CS-210 Homework 9

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1 Problem 1

Collaborated with: None
Proving: a mod m = b mod m iff m|(a-b)
m|(a-b) means $\exists$n $\in$ Z such that a-b = nm
The Division Theorem tells us:
Let c be an integer and d a positive integer, then there are unique integers q and r, with 0 $\leq$ r $<$ d such that c = dq + r
Since a and b are both integers and m a positive integer the Division Theorem will apply on them such that:
a=m$q_1 + r_1$ and b=m$q_2 + r_2$ where 0 $\leq r_1 <$ m and 0 $\leq r_2 <$ m
Putting this into a - b= nm:
a = b + nm
a = (m$q_2 + r_2$) + nm
a = m (q_2 + n) + r_2
But we also know that a=m$q_1 + r_1$
So we can deduce that $r_1 = r_2$ and $q_1 = q_2$ + n
Since a and b have the same remainders when divided with m, they'll belong to the same residual class mod m and a mod m = b mod m
As the remainders are same we can write a and b in the forms a=m$q_1 + r_1$ and b=m$q_2 + r_1$
So a - b = (m$q_1 + r_1$) - (m$q_2 + r_1$)
a-b= m ($q_1 - q_2$)
a-b= m ($q_2 + n - q_2$)
a-b= mn
Hence proved a mod m = b mod m iff m|(a-b)

2 Problem 2

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1. If we take any integer x then the sum of three consecutive numbers with x as the first term will be:
   x + (x+1) + (x+2) = 3x + 3
   = 3(x+1)
   Since $\forall$x 3|3(x+1) as 3(x+1) = 3(y) where $\exists$y $\in$ Z such that y = x+1 hence the statement is proved.
2. For any two even numbers a and b, a can be written as 2x and b as 2y
   So the product of a and b can be written as 4xy
   Since $\forall$x,y 4|4xy as 4xy = 4(z) where $\exists$z $\in$ Z such that z = xy hence the statement is proved.
3. For any 4 consecutive integers any two alternate integers will be odd and the other two alternate will be even. If we take x to be the first even integer then x+2 will be the second. Since both are even we can write then as x=2a and x+2 = 2a + 2= 2(a+1). We can see that a and a+1 will have opposite parities. So one even integer from our four consecutive integers will be a product of 2 and an odd integer and the other will be a product of 2 with an even integer. From this the first will be divisible

by 2 as we know:
$\forall c\ 2|2c$ as $2c = 2(y)$ where $\exists y \in Z$ such that $y = c$
And for the second, 2 multiplied with any even integer will be divisible by 4 using the proof presented in part 2(b)
So $\forall y\ 4|4d$ as $4d = 4(z)$ where $\exists z \in Z$ such that $z = d$
This means that we can write one even integer as 2s and the other as 4t
So the product of any 4 consecutive integers will be:
e*(e+1)*(e+2)*(e+3)=2s*(e+1)*4t*(e+3) if the first integer is even or e*2s*(e+2)*4t if the first integer is odd
In either case we will get a result in the form 8w where w is an integer which will always be divisible by 8 as
$\forall w\ 8|8w$ as $8w = 8(z)$ where $\exists z \in Z$ such that $z = w$

4. If we take a=5, b=3 and m=2 then (5-3) mod 2 =0
   But 5 mod 2 = 1 and 3 mod 2 = 1 which means they don't have remainders zero like a-b hence the statement is disproved.

5. If n an is odd integer we can write it as 2k+1 where k is an integer
   So $3n + 3 = 3(2k + 1) + 3$
   $= 6k + 3 + 3$
   $= 6k+6$
   $= 6(k+1)$
   Since $\forall k\ 6|6(k+1)$ as $6(k+1) = 6(z)$ where $\exists z \in Z$ such that $z = k+1$ hence the statement is proved.