1. The Graph used to represent this game is directed, cyclic graph (G) with no of nodes 5 and no of edges 10.

Vertices (v) = {spock, scissors, paper, rock, lizard}

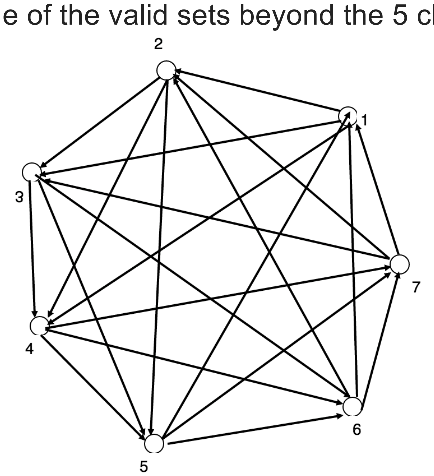
Edges (e)= {spock smashes scissors, scissors cuts paper, paper disproves spock , paper covers rock, rock crushes lizard, lizard eats paper, lizard poisons spock , spock vaporizes rock, rock crushes scissors, Scissor decapitates lizard}

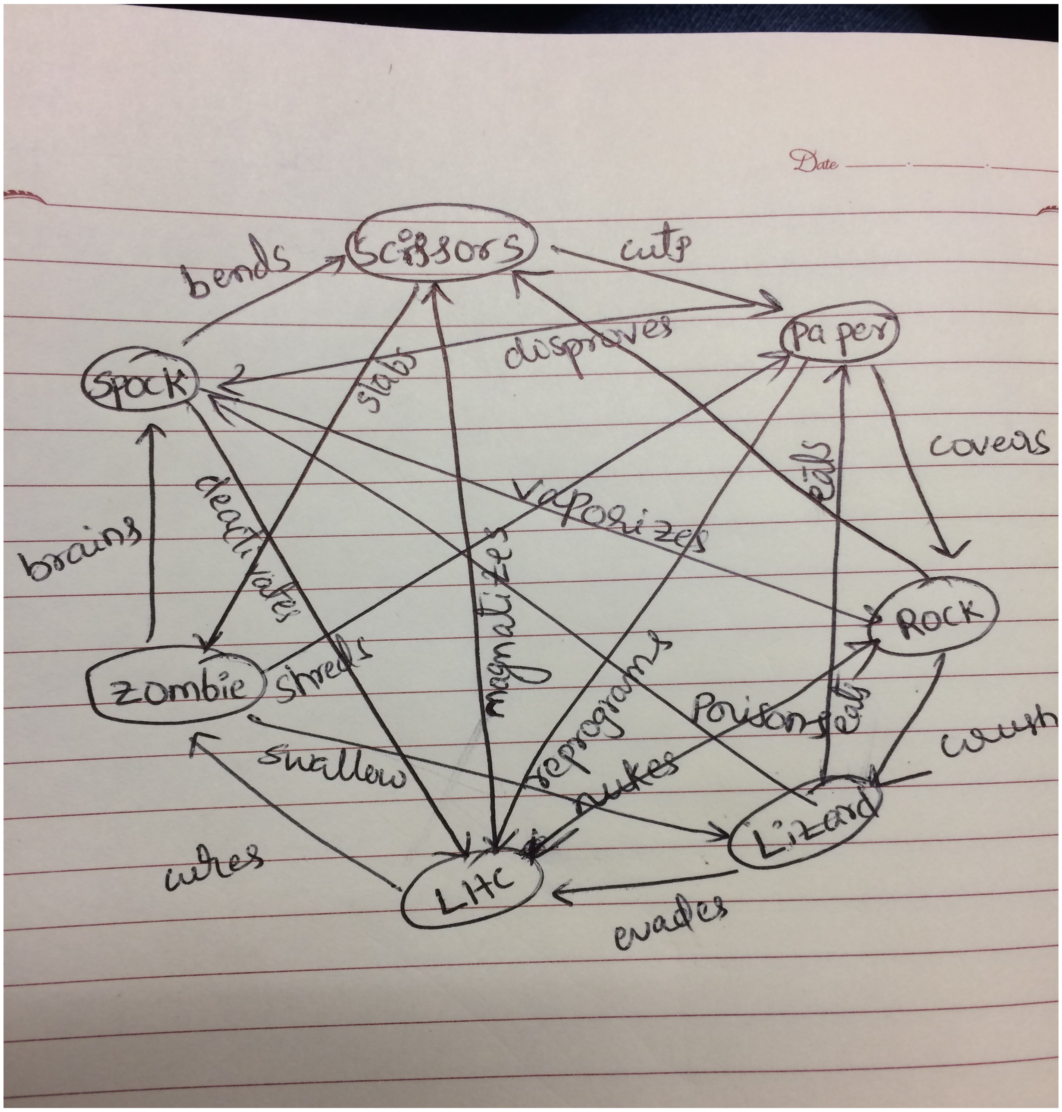
e1: spock smashes scissors
direction: spock → scissors
e2: scissors cuts paper
direction: scissors → papar
e3: paper covers rock
direction: paper → rock
e4:rock crushes lizard
direction: rock→ lizard
e5: lizard poisons spock
direction: lizard→ spock
e6: spock vaporizes rock
spock→rock
e7: rock crushes scissors
rock→scissors
e8: scissors decapitates lizard
scissors→ lizard
e9: lizard eats paper
lizard→paper
e10: paper disproves spock
paper→spock

b) In-degree of each node must be equals to out-degree.

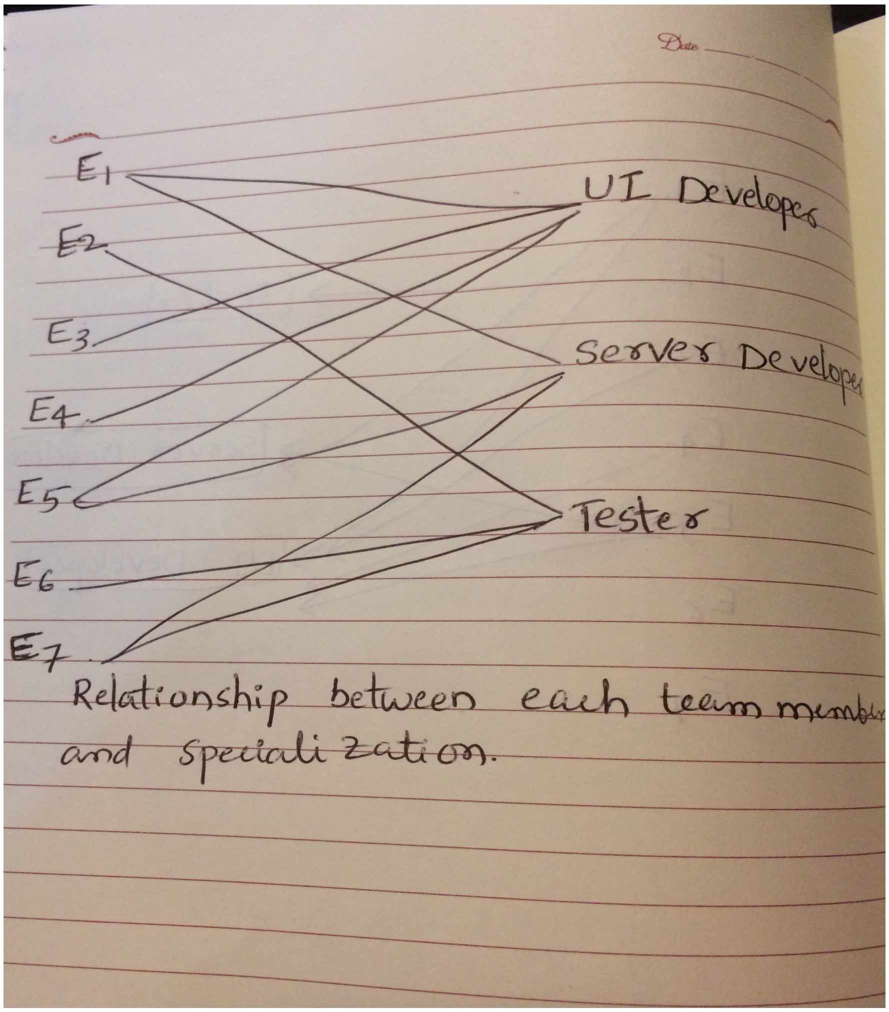
c) It is not possible to have version with four choices. Since the rule of the game is:
Rules: Game must have an odd number (n) of choices, each choice should win and lose (n-1)/2 other choices.

d) One of the valid sets beyond the 5 choices is game with seven choices.

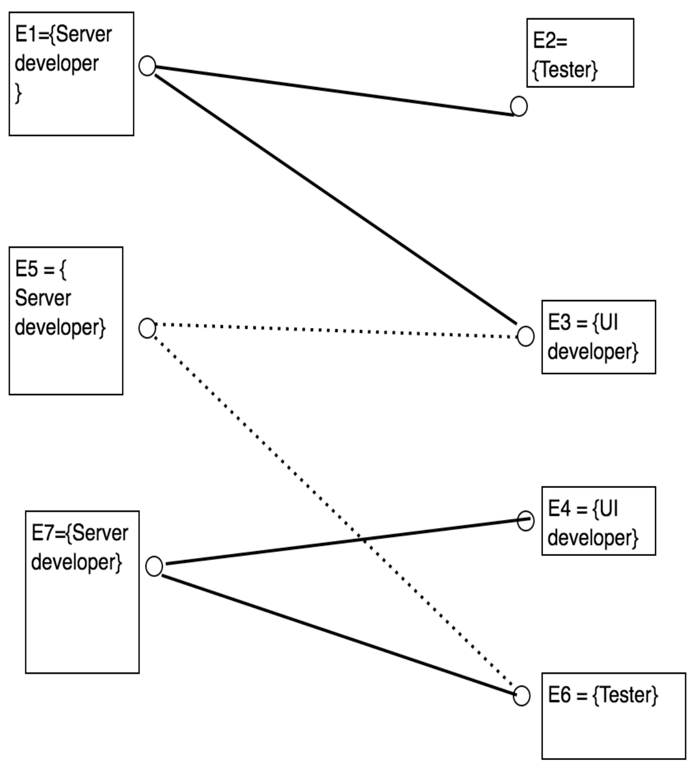



Date
Scissors
Paper
Spock
Rock
Zombie
Lizard
LHC
bends
cuts
disproves
stabs
covers
eats
vaporizes
brains
deactivates
magnatizes
shreds
reprograms
Poisons
nukes
swallow
wires
evades

2. a) We can draw a bi-partite graph K{m,n}
E1→Server Developer, UI developer E2→ Tester,   E3→ UI Developer
E4→UI Developer,E5→ UI Developer,Server Developer   E6→Tester,   E7→Server Developer,Tester



Relation between Team members.



b) At the same time this group of employees can have two teams.

E1→Server Developer, E2→ Tester, E3→ UI Developer
E4→UI Developer, E6→Tester, E7→Server Developer

Team1= E1,E2,E3
Team2= E4,E6,E7

Question 3:

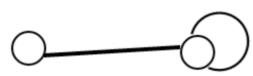
1.8) No of jobs-5 (j1,j2,j3,j4,j5)
No of Applicants -7 (A1,A2,A3,A4,A5,A6,A7)
Qualified Applicants for each job is:
J1: A2
J2: A2,A3,A5
J3: A3,A5
J4: A2
J5: A1,A4,A6,A7
No. Company can not meet its hiring position need with qualified applicants.
Since A2 is the only applicant can fulfill the J1,J4 positions.

1.14) N_n, P_n,C_n,K_n relation in terms of subgraph:

N_n is sub graph of P_n, C_n,K_n
P_n is sub graph of C_n,K_n
C_n is sub graph of K_n
➔ N_n subset of P_n subset of C_n subset of K_n

1.15) K_n is a subgraph of $K_{l,m}$ for values n<=2. For any values n>3, K_n graph contains cycle where $K_{l,m}$ does not contain any cycle.

1.21) There is no such simple graph each with distinct degree. Since, a simple graph will have at least two vertices with same degree. And there is a possibility to exist a general graph with vertices having distinct degree.



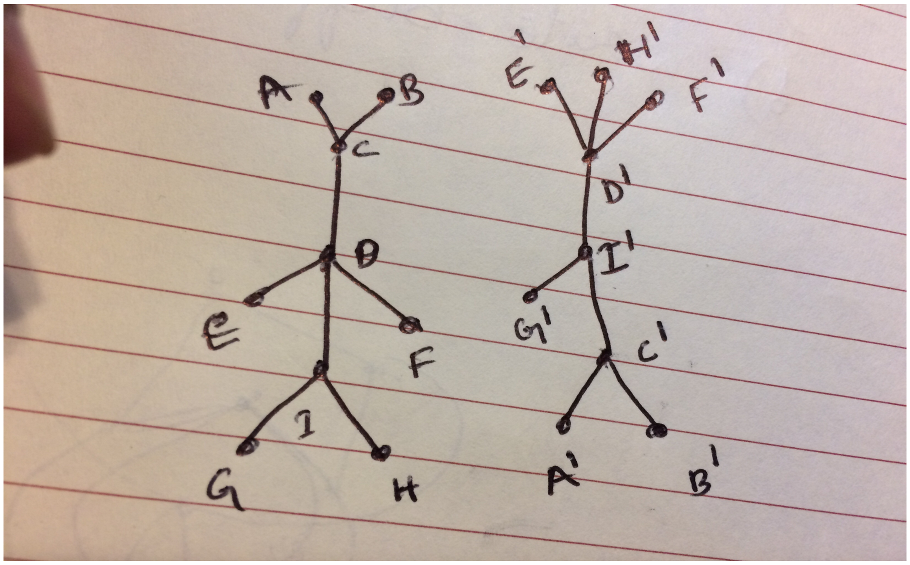
An Example for general graph with distinct degree.

1.22)
Possible degrees are: 2,3,4,6,8
No of degree 2 vertices:4
No of degree 3 vertices= 8
No of degree 4 vertices =20
No of degree 6 vertices: 16
No of degree 8 vertices: 16

No of edges= Sum of degrees/2
=(2*4+3*8+4*20+6*16+8*16)/2 = 168 edges

Question 4:
2.14) Two graphs are said to be isomorphic to each other if they follow below properties.
i. Two graphs must have same number of vertices
ii. Two graphs must have same number of edges
iii. the same degrees for corresponding vertices
iv. the same number of connected components
v. the same number of loops
vi. And there exist f:v→v' & g: e→e' such that each element(v) in Graph(G1) associates exactly one element(v') in Graph(G2) and similarly, each edge(e) in Graph(G1) associates exactly one edge(e') in Graph(G2).
vii. pairs of connected vertices must have the corresponding pair of vertices connected.

G1 & G2 are not isomorphic, since pairs of connected vertices is not having same corresponding pair of vertices, vertex D in graph G1 is connected I, C ,E,F having degree 3,3,1,1 but in G2, D' is connected to I',E',H',F' having degree 3,1,1,1 .



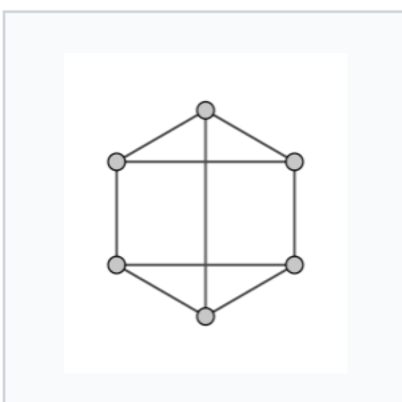
Similarly, neither of the given graphs are isomorphic.

2.16) Graphs are not isomorphic.

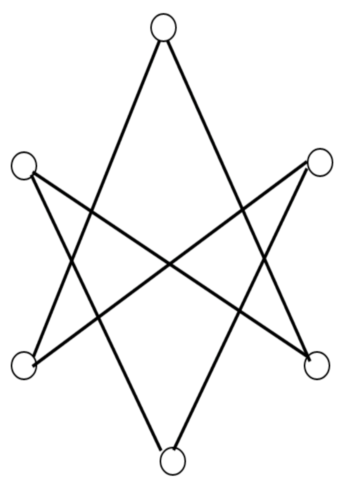
- G1 is not isomorphic to G2, since degree sequence is different.
- G2 is not isomorphic to G3, since degree sequence is different.
- G1 is not isomorphic to G3, since pairs of connected vertices is not having same corresponding pair of vertices

2.22) Proof for complement of every regular graph is regular.

Proof by contradiction, Assume complement of every regular graph is not regular. But,



3-regular graph



Complement of regular graph is also regular

Our assumption is proved wrong. Hence, complement of every k-regular graph is n-k-1 regular.

2.23) Let us consider G_n and G'_n isomorphic, if G_n & G'_n is complement these must have same number of edges.

Edges in G_n+ Edges in G'_n= Edges in K_n

$E(K_n)=n(n-1)/2$

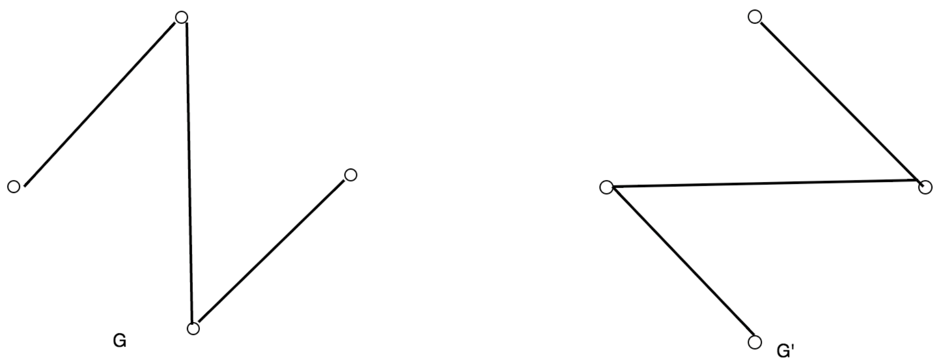
$E(G)=E(G')=n(n-1)/4$

This is only possible for n or n-1 divisible by 4.

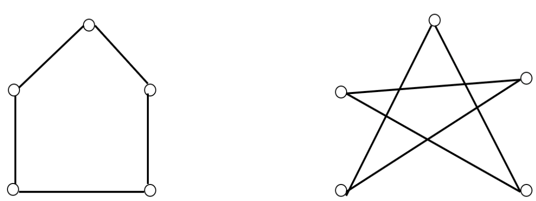
$n=4k$ or $n-1=4k \Rightarrow n=4k+1$

In conclusion for values of n=4k or 4k+1 where k is some natural number, G &G' are isomorphic.

Simple graphs on 4 &5 vertices that are isomorphic:



Graph (G) with an order 4 and its self-complementary graph(G')



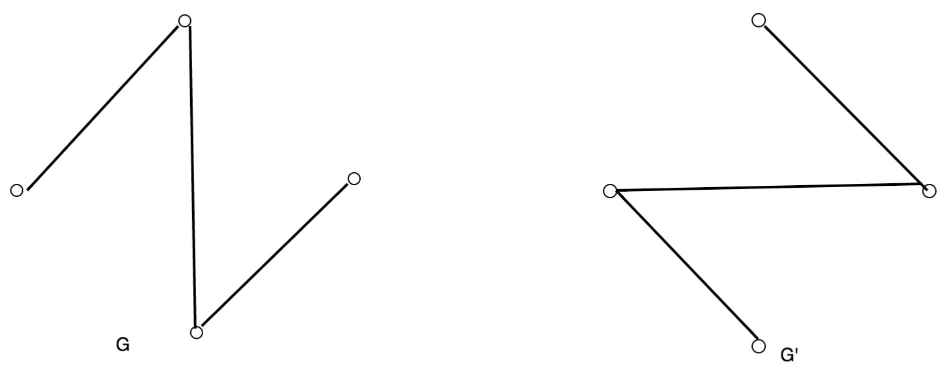
Graphs with order of 5 and self complementary graphs

2.39) Tournament diagraph: it is a directed graph whose underlying graph is complete graph.
For n=2; two tournament graphs are possible whose vertices one is having in-degree=0 and other’s out degree =0.
For n=3; six tournament graphs are possible whose vertices one is having in-degree=0 and other’s out degree =0.
In general, for a graph with n vertices n*n-1 tournament graphs are possible whose vertices one is having in-degree=0 and other’s out degree =0.

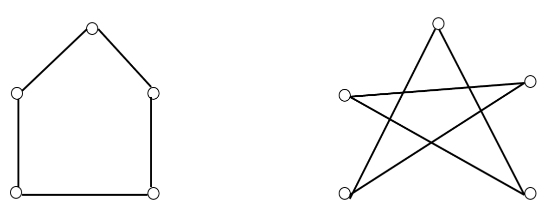
5) A self-complementary graph is a graph that is isomorphic to its complement. And a self- complementary graph should have exactly half of the total no of possible edges.

$$n(E)= n(n-1)/4$$

a) Self-complementary graphs exist for graphs of order 4 and 5



Graph (G) with an order 4 and its self-complementary graph(G')



Graphs with order of 5 and self complementary graphs

b) A graph with six-edges can not have a self-complementary graph. Since, a graph and its complementary graph must have same number of edges. A complete graph of 6 can has total number edges 15 and this fifteen cannot be divided into equally.
If a graph G with order n, have a self-complementary graph then n(n-1) must be divisible by 4 so n mod 4 should equals to 0 or 1.