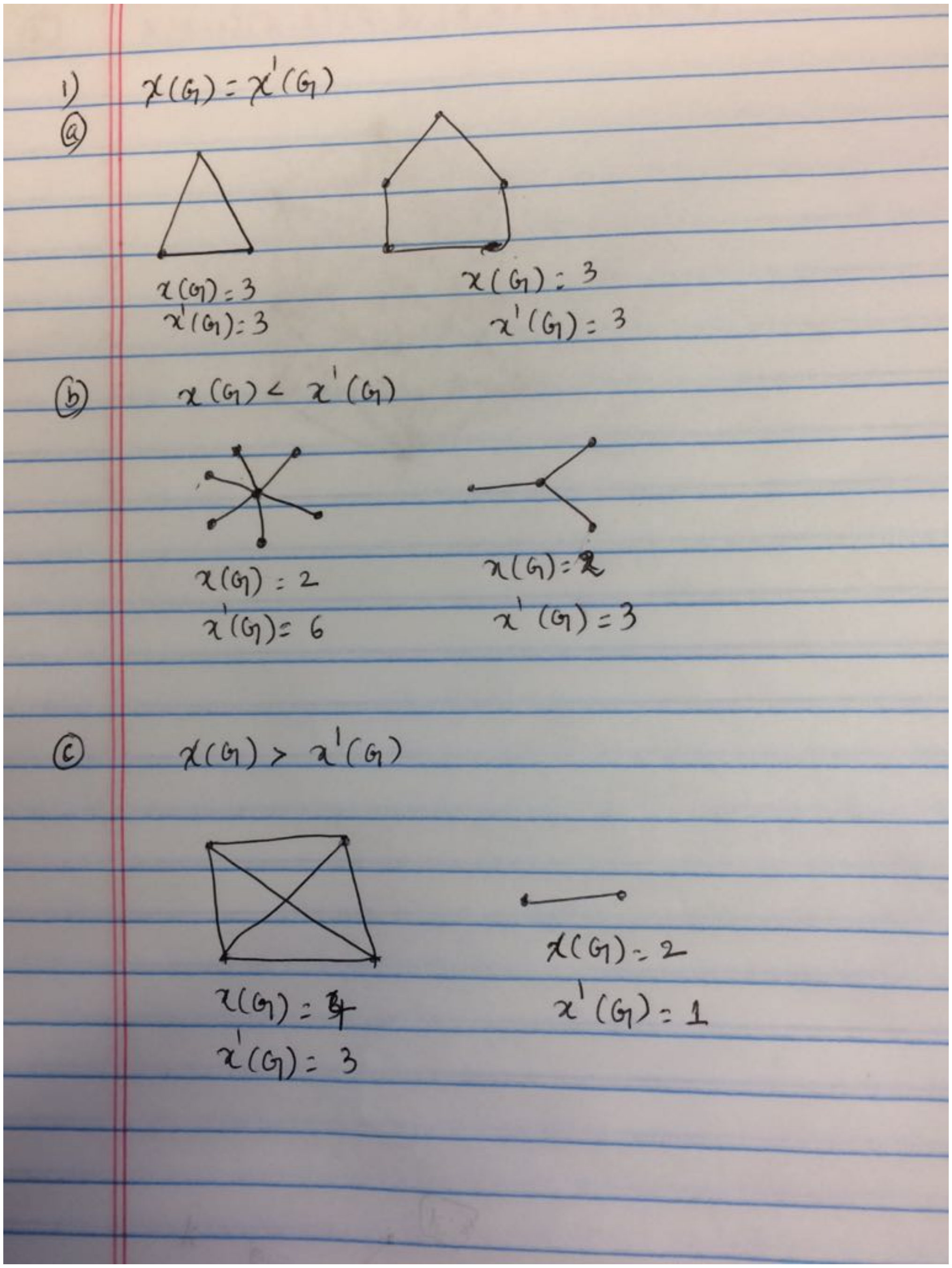
Assignment 7

1)



1) $\chi(G) = \chi'(G)$

ⓐ

$\chi(G) = 3$
$\chi'(G) = 3$

$\chi(G) = 3$
$\chi'(G) = 3$

ⓑ $\chi(G) < \chi'(G)$

$\chi(G) = 2$
$\chi'(G) = 6$

$\chi(G) = 2$
$\chi'(G) = 3$

ⓒ $\chi(G) > \chi'(G)$

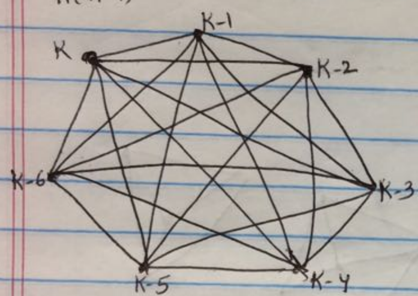
$\chi(G) = 4$
$\chi'(G) = 3$

$\chi(G) = 2$
$\chi'(G) = 1$

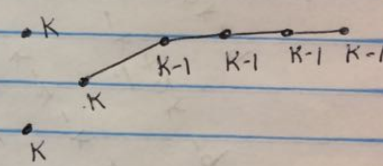
2)

(2) $K(K-1)(K-2)(K-3)(K-4)(K-5)(K-6)$

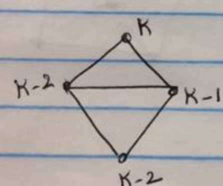
(a.)



(c) $K^3(K-1)^4$

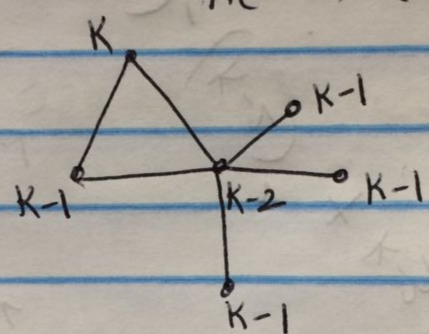


(b) $K(K-1)(K-2)^2$



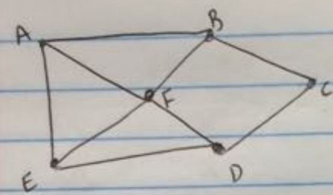
$K(K-1)^4(K-2)$

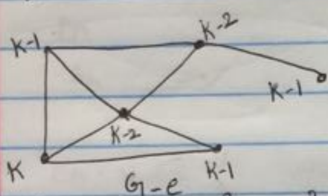
(d)

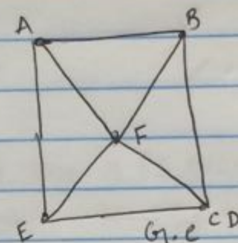


3)

3

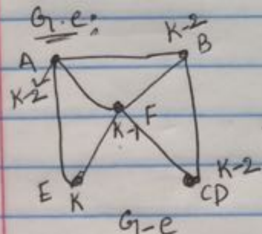


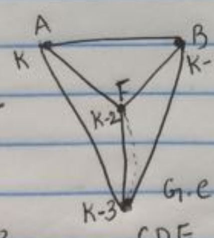




(i) → $G-e = K(K-1)^3(K-2)^2$

(ii) G.e:





G-e: $K(K-1)(K-2)^3$

G.e: $K(K-1)(K-2)(K-3)$

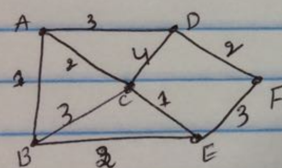
chromatic polynomial:

$$\chi(G) = \chi(G-e) - \chi(G.e)$$

$$= K(K-1)^3(K-2)^2 - \left[K(K-1)(K-2)^3 - K(K-1)(K-2)(K-3)\right]$$

③. $\chi(G) = K(K-1)(K-2)\left[(K-1)^2(K-2) - (K-2)^2 + (K-3)\right]$

$\chi(G) = 3$



$\chi'(G) = 4$

$\Delta(G) = 4$

∴ Graph is Δ-edge-colorable (class one)

4)

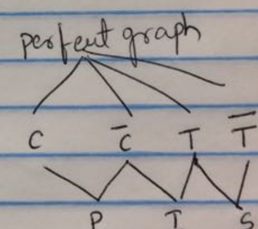
4) a)

	chordal	comparable	Interval	split
K_n	✓	✓	✓	✓
C_n	X	✓/X	x	x
P_n	✓	✓	✓	x
S_n	✓	✓	✓	✓

C_n: C_n is comparable for n is even (∵ even cycle)

C_n: C_n is not comparable for 'n' is odd (∵ odd cycle).

C_n: is chordal if $n \leq 3$



5)

(5) 8.19 Let G be simple graph.

$w(G) \rightarrow$ no. of vertices largest clique subgraph of G

$w(G) + w(\bar{G}) \leq n+1$

Consider, max clique size of a subgraph of G is $K = w(G)$

we Know that,

$w(G) \leq \chi(G) \leq \chi(\bar{G})+1$

and $w(\bar{G}) \leq \chi(\bar{G}) \leq \chi(\bar{G})+1$

$$w(G) + w(\bar{G}) \leq \chi(\bar{G}) + \chi(G) + 2$$
$$\leq d_G(w) + d_{G}(u) + 2$$
$$\leq (n-1)+2$$

$w(G) + w(\bar{G}) \leq n+1$

(5) 8.20

$\chi(G) + \chi(\bar{G}) \leq n+1$

Let 'G' be a simple graph on 'n' vertices and its complement $\bar{G}$. 'u' be a vertex of G and $\bar{G}$.

If $G' = G - u$ then $\chi(G) \leq \chi(G') + 1$ and

$\chi(\bar{G}) \leq \chi(\bar{G}') + 1$

if equality holds in both cases, then

$\chi(G') \leq d_G(u)$ and $\chi(\bar{G}) \leq d_{\bar{G}}(u)$

$$\Rightarrow \chi(G) + \chi(\bar{G}) \leq \chi(\bar{G}) + 1 + \chi(\bar{G}') + 1$$
$$\leq d_G(u) + 1 + \chi(G) + 1$$
$$\leq d_G(u) + 1 + d_G(u) + 1$$
$$\leq d_{\bar{G}}(u) + d_G(u) + 2 = n-1+2$$

$\chi(G) + \chi(\bar{G}) \leq n+1$