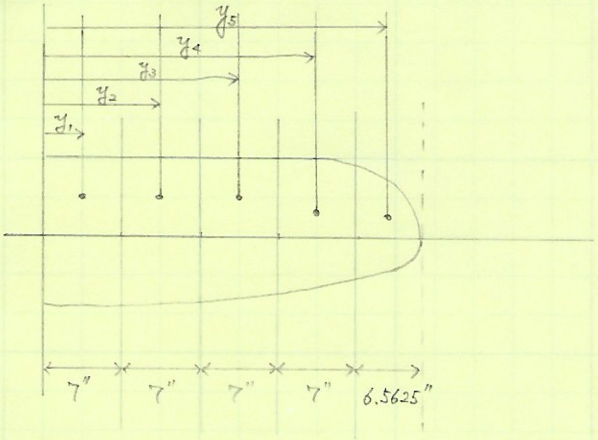
Directional Static Stability

Wing :



$A = \frac{b^2}{S} = \frac{(72.5)^2}{668.36} = 7.86 > 4$

$\Gamma = 2$ deg

$C_{D_{\alpha,\ell}} = 0.53$ /rad

$W = 5.340$ lb

fuselage width = $3\frac{3}{8}$"

$b/2 = \frac{72.5 - 3\frac{3}{8}}{2} = 34.5625$"

Section	$\bar{c}$ (in)	y (in)	Δy (in)
1	11.375	3.5	7
2	10.875	10.5	7
3	10.125	17.5	7
4	8.5	24.5	7
5	6.5	31.28	6.5625

for altitude = 0 ft

$\rho_{SL} = 2.3769 \times 10^{-3}$ slugs/ft³

flight speed = 10 m/s = 33.0846457 ft/s

assuming $C_{D_{\alpha,\ell}}$ is constant

$$(C_{n\beta})_\Gamma = -\frac{2\Gamma}{Sb}\left(C_L - C_{D_{\alpha,\ell}}\right)\int_0^{\frac{b}{2}} c(y)\,y\,dy$$

$$C_L = \frac{L}{\bar{q}S} = \frac{W}{\bar{q}S} = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{5.340 \text{ lb}}{\frac{1}{2}\underset{\text{slugs/ft}^3}{(2.3769\times10^{-3})}\underset{\text{ft}^2/\text{s}^2}{(33.0846457^2)}\underset{\text{ft}^2}{(668.36\times\frac{1}{12^2})}}$$

$$= \underset{\text{rad}}{0.884421736}$$

$$(C_{n\beta})_\Gamma = -\frac{2\left(2\times\frac{\pi}{180}\right)}{\underset{\text{in}^2}{(668.36)}\underset{\text{in}}{(72.5)}}\underset{/\text{rad}}{(0.884421736 - 0.53)}\underset{\text{in}^3}{\left\{\begin{array}{l}11.375(3.5)(7) + 10.875(10.5)(7)\\ + 10.125(17.5)(7) + 8.5(24.5)(7)\\ + 6.5(31.28)(6.5625)\end{array}\right\}}$$

$$= -(0.000001441)(0.354421736)(5110.35)$$

$$= -0.002609516 \text{ /rad}$$

Mach #	V (m/s)	C_L	$(C_{n\beta})_\Gamma$ (/rad)
0.029	10	0.884421736	-0.002609516
0.04408	15	0.399723601	0.000959191
0.05878	20	0.224844525	0.002246781
0.07348	25	0.143900496	0.002842751
0.08817	30	0.0999309	0.003166488

Directional stability (Fus.)

$$(C_{n\beta})_f = -K_N K_{Rl} \left(\frac{S_{BS}}{S_w}\right)\left(\frac{l_f}{b}\right)$$

K_{Ni} (in)

$l_f = 46.25$ $x_m = 11.5625$ $S_w =$

$h_1 = 7.625$ $S_{B,S} = 182.109$

$h = 7.8125$ $b_{f_{max}} = 3.6875$

$h_2 = 3.9375$

$S_{B,S} \cong h_2 \cdot l_f = (3.9375)(46.25)$

$S_{B,S} \cong 182.109$ (in²)

K_{Ni}

① $\frac{x_m}{l_f} = \frac{11.5625}{46.25} = 0.25$

② $\frac{l_f^2}{S_{B,S}} = \frac{46.25^2}{182.109} = 11.7461$

3) $\sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{7.625}{3.9375}} = 1.39158$

④ $\frac{h}{b_{f,max}} = \frac{7.8125}{3.6875} = 2.11864$

$K_N = 0.0009$

K_{Rl}:

$M_{0.0294}$ $V = 32.84$ (ft/s) $\nu = 15.7764(10^{-5})$

$R_e = \frac{(32.84)(46.25)}{1.57764(10^{-4})} = 9.62723(10^6) \Rightarrow K_{Rl} = 1.45$

$M_{0.044}$ $V = 49.21$

$R_e = 14.4262(10^9) \Rightarrow K_{Rl} = 1.55$

Directional stability (F-5)

$$S = \frac{b}{2} C_r (1 + \lambda)$$

$$= \frac{72.5}{2}(11.9375)(1 + 0.5445) = 668.358 (in^2)$$

$M_{0.059}$ $V = 65.61$

$R_e = 19.234(10^6) \Rightarrow K_{Rl} = 1.625$

$M_{0.07347}$ $V = 82.02$

$R_e = 24.045 \Rightarrow K_{Rl} = 1.67$

$M_{0.08816}$ $V = 98.43$

$R_e = 28.855(10^6) \Rightarrow K_{Rl} = 1.69$

$$\frac{S_{BS}}{S_w} = \frac{182.109}{668.358} = 0.272472$$

$$\frac{l_f}{b} = \frac{46.25}{72.5} = 0.637931$$

for $M = 0.0294$

$$(C_{n\beta})_f = -(0.0009)(1.45)(0.272472)(0.637931) = \frac{-0.000227}{deg}$$

$$\boxed{(C_{n\beta})_f = -0.000227/deg}$$

M #	$(C_{n\beta})_f$ (1/deg)
0.0294	-0.000227
0.04408	-0.000242
0.05877	-0.000254
0.07347	-0.000261
0.08816	-0.000264

Directional Static Stability

Vertical Tail: $(C_{n\beta})_v = K C_{L_{\alpha,v}} \left(1 + \frac{\partial\sigma}{\partial\beta}\right) \eta_v \bar{V}_v$

$\frac{b_v}{2r_1} = \frac{9.1875\text{ in}}{2(1.98\text{ in})} = 2.320075758 \longrightarrow K = 0.8$ (graph fig. 3.75)

assume $\eta_v = \frac{\bar{q}_v}{\bar{q}} = 0.9$

$S_v = \frac{b_v}{2} C_r(1+\lambda) = \frac{9.1875}{2}(8.9)(1+0.42247191)$

$= 58.156875\text{ in}^2$

$\bar{V}_v = \frac{S_v}{S}\left(\frac{l_v}{b}\right) = \frac{58.156875\text{ in}^2}{668.36\text{ in}^2}\left(\frac{25.338\text{ in}}{72.5\text{ in}}\right) = 0.030410596$

$\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v = \left(0.724 + \frac{3.06\frac{S_v}{S}}{1+\cos\Lambda_{c/4,w}} + \frac{0.4 z_w}{d_{f,max}} + 0.009 A_w\right) 0.9$

$= \left\{0.724 + \frac{3.06\frac{58.156875}{668.36}}{1+\cos(-0.037503432)} + \frac{0.4(-4\text{ in})}{7.8125\text{ in}} + 0.009(7.86)\right\} 0.9$

$\tan\Lambda_{c/4,w} = \tan\Lambda_{LE,w} - \frac{4(0.25)(1-\lambda_w)}{A_w(1+\lambda_w)} = \tan(0) - \frac{4(0.25)\left(1-\frac{6.5}{11.9375}\right)}{7.86\left(1+\frac{6.5}{11.9375}\right)}$

$= -0.037521025$ rad

$\Lambda_{c/4,w} = -0.037503432$ rad

$\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v = 0.650806827$

vertical tail

$C_{L_{\alpha,v}} = \frac{2\pi A_{eff,v}}{2+\sqrt{\frac{A_w^2\beta^2}{K^2}\left(1+\frac{\tan^2\Lambda_{c/2,w}}{\beta^2}\right)+4}}$ ← wing geometry

$A_{eff,v} = \left(\frac{A_v(B)}{A_v}A_v\right)\left[1+K_H\left(\frac{A_v(HB)}{A_v(B)}-1\right)\right]$

$A_v = \frac{b_v^2}{S_v} = \frac{9.1875^2}{58.156875} = 1.451421801$

$\lambda_v = \frac{3.76\text{ in}}{8.90\text{ in}} = 0.42247191 \leq 0.6$

$\frac{S_t}{S_v} = \frac{146.764\text{ in}^2}{58.156875\text{ in}^2} = 2.523588814$

$\frac{z_H}{b_v} = \frac{-1\text{ in}}{9.1875\text{ in}} = -0.108843537$

$\frac{x}{c_v} = \frac{4.927809373\text{ in}}{6.677809373\text{ in}} = 0.737938012$

$\frac{A_v(B)}{A_v} = 1.57$ (fig. 3.77)

$K_H = 1.18$ (fig. 3.79)

$\frac{A_v(HB)}{A_v(B)} = 1.12$ (fig. 3.78)

$c_v = \frac{2}{3}C_r\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) = \frac{2}{3}(8.90)\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) = 6.677809373$ in

$A_{eff,v} = (1.57)(1.451421801)\left\{1+(1.18)(1.12-1)\right\} = 2.601400711$

Vertical Tail cont. :

$A_w = 7.86$

$\beta = \sqrt{1-M^2} = \sqrt{1-(0.0294)^2} = 0.999567727$

$K \approx 1$

$$\tan\Lambda_{c/2,w} = \tan(\Lambda_{LE,w}) - \frac{4(0.5)(1-\lambda)_w}{A_w(1+\lambda_w)} = \tan(0) - \frac{4(0.5)\left(1-\frac{6.5}{11.9375}\right)}{7.86\left(1+\frac{6.5}{11.9375}\right)}$$

$$= -0.075042049$$

$$C_{L\alpha,v} = \frac{2\pi A_{eff,v}}{2+\sqrt{\frac{A_w^2\beta^2}{K^2}\left(1+\frac{\tan^2\Lambda_{c/2,w}}{\beta^2}\right)+4}}$$

$$= \frac{2\pi(2.601400711)}{2+\sqrt{\frac{(7.86^2)(0.999567727^2)}{1^2}\left(1+\frac{(-0.075042049)^2}{0.999567727^2}\right)+4}}$$

$$= 1.613755748 \ /\text{rad}$$

$$(C_{n\beta})_v = K C_{L\alpha,v}\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v \bar{V}_v$$

$$= (0.8)(1.613755748)(0.650806827)(0.030410596)$$

$$= 0.025550819 \ /\text{rad}$$

Mach #	v (m/s)	$C_{L\alpha,v}$ (/rad)	$(C_{n\beta})_v$ (/rad)
0.029	10	1.613755748	0.025550819
0.04408	15	1.614409283	0.025561166
0.05878	20	1.615326105	0.025575682
0.07348	25	1.616508207	0.025594399
0.08817	30	1.617955863	0.02561732

Summary of Directional Static Stability

Mach #	$(C_{n\beta})$ (/rad)
0.029	0.009935161
0.04408	0.012654778
0.05878	0.013269335
0.07348	0.013482952
0.08817	0.013657722