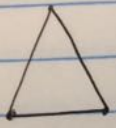


Assignment 7


1)

1) $\chi(G) = \chi'(G)$

(a)




$\chi(G) = 3$
 $\chi'(G) = 3$

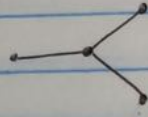


$\chi(G) = 3$
 $\chi'(G) = 3$

(b) $\chi(G) < \chi'(G)$

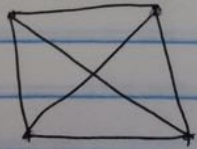


$\chi(G) = 2$
 $\chi'(G) = 6$




$\chi(G) = 2$
 $\chi'(G) = 3$

(c) $\chi(G) > \chi'(G)$



$\chi(G) = 4$
 $\chi'(G) = 3$

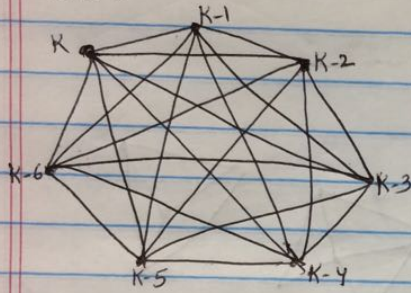


$\chi(G) = 2$
 $\chi'(G) = 1$

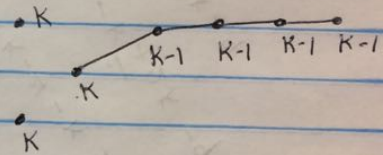
2)

(2) $K(K-1)(K-2)(K-3)(K-4)(K-5)(K-6)$

(a)

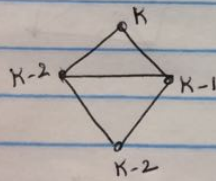


(c) $K^3(K-1)^4$



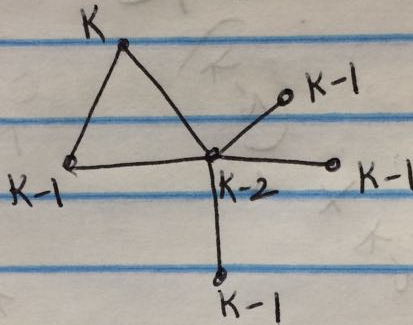
(b)

$K(K-1)(K-2)^2$



(d)

$K(K-1)^4(K-2)$



3)

③

Diagram of graph G with vertices A, B, C, D, E and internal vertices F, G . The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices, and a vertex G connected to F .

Diagram of graph $G-e$ (removing edge FG). The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices. Vertex degrees are: $\deg(A)=2, \deg(B)=2, \deg(C)=2, \deg(D)=2, \deg(E)=2, \deg(F)=5$.

(i) $\rightarrow G-e = K(K-1)^3(K-2)^2$

Diagram of graph G with vertices A, B, C, D, E and internal vertices F, G . The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices, and a vertex G connected to F . Vertex degrees are: $\deg(A)=2, \deg(B)=2, \deg(C)=2, \deg(D)=2, \deg(E)=2, \deg(F)=5, \deg(G)=1$.

Diagram of graph $G-e$ (removing edge FG). The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices. Vertex degrees are: $\deg(A)=2, \deg(B)=2, \deg(C)=2, \deg(D)=2, \deg(E)=2, \deg(F)=5$.

(ii) $G-e = K(K-1)(K-2)^3$

Diagram of graph G with vertices A, B, C, D, E and internal vertices F, G . The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices, and a vertex G connected to F . Vertex degrees are: $\deg(A)=2, \deg(B)=2, \deg(C)=2, \deg(D)=2, \deg(E)=2, \deg(F)=5, \deg(G)=1$.

Diagram of graph $G-e$ (removing edge FG). The graph is a cycle $A-B-C-D-E-A$ with a central vertex F connected to all cycle vertices. Vertex degrees are: $\deg(A)=2, \deg(B)=2, \deg(C)=2, \deg(D)=2, \deg(E)=2, \deg(F)=5$.

$G-e = K(K-1)(K-2)(K-3)$

chromatic polynomial:

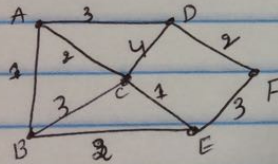
$$\chi(G) = \chi(G-e) - \chi(G \cdot e)$$

$$= K(K-1)^3(K-2)^2 - [K(K-1)(K-2)^3 - K(K-1)(K-2)(K-3)]$$

③.

$$\chi(G) = K(K-1)(K-2) \left[(K-1)^2(K-2) - (K-2)^2 + (K-3) \right]$$

$$\chi(G) = 3$$



$$\chi'(G) = 4$$

$$\Delta(G) = 4$$

\therefore Graph is Δ -edge-colorable (class one)

4)

4)
a)

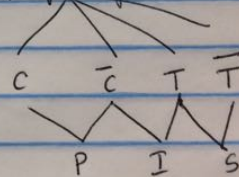
	chordal	comparable	Interval	Split
K_n	✓	✓	✓	✓
C_n	x	✓/x	x	x
P_n	✓	✓	✓	x
S_n	✓	✓	✓	✓

C_n : C_n is comparable for n is even (\because even cycle)

C_n : C_n is not comparable for n is odd (\because odd cycle).

C_n : is chordal iff $n \leq 3$

perfect graph



5)

(5)
8.19

Let G be simple graph.

$w(G) \rightarrow$ no. of vertices largest clique subgraph of G

$$w(G) + w(\bar{G}) \leq n + 1$$

Consider, max clique size of a subgraph of G is $k = w(G)$

we know that,

$$w(G) \leq \chi(G) \leq \chi(\bar{G}) + 1$$

$$\text{and } w(\bar{G}) \leq \chi(\bar{G}) \leq \chi(G) + 1$$

$$w(G) + w(\bar{G}) \leq \chi(\bar{G}) + \chi(G) + 2$$

$$\leq d_G(u) + d_{\bar{G}}(u) + 2$$

$$\leq (n-1) + 2$$

$$w(G) + w(\bar{G}) \leq n + 1$$

(5) 8.20

$$\chi(G) + \chi(\bar{G}) \leq n + 1$$

Let ' G ' be a simple graph on ' n ' vertices and its complement \bar{G} . ' u ' be a vertex of G and \bar{G} .

If $G' = G - u$ then $\chi(G) \leq \chi(G') + 1$ and

$$\chi(\bar{G}) \leq \chi(\bar{G}') + 1$$

If equality holds in both cases, then

$$\chi(G') \leq d_G(u) \text{ and } \chi(\bar{G}') \leq d_{\bar{G}}(u)$$

$$\Rightarrow \chi(G) + \chi(\bar{G}) \leq \chi(\bar{G}') + 1 + \chi(G') + 1$$

$$\leq d_G(u) + 1 + \chi(G) + 1$$

$$\leq d_G(u) + 1 + d_G(u) + 1$$

$$\leq d_G(u) + d_G(u) + 2 = n - 1 + 2$$

$$\chi(G) + \chi(\bar{G}) \leq n + 1$$