OPIM 5603-B14 — Statistics in Business Analytics
Fall 2018, University of Connecticut

Homework 6 - v1

Instructions: Please complete the following questions and submit them as an RNotebook (as an `Rmd` file) via the submission link on HuskyCT. You must submit the assignment by the time and due date listed on the course syllabus. Failure to submit a file by the deadline will result in a score of 0 on the assignment.

Set the heading of the RNotebook as an `html_document`, with a table of contents and without numbered sections. Add your name and a date to the header as well. The solution to each problem should be a separate section (specified by #), and each subproblem should be set as a subsection (specified as ##). For example, for Problem 2, you should have a section titled Problem 2, specified by:

```
# Problem 2
```

in your RNotebook. Also, for subproblem b in Problem 2, you should have a subsection, specified by:

```
## Problem 2b
```

As with all course material, the problems appearing in this homework assignment are taken from the instructor's real-world experiences, from other courses taught at the University of Connecticut, and from the sources listed in the course syllabus.

Note that R code submitted should work independent of the data that sits in the data structure. For example, suppose there was a vector `r_vec` with the values $(1, 2, 6)$ and the problem asks for you to create R code to create a vector `answer` which doubles each element of `r_vec`. The answer

$$\texttt{answer} \leftarrow \mathrm{c}(2, 4, 12)$$

would be given no credit. The answer

$$\texttt{answer} \leftarrow 2^{*}\texttt{r\_vec}$$

would be an appropriate answer.

You must show all steps in your solution. For example, if a problem asks for the expected value of a random variable that is binomially distributed with $n = 10$ and $\pi = 0.3$, and you simply write

$$3,$$

this will be given no credit. However,

$$10 * 0.3$$

would be given credit.

If you have any questions, please submit them via email to the instructor and/or the teaching assistant prior to submitting your solution.

Problem 1 (30 points)

Suppose your portfolio consists of two investments, A and B. Investment A has an expected return of 10% and investment B has an expected return of 15%. Suppose the volatility (i.e., standard deviation) of investment A is 25% and that of investment B is 32%.

a. What is the minimum value for the correlation coefficient between these investments?

b. What is the maximum value for the covariance between these two investments?

c. Suppose the correlation coefficient between the two investments is 0.6, and that you have 60% of your portfolio invested in A and 40% of your portfolio invested in B. What is the expected return of your portfolio?

d. Suppose the correlation coefficient between the two investments is 0.6, and that you have 60% of your portfolio invested in A and 40% of your portfolio invested in B. What is the variance of your portfolio?

Problem 2 (40 points)

In this problem we will explore the Central Limit Theorem.

a. Let X be a uniformly distributed random variable, with mean 35 and standard deviation 2. Generate 10,000 random samples of size 4 from this distributions. What is the average of the sample means? What is the standard deviation of the sample means?

b. Consider the same random variable X, but now generate 10,000 random samples of size 40. What is the average of the sample means? What is the standard deviation of the sample means?

c. Now consider a new random variable, Y, which is normally distributed with mean 35 and standard deviation 2. Generate 10,000 random samples of size 4 from this distributions. What is the average of the sample means? What is the standard deviation of the sample means?

d. Consider the same random variable Y, but now generate 10,000 random samples of size 40. What is the average of the sample means? What is the standard deviation of the sample means?

e. Discuss the results from the previous parts, specifically relating them to the Central Limit Theorem. Explain which of the four parts the Central Limit Theorem applies to, and measure how close or far the theoretic mean and standard deviation is from the simulated one.

Problem 3 (30 points)

In this problem we will examine how good Chebyhev's rule is at approximating ranges for random variables.

a. Consider a random variable X that is uniformly distributed with mean 10, min 8, and max 12. What percentage of the distribution is within 2 standard deviations from the mean?

b. Consider a random variable Y that is uniformly distributed with mean 10 and standard deviation 3. What percentage of the distribution is within 2 standard deviations from the mean?

c. Consider the random variable Z which takes value -2 with probability $\frac{1}{98}$, takes value 2 with probability $\frac{1}{98}$, and takes values 0 with probability $\frac{96}{98}$. What percentage of the distribution is within 2 standard deviations of the mean?

d. Discuss the results from the previous parts, relating them to Chebychev's inequality.