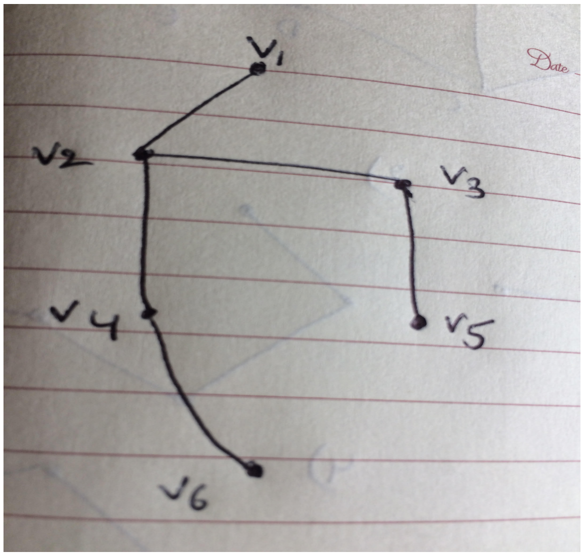
Assignment 3

1

a) The equivalent graph problem for the given cable connection problem with same cost for connecting any two houses is finding a spanning tree of a graph



b) Finding a Minimum Spanning tree for a graph is a graph problem. It models accurately the given cable connection problem. Since, Minimum Spanning Tree is a connected spanning tree that includes all nodes together with minimal total weight for its edges. In the given problem nodes implies the houses and weight corresponds to expense to install cable between two houses.

c) In order to solve above problem we can use:
(i). Prims algorithm   or
(ii). Kruskal algorithm

The main steps of the Kruskal's Algorithm are as follows:

- Sort the edges by weight in ascending order: least weight first and heaviest last.
- Choose the minimum edge from the list that is not added to graph before, add this chosen edge to the tree, only if adding so does not make a cycle.
- Stop the process whenever n - 1 edges have been added to the tree

The following are the main 3 steps of the Prim's Algorithm:

- Begin with any random vertex, which will be suitable and add it to the tree.
- Find neighbor edges of the lowest weight, which connect any vertex in the tree to any vertex that is not in the tree. Note that, we don't have to form cycles.
- Stop when n – 1 no of edges has been added to the tree.

d) Applying Kruskal algorithm to find a minimum spanning tree:

Step1: Sort the edges with cost minimum to maximum

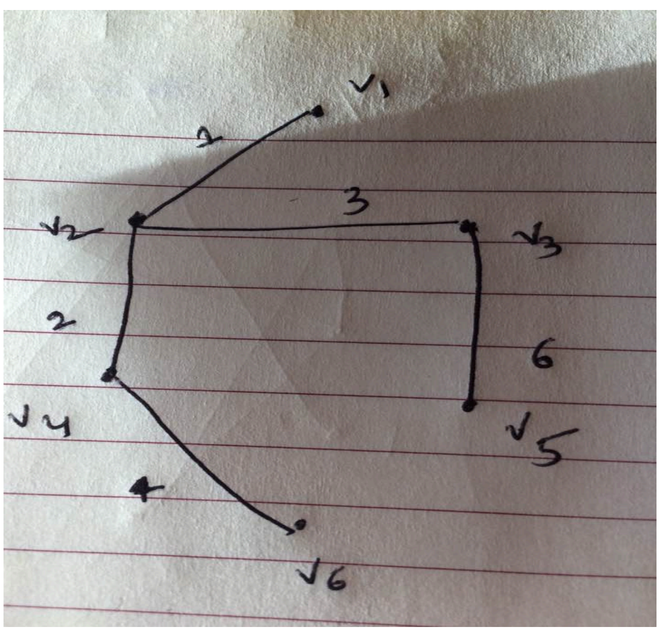
Edge	Between vertices	cost
E1	v1v2	1
E2	v2v4	2
E3	v2v3	3
E4	v4v6	4
E5	v3v5	6
E6	v1v3	7
E7	v4v5	9
E8	v4v3	11
E9	v5v6	12

Step 2:

- Choose the minimum edge from the list that is not added to graph before, add this chosen edge to the tree, only if adding so does not make a cycle.

Selected edges are: E1,E2,E3,E4,E5

Step 3: halt when n-1 edges are added to tree.



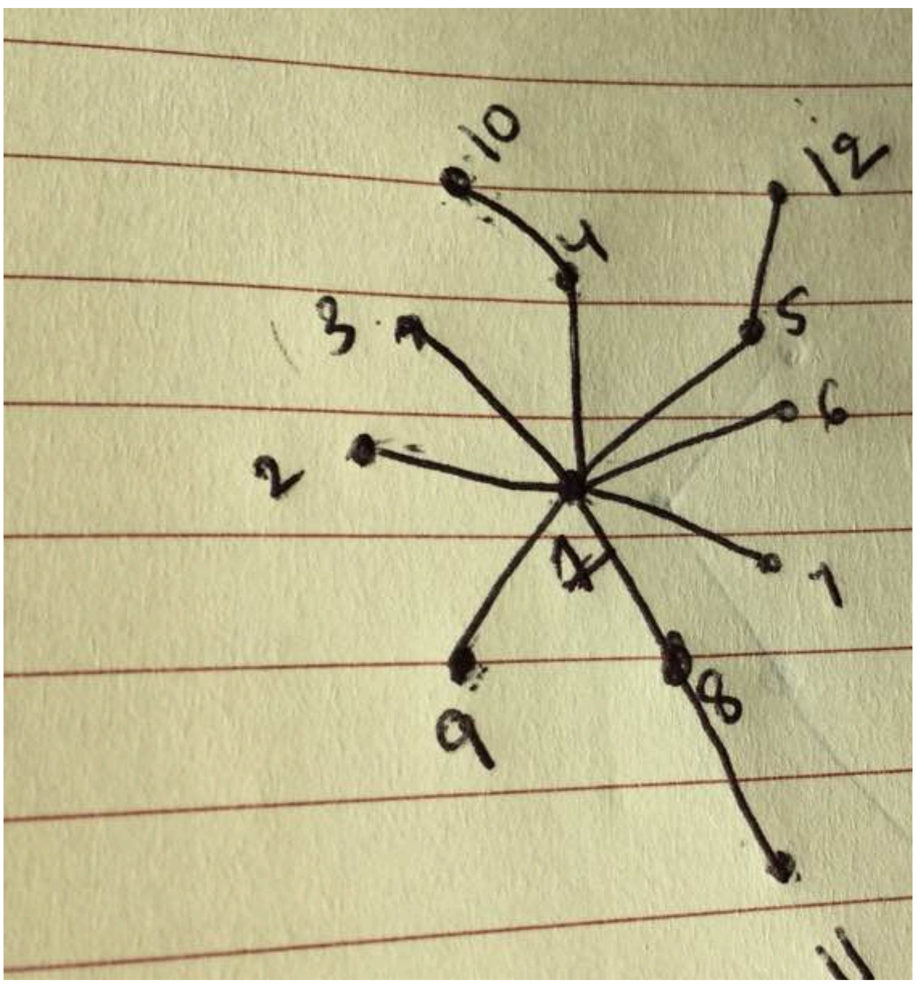
2)

a) The diameter of the graph can be used to determine whether company website can be easily navigable or not. The diameter has to be at most 4 for any node in the graph. Where node represents each web page and edges represents navigation among them. We need to check each page is navigable with In the 4 clicks from every other webpage that is eccentricity. The diameter is maximum eccentricity in the graph.

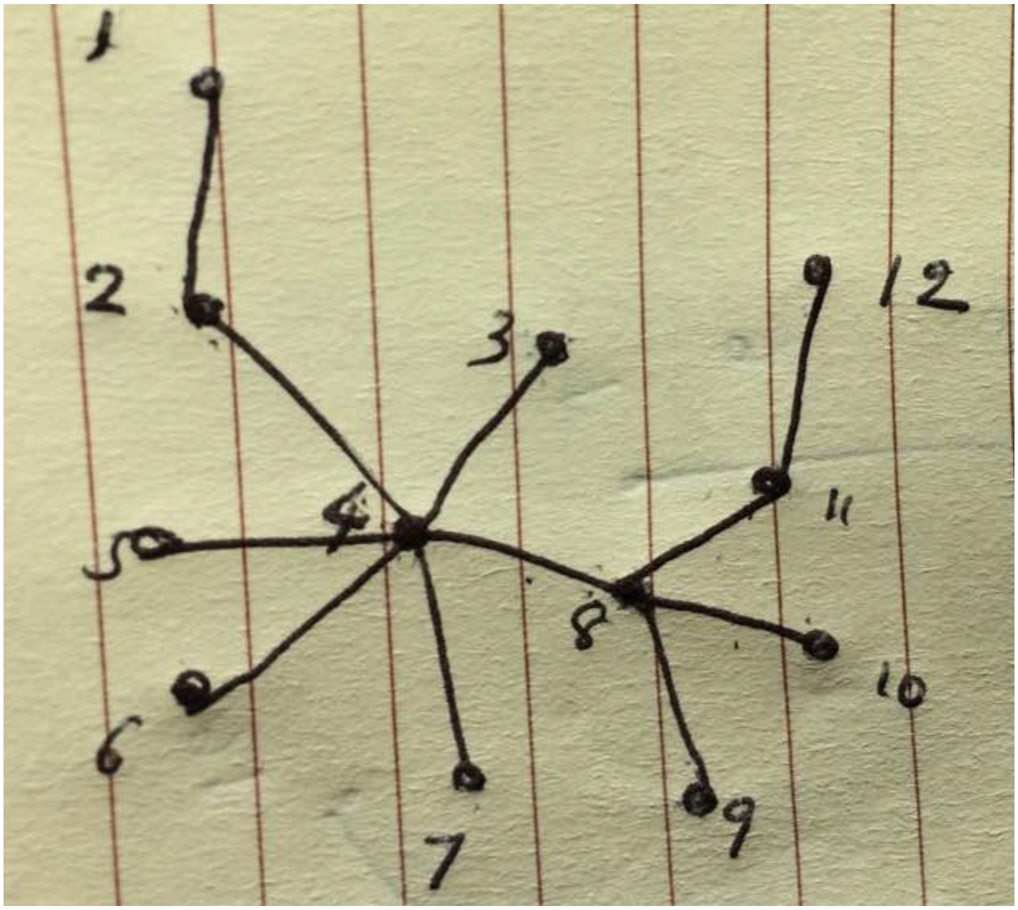
b) Center of the graph parameter can be used to determine which page can be chosen for HOMEPAGE. Since, we are concentrating on the easily navigable website that means we should be able to visit to any page with in minimum number clicks from home page. That is similar to graph property center (minimum eccentricity) guarantees to traverse to other node with in minimum no of clicks.

c) An Example for easily navigable graph, with single center for webpages is:

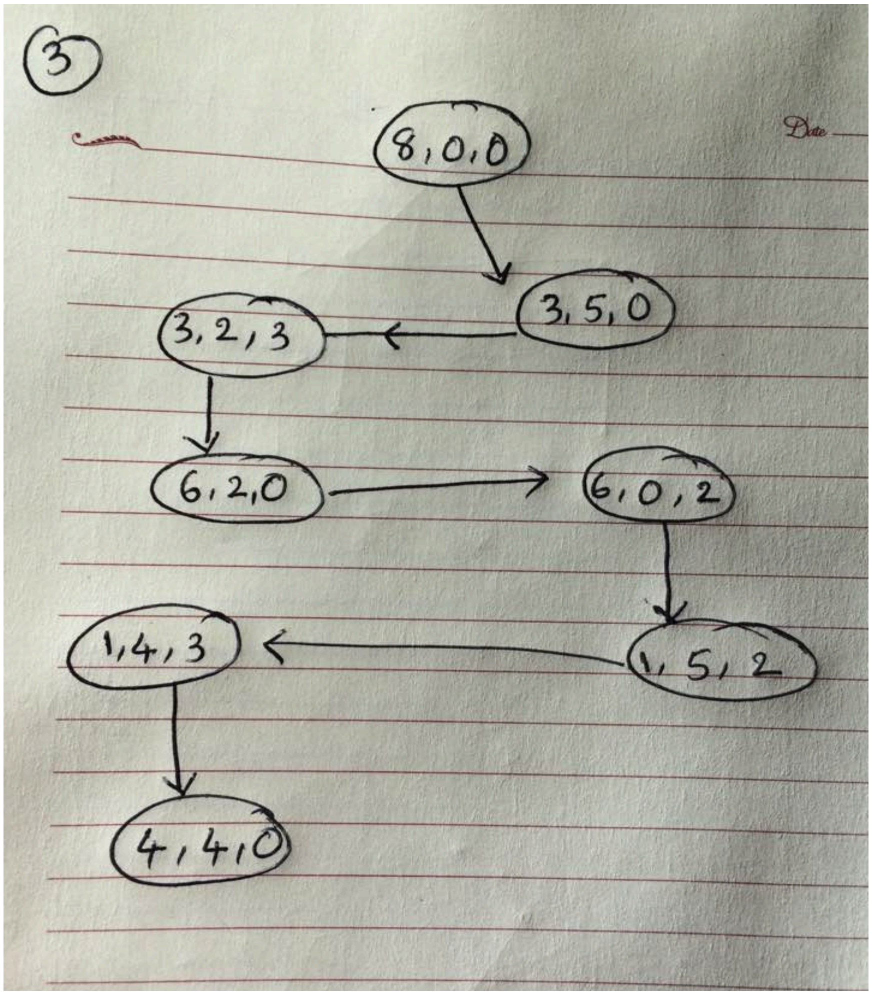
center is : 1



d) Example for pages a website that is not easily navigable and has more than one option for a home page: Possible centers/Homepages are 4 and 8



3) We can represent given problem as directed graph with nodes that represents the state of the cans at any point of time, edges show exchange of milk among the cans that leads to another state.

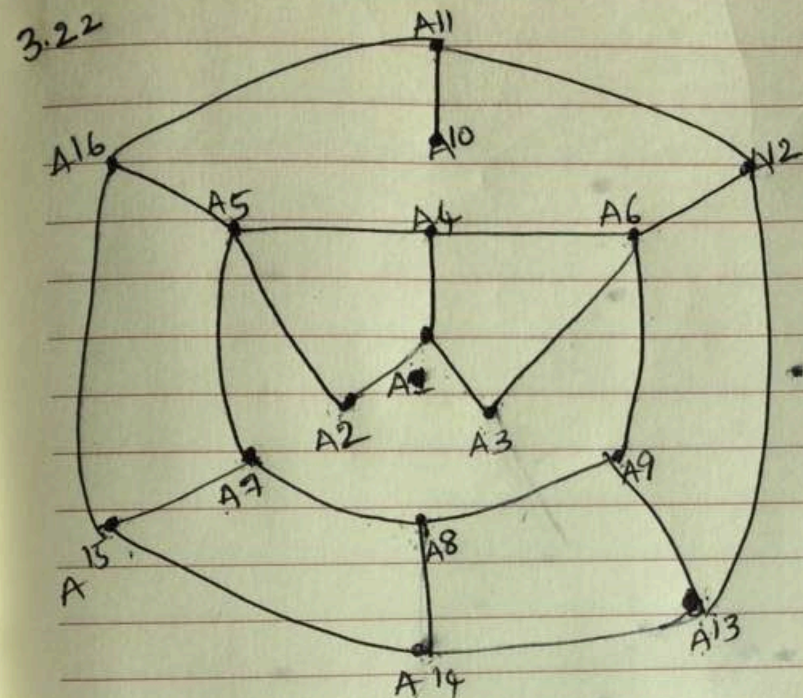


4)

a)

4 a.

3.2.2



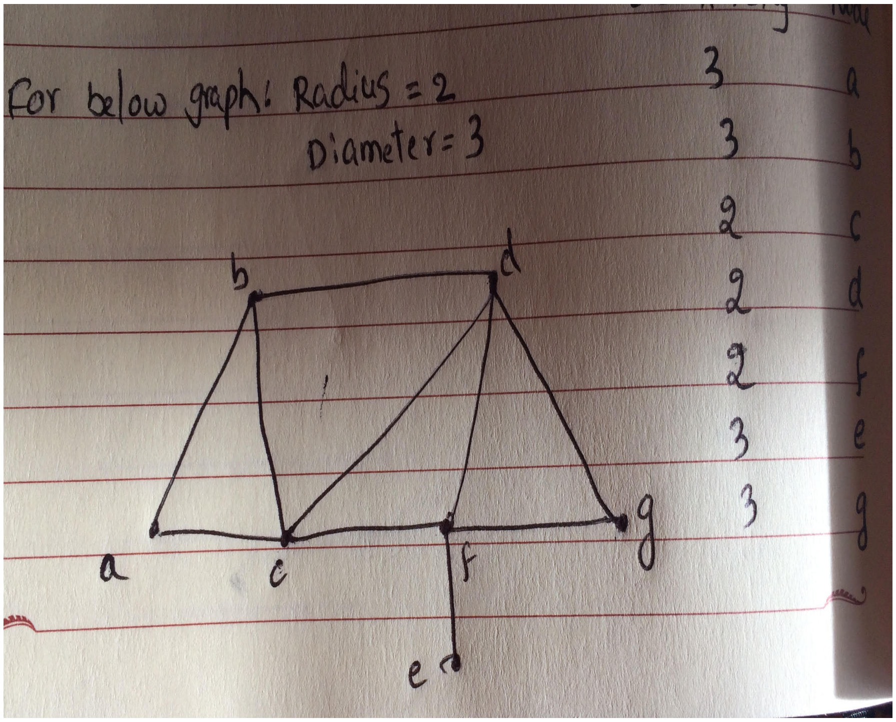
Node	eccentricity
A_1 →	5
A_2 →	5
A_3 →	5
A_4 →	4
A_5 →	4
A_6 →	4
A_7 →	4
A_8 →	5
A_9 →	4
A_{10} →	5
A_{11} →	4
$A_{..}$ →	4

Nodes	eccentricity
A_{13}	4
A_{14}	5
A_{15}	4

Radius(G) = 4

Diameter = 5

b) If all the nodes in the graph has the same eccentricity then graph will have same radius and diameter. If the graph is a complete graph it will have same radius and diameter.



Eccentricity	Nodes
3	a
3	b
2	c
2	d
3	e
2	f
3	g

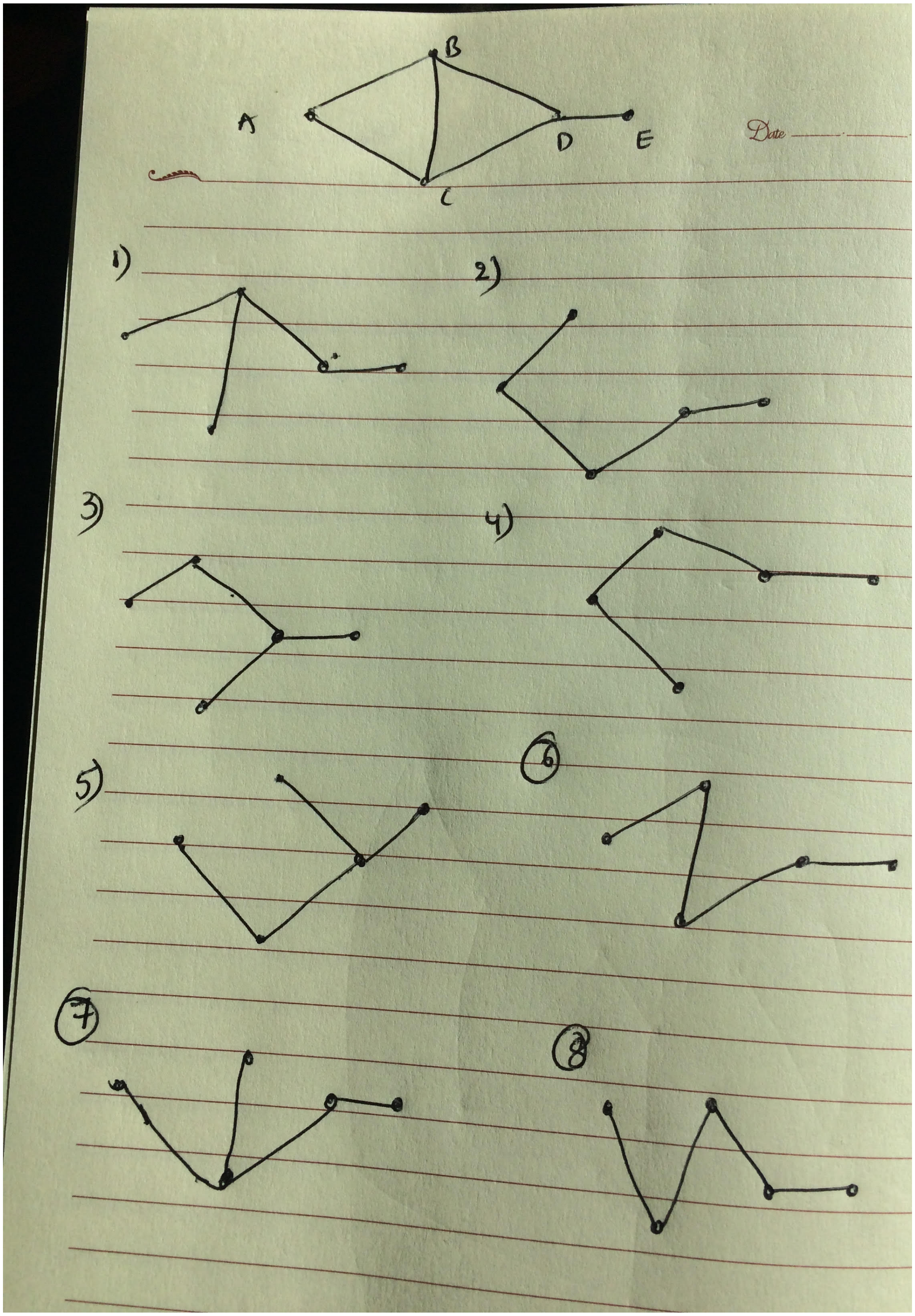
Radius is 2 :Diameter is 3

Center is : nodes with minimum eccentricity(c,d,f).

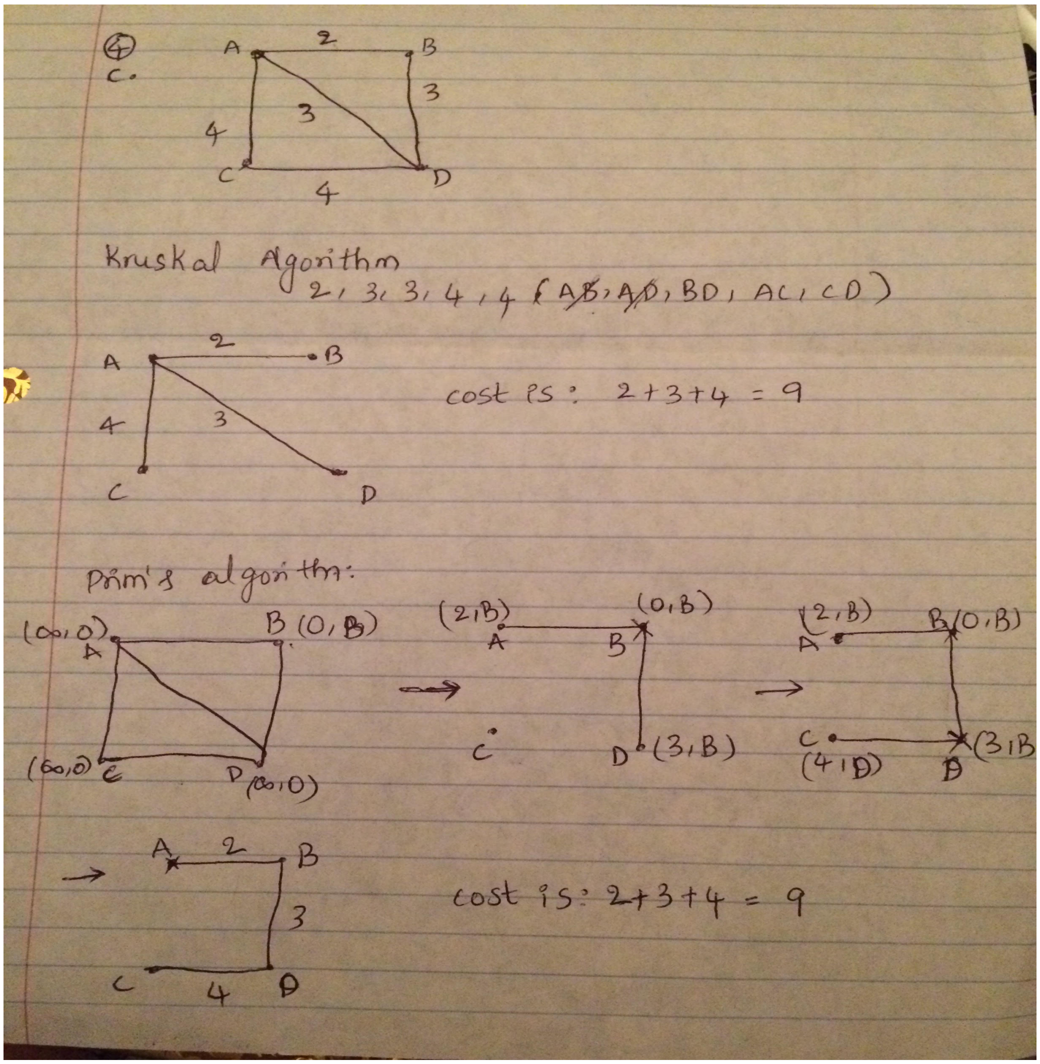
c)

i) The number of spanning trees for a given graph are 8

ii) All spanning trees are:

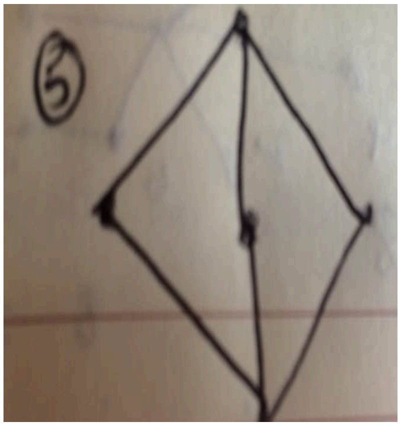


iii) If the Graph has cost on edges is unique for each edge then both Prim's and Kruskal gives the same minimum cost spanning. If the costs are equal then it may give different minimum spanning trees with the same cost, below is the example for such case:



5) There are many ways to solve this. One method is to note that $K_{2,3}$ has 5 vertices and 6 edges. Any spanning tree of $K_{2,3}$ would have exactly four edges and there are 6C2 = 15 ways to remove two edges. But this is an over counting, since removing just any two edges doesn't work. You can see from the following representation of τ ($K_{2,3}$) that the pairs of edges that don't produce a tree are those incident to the same vertex in the bipartition set of order 3.

There are 3 such pairs. Hence, τ (K2,3) = 15 − 3 = 12.



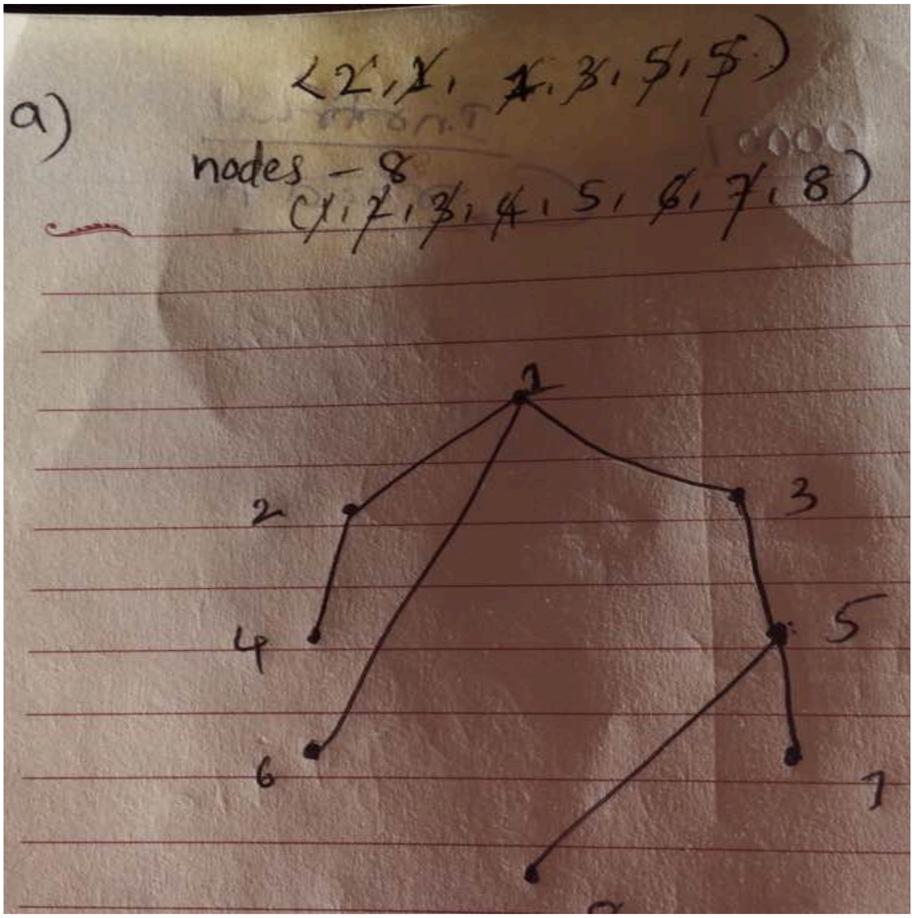
And other way is, the number of spanning trees for complete bipartite graph $K_{m,n}$ is : $n^{m-1}m^{n-1}$.

Therefore,

K2,3 has 2^2*3=12 spanning trees

6)

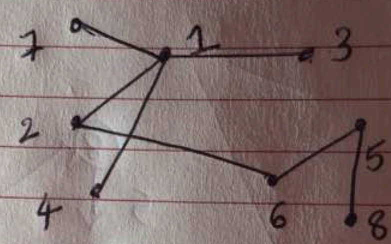
a)



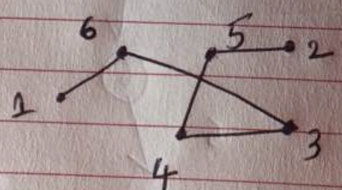
6.b&c)

(b) $\langle \cancel{1}, \cancel{1}, \cancel{1}, \cancel{2}, \cancel{6}, \cancel{5} \rangle$

nodes: $\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, 5, \cancel{6}, \cancel{7}, 8$

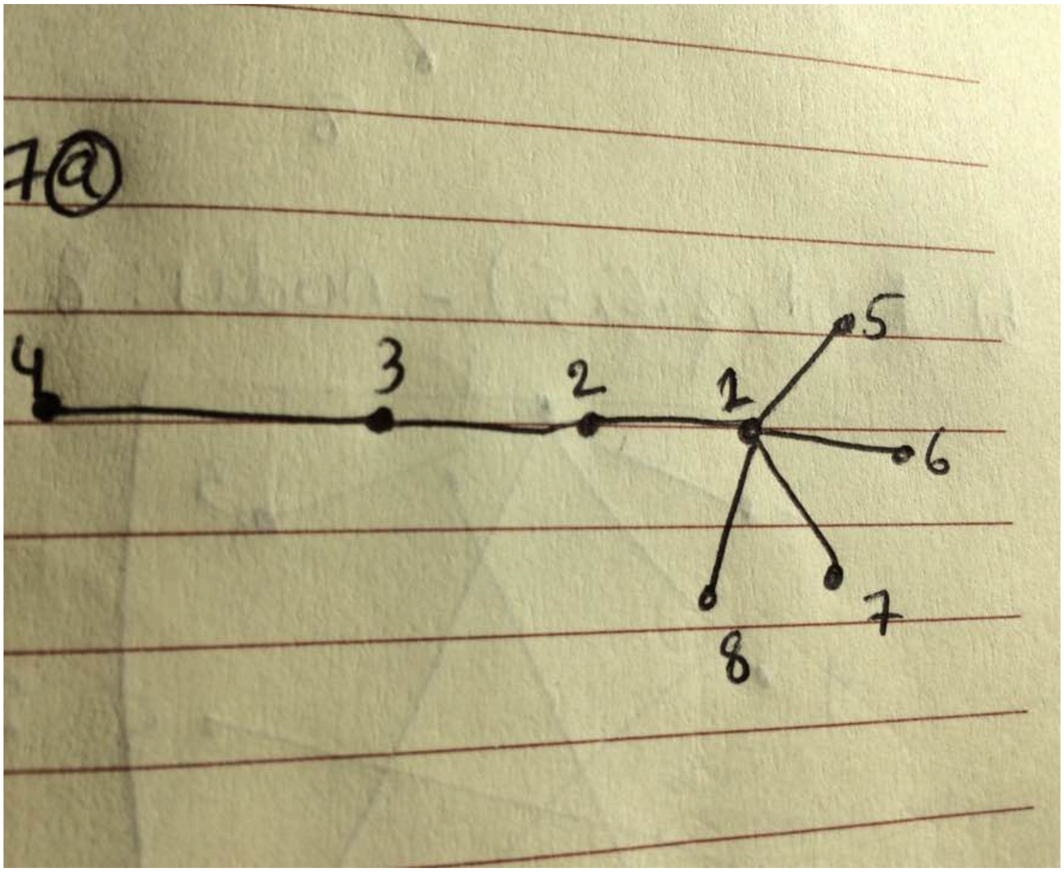


(c) $\langle \cancel{6}, \cancel{5}, \cancel{4}, \cancel{3} \rangle$ nodes $\langle \cancel{1}, \cancel{2}, 3, \cancel{4}, \cancel{5}, 6 \rangle$



7

a)



b) We can't draw a tree with given degree sequence. Since no of edges for any given tree with n vertices must be n-1. Here, from the given sequence no of vertices are 8 hence edges must be 7 but in the given graph,

by Hand shaking lemma E(G)= sum of degrees/2➔ 8. Since no of edges are not n-1(7) we can't draw with this degree sequence.