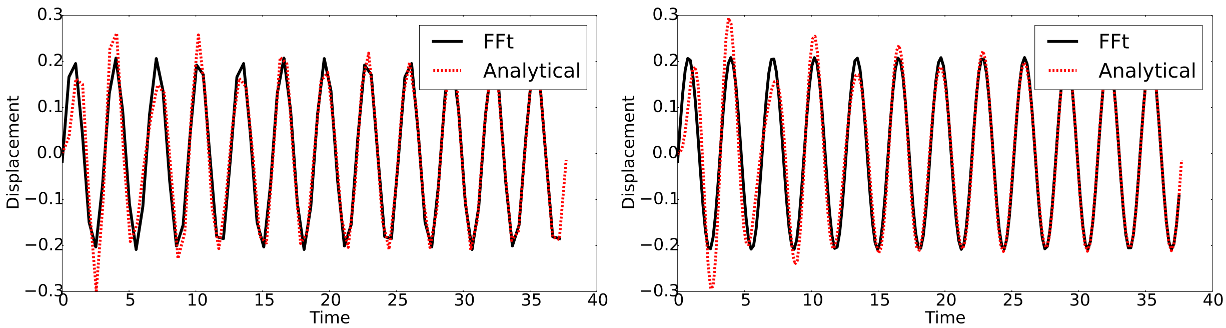
Linear oscillating system response attained using Harmonic Balance Method (HBM), coincides well with the analytical solution.

2.2 Nonlinear Oscillator

(Problem 2.45 on page 140 of Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods (Nayfeh))

$$\ddot{x} + 2\mu\dot{x} + \frac{g}{R}\sin x - \alpha^2 \sin x \cos x = \sin 2t \tag{2}$$

mu = 0.1 g = 9.81 R = 1.0 alpha = 1



(a) n = 75 (b) n = 199

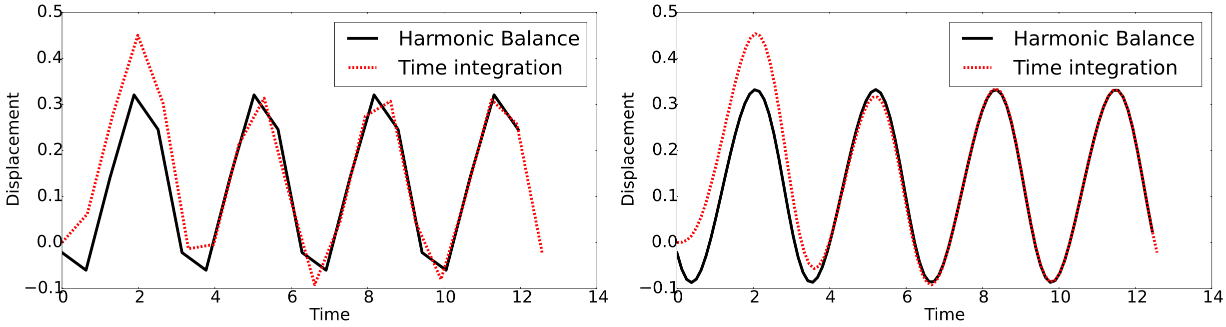
Figure 2: Comparison between HB and analytical result.

From Figure 2, it is observed that HBM is in good agreement with the analytical solution at both low and high order sampling. The response converges and coincides with analytical solution.

2.3 Parametrically excited Duffing Oscillator

$$\ddot{x} + x + \epsilon\left[2\mu\dot{x} + \alpha x^3 + 2kx\cos\omega t\right] = \sin 2t \tag{3}$$

ϵ = 1.0 μ = 1.0 α = 1.0 k = 1.0 ω = 2.0



(a) n = 20 (b) n = 99

Figure 3: Comparison between HB and analytical result.

Similar convergence trend is observed for parametrically excited nonlinear oscillating system.

2.4 Hypersonic Flutter

The governing equation can be written as

$$m\ddot{\theta} + k\theta = f(\theta, t) \tag{4}$$

where m is the mass of airfoil, k is the stiffness, θ is the angle of attack and f is the aerodynamic load. We use the Piston Theory to calculate the aerodynamic load acting on the airfoil. The pressure distribution on the airfoil can be written as follows

$$\frac{P(x,t)}{P_\infty} = \left(1 + \frac{\gamma - 1}{2}\frac{v_n}{a_\infty}\right)^{\frac{2\gamma}{\gamma-1}} \tag{5}$$

where $P(x,t)$ is the pressure at point x on the airfoil, P_∞ is the free stream pressure, γ is the ratio of specific heats and a_∞ is the speed of sound. v_n is calculated using the following equation.

$$v_n = \frac{\partial Z(x,t)}{\partial t} + V\frac{\partial Z(x,t)}{\partial x} \tag{6}$$

where V is the free stream velocity and $Z(x,t)$ is the position of airfoil surface. The position of airfoil surface can be related to the angle of attack using the following equation:

$$Z(x,t) = M_r\left(\theta(t)\right) Z_0(x) \tag{7}$$

where M_r is the rotation matrix, and $Z_0(x)$ is the initial shape of the airfoil. As can be seen here, the location of surface at time t only depends on the angle of attach at that time, $\theta(t)$. The rotation matrix is defined as

$$M_r = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

By expanding Equation (5), and substituting for $Z(x,t)$ from Equation (7), Equation (4) can be rewritten as

$$m\ddot{\theta} + k\theta = \oint_{airfoil} P_\infty \left[\gamma\frac{v_n}{a_\infty} + \frac{\gamma(\gamma+1)}{4}\left(\frac{v_n}{a_\infty}\right)^2 + 1\right] ds \tag{8a}$$

$$v_n = \frac{\partial M_r(\theta)}{\partial t} Z_0(x) + V M_r(\theta)\frac{\partial Z_0(x)}{\partial x} \tag{8b}$$

Equation (8) needs to be solved for θ using Harmonic Balance method.

References