

# Aeroelastic Analysis of Hypersonic Double-wedge Lifting Surface

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## Abstract

## 1 Remonstrations Results

### 1.1 Linear MCK System

$$\ddot{x} + \dot{x} + x = \sin 2t \quad (1)$$

### 1.2 Nonlinear Oscillator

(Problem 2.45 on page 140 of Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods (Nayfeh))

$$\ddot{x} + 2\mu\dot{x} + \frac{g}{R} \sin x - \alpha^2 \sin x \cos x = \sin 2t \quad (2)$$

### 1.3 Parametrically excited Duffing Oscillator

$$\ddot{x} + x + \epsilon [2\mu\dot{x} + \alpha x^3 + 2kx \cos \omega t] = \sin 2t \quad (3)$$

### 1.4 Hypersonic Flutter

The governing equation can be written as

$$m\ddot{\theta} + k\theta = f(\theta, t) \quad (4)$$

where  $m$  is the mass of airfoil,  $k$  is the stiffness,  $\theta$  is the angle of attack and  $f$  is the aerodynamic load. We use the Piston Theory to calculate the aerodynamic load acting on the airfoil. The pressure distribution on the airfoil can be written as follows

$$\frac{P(x, t)}{P_\infty} = \left(1 + \frac{\gamma - 1}{2} \frac{v_n}{a_\infty}\right)^{\frac{2\gamma}{\gamma - 1}} \quad (5)$$

where  $P(x, t)$  is the pressure at point  $x$  on the airfoil,  $P_\infty$  is the free stream pressure,  $\gamma$  is the ratio of specific heats and  $a_\infty$  is the speed of sound.  $v_n$  is calculated using the following equation.

$$v_n = \frac{\partial Z(x, t)}{\partial t} + V \frac{\partial Z(x, t)}{\partial x} \quad (6)$$

where  $V$  is the free stream velocity and  $Z(x, t)$  is the position of airfoil surface. The position of airfoil surface can be related to the angle of attack using the following equation:

$$Z(x, t) = M_r(\theta(t)) Z_0(x) \quad (7)$$

where  $M_r$  is the rotation matrix, and  $Z_0(x)$  is the initial shape of the airfoil. As can be seen here, the location of surface at time  $t$  only depends on the angle of attach at that time,  $\theta(t)$ . The rotation matrix is defined as

$$M_r = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

By expanding Equation (5), and substituting for  $Z(x, t)$  from Equation (7), Equation (4) can be rewritten as

$$m\ddot{\theta} + k\theta = \oint_{airfoil} P_\infty \left[ \gamma \frac{v_n}{a_\infty} + \frac{\gamma(\gamma+1)}{4} \left( \frac{v_n}{a_\infty} \right)^2 + 1 \right] ds \quad (8a)$$

$$v_n = \frac{\partial M_r(\theta)}{\partial t} Z_0(x) + V M_r(\theta) \frac{\partial Z_0(x)}{\partial x} \quad (8b)$$

Equation (8) needs to be solved for  $\theta$  using Harmonic Balance method.