**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Given : The avg servicing time *μ* = 45

The standard deviation is *σ* = 8

Given x value = 1hr/60 mins

As the service manager starts transmissions after 10min from the drop off avg servicing time *μ* = 45+10 = 55min

Z = (x - *μ)* / *σ = (60-55)/8 = 5/8 = 0.625*

Normal Distribution N given as =N(x,*μ,σ* )

*In python probability p(x) is given using stats.norm.cdf(*x,*μ,σ)*

*Probability p(service manager meeting commitment) is given as  
stats.norm.cdf(60,55,8) =* 0.734

*Probability p(service manager not meeting commitment) is given as  
1-stats.norm.cdf(60,55,8) =* 0.266

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

Given N = 400

*μ* = 38

*σ* =6

P(38<x<44) = p(x<44) - P(x>38)

Formula

P(x<44) = stats.norm.cdf(44,38,6) = 0.8413447460685429

P(x>38) =1- stats.norm.cdf(38,38,0) = 0.5

P(38<x<44) = p(x<44) -p(x>38) = 0.3413447460685429

P(x>44) = 1- stats.norm.cdf(44,38,6)= 0.15865525393145707

No. of clerical employees b/w 38 and 44 ages are = (34.13/100)\*400=136.52 I.e 137out of 400

No. of clerical employees above 44 ages are = (15.86/100)\*400 = 63.44 I.e 63 out of 400

Ans: As the no employees b/w ages 38 and 44 are greater than no. employees above 44years. Hence the given statement is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: p(x<30) = p(30,38,6) = stats.norm.cdf(30,38,6) = 0.09121

No. of employees under the age of 30 are =9.12/100\*400 = 36.48 I.e 36 employees out of 400 are expected to be attracted to the training program.Hence the given statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Given *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) two independent normal random variables then

*X1 +X*2 can be given as: *X1 +X*2 ~N(μ1 +μ2,σ12+σ22) and

*X1 - X*2 can be given as: *X1 - X*2 ~N(μ1 - μ2,σ12+σ22)

Similarly if Z = aX+bY where X and Y are normal random variables as defined above then

can be defined as linear Combination of x and y I.e

Z ~N(aμ1 +bμ2,(a\*σ1)2+(b\*σ2)2)

Then 2X1 can be given as: 2X1~(2μ,4σ2)

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 µ, 2σ^2 )

Therefore the difference b/w 2X1 and X1+X2 that the σ value is doubled whereas µ value remains same.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Given: p(a<x<b) = 0.99 ,m ean =100,standardDeviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

A.

Ans) Z value at 95% probability is given as z = stats.norm.ppf(0.975) =1.96

Z value at 5% probability is given as z = stats.norm.ppf(0.025)=-1.96

Annual profit normal distribution is given as N(12,5)

Z = (x-mean)/sd = z = (x-12)/5 =>x = 5z+12

Min val a = 5(-1.96)+12 = 2.2

Max val b = 5(1.96)+12 = 21.8

A in rupees = 2.2\*45 = 99

B in rupees = 21.8\*45 = 981

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:) Z value at 5th % is given as Z = stats.norm.ppf(0.05)=-1.645

X = 5z+12 = 5\*(-1.645)+12 = 3.775\*45 = 169.875

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:) Z value at 5th % is given Z = stats.norm.ppf (0.05)=1.645

X of Profit1 = 3z+5 = 3\*(-1.645)+5 = 0.065\*45 = 2.945

Z value at 5th % is given Z = stats.norm.ppf (0.05)=1.645

X of Profit2 = 4z+7 = 4\*(-1.645)+7 = 0.42\*45 = 18.9

According to X value Division 1 is having a large probability of making loss.