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409 Hw2 Group  
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Part 1: Convert (1) Develop Grammar (2) convert to CNF (use this for input in program)

1) grammar for language  $\{a^n b^n | n \geq 0\}$   
 $\rightarrow$  same number of a's and b's or  $\epsilon$ .

①  $S \rightarrow aPb | \epsilon$   
 $P \rightarrow aPb | \epsilon$   
 ex)  $S \rightarrow aPb \rightarrow aabb$   
 CFG (4 tuples) rules  $\{V, \Sigma, R, S\}$   
 CFG,  $G = \{ \{S, P\}, \{a, b\}, R, A \}$  where  $R$  is  $S \rightarrow aPb | \epsilon$   
 $P \rightarrow aPb | \epsilon$

② Convert CFG  $\rightarrow$  CNF  
 length3 chop down  
 $S \rightarrow aPb | \epsilon$   
 $P \rightarrow aPb | \epsilon$

Chop down  
 $S \rightarrow aZ_1 | \epsilon$   
 $Z_1 \rightarrow Pb$   
 $P \rightarrow aPb | \epsilon$   
 $\epsilon$  eliminate  
 $S \rightarrow aZ_1 | \epsilon$   
 $Z_1 \rightarrow Pb | b$   
 $P \rightarrow aP$   
 fix rhs length 2  
 $S \rightarrow XaZ_1 | \epsilon$   
 $Z_1 \rightarrow PXb | b$   
 $P \rightarrow XaXb$   
 $Xa \rightarrow a$   
 $Xb \rightarrow b$   
 CNF.

2. Grammar for language  $\{a^i b^j c^k | i=j \text{ or } j=k\}$   
 $S \rightarrow S_1 | S_2$   
 $S_1 \rightarrow A_1 B_1$   
 $A_1 \rightarrow aA_1 b | \epsilon$   
 $B_1 \rightarrow cB_1 | \epsilon$   
 $S_2 \rightarrow A_2 B_2$   
 $A_2 \rightarrow aA_2 | \epsilon$   
 $B_2 \rightarrow bB_2 c | \epsilon$   
 Grammar CFG

$S_1 = \text{same \# of a's and b's}$   
 $S_1 \rightarrow A_1 B_1$   
 $A_1 \rightarrow aA_1 b | \epsilon$   
 $B_1 \rightarrow cB_1 | \epsilon$   
 $S_2 = \text{same \# of b's and c's}$   
 $S_2 \rightarrow A_2 B_2$   
 $A_2 \rightarrow aA_2 | \epsilon$   
 $B_2 \rightarrow bB_2 c | \epsilon$

// In our program  
 we will type  
 A as Xa  
 Z as Z1  
 B as Xb

so CNF will be  
 $S \rightarrow AZ | \epsilon$   
 $Z \rightarrow PB | b$   
 $P \rightarrow AB$   
 $A \rightarrow a$   
 $B \rightarrow b$

<chopdown>

convert CFG  $\rightarrow$  CNF

$$S \rightarrow S_1 S_2$$

$\epsilon$  eliminate for  $A_1$

unity rule eliminate

$$S_1 \rightarrow A_1 B_1$$

$$S \rightarrow S_1 S_2$$

$$S \rightarrow A_1 B_1 C | A_2 B_2 | A A_2 | a$$

$$A_1 \rightarrow a Z_1 (\epsilon)$$

$$S_1 \rightarrow A_1 B_1 | B_1 A_1$$

$$S_1 \rightarrow A_1 B_1 | C B_1 | C | a Z_1$$

$$Z_1 \rightarrow A_1 b$$

$$A_1 \rightarrow a Z_1$$

$$Z_1 \rightarrow A_1 B_1 | b$$

$$B_1 \rightarrow c B_1 (\epsilon)$$

$$Z_1 \rightarrow A_1 b | b$$

$$B_1 \rightarrow C B_1 | C$$

$$S_2 \rightarrow A_2 B_2$$

$$B_1 \rightarrow C B_1 | C$$

$$S_2 \rightarrow A_2 B_2 | b Z_2 | a A_2 | a$$

$$A_2 \rightarrow a A_2 (\epsilon)$$

$$S_2 \rightarrow A_2 B_2 | B_2 | A_2$$

$$A_2 \rightarrow a A_2 | a$$

$$B_2 \rightarrow b Z_2 (\epsilon)$$

$$A_2 \rightarrow a A_2 | a$$

$$B_2 \rightarrow b Z_2$$

$$Z_2 \rightarrow B_2 C$$

$$B_2 \rightarrow b Z_2$$

$$Z_2 \rightarrow B_2 C | C$$

write down nicely

<fix rhs length more than 2>

$$S \rightarrow A_1 B_1 | C B_1 | C | a Z_1 | A_2 B_2 | a A_2 | a | b Z_2$$

$$S_1 \rightarrow A_1 B_1 | C B_1 | C | a Z_1$$

$$S \rightarrow A_1 B_1 | X_c B_1 | C | X_a Z_1 | A_2 B_2 | X_a A_2 | a | X_b Z_2$$

$$A_1 \rightarrow a Z_1$$

$$S_1 \rightarrow A_1 B_1 | X_c B_1 | C | X_a Z_1$$

$$Z_1 \rightarrow A_1 b$$

$$A_1 \rightarrow X_a Z_1$$

$$B_1 \rightarrow C B_1 | C$$

$$Z_1 \rightarrow A_1 b | b$$

$$S_2 \rightarrow A_2 B_2 | b Z_2 | a A_2 | a$$

$$B_1 \rightarrow X_c B_1 | C$$

$$A_2 \rightarrow a A_2 | a$$

$$S_2 \rightarrow A_2 B_2 | X_b Z_2 | X_a A_2 | a$$

$$B_2 \rightarrow b Z_2$$

$$A_2 \rightarrow X_a A_2 | a$$

$$Z_2 \rightarrow B_2 C | C$$

$$B_2 \rightarrow X_b Z_2$$

$$Z_2 \rightarrow B_2 X_c | C$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

$$X_c \rightarrow c$$

CNF



3. grammar for language  $\{x_1 \# x_2 \# \dots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^+, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$ . - sipser textbook 2.6d.

String contains number of a's and b's. a's and b's are separated by #

ex) a#b#a

CFG

$$S \rightarrow P \mid P \# Y \mid Y \# P \mid Y \# P \# Y$$

$$P \rightarrow aPa \mid bPb \mid \#Y\# \mid \#$$

$$Y \rightarrow aY \mid bY \mid \#Y \mid \epsilon$$

convert to CNF

<ε eliminate>

$$S \rightarrow P \mid P \# Y \mid Y \# P \mid Y \# P \# Y \mid P \# \mid \#P \mid \#P \#$$

$$P \rightarrow aPa \mid bPb \mid \#Y\# \mid \# \mid \# \#$$

$$Y \rightarrow aY \mid bY \mid \#Y \mid \#$$

ex.  $S \rightarrow P \rightarrow aPa \rightarrow a\# \# a$

ex.  $S \rightarrow P \# Y \rightarrow aPa \# Y \rightarrow a\# \# a \# b$

$S \rightarrow P \mid P \# Y \mid Y \# P \mid Y \# P \# Y$  // take P and Y as substring

$P \rightarrow aPa \mid bPb \mid \#Y\# \mid \#$  // P will generate matching pair of a's b's and #'s For example aPa, bPb

$Y \rightarrow aY \mid bY \mid \#Y \mid \epsilon$  watch. watch

Y will generate just a's or just b's or just #'s or ε.

① <ε> eliminate.

$$S \rightarrow P \mid P \# Y \mid Y \# P \mid Y \# P \# Y \mid P \# \mid \#P \mid \#P \# \mid Y \# P \# \mid \#P \# Y$$

$$P \rightarrow aPa \mid bPb \mid \#Y\# \mid \# \mid \# \#$$

$$Y \rightarrow aY \mid bY \mid \#Y \mid a \mid b \mid \#$$

<unit rule>

<Chown down length 2>

$$z_5 \quad z_4 z_5 \mid z_2 z_1.$$

$S \rightarrow P_1 P_2 \mid P_2 \mid Y_2 Z_2 \mid P\# \mid \#P \mid \# \mid Z_2 \mid Z_5 \mid Z_{17}$

$$Z_1 \rightarrow \text{H}^+$$
$$Z_2 \rightarrow \#p$$
$$Z_3 \rightarrow Z_2 Z_1$$
$$Z_4 \rightarrow Y \#$$
$$Z_5 \rightarrow P\#$$
$$p \rightarrow a \wedge s \mid b \vee$$

$P \rightarrow a^2 b^7 \#^4 \# \#$

$$Z_6 \rightarrow Pa$$
$$Z_7 \rightarrow P_6$$
$$Y \rightarrow aY \mid bY \mid \#Y \mid a \mid b \mid \#.$$

<unit rule>.

$S \rightarrow aZ_6 | bZ_7 | \#Z_4 | \# | \#\# | pZ_1 | pZ_2 | rZ_3 | p\# | \#r$

$$\#Z_5 | Z_4 Z_5 | Z_2 Z_1$$
$$Z_1 \rightarrow \#^x$$
$$Z_2 \rightarrow \#P$$
$$Z_n \rightarrow Z_1 Z_2$$
$$\mathbb{Z}_4 \rightarrow \mathbb{F}_4$$
$$Z_3 \rightarrow P\#$$
$$P \rightarrow a z_6 | b z_7 | \# z_4 | \# | \# \#$$
$$Z_6 \rightarrow Pa$$
$$Z_7 \rightarrow P_6$$
$$Y \rightarrow aY \mid bY \mid \#Y \mid a \mid b \mid \#$$



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<Fix rhs length 2>

$S \rightarrow x_a z_6 \mid x_b z_7 \mid w z_4 \mid \# \mid w w \mid p z_1 \mid p z_2 \mid y z_3 \mid p w \mid w p \mid w z_5 \mid z_4 z_5 \mid z_2 z_1$

$z_1 \rightarrow w y$

$z_2 \rightarrow w p$

$z_3 \rightarrow z_2 z_1$

$z_4 \rightarrow y w$

$z_5 \rightarrow p w$

$p \rightarrow x_a z_6 \mid x_b z_7 \mid w z_4 \mid \# \mid w w$

$z_6 \rightarrow p x_a$

$z_7 \rightarrow p x_b$

$y \rightarrow x_a y \mid x_b y \mid w y \mid a \mid b \mid \#$

$x_a \rightarrow a$

$x_b \rightarrow b$

$w \rightarrow \#$

Question 4 Grammar in Sipser Question 2.13.

Let  $G = (V, \Sigma, R, S)$  be following grammar.  $V = \{S, T, U\}$ ;  $\Sigma = \{0, 1\}$  and

$R$  is  $S \rightarrow TT|U$

new start state  
chop down  
eliminate  
unity eliminate

$T \rightarrow 0T|T0|1\#$  ①

②

$U \rightarrow 0U0|1\#$  <chop down length 2> <unity eliminate>

CFG

$S \rightarrow TT|U$

$S \rightarrow TT|Z_1Z_2|1\#$

convert to CNF

$T \rightarrow 0T|T0|1\#$

$T \rightarrow 0T|T0|1\#$

①  $U \rightarrow Z_1Z_2|1\#$

$U \rightarrow Z_1Z_2|1\#$

$Z_1 \rightarrow 0U$

$Z_1 \rightarrow 0U$

$Z_2 \rightarrow 00$

$Z_2 \rightarrow 00$

<fix rhs length 2>

$S \rightarrow TT|Z_1Z_2|1\#$

$Z_1 \rightarrow 0U$   $WU$

$T \rightarrow WT|TW|1\#$

$Z_2 \rightarrow WW$  (CNF)

$U \rightarrow Z_1Z_2|1\#$

$W \rightarrow 0$

~~$W \rightarrow 1$~~