OPIM 5603-B14 — Statistics in Business Analytics
Fall 2018, University of Connecticut

Homework 5 - v1

Instructions: Please complete the following questions and submit them as an RNotebook (as an `Rmd` file) via the submission link on HuskyCT. You must submit the assignment by the time and due date listed on the course syllabus. Failure to submit a file by the deadline will result in a score of 0 on the assignment.

Set the heading of the RNotebook as an `html_document`, with a table of contents and without numbered sections. Add your name and a date to the header as well. The solution to each problem should be a separate section (specified by #), and each subproblem should be set as a subsection (specified as ##). For example, for Problem 2, you should have a section titled Problem 2, specified by:

```
# Problem 2
```

in your RNotebook. Also, for subproblem b in Problem 2, you should have a subsection, specified by:

```
## Problem 2b
```

As with all course material, the problems appearing in this homework assignment are taken from the instructor's real-world experiences, from other courses taught at the University of Connecticut, and from the sources listed in the course syllabus.

Note that R code submitted should work independent of the data that sits in the data structure. For example, suppose there was a vector `r_vec` with the values $(1, 2, 6)$ and the problem asks for you to create R code to create a vector `answer` which doubles each element of `r_vec`. The answer

$$\texttt{answer} \leftarrow \mathrm{c}(2, 4, 12)$$

would be given no credit. The answer

$$\texttt{answer} \leftarrow 2^{*}\texttt{r\_vec}$$

would be an appropriate answer.

You must show all steps in your solution. For example, if a problem asks for the expected value of a random variable that is binomially distributed with $n = 10$ and $\pi = 0.3$, and you simply write

$$3,$$

this will be given no credit. However,

$$10 * 0.3$$

would be given credit.

If you have any questions, please submit them via email to the instructor and/or the teaching assistant prior to submitting your solution.

Problem 1 (30 points)

Suppose customers arrive to an ice cream shop at a rate of 1 every 150 seconds

a. What is the probability that fewer than 6 customers arrive in the first 10 minutes that the store is open?

b. Every time a customer purchases ice cream, the number of scoops they order is 1 plus a random number, which is binomial distributed with size 2 and probability 0.5 (so that each customer orders 1, 2, or 3 scoops). What is the probability mass function for the number of scoops of ice creams a randomly chosen customer will purchase?

c. How many scoops of ice cream do you expect the ice cream shop to sell in the first hour that they are open?

Problem 2 (30 points)

Cars arrive to a toll plaza at a rate of 3 per minute and trucks arrive to the toll plaza at a rate or 1.5 per minute.

a. How many cars do you expect to arrive in the first 10 minutes?

b. What is the probability that no truck arrives in the next minute?

c. What is the standard deviation of the number of vehicles that arrive in the first 20 minutes?

d. Cars pay \$3.00 and trucks pay \$6.50 to pass through the toll. What is the probability that the toll brings in revenue of over \$1000 in the next hour?

Problem 3 (30 points)

The exponential distribution is a continuous distribution, often used to model the length of time between two occurrences of an event (the time between arrivals). The PDF of the distribution is

$$f(x) = \lambda e^{-\lambda x}, \text{ for all } x > 0.$$

There is a single parameter to this distribution, λ, which is the rate of arrival per time period. For example, if trucks arrive to a toll booth at a rate of 10 trucks per minute, then we can model the time between the arrival of consecutive trucks as an exponential random variable with $\lambda = 10$. For this distribution, the mean μ is $\frac{1}{\lambda}$ and the standard deviation σ is $\frac{1}{\lambda}$.

`R` has the following four functions that can be used to calculate probabilities associated with randomly variables that are exponentially distribution: `dexp(x, rate = 1, log = FALSE)`, `pexp(q, rate = 1, lower.tail = TRUE, log.p = FALSE)`, `qexp(p, rate = 1, lower.tail = TRUE, log.p = FALSE)`, and `rexp(n, rate = 1)`, with interpretations similar to those that we have seen in class for other continuous distributions. Namely, `dexp` calculates the value of the PDF at a particular value, `pexp` calculates the CDF at a particular value, `qexp` calculates percentiles, and `rexp` samples from the distribution.

a. A common use of the exponential distribution is to model the lifetime of a light bulb. Suppose that the lifetime of a light bulb is exponentially distribution with *mean* $\mu = 5$ years. Define a random variable for the lifetime of a single light bulb, and find the probability that the lifetime of a light bulb is more than 7 years.

b. Find the probability that the lifetime of a light bulb is more than 4 years.

c. Suppose that a light bulb has lasted for 6 years. What is the probability that its remaining lifetime is more than 4 years?

d. Suppose that a light bulb has lasted for 10 years. What is the probability that its remaining lifetime is more than 4 years?

e. Do you see a trend in the previous three questions? Explain.

Problem 4 - Practice Problem from Class (10 points)

Cruz Inc. manufactures steel beams that are frequently used to reinforce and replace old structural channels in large and spacious rooms. From past experience, it is known that the manufacturing process has some variability. Any given steel beam created has length randomly distributed according to a uniform random variable with minimum length 8.7 inches, and maximum length 9.1 inches.

a. What is the probability that a steel beam created by Cruz Inc. is 8.5 inches long?

b. What is the probability that a steel beam created by Cruz Inc. is 8.9 inches long?

c. What is the probability that a steel beam created by Cruz Inc. is between 8.92 and 9.01 inches?

d. What is the probability that a steel beam created by Cruz Inc. is between 8.5 and 9.01 inches?

e. Cruz Inc. is considering switching to a new trucking company, Gopal LLC, that will handle the logistics of transporting the manufactured steel beams to Cruz Inc.'s customers. However, the truck operated by Gopal LLC can only carry a collection of steel beams that have a total length of under 900 inches. Cruz Inc. wants to ensure that with 90% certainty 101 steel beams can be shipped on a single truck. How can you estimate the probability that 101 manufactured beams fit on a single truck? Do you think Cruz Inc. will be comfortable using Gopal LLC for its steel beam transportation? Why or why not? Provide numerical sampling analysis to support your decision.