

category	Email text
Not spam	"Hi there, how are you?"
Not spam	"Meeting at 3PM tomorrow"
Not spam	"Please send the report"
spam	"win a free prize now!"
spam	"claim your discount today"
spam	"Limited time offer: click here"
?	"free meeting tomorrow" (To classify)
?	"claim your free prize" (To classify)

Soln ① Total unique words in spam: 14

Total unique words in not spam: 14

Vocabulary Size = 28

using smoothing:

Now, To classify "free meeting tomorrow"  
we need to find;

$$P(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{meeting} | \text{spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{tomorrow} | \text{spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{free} | \text{Notspam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{meeting} | \text{Notspam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{tomorrow} | \text{notspam}) = \frac{1+1}{14+28} = \frac{2}{42}$$



## # Prior probabilities

$$\begin{aligned} P(\text{spam}) &= \frac{3}{6} = 0.5 \\ P(\text{notspam}) &= \frac{3}{6} = 0.5 \end{aligned} \quad \left\{ \begin{array}{l} (3 \text{ not spam and } 3 \text{ spam}) \\ \text{emails, total } 6 \end{array} \right\}$$

Now,

$$P(\text{spam} | \text{free, meeting, tomorrow}) \propto P(\text{spam}) \times P(\text{free} | \text{spam}) \times P(\text{meeting} | \text{spam}) \times P(\text{tomorrow} | \text{spam})$$

$$\propto 0.5 \times \frac{2}{42} \times \frac{1}{42} \times \frac{1}{42}$$

$$\approx 0.00001349$$

$$P(\text{notspam} | \text{free, meeting, tomorrow}) \propto P(\text{notspam}) \times P(\text{free} | \text{notspam}) \times P(\text{meeting} | \text{notspam}) \times P(\text{tomorrow} | \text{notspam})$$

$$\approx 0.5 \times \frac{2}{42} \times \frac{1}{42} \times \frac{2}{42}$$

$$\approx 0.00002699$$

Since  $P(\text{notspam}) > P(\text{spam})$ , the email is not spam.

$$\text{Normalization} = \frac{0.00002699}{0.00002699 + 0.00001349} \times 100\%$$

$$= \frac{0.00002699}{0.00004048} \times 100\% = 66.67\% \text{ not spam}$$



② for email "claim your free prize"

using smoothing

$$P(\text{claim} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{your} | \text{spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{free} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{prize} | \text{spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{claim} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{your} | \text{not spam}) = \frac{1+1}{14+28} = \frac{2}{42}$$

$$P(\text{free} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

$$P(\text{prize} | \text{not spam}) = \frac{0+1}{14+28} = \frac{1}{42}$$

Now,

$$P(\text{spam} | \text{claim, your, free, prize}) \propto P(\text{spam}) \times P(\text{claim} | \text{spam}) \times P(\text{your} | \text{spam}) \times P(\text{free} | \text{spam}) \times P(\text{prize} | \text{spam})$$

$$\approx 0.5 \times \frac{2}{42} \times \frac{1}{42} \times \frac{2}{42} \times \frac{2}{42}$$

$$\approx \cancel{0.0000026}$$

$$\approx \frac{4}{3111696} \approx 0.0000012855$$



$$P(\text{not spam} | \text{claim, your, free, prize}) \propto P(\text{not spam}) \times P(\text{claim} | \text{not spam}) \times P(\text{your} | \text{not spam}) \times P(\text{free} | \text{not spam}) \times P(\text{prize} | \text{not spam})$$

$$\approx 0.5 \times \frac{1}{42} \times \frac{1}{42} \times \frac{2}{42} \times \frac{1}{42}$$

$$\approx \frac{1}{3111696}$$

$$\approx 0.0000003213$$

Since  $P(\text{spam}) > P(\text{not spam})$ , the email "claim your free prize" is spam

$$\text{Normalize} = \frac{0.0000012855}{0.0000012855 + 0.0000003213}$$

$$= \frac{0.0000012855}{0.0000016068}$$

$$= 0.800110$$

$$= 80.01\% \text{ spam}$$