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Static Longitudinal Stability Calculation (English Units)

7-28-18

Sec	Δx	b_f	b_f^2	x_ℓ	x_ℓ / c_{re}	x_ℓ / ℓ_h	$\partial\varepsilon/\partial\alpha$	$1+\partial\varepsilon/\partial\alpha$
1	2.21	3	9	9.945	1.39578		0.2	
2	2.21	3.5	12.25	7.735	1.08561		0.275	
3	2.21	3.375	11.39063	5.525	0.77544		0.375	
4	2.21	3.625	13.14063	3.315	0.46526		0.41	
5	2.21	3.6875	13.59766	2.21	0.31018		1.9	
6	2.25	3.375	11.39063	1.125		0.05232		0.031324
7	2.25	3.25	10.5625	3.375		0.15698		0.093983
8	2.25	3.0625	9.378906	5.625		0.26163		0.156636
9	2.25	2.8125	7.910156	7.875		0.36628		0.21929
10	2.25	2.5625	6.566406	10.125		0.47093		0.281943
11	2.25	2.1875	4.785156	12.375		0.57558		0.344597
12	2.25	1.875	3.515625	14.625		0.68023		0.40725
13	2.25	1.5	2.25	16.875		0.78488		0.469903
14	2.25	0.8125	0.660156	19.125		0.88953		0.532557
15	3	0.25	0.0625	21.75		1.01163		0.605657

$$\frac{d\varepsilon}{d\alpha} = 4.44\left[K_A K_\lambda K_H\left(\cos\Lambda_{c/4}\right)^{1/2}\right]^{1.19}$$

$$K_A = \frac{1}{A} - \frac{1}{1+A^{1.7}} = \frac{1}{7.86} - \frac{1}{1+(7.86)^{1.7}}$$

$K_A = 0.098031$

$$K_\lambda = \frac{10-3\lambda}{7} = \frac{10-3(0.5445)}{7}$$

$K_\lambda = 1.1952$

$$K_H = \frac{1-\frac{h_H}{b}}{\left(2\frac{\ell_h}{b}\right)^{1/3}} = \frac{1-\frac{3.5}{72.5}}{\left(2\frac{(21.5)}{(72.5)}\right)^{1/3}}$$

$K_H = 1.1327$

$$\tan\Lambda_{c/4} = \cancel{\tan\Lambda_{LE}} - \left(\frac{\bar{c}_r - \bar{c}_t}{2b}\right) \rightarrow \tan\Lambda_{c/4} = -\left(\frac{11.9375-6.5}{2(72.5)}\right)$$

no sweep angle

$\tan\Lambda_{c/4} = -0.0375$ $\qquad \Lambda_{c/4} = \tan^{-1}(-0.0375)$

$\Lambda_{c/4} = -0.03748\,2437$

WING

$$A = \frac{b^2}{S} = \frac{(72.5)^2}{668.36} = 7.86$$

$$S = \frac{b}{2}C_r(1+\lambda) = 668.36\,in^2$$

$$\lambda = \frac{C_t}{C_r} = \frac{6.5}{11.9375} = 0.5445$$

$$\bar{c} = \frac{2}{3}C_r\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) = 9.486$$

HORIZONTAL TAIL

$$A_t = \frac{b_t^2}{S_t} = \frac{(25.25)^2}{146.7648} = 4.344$$

$$S_t = \frac{b_t}{2}C_{r_t}(1+\lambda_t) = 146.764\,in^2$$

$$\lambda_t = \frac{C_{t_t}}{C_{r_t}} = \frac{4.5}{7.125} = 0.63157$$

$$\frac{d\varepsilon}{d\alpha} = 4.44\left[(0.098031)(1.1952)(1.1327)\left(\cos(-0.037482437)\right)^{1/2}\right]^{1.19}$$

$$\frac{d\varepsilon}{d\alpha} = 4.44\left[0.132668049\right]^{1.19} \qquad \frac{d\varepsilon}{d\alpha} = 0.401305322$$

$$C_{m_{\alpha,f}} = \frac{\pi}{2S\bar{c}} \sum_{i=1}^{15} b_f^2 \left(1 + \frac{\partial\varepsilon}{\partial\alpha}\right)\Delta x$$

$$C_{m_{\alpha,f}} = \ldots \frac{\pi}{2(668.36)(9.486)} \left[\begin{array}{l} (9 \times 2.1 \times 2.21) + (12.25 \times 1.275 \times 2.21) + (11.39063 \times 1.375 \times 2.21) \\ + (13.14063 \times 1.41 \times 2.21) + (13.59766 \times 2.9 \times 2.21) + \ldots \\ (11.39063 \times 0.031324 \times 2.25) + (10.5625 \times 0.093983 \times 2.25) + \ldots \\ (9.378906 \times 0.156636 \times 2.25) + (7.910156 \times 0.21929 \times 2.25) + \ldots \\ (6.566406 \times 0.281943 \times 2.25) + (4.785156 \times 0.344597 \times 2.25) + \ldots \\ (3.515625 \times 0.40725 \times 2.25) + (2.25 \times 0.469903 \times 2.25) + \ldots \\ (0.660156 \times 0.532557 \times 2.25) + (0.0625 \times 0.605657 \times 3) \end{array}\right]$$

$$C_{m_{\alpha,f}} = (0.000247757)(245.7188226) \Rightarrow \boxed{C_{m_{\alpha,f}} = 0.060878558/\text{rad}}$$

$$C_{m_\alpha} = C_{m_{\alpha,f}} + C_{L_{\alpha,w}}\bar{X}_a - C_{L_{\alpha,t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta\bar{V}$$

$T_{SL} = 518.69°R$

* 10 m/s → 30 m/s increments of 5

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T_{SL}}} = \frac{32.8084}{1116.288} = 0.029 \text{ (for 10 m/s)}$$

Mach #	$C_{L_{\alpha,w}}$	$C_{L_{\alpha,t}}$	C_{m_α}
0.029	4.876/rad	4.012/rad	-1.45038/rad
0.04408	4.878/rad	4.014/rad	-1.45072/rad
0.05878	4.881/rad	4.016/rad	-1.450848/rad
0.07348	4.884/rad	4.018/rad	-1.450973/rad
0.08817	4.889/rad	4.021/rad	-1.451058/rad
0	4.874/rad	4.012/rad	-1.45079/rad

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{\text{Mach } 0.029} = (0.401305322) \times \frac{(4.876)}{(4.874)}$$

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{\text{mach } 0.029} = 0.4014699$$

$$C_{L_{\alpha,w}} = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{K^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{\beta^2}\right) + 4}}$$

$$\beta = \sqrt{1 - (0.029)^2}$$

$$\beta = 0.9995$$

$$C_{L_{\alpha,w}} = \frac{2\pi(7.86)}{2 + \sqrt{\frac{(7.86)^2(0.9995)^2}{(1)^2}\left(1 + \frac{(-0.07504)^2}{(0.9995)^2}\right) + 4}}$$

$$\tan\Lambda_{c/2} = \cancel{\tan\Lambda_{LE}} - \frac{4(0.5)(1-\lambda)}{(7.86)(1+\lambda)}$$

no sweep angle

$$\tan\Lambda_{c/2} = \frac{4 \times 0.5 \times 0.4555}{(7.86)(1.5445)}$$

$$\tan\Lambda_{c/2} = -0.07504$$

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$$C_{L\alpha,w} = \frac{49.385}{2+\sqrt{6.1717(1.0056)+4}} \Rightarrow C_{L\alpha,w} = 4.876\,/\text{rad}$$

$$C_{L\alpha,t} = \frac{2\pi A_t}{2+\sqrt{\frac{A_t^2\beta^2}{K^2}\left(1+\frac{\tan^2\Lambda_{c/2}}{\beta^2}\right)+4}} = \frac{2\pi(4.344)}{2+\sqrt{\frac{(4.344)^2(0.9995)^2}{(1)^2}\left(1+\frac{(-0.1039655)^2}{(0.9995)^2}\right)+4}}$$

$$\tan\Lambda_{c/2} = \cancel{\tan\Lambda_{LE}} - \frac{4(0.5)(1-\lambda_t)}{(4.344)(1+\lambda_t)} \Rightarrow \tan\Lambda_{c/2} = -0.1039655$$

no sweep angle

$$C_{L\alpha,t} = 4.012\,/\text{rad}$$

$$\bar{X}_a = \bar{X}_{cg} - \bar{X}_{ac} = (0.24119)-(0.25) = -0.0081$$

$$\bar{V} = \frac{l_t}{\bar{c}}\cdot\frac{S_t}{S_w} \Rightarrow \frac{(31.1495)}{(9.486)}\cdot\frac{(146.76)}{(668.35)} \Rightarrow \bar{V} = 0.72106$$

$$C_{m\alpha} = (0.060878558/\text{rad}) + (4.876/\text{rad})(-0.0081) - (4.012/\text{rad})\ldots$$
$$(1-0.401469)(0.85)(0.72106)$$

$$C_{m\alpha} = 0.021382958 - 1.471763 \qquad C_{m\alpha} = -1.4503800\,/\text{rad}$$

$$\frac{dC_m}{dC_L} = \bar{X}_{cg} - \bar{X}_{ac} + \left(\frac{dC_m}{dC_L}\right)_f - \frac{C_{L\alpha,t}}{C_{L\alpha,w}}\left(1-\frac{d\varepsilon}{d\alpha}\right)\eta\bar{V}$$

$$\left(\frac{dC_m}{dC_L}\right)_f = C_{m\alpha_f}\cdot\frac{1}{C_{L\alpha}} = (0.060878558)\left(\frac{1}{4.876+4.012}\right)$$

$$\left(\frac{dC_m}{dC_L}\right)_f = 0.006849523$$

$$\frac{dC_m}{dC_L} = (0.24119)-(0.25)+(0.006849523) - \frac{4.012}{4.876}(0.5985301)(0.85)(0.72106)$$

$$\frac{dC_m}{dC_L} = -0.001960477 - 0.303183749 \Rightarrow \frac{dC_m}{dC_L} = -0.305144252$$

$$N = \bar{X}_{cg} - \frac{dC_m}{dC_L} \Rightarrow N = 0.24119 - (-0.305144252) \qquad N = 0.546334252$$

@ 10 m/s

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Mach #	Neutral Pt.
0.029	0.54633452
0.04408	0.54493177
0.05878	0.54478057
0.07348	0.544612304
0.08817	0.544321329
0	0.545195113