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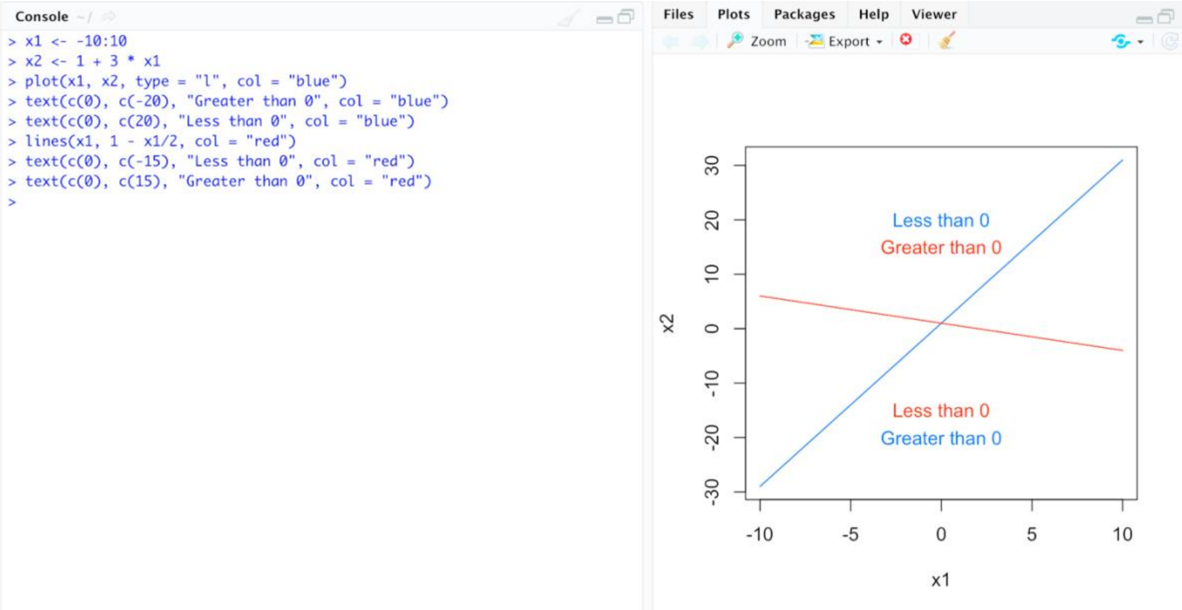
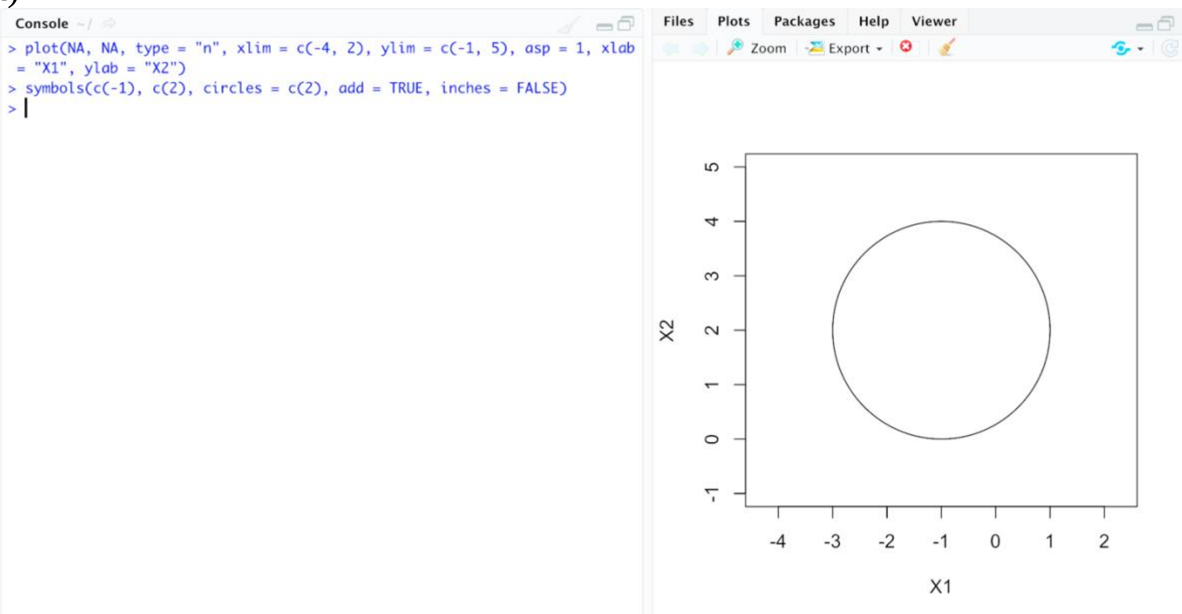
Assignment8

1.



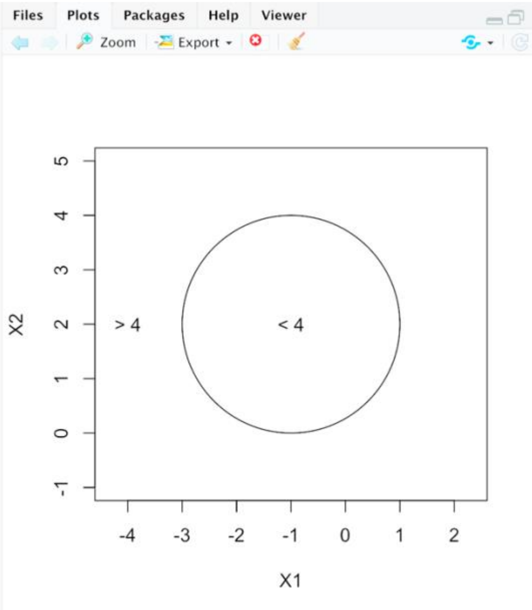
2.

a)



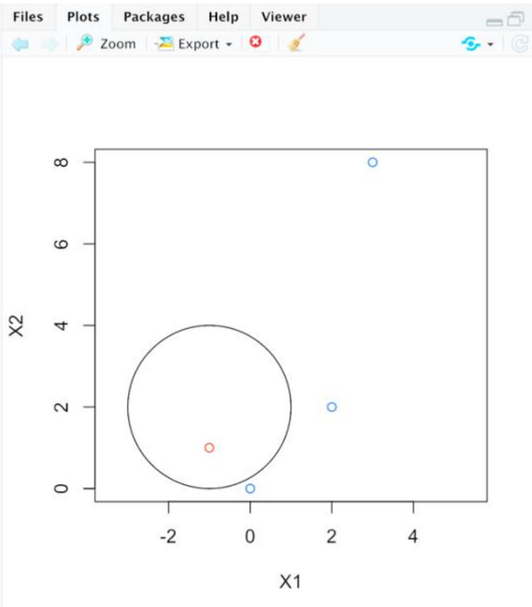
b)

```
Console ~/
> plot(NA, NA, type = "n", xlim = c(-4, 2), ylim = c(-1, 5), asp = 1, xlab
 = "X1", ylab = "X2")
> symbols(c(-1), c(2), circles = c(2), add = TRUE, inches = FALSE)
> text(c(-1), c(2), "< 4")
> text(c(-4), c(2), "> 4")
>
```



c)

```
Console ~/
> plot(c(0, -1, 2, 3), c(0, 1, 2, 8), col = c("blue", "red", "blue", "blu
e"),
+      type = "p", asp = 1, xlab = "X1", ylab = "X2")
> symbols(c(-1), c(2), circles = c(2), add = TRUE, inches = FALSE)
>
```

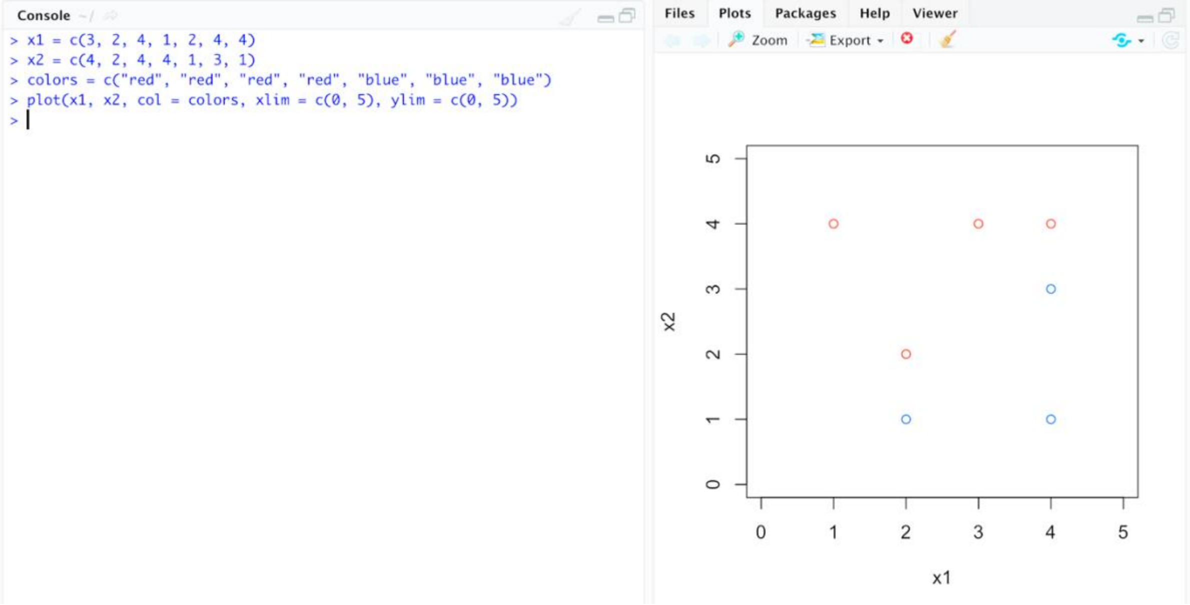


d)

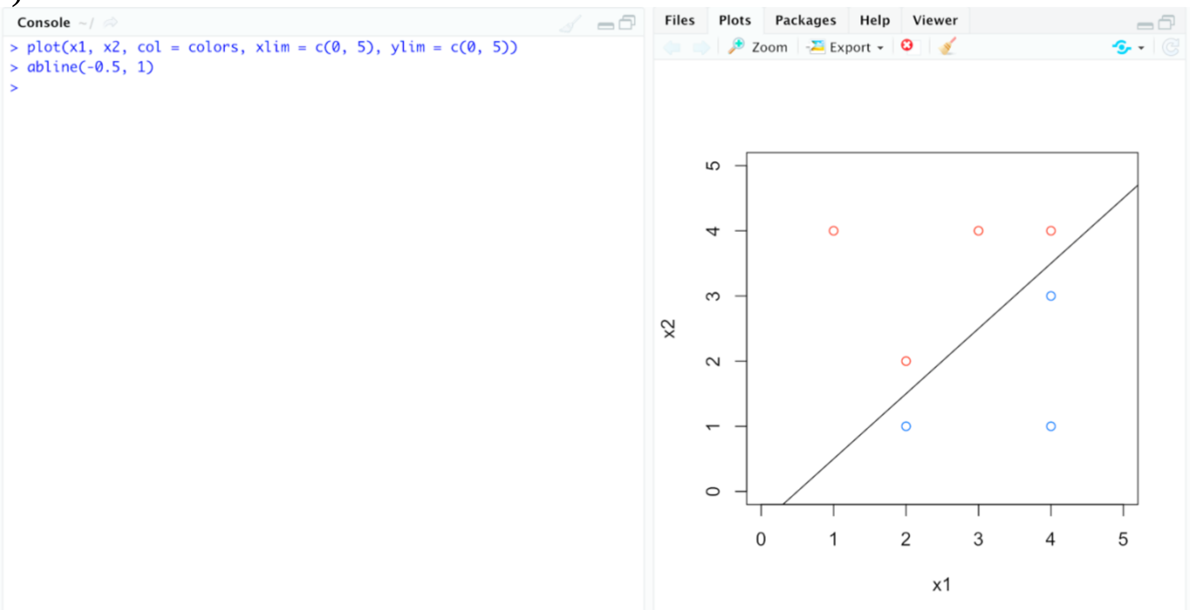
```
Console ~/
> (1+X1)2 +(2-X2)2 =4
Error: unexpected numeric constant in "(1+X1)2"
> this may expand the equation of decision boundary X21+X22+2X1-4X2+1=0
  which is linear interms of X1
  , X21
  , X2
  and  x22
```

3.

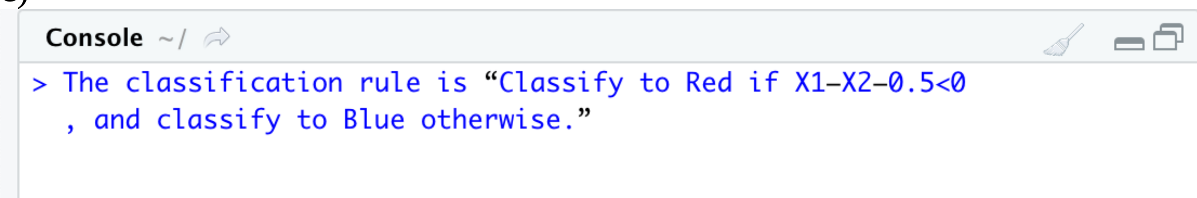
a)


Console ~/
> x1 = c(3, 2, 4, 1, 2, 4, 4)
> x2 = c(4, 2, 4, 4, 1, 3, 1)
> colors = c("red", "red", "red", "red", "blue", "blue", "blue")
> plot(x1, x2, col = colors, xlim = c(0, 5), ylim = c(0, 5))
>
Files
Plots
Packages
Help
Viewer
Zoom
Export
x2
x1
0
1
2
3
4
5

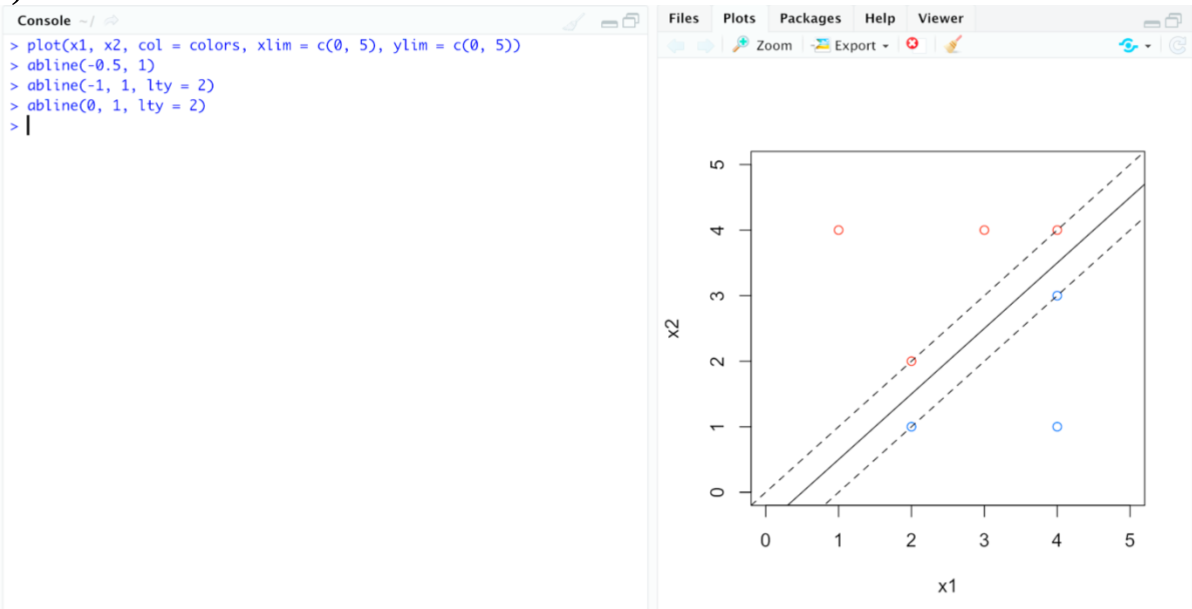
b)


Console ~/
> plot(x1, x2, col = colors, xlim = c(0, 5), ylim = c(0, 5))
> abline(-0.5, 1)
>
Files
Plots
Packages
Help
Viewer
Zoom
Export
x2
x1
0
1
2
3
4
5

c)


Console ~/
> The classification rule is “Classify to Red if X1–X2–0.5<0
, and classify to Blue otherwise.”

d)



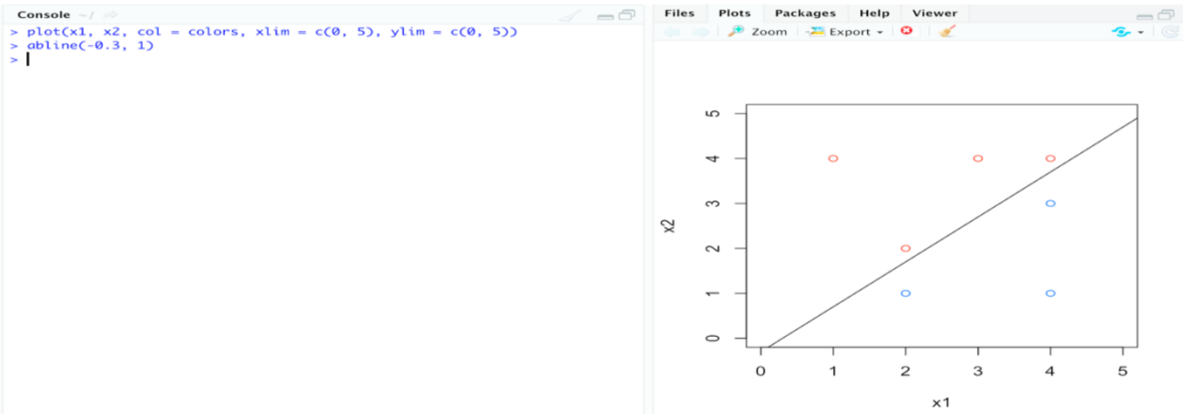
e)

```
Console ~/
> The support vectors are the points (2,1)
  , (2,2)
  , (4,3)
  and (4,4)
```

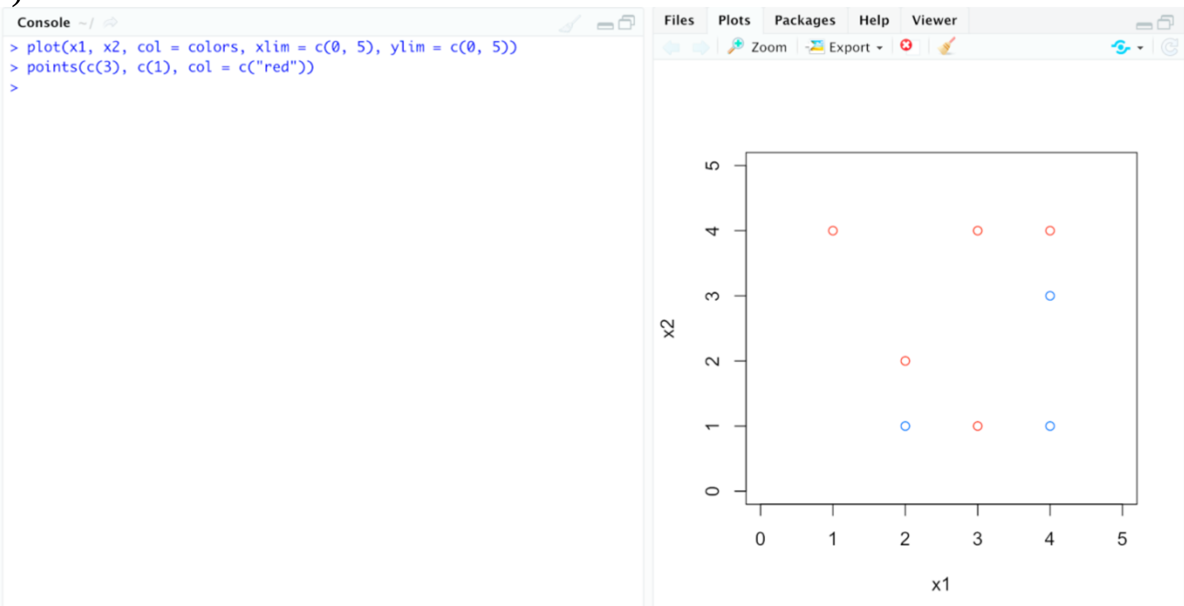
f)

```
Console ~/
> By examining the plot, it is clear that if we moved the observation (4,1
  )
  , we would not change the maximal margin hyperplane as it is not a
  support vector.
```

g)



h)


Console
> plot(x1, x2, col = colors, xlim = c(0, 5), ylim = c(0, 5))
> points(c(3), c(1), col = c("red"))
>
Files
Plots
Packages
Help
Viewer
Zoom
Export
x2
x1
0
1
2
3
4
5

(1)

4) a $(-2,2)$ $(1,1)$ given points

$$L(\lambda,\nu) = \lambda_1+\lambda_2+\lambda_1\lambda_2 \langle \bar{x}_1,\bar{x}_2\rangle - \tfrac{1}{2}\lambda_1^2 \langle \bar{x}_1,\bar{x}_1\rangle - \tfrac{1}{2}\lambda^2 \langle \bar{x}_2,\bar{x}_2\rangle$$
$$-\nu\lambda_1+\nu\lambda_2$$

Gradient of dual lagrangian

$$\frac{\delta}{\delta\lambda_1} L(\lambda,\nu) = 1+\lambda_2\langle \bar{x}_1,\bar{x}_2\rangle - \lambda_1\langle \bar{x}_1,\bar{x}_1\rangle - \nu = 0$$

$$\delta/\delta\lambda_2\, L(\lambda,\nu) = 1+\lambda_1\langle \bar{x}_1,\bar{x}_2\rangle - \lambda_2\langle \bar{x}_2\bar{x}_2\rangle + \nu = 0$$

$$\delta/\delta\lambda\, L(\lambda,\nu) = -\lambda_1+\lambda_2 = 0$$

suppose $\bar{x}_1 = (-2,2)$
$\bar{x}_2 = (1,1)$

we can write

$$1+\lambda_2\langle(-2,2)(1,1)\rangle - \lambda_1\langle(-2,2)(-2,2) - \nu = 0$$
$$1+\lambda_1\langle(-2,2)(1,1) - \lambda_2\langle(1,1)(1,1)\rangle + \nu = 0$$
$$-\lambda_1+\lambda_2 = 0$$
$$\lambda_1 = \lambda_2$$

So we can replace λ_1 and λ_2 with λ

$$1+\lambda\langle(-2,2)(1,1)\rangle - \lambda\langle(-2,2)(-2,2) - \nu = 0$$
$$1+\lambda(-2+2) - \lambda(4+4) - \nu = 0$$
$$1-8\lambda-\nu = 0 \rightarrow (1)$$

$$1+\lambda\langle(-2,2)(1,1)\rangle = \lambda\langle(1,1)(1,1)\rangle + \delta = 0$$
$$1+\lambda(-2+2) - \lambda(1+1) + \nu = 0$$
$$1-2\lambda+\nu = 0 \quad - (2)$$

Solving
$$1-8\lambda-\nu = 0$$
$$1-2\lambda+\nu = 0$$
$$2-10\lambda = 0$$

$$\boxed{\lambda = 1/5}$$

$\bar{\omega} - \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 = 0$

$\omega = \lambda_1 \bar{x}_1 - \lambda_2 \bar{x}_2$

$\bar{\omega} = 1/5(2,2) - 1/5(1,1)$

$= \left(\frac{-2}{5}, 2/5\right) - (1/5, 1/5)$

$\bar{\omega} = \underline{(-3/5, 1/5)}$

$b = 1 - \langle(-3/5, 1/5)(-2,2)\rangle$

$1 - (-3/5(-2) + 1/5(2))$

$= \frac{-3}{5}$

$b = 1 - \langle(-3/5, 1/5)(1,1)\rangle$

$= 1 - (-3/5 + 1/5)$

$= 1 - (-2/5)$

$= -2/5$

4.b) Given, $(1,1), (4,3)$

$L(\lambda, \gamma) = \lambda_1 + \lambda_2 + \lambda_1\lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle - 1/2 \lambda_1^2 \langle \bar{x}_1, \bar{x}_1 \rangle - 1/2 \lambda_2^2 \langle \bar{x}_1, \bar{x}_2 \rangle$
$- \gamma\lambda_1 + \gamma\lambda_2$

$\delta/\delta\lambda_1 \, L(\lambda, \gamma) = 1 + \lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle - \lambda_1 \langle \bar{x}_1, \bar{x}_1 \rangle - \gamma = 0$

$\delta/\delta\lambda_2 \, L(\lambda, \gamma) = 1 + \lambda_1 \langle \bar{x}_1, \bar{x}_2 \rangle - \lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle + \gamma = 0$

$\delta/\delta\gamma \, L(\lambda, \gamma) = -\lambda_1 + \lambda_2 = 0$

$x_1 = (1,1), \; x_2 = (4,3)$

we can write

$1+\lambda_2 <(1,1)(4,3)> - \lambda_1 <(1,1)(1,1)> - \gamma = 0$

$1+\lambda_1 <(1,1)(4,3)> - \lambda_2 <(4,3)(4,3)> + \gamma = 0$

$\lambda_1 = \lambda_2$

So

$1+\lambda(1,1)(4,3)) - \lambda<(1,1)(1,1)> - \gamma = 0$

$1+\lambda(4+3) - \lambda(1+1) - \gamma = 0$

$1+5\lambda - \gamma = 0 \quad - (1)$

$1+\lambda(4+3) - \lambda(16+9) + \gamma = 0$

$1+7\lambda - 26\lambda + \gamma = 0$

$1-18\lambda + \gamma = 0 \quad - (2)$

$1+5\lambda - \gamma = 0$

$1-18\lambda + \gamma = 0$

$\lambda = 2/13$

$\bar{\omega} = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 = 0$

$\omega = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2$

$\bar{\omega} = \frac{2}{13}(1,1) - 2/13(4,3)$

$= (-6/13, -4/13)$

$b = 1 - <(\frac{-6}{13}, -4/13)(1,1)>$

$= 1 + \frac{10}{13} \Rightarrow \frac{23}{13}$

$b = 1 - <(\frac{-6}{13}, \frac{-4}{13})(4,3)>$

$= 1 + 36/13 = \frac{49}{13}$

c) $(2,-2)$ $(-1,+1)$

$$L(\lambda, \gamma) = \lambda_1 + \lambda_2 + \lambda_1\lambda_2 \langle \bar{x}_1, \bar{x}_2 \rangle - \tfrac{1}{2}\lambda_1^2 \langle \bar{x}_1\, \bar{x}_1 \rangle - \tfrac{1}{2}\lambda_2^2 (\bar{x}_2\, \bar{x}_2)$$
$$- \gamma\lambda_1 + \gamma\lambda_2$$

gradient of dual lagrangian

$$\frac{\delta}{\delta\lambda_1} L(\lambda, \gamma) = 1 + \lambda_2 \langle \bar{x}_1 \bar{x}_2 \rangle - \lambda_1 (\bar{x}_1\, \bar{x}_1 \rangle - \gamma = 0$$

$$\delta/\delta\lambda_2\, L(\lambda, \gamma) = 1 + \lambda_1 \langle \bar{x}_1\, \bar{x}_2 \rangle - \lambda_2 (\bar{x}_2\, \bar{x}_2 \rangle + \gamma = 0$$

$$\delta/\delta\gamma\, L(\lambda, \gamma) = -\lambda_1 + \lambda_2 = 0$$

$$x_1 = (2, -2), \quad \lambda_2 (-1, 1)$$

we can write

$$1 + \lambda_2 \langle (2,-2)(-1,1) - \lambda_1 \langle (2,-2)(2,-2) \rangle - \gamma = 0$$
$$1 + \lambda_1 \langle (2,-2)(-1,1) - \lambda_2 \langle (-1,1)(-1,1) \rangle + \gamma = 0$$

$$\lambda_1 = \lambda_2$$

$$1 + \lambda(-2 - 2) - \lambda(4+4) - \gamma = 0$$

$$1 - 4\lambda - 8\lambda - \gamma = 0$$

$$1 - 12\lambda - \gamma = 0 \quad - (1)$$

$$1 + \lambda(-2-2) - \lambda(1+1) + \gamma = 0$$

$$1 - 6\lambda + \gamma = 0 \quad - (2)$$

$$1 - 12\lambda - \gamma = 0$$
$$1 - 6\lambda + \gamma = 0$$
$$\lambda = 1/9$$

$$\bar{w} - \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 = 0$$

$$w = \lambda_1 \bar{x}_1 - \lambda_2 \vec{x}_2$$

$$\bar{w} = \frac{1}{9}(2,-2) - \frac{1}{9}(-1,1)$$

$$\left(\frac{2}{9}, \frac{-2}{9}\right) - \left(\frac{-1}{9}, \frac{1}{9}\right)$$

$$\left(\left(\frac{3}{9}\right), \left(\frac{-3}{9}\right)\right)$$

$$b = 1 - \langle \left(\frac{3}{9}, -\frac{3}{9}\right)(2,-2) \rangle$$

$$= 1 - \left(\frac{6}{9} + \frac{6}{9}\right)$$

$$= -\frac{3}{9}$$

$$b = 1 - \langle \left(\frac{3}{9}, -\frac{3}{9}\right)(1,1) \rangle$$

$$= 1 - \left(\cancel{\frac{3}{9}} - \cancel{\frac{3}{9}}\right)$$

$$= 1$$