University of Washington
EE 547 (PMP) Project Assignment
Autumn – 2017

MinSeg Robot

Project Report

Assigned: 10/31/17
Due: 12/05/17
Professor Linda Bushnell [UW EE]

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1. Introduction

The goal of the project is to implement concepts learned in the class to develop a linear state space model of the MinSeg robot, and to control the movement and body attitude by stabilizing feedback controller. The goal of stabilizing is to make the minseg bot balance in an upright position.

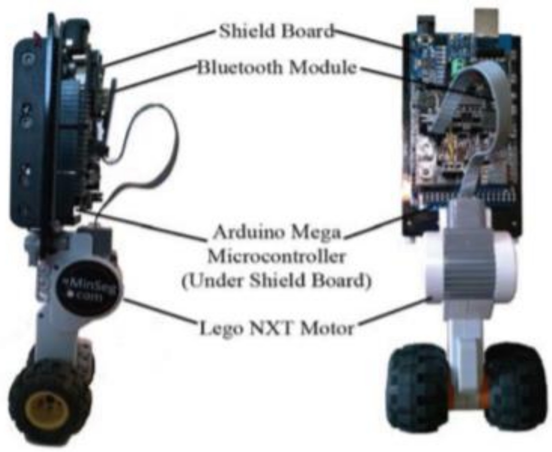


Figure 1: Minseg Robot

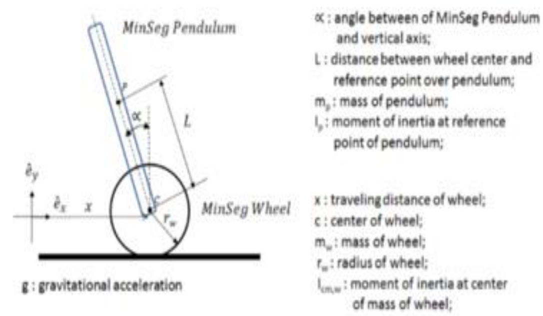


Figure 1: Mathematical model of Minseg around zero-equilibrium state

2. A Dynamical Model of Minseg Robot

The equations of motion by Newton's law of the system depicted in Fig(2) are:

$$\begin{bmatrix} -(I_p + m_p L^2) & m_p L \cos\alpha \\ m_p L\, r_w^2 \cos\alpha & -(I_{cm,w} + m_w r_w^2 + m_p r_w^2) \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} T_m - m_p L g \sin\alpha \\ T_m r_w + m_p L\, r_w^2 \dot{\alpha}^2 \cos\alpha \end{bmatrix} \quad (1)$$

And the torque Tm was provided by DC motor of the minseg. The relation is as given below :

$$T_m = \frac{k_t}{R} V + \frac{k_t k_b}{R r_w} \dot{x} + \frac{k_t k_b}{R} \dot{\alpha} \quad (2)$$

3. Linear Dynamical Model of Minseg Robot

Step 1: Linearizing the nonlinear system of minseg around the equilibrium point

Using (1) and (2) and by linearizing around equilibrium point , we get the state-space representation of this system is as follows:

ẋ = A x + B u (3)

y = C x + D u

The state variables, x, is defined as x: = [α α̇ x $\dot{x}$]^T, the input vector as u:= V and the output vector as y:= x =[α $\dot{\alpha}$ x $\dot{x}$]^T

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{gLm_p\,(i_{cmw}+(m_p+m_w)r_{w^2})}{i_{cmw}(i_p+L^2m_p)+(L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2}} & -\frac{k_bk_t(i_{cmw}+r_w(m_wr_w+m_p\,(L+r_w)))}{R(i_{cmw}(i_p+L^2m_p)+L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2})} & 0 & -\frac{k_bk_t(i_{cmw}+r_w(m_wr_w+m_p\,(L+r_w)))}{Rr_w(i_{cmw}(i_p+L^2m_p)+L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2})} \\ 0 & 0 & 0 & 1 \\ \frac{gL^2m_{p^2}r_{w^2}}{(i_{cmw}(i_p+L^2m_p)+(L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2}} & -\frac{k_bk_tr_w(i_p+Lm_p\,(L+r_w)))}{R(i_{cmw}(i_p+L^2m_p)+L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2})} & 0 & -\frac{k_bk_t(i_p+Lm_p\,(L+r_w)))}{R(i_{cmw}(i_p+L^2m_p)+L^2m_p\,m_w+i_p\,(m_p+m_w))r_{w^2})} \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ -\frac{k_t(i_{cmw}+r_w(m_w r_w+m_p\ (L+r_w)))}{R(i_{cmw}(i_p+L^2 m_p)+L^2 m_p\, m_w+i_p\,(m_p+m_w))r_{w^2})} \\ 0 \\ -\frac{k_t r_w(i_p+L m_p\ (L+r_w)))}{R(i_{cmw}(i_p+L^2 m_p)+L^2 m_p\, m_w+i_p\,(m_p+m_w))r_{w^2})} \end{bmatrix},\ C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ D=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step 2: The physical parameters of MinSeg:

Parameter	Measurement of bot 1	Measurement of bot 2 (without batteries)	Measurement of bot 2 (with batteries)
g	9.81 meter / second^2	9.81 meter / second^2	9.81 meter / second^2
L	0.115 meter	0.115 meter	0.122 meter
m_p	0.3455 kg	0.1998 kg	0.3383 kg
i_p	0.00615 kg –m^2	2.3983477 kg –m^2	2.5172996 kg –m^2
m_w	0.0357 kg	0.0358 kg	0.0358 kg
i_{cmw}	1.49433x10^{-4} kg –m^2	1.74047 x 10^{-5} kg –m^2	1.74047 x 10^{-5} kg –m^2
r_w	0.0215 meter	0.018 meter	0.018 meter
R	5.2628 ohms	5.2628 ohms	5.2628 ohms
k_b	0.4953 volts second / radian	0.4953 volts second / radian	0.4953 volts second / radian
k_t	0.3233 Nm/A	0.3233 Nm/A	0.3233 Nm/A

Formulas used in the experiment to find the values of parameters:

1. Radius of the wheel (r_w) → radius = Circumference of wheel/2*π

2.Mass moment of the wheel (i_{cmw}) → $i_{cmw} = r_{w^2}\left[\frac{m_w}{a_x}\sin\theta - m_w\right]$ and $I_{cmw} = \frac{r^2 m(2gh-v^2)}{v^2}$

Rolling the wheel down the ramp with an angle θ to find the acceleration of the wheel under influence of gravity a_x. The acceleration is then used to calculate i_{cmw} using the above formula.

3. Length of inverted pendulum (L) →
   i. Calculating the center of gravity of the system by suspending it from various points and finding the intersection.
   ii. Measuring the distance from the center of gravity found in step (i) to center of wheel.

4. Mass moment of Inverted pendulum (i_p) → Suspending the MinSeg as a bifilar pendulum and applying a very small torque to get period of oscillation (T) to find mass moment by the equation: $\boldsymbol{i_p} = r_{w^2}\left[\frac{m_p\, g\, b^2 T^2}{\boldsymbol{4\pi^2 L_{string}}}\right]$ and $I_p = \frac{gLm_{total}T^2}{4\pi^2}$

By substituting these values in equation (3), the values of A, B, C, D for linearized model of MinSeg robot are as given below :

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 45.9353 & -13.0049 & 0 & -604.8800 \\ 0 & 0 & 0 & 1 \\ 2.5942 & -2.7460 & 0 & -127.7216 \end{bmatrix} \quad B=\begin{bmatrix} 0 \\ -26.2567 \\ 0 \\ -5.5441 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 3: Transfer Function of the linearized system.

For an LTI system in state space model, the transfer function is given by $\mathrm{G(s)} = \mathrm{C}(sI - A)^{-1}\mathrm{B} + \mathrm{D}$

$$G(s) = \begin{vmatrix} \dfrac{-26.26\,s + 1.54x10^{-12}}{s^3 + 140.7\,s^2 - 45.94\,s - 4298} \\ \dfrac{-26.26\,s^2 + 1.54x10^{-12}s}{s^3 + 140.7\,s^2 - 45.94\,s - 4298} \\ \dfrac{-5.544\,s^2 - 2.955x10^{-14}\,s + 186.6}{s^4 + 140.7\,s^3 - 45.94\,s^2 - 4298s} \\ \dfrac{-5.544\,s^2 - 2.462x10^{-14}\,s + 186.6}{s^3 + 140.7\,s^2 - 45.94\,s - 4298} \end{vmatrix}$$

The impulse response of the system is given in Fig. We see that the states are unstable.

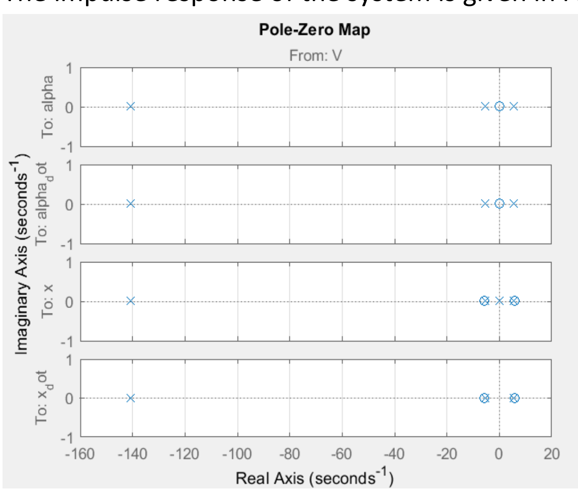


Figure 3: Poles and Zero of Open Loop model of MinSeg

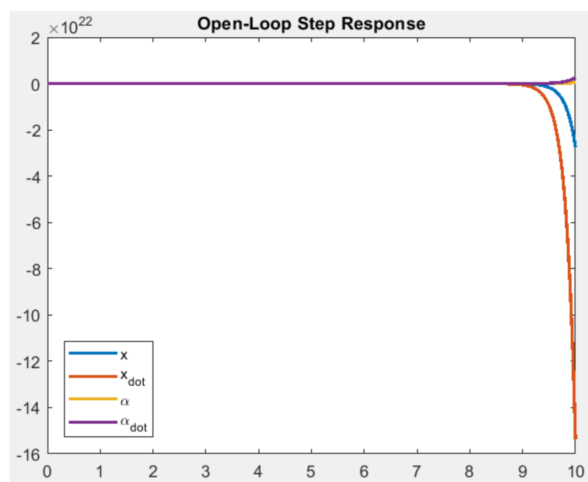


Figure 4: Step response of Open Loop model of MinSeg

Stability

Step 4,5,6:

The characteristic polynomial of the LTI state space model is given by det (λI- A) = 0. The characteristic polynomial of the MinSeg is given by:

$$\Delta(\lambda) = s^4 + 140.7s^3 - 45.94s^2 - 4298$$

The poles or the eigen values of the MinSeg are the roots of Characteristic Polynomial and are as below:

$$\lambda = [\ 0 \quad -140.8360 \quad 5.5791 \quad -5.46960\ ]$$
$$Poles_{minseg} = [\ 0 \quad -140.8360 \quad 5.5791 \quad -5.46960\ ]$$

The stability of Minseg model is evaluated by the eigen values of open loop system. As all of the eigen values of the system do not have negative real parts, the system is not asymptotically stable. (poles at 0 and 5.5791) and not BIBO stable. As all of the eigen values of the system do not have 0 real parts, the system is not marginally stable.

4 Controllability and Observability

The controllability matrix is formed by matrices A and B as $\mathcal{C}$ = [B AB A^2B . . . . $A^{n-1}B$] and the observability matrix is given by matrices A and C as $\mathbb{O}$ = [C CA CA^2 . . . C A^{n-1}]^T For the system to be completely controllable and completely observable the rank of C and O should match the number of rows of matrix A respectively.

In this case rank of Controllability Matrix ($\mathcal{C}$)and rank of Observability matrix ($\mathbb{O}$) were 4, same as number of rows of matrix A. Therefore, the system is Controllable and Observable.

$$\mathcal{C} = \begin{bmatrix} 0 & -26.25665 & 3695.007 & -521191.5 \\ -26.25665 & 3695.007 & -521191.5 & 73402344.0 \\ 0 & -5.544143 & 780.2079 & -109864.0 \\ -5.544143 & 780.2079 & -109864 & 15472799.0 \end{bmatrix}$$

$$\mathbb{O} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 45.93 & -13 & 0 & -604.8 \\ 0 & 0 & 0 & 1 \\ 2.59 & -2.74 & 0 & -127.72 \\ 45.93 & -13 & 0 & -604.87 \\ -2166.57 & 1876.07 & 0 & 8.51x10^4 \\ 2.59 & -2.74 & 0 & -127.72 \\ -457.47 & 389.03 & 0 & 1.79x10^4 \\ -2166.57 & 1876.07 & 0 & 8.51x10^4 \\ 3.07x10^5 & -2.6x10^5 & 0 & -1.2x10^7 \\ -457.47 & 389.03 & 0 & 1.79x10^4 \\ 6.44x10^4 & -5.48x10^4 & 0 & -2.53x10^6 \end{bmatrix}$$

As the system is controllable, we change the axis or states of the system to get its controllable canonical form (CCF)

$$A_{CCF} = \begin{bmatrix} -140.7 & 45.94 & 4298 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B_{CCF} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C_{CCF} = \begin{bmatrix} 0 & -26.26 & 0 & 0 \\ -26.26 & 0 & 0 & 0 \\ 0 & -5.544 & 0 & 186.6 \\ -5.544 & 0 & 186.6 & 0 \end{bmatrix} \quad D_{CCF} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

As the system is observable, we change the axis or states of the system to get its observable canonical form(OCF)

$$A_{OCF}=\begin{bmatrix}0&0&0&1.017e-07\\1&0&0&4298\\0&1&0&45.94\\0&0&1&-140.7\end{bmatrix}\quad B_{OCF}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\quad C_{OCF}=\begin{bmatrix}0&-26.26&3695&-5.212e+05\\-26.26&3695&-5.212e+05&7.34e+07\\0&-5.544&780.2&-1.099e+05\\-5.544&780.2&-1.099e+05&1.547e+07\end{bmatrix}\quad D_{OCF}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

5. State Estimator

State Estimators would help to determine states which cannot be measured on physical system due to cost or issues with sensor mounting or placement etc. Even if in MinSeg all the chosen states could be measured by sensors, the state observer is designed to verify the system.

To develop a closed-loop state estimator (full-dimensional observer) for the open-loop system such that the poles of the observer are stable and that the dynamics of the observer is at least 6-8times faster than the dynamics of the linearized model, the parameters considered to choose desired poles are:

i. Near to the open loop poles (to quickly achieve steady state)
ii. Not very negative (to reduce control effort)

The design was developed to make the observer to be 6times faster than the MinSeg model. Different set of poles were tried and tested and the best response out of all sets was provided by placing the poles at:

$$Poles_{observer} = [\,-6\,, -650,\ -30,\ -20]$$

Poles placed are the eigen values of (A'-C'K)' where L is the Observer gain given by L = K'

$$L=\begin{bmatrix}6&1&0&0\\45.9353&636.9951&0&-604.88\\0&0&30&1\\2.5942&-2.7460&0&-107.7216\end{bmatrix}$$

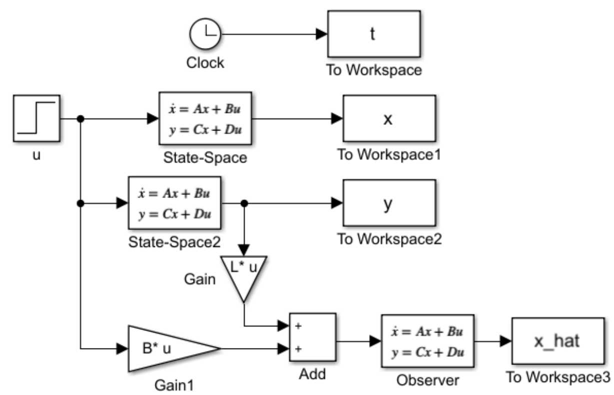


Figure 5: Observer Model

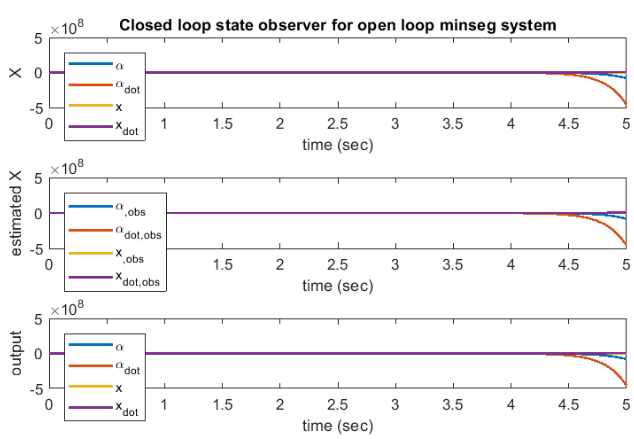


Figure 6: Observer Performance

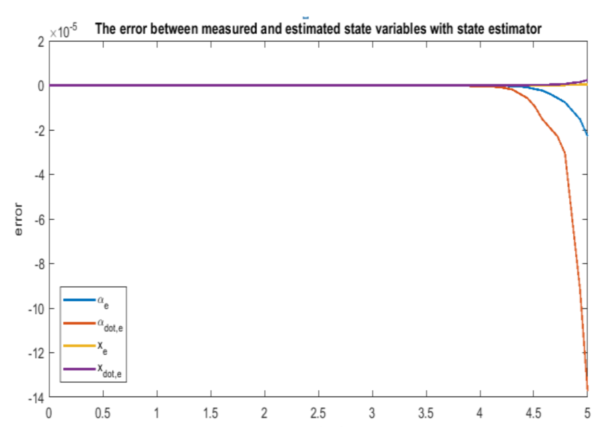


Figure 7: The error between actual and estimated state variables using a state estimator

6. Feedback Control

For the proportional feedback controller gain K, the poles of closed loop system were chosen to provide 6 times faster dynamics than the original open loop MinSeg model. The result of proportional feedback gain K is given below :

K = [-4070.85, -692.594, 12543.1, 3178.12]

The state space system of closed loop system is changed due to the closed loop feedback and the stability of the system is given by the new characteristic polynomial . The new A matrix of the state space model is given by A_{CL} = A-B*K.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.608e+5 & -1.82e+4 & 3.293e+5 & 8.284e+4 \\ 0 & 0 & 0 & 1 \\ -2.257e+4 & -3843 & 6.954e+4 & 1.749e+4 \end{bmatrix} \quad B=\begin{bmatrix} 0 \\ -26.26 \\ 0 \\ -5.544 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The eigen values of A_{CL} are given below:

$$\Delta_{CL}(\lambda) = s^4 + 706s^3 + 37300s^2 + 588600s + 2.34 * 10^6$$

$$Poles_{CLminseg} = [\ -650\ ,\quad -30,\ \ -20, -6]$$

As the poles of closed loop have negative real parts and lie on the left half of s-plane, the closed loop system of MinSeg is Asymptotically stable.

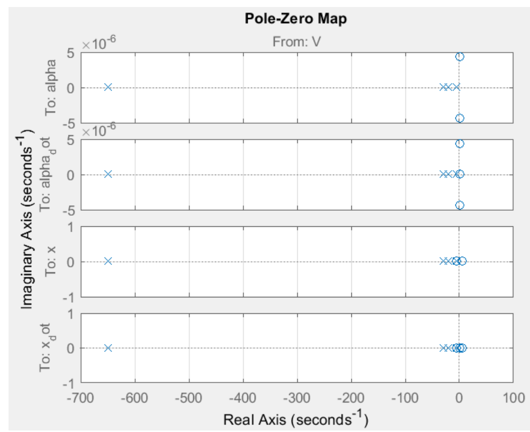


Figure 8: Poles designed for stability of MinSeg Robot

To demonstrate the proportional controller, the Simulink model is deigned as below with performance of driving the states to stability.

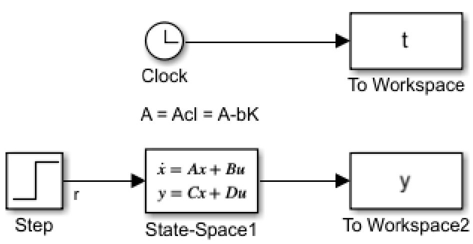


Figure 9: Proportional Controller

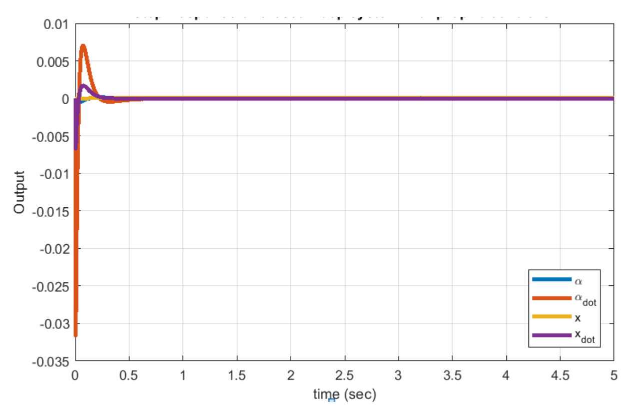


Figure 10: Step response of system with proportional controller

7. Feedback Control using State Estimator

Finally, combining the state estimator and feedback controller in single Simulink model to check the performance of combined system:

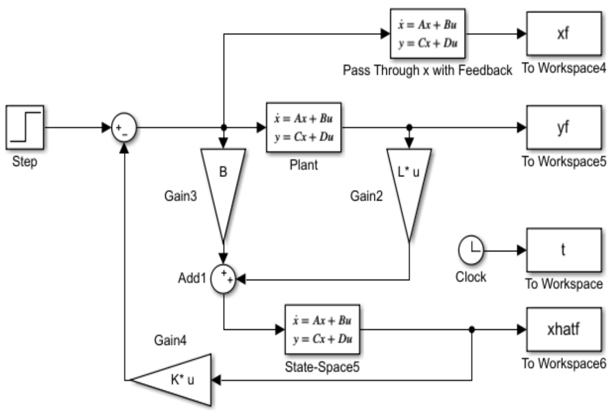


Figure 11: Proportional Controller with Estimator

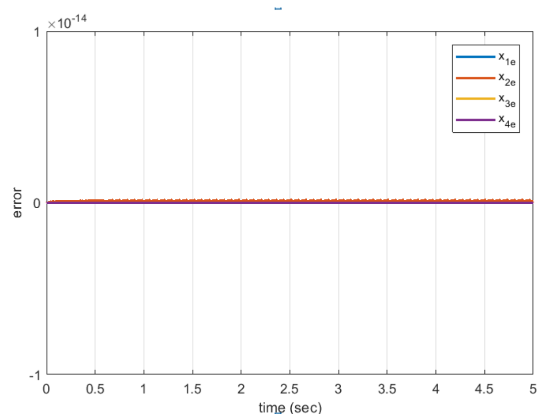


Figure 12: Error measured between x and $\hat{x}$

With the proportional feedback with the estimator the estimator error has reduced by 10^{10} times. The error is small when a closed loop feedback is used to correct the error.

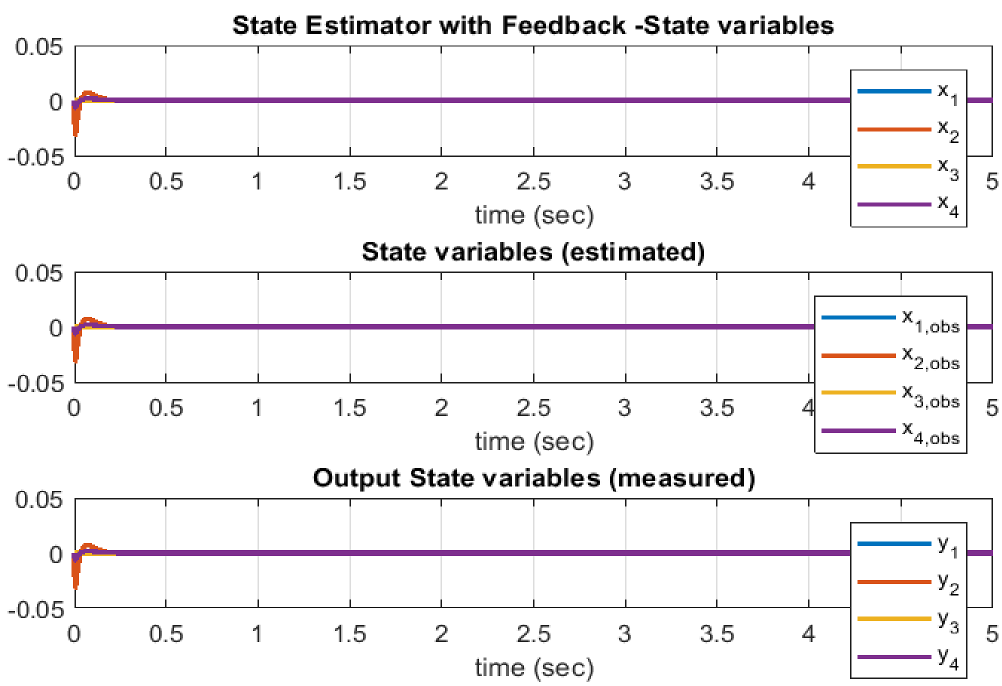


Figure 13: Step Response of Proportional controller with Estimator Robot

8. MinSeg Robot Implementation

Hardware Setup

The MinSeg consists of an Arduino Mega 2560 system-on-a-chip with MinSeg shield M1V4 (Uno/Mega Compatible). It has a DC motor with wheel and encoder (LEGO NXT).The board has many useful sensors by which physical parameters could be read. In this case, to balance the MinSeg we made use of many sensor data from Gyroscope (3 axis gyroscope supported), Accelerometer (3 axis Accelerometer supported) and Encoder. The Gyro helps in measurement of angular position and rate about the x, y and z axis, while, the encoder helps in measuring the displacement from wheel rotation and hence the velocity of the MinSeg robot.

Software Setup

The drivers were procured from minseg.com to interface the sensors with the Arduino chip. The Arduino support package for MATLAB and Simulink was procured from mathworks.com. The weekly lab sessions helped us build modules for each software component.

Development

The work for this project started by measuring the physical parameters (as given in table in step2) – mass of wheel, mass of pendulum, length of pendulum, radius of wheel etc. The weight of wheel and weight of pendulum was measured by the spice weighing scale. The center of gravity of the pendulum was found by suspending the pendulum from 3 orientations and finding out the intersection point. The scale was used to measure the length of pendulum from center of gravity of pendulum to center of wheel. The moment of inertia of the wheel is found out by conducting an experiment by rolling the wheel from slightly inclined ramp. The mass moment of pendulum is found by suspending it as a bifilar pendulum and applying a slight torque to find the time period of oscillation.

The components made in the lab sessions were tweaked and calibrated to make individual modules of gyroscope, accelerometer and encoder. The gyroscope was meticulously calibrated to get the exact 90-degree angle when the pendulum was at equilibrium position and the slightest deviation to be observed by using a good gain factor. A dc factor was observed in the readings and that was nullified by using appropriate Bias reduction factor. A module of Complimentary filter was formed to read exact angular velocity and therefore to deduce angular movement and to convert the raw gyroscope data with fluctuations to effective gyroscope information. In addition, the accelerometer information was used to precisely know the position and velocity of the angular movement of the MinSeg model. The Encoder block was used to measure the translational movement, position and hence measure the velocity of the robot.

The processed information from sensors is used to feed the LQR controller for α(angular position), $\dot{\alpha}$ (angular velocity) , x (position) , $\dot{x}$(velocity) by the gain (KLQR) .The LQR controller is designed to have a closed loop stability for the open loop MinSeg model. The tradeoff between control accuracy and control effort is managed by choosing appropriate values for Q and R respectively where Q and R are positive definite matrices.
The values chosen for Q and R to get KLQR are provided below:

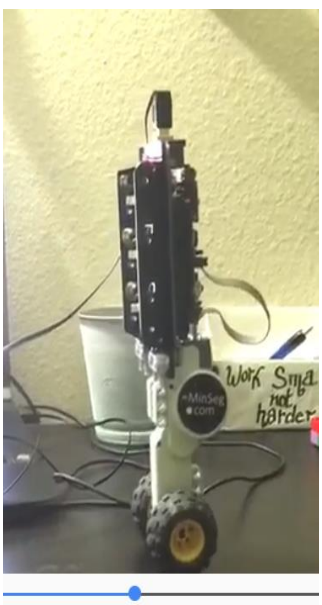
$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \qquad R = 1 \quad \rightarrow \quad KLQR = [10,\ 52.6047,\ -88.2625,\ -17.5694]$$

The gain range was not exact and there were many hit, and trial data tried on the bot. The MinSeg bot is balanced in tethered mode (powered and connected to Simulink by USB) working in external mode of Simulink and worked consistently. The gain(KLQR) used to balance the MinSeg robot :

KLQR = [10, 42.6047, -68.2625, -12.5694]

Result

Although there was initial tension to balance the robot, we experimented a lot using different K values for the LQR controller and the MinSeg balanced quite well for our team. We could balance the MinSeg in tethered mode consistently by the design developed and deployed by Simulink. We could detect exact 90-degree crossing of the robot's angular position and could control precisely the feedback control to prevent the robot toppling.



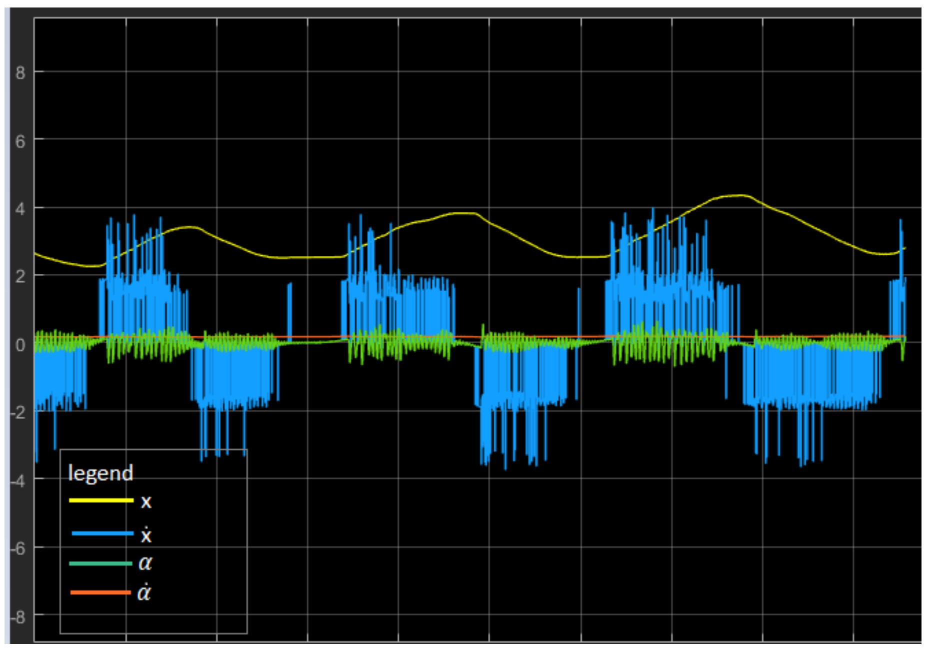


Figure 14: Balancing the MinSeg Robot

Figure 15: Data captured while the MinSeg was balanced

Conclusion

The MinSeg is designed to be balanced considering its natural behavior of instability. A feedback controller is designed to make its behavior stable. The interfacing to the robot was done owing to rich abstraction layer of Arduino. The MinSeg system was perfect canvas to put practical use of the theory studies in Linear Systems Theory course.

In conclusion, the team observed that important factor in balancing the MinSeg is to use a appropriate gain matrix (K_{LQR}). To calibrate gyro meter precisely plays an important role in controlling the MinSeg. All this is given only if the physical parameters of the robot are accurately measured.