Consider the equation for anomalous change in velocity with MSE 1.43.

$$1.\ f = \varkappa V\left(\frac{R_E}{R_E+1}\right)^{\beta} |cos(2I)|^{\gamma} sin(2\phi)$$

We propose an improved equation of the form

$$2.\ \varDelta V = c_1 f + c_2 g$$

Suppose the function g is a linear combination of 6 predictors. Predictor coefficients are computed via regularised least-squares regression of 6 anomaly cases and 3 null cases, minimising

$$3.\ \frac{1}{18}\sum_{i=1}^{9}\left(\varDelta V_i - \beta_0 - x_i^T\beta\right)^2 + \lambda\sum_{j=1}^{6}|\beta_j|$$

However, all coefficients beta(i) go to 0. Repeating for square inverses of all predictors, non-zero betas are assigned to altitude and azimuthal velocity. Regarding azimuthal velocity, consider the equation for drag force.

$$4.\ F_d = \frac{1}{2}\varrho u^2 C_D A \propto u^2\varrho \qquad \text{-->}\ \varDelta V = c_1 f + c_2\cdot\frac{1}{F_d} = c_1 f + c_2 u^{-2}\varrho^{-1}$$

We hypothesise that the satellite experiences some anomalous atmospheric drag. A basic model of altitude-dependant atmospheric density is given.

$$5.\ \varrho = \varrho_0 e^{-\frac{h}{H}} \propto e^{-h}$$

However, exponential dependance is implausible; the proposed equation blows up. The MSIS-E-90 atmospheric model is an empirical alternative modelled approximately as follows.

$$6.\ \varrho = 7\cdot 10^{7}\cdot h^{-7.2} \propto h^{-7.2}$$

Substituting the expressions for ϱ and u into 2) and solving for c1 and c2 via multiple linear regression yields

$$7.\ \varDelta V = 0.771\cdot u^{-2}\cdot h^{7.2}$$

with c1 set to 0 and MSE of 24.99. Significant MSE increase suggests a fluid density dependance on some unexpected phenomenon. We propose that satellites experience enhanced drag when passing through high density plasma “patches” found in the polar ionosphere (80-1000km). Hosokawa et al report (fig. 1)

$$8.\ Intensity \propto x \text{ -->}\ \varrho \propto x$$

Rewriting ϱ in terms of satellite inclination I (fig. 2)

$$9.\ \varrho \propto x = cos(I)$$

Substituting the new expression for ϱ into 2) and solving for c1 and c2 via multiple linear regression yields

$$10.\ \Delta V = 1.0885 \cdot \left[\varkappa V \left(\frac{R_E}{R_E + 1} \right)^{\beta} |cos(2I)|^{\gamma} sin(2\phi) \right] - 13.3416 \cdot \left[u^{-2} \cdot cos^{-1}(I) \right]$$

with MSE 0.7796.

Constants

$R_E = Radius\ of\ Earth$

$I = Orbital\ Inclination$

$\beta = LASSO\ coefficients$

$x_i = predictors$

$\lambda = regularization\ parameter$

$\Delta V = anomalous\ velocity\ change$

$\beta = 21.8739$

$C_D = drag\ coefficient$

$A = cross\text{-}sectional\ area$

$\varrho = fluid\ density$

$u = velocity$

$H = earth\ scale\ height$

$\varkappa = 16.2247e{-6}$

$\gamma = 1.1537$