Aeroelastic Analysis of Hypersonic Double-wedge Lifting Surface

Koorosh Gobal[1], Anusha Anisetti[1], and Vana Naga Samyuktha Nuthy[1]

[1]Department of Mechanical and Materials Engineering, Wright State University

Abstract

1 Numerical Method

The governing equation can be written as

$$m\ddot{\theta} + k\theta = f(\theta, t) \tag{1}$$

where m is the mass of airfoil, k is the stiffness, θ is the angle of attack and f is the aerodynamic load. We use the Piston Theory to calculate the aerodynamic load acting on the airfoil. The pressure distribution on the airfoil can be written as follows

$$\frac{P(x,t)}{P_\infty} = \left(1 + \frac{\gamma - 1}{2}\frac{v_n}{a_\infty}\right)^{\frac{2\gamma}{\gamma-1}} \tag{2}$$

where $P(x,t)$ is the pressure at point x on the airfoil, P_∞ is the free stream pressure, γ is the ratio of specific heats and a_∞ is the speed of sound. v_n is calculated using the following equation.

$$v_n = \frac{\partial Z(x,t)}{\partial t} + V\frac{\partial Z(x,t)}{\partial x} \tag{3}$$

where V is the free stream velocity and $Z(x,t)$ is the position of airfoil surface. The position of airfoil surface can be related to the angle of attack using the following equation:

$$Z(x,t) = M_r\left(\theta(t)\right) Z_0(x) \tag{4}$$

where M_r is the rotation matrix, and $Z_0(x)$ is the initial shape of the airfoil. As can be seen here, the location of surface at time t only depends on the angle of attach at that time, $\theta(t)$. The rotation matrix is defined as

$$M_r = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

By expanding Equation (2), and substituting for $Z(x,t)$ from Equation (4), Equation (1) can be rewritten as

$$m\ddot{\theta} + k\theta = \oint_{airfoil} P_\infty \left[\gamma\frac{v_n}{a_\infty} + \frac{\gamma(\gamma+1)}{4}\left(\frac{v_n}{a_\infty}\right)^2 + 1\right] ds \tag{5a}$$

$$v_n = \frac{\partial M_r(\theta)}{\partial t} Z_0(x) + V M_r(\theta)\frac{\partial Z_0(x)}{\partial x} \tag{5b}$$

Equation (5) needs to be solved for θ