Foundation of

Data Science and Analytics

Test of Hypotheses

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Introduction

- A criminal trial is an example of hypothesis testing without the statistics.
- In a trial a jury must decide between two hypotheses. The null hypothesis is

H_0: The defendant is innocent

- The alternative hypothesis or research hypothesis is

H_1: The defendant is guilty

- The jury does not know which hypothesis is true. They must make a decision on the basis of evidence presented.

Nonstatistical Hypothesis Testing…

- In the language of statistics convicting the defendant is called *rejecting the null hypothesis in favor of the alternative hypothesis*. That is, the jury is saying that there is enough evidence to conclude that the defendant is guilty (i.e., there is enough evidence to support the alternative hypothesis).

- If the jury acquits it is stating that *there is not enough evidence to support the alternative hypothesis*. Notice that the jury is not saying that the defendant is innocent, only that there is not enough evidence to support the alternative hypothesis. That is why we never say that we accept the null hypothesis.

Nonstatistical Hypothesis Testing

- There are two possible errors:
  - A Type I error occurs when we reject a true null hypothesis. That is, a Type I error occurs when the jury convicts an innocent person.
  - A Type II error occurs when we don’t reject a false null hypothesis. That occurs when a guilty defendant is acquitted.

Type I error: Reject a true null hypothesis

Type II error: Do not reject a false null hypothesis.

P(Type I error) = α

P(Type II error) = β

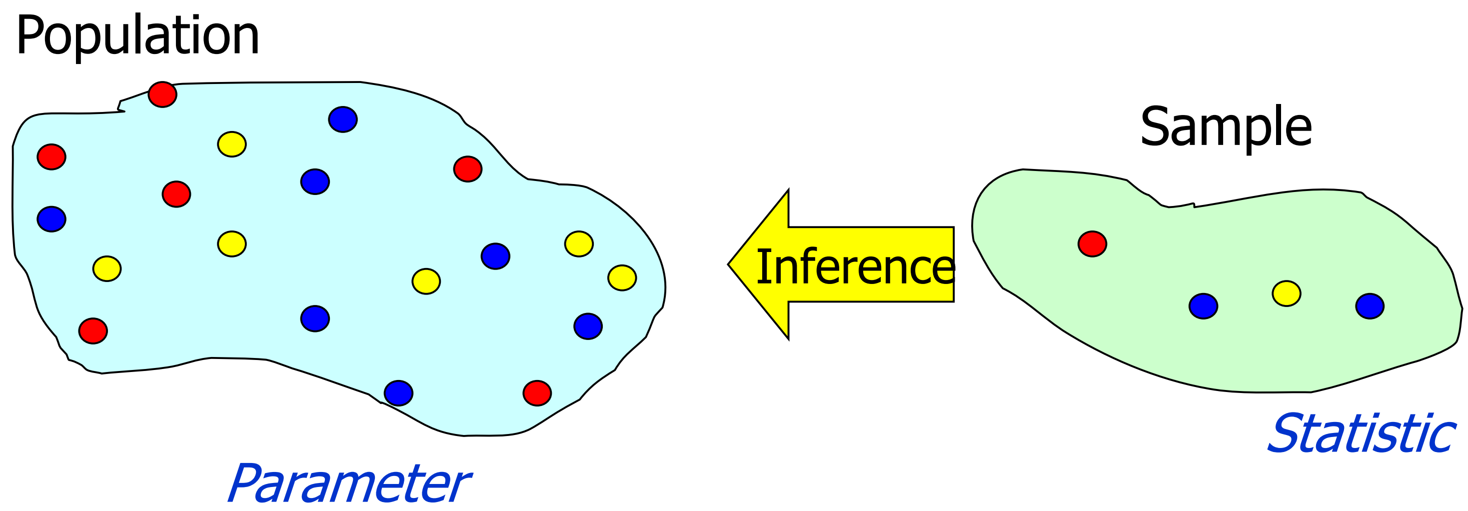
Outcomes and Probabilities

Key:
Outcome
(Probability)

Possible Hypothesis Test Outcomes

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**No error** **(1 - α)**	**Type II Error** **(β)**
Reject **H_0**	**Type I Error** **(α)**	**No Error** **(1 - β)**

Estimation



- Hypothesis testing allows us to determine whether enough statistical evidence exists to conclude that a **belief** (i.e. ***hypothesis***) about a parameter is <u>supported</u> by the data.

Concepts of Hypothesis Testing

- There are **two** hypotheses. One is called the ***null hypothesis*** and the other the ***alternative*** or ***research hypothesis***. The usual notation is:

pronounced
H "nought"

H_0: — *the 'null' hypothesis*

H_1: — *the 'alternative' or 'research' hypothesis*

- The null hypothesis (H_0) will always state that the ***parameter equals the value*** specified in the alternative hypothesis (H_1)

Concepts of Hypothesis Testing

- Rather than estimate the mean demand, if operations manager wants to know ***whether the mean is different from 350 units.*** We can rephrase this request into a test of the hypothesis:

$$H_0: \quad = 350$$

Thus, our research hypothesis becomes:

$$H_1: \quad \neq 350$$

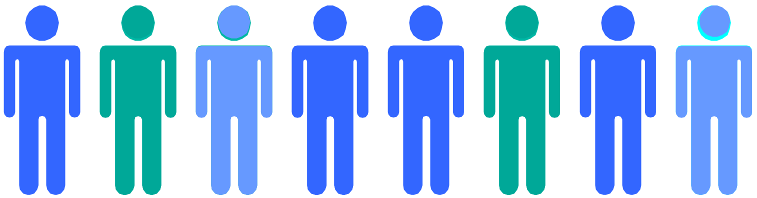
This is what we are interested in determining…

- The **goal** of the process is to determine ***whether there is enough evidence*** to infer that the alternative hypothesis is true.

Hypothesis Testing Process

Claim: **the population mean age is 50. (Null Hypothesis:**

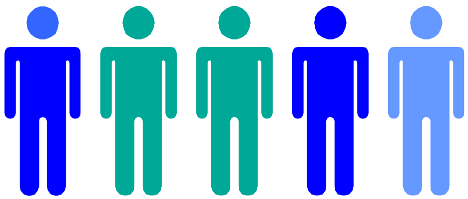
$$H_0: \mu = 50$$ **)**



Population

Now select a random sample

Is $\bar{X} = 20$ likely if $\mu = 50$?



Sample

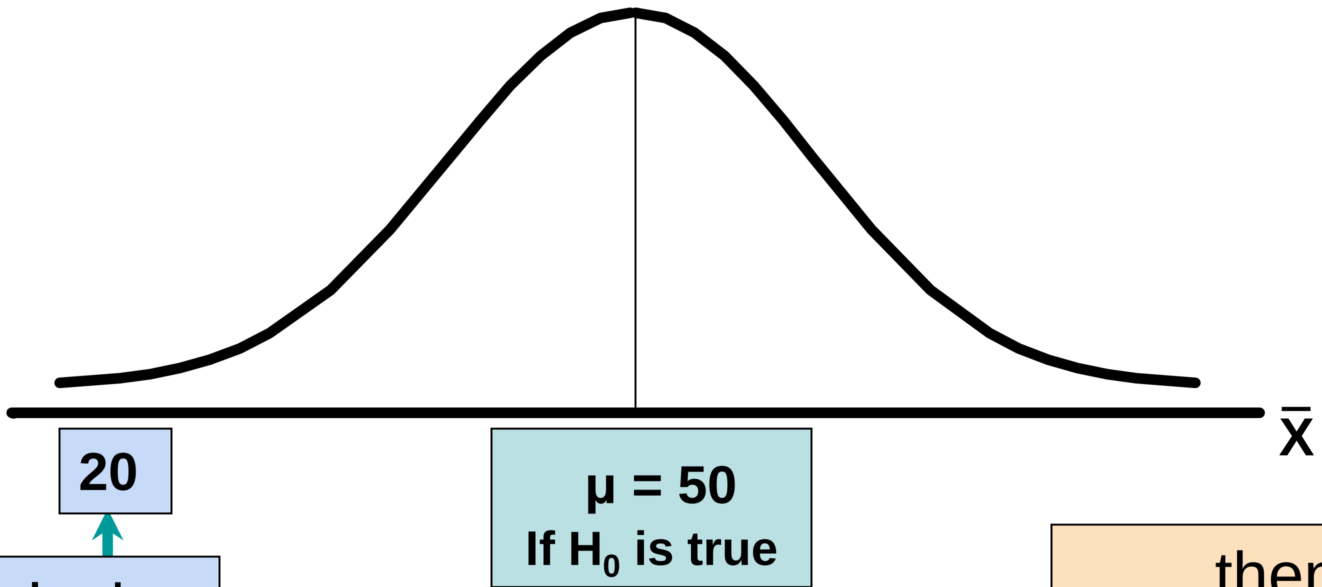
Suppose the sample mean age is 20: $\bar{X} = 20$

If not likely,

REJECT Null Hypothesis

Reason for Rejecting H_0

Sampling Distribution of $\bar{X}$



If it is unlikely that we would get a sample mean of this value ...

... if in fact this were the population mean…

... then we reject the null hypothesis that μ = 50.

Level of Significance, α

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance and the Rejection Region

Level of significance = α

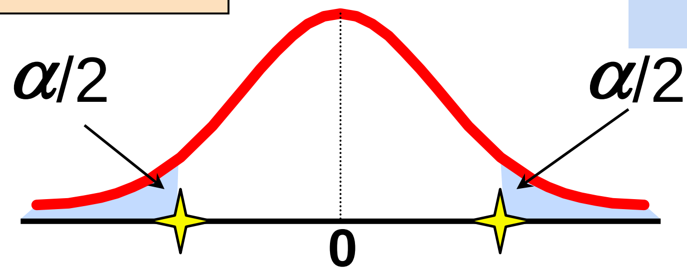
Represents critical value

Rejection region is shaded

H_0: $\mu = 3$

H_1: $\mu \neq 3$

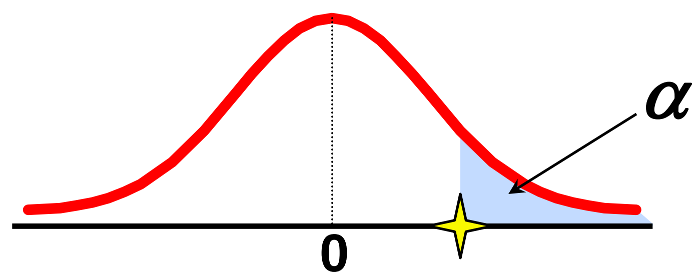
Two-tail test



H_0: $\mu \leq 3$

H_1: $\mu > 3$

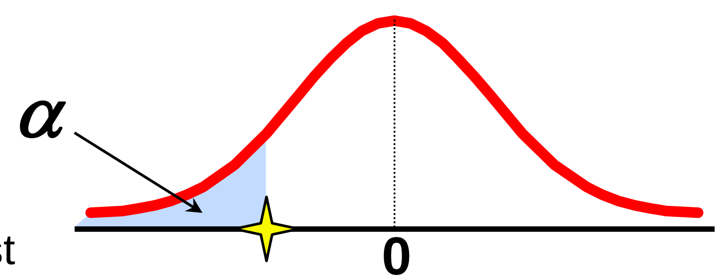
Upper-tail test



H_0: $\mu \geq 3$

H_1: $\mu < 3$

Lower-tail test



Hypothesis Tests for the Mean

For samples of size > 30

$$Z = \frac{\overline{X} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

Two tailed

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

One tailed

$$H_0 : \mu = \mu_0$$
$$H_a : \mu > \mu_0 \text{ or}$$
$$\mu < \mu_0$$

For samples of size < 30

$$T = \frac{\overline{X} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

Two tailed

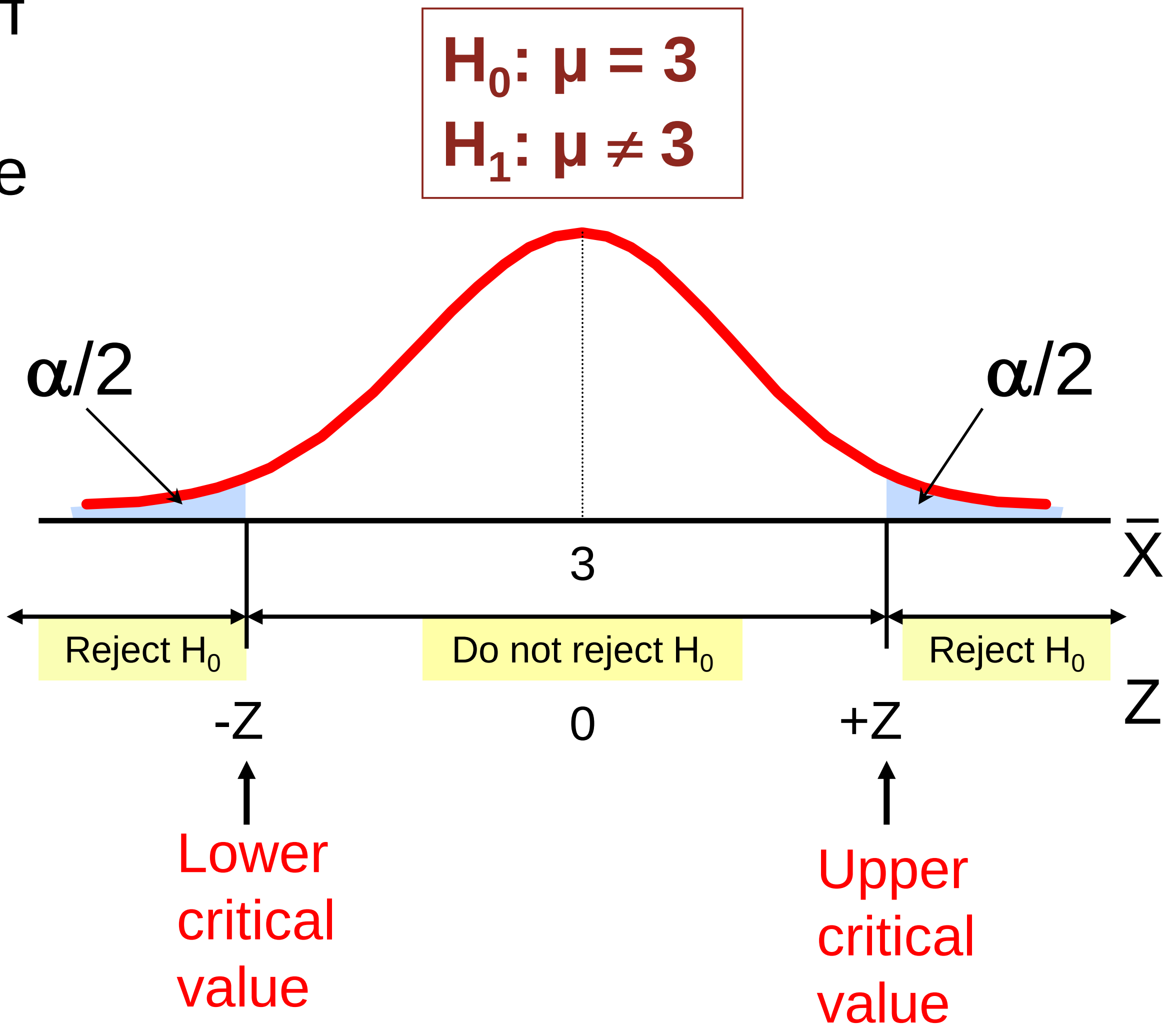
$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

One tailed

$$H_0 : \mu = \mu_0$$
$$H_a : \mu > \mu_0 \text{ or}$$
$$\mu < \mu_0$$

Two-Tail Tests

There are two cutoff values (critical values), defining the regions of rejection



Example

Test the claim that the true mean # of TV sets in US homes *is different from 3 units*
(Assume s = 0.8)

1. State the appropriate null and alternative hypotheses
   – H_0: $\mu = 3$ H_1: $\mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   – Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

Example

3. Determine the appropriate technique
   - σ is known so this is a Z test.
4. Determine the critical values
   - For α = 0.05 the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are

     n = 100,   X = 2.84 (σ = 0.8 is assumed known)

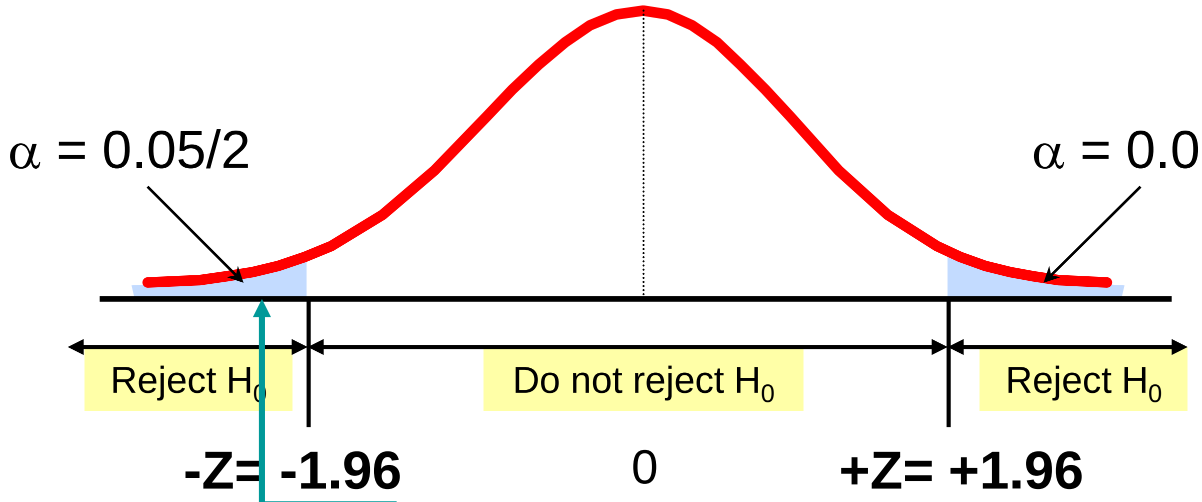
   So the test statistic is:

$$Z = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

Example

6. Is the test statistic in the rejection region?

Reject H_0 if $Z < -1.96$ or $Z > 1.96$; otherwise do not reject H_0



Here, $Z = -2.0 < -1.96$, so the test statistic is in the rejection region

Since $Z = -2.0 < -1.96$, we <u>reject the null hypothesis</u> and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3

p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

If p-value $< \alpha$, reject H_0
If p-value $\geq \alpha$, do not reject H_0

Example

- Example: How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is μ = 3.0?

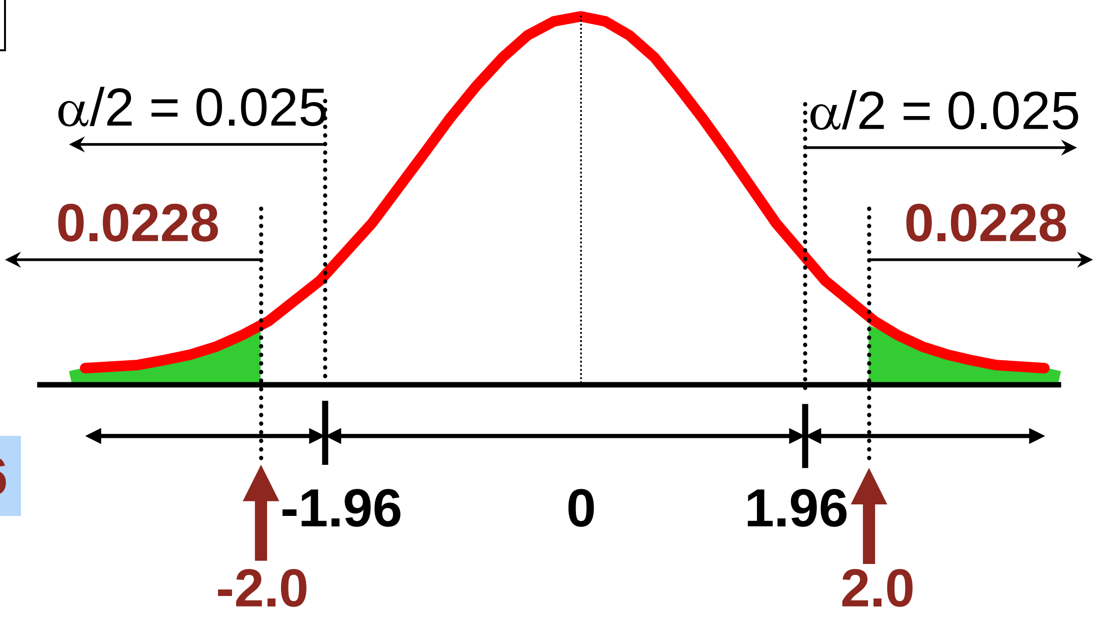
$\overline{X}$ = 2.84 is translated to a Z score of Z = -2.0

$$P(Z < -2.0) = 0.0228$$

$$P(Z > 2.0) = 0.0228$$

p-value

= 0.0228 + 0.0228 = 0.0456



Connection to Confidence Intervals

- For $\bar{X} = 2.84$, $s = 0.8$ and $n = 100$, the 95% confidence interval is:

$$\boxed{2.84 - (1.96)\frac{0.8}{\sqrt{100}} \quad \text{to} \quad 2.84 + (1.96)\frac{0.8}{\sqrt{100}}}$$

$$\boxed{2.6832 \le \mu \le 2.9968}$$

- Since this interval does not contain the hypothesized mean (3.0), we reject the null hypothesis at $\alpha = 0.05$

Example

The average cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in $\bar{X}$ = $172.50 and

s = $15.40. Test at the

α = 0.05 level.

(Assume the population distribution is normal)

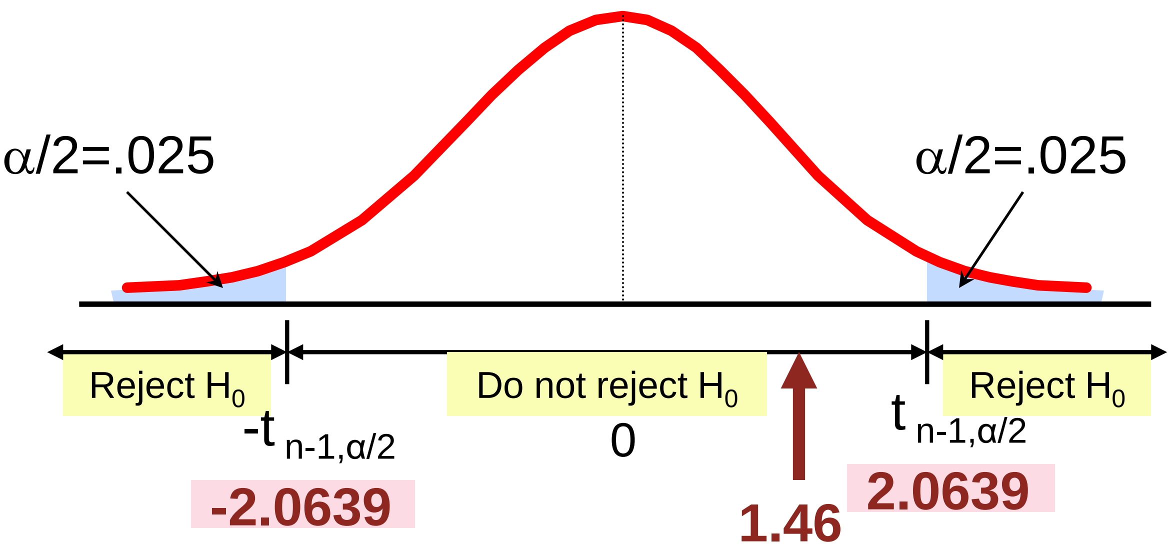
H_0: $\mu = 168$

H_1: $\mu \neq 168$

Example (cont.)

H_0: $\mu = 168$

H_1: $\mu \neq 168$



$\alpha = 0.05$

- $n = 25$
- σ is unknown, so use a t statistic
- Critical Value:

$t_{24} = \pm 2.0639$

$$t_{n-1} = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than $168

Connection to Confidence Intervals

For $\overline{X} = 172{,}5$, $s = 15.4$ and $n = 25$,

the 95% confidence interval is :

$$166.14 \le \mu \le 178.86$$

Since this interval contains the Hypothesized mean (168), we **do not reject the null** hypothesis at $\alpha = 0.05$

Hypothesis Tests for the Proportion

$$p = \frac{X}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

Two tailed

$$H_0 : p = p_0$$
$$H_a : p \neq p_0$$

One tailed

$$H_0 : p = p_0$$
$$H_a : p > p_0 \text{ or } p < p_0$$

Example

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the α = 0.05 significance level.

H_0: $p = 0.08$

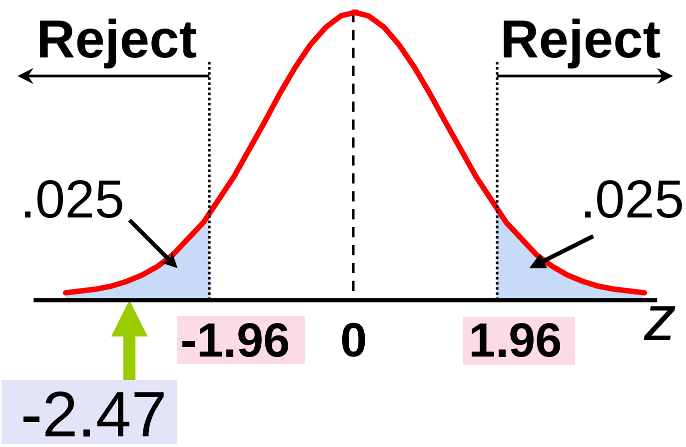
H_1: $p \neq 0.08$

Example (cont.)

Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{.05-.08}{\sqrt{\dfrac{.08(1-.08)}{500}}} = -2.47$$

Critical Values: ± 1.96



Decision:

Reject H_0 at α = 0.05

Conclusion:

There is sufficient evidence to reject the company's claim of 8% response rate.

Testing the Two Population Means: Independent Samples

Large Sample, σ Unknown

$$Z = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

Large Sample, σ known

$$Z = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Small Sample

$$\sigma_1^2 = \sigma_2^2$$

$$T = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Testing the Two Population Means: Independent Samples

Small Sample

$$\boxed{\sigma_1^2 \neq \sigma_2^2}$$

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}}}$$

$$n_1 = n_2 = n \Rightarrow \qquad \upsilon = n_1 + n_2 - 2$$

$$n_1 \neq n_2 \qquad \Rightarrow \qquad \boxed{\nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}}$$

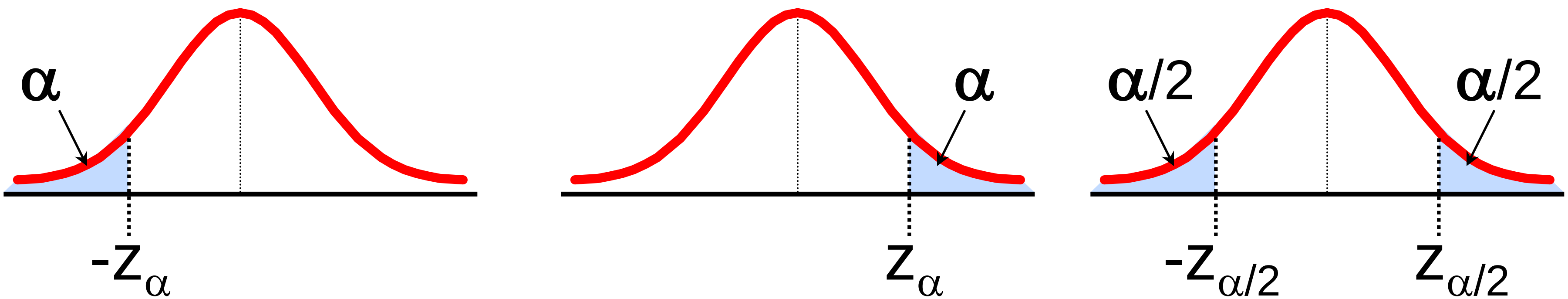
Testing the Two Population Means: Independent Samples

Lower-tail test:	Upper-tail test:	Two-tail test:
H_0: $\mu_1 \geq \mu_2$ H_1: $\mu_1 < \mu_2$	H_0: $\mu_1 \leq \mu_2$ H_1: $\mu_1 > \mu_2$	H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 \neq \mu_2$
i.e.,	i.e.,	i.e.,
H_0: $\mu_1 - \mu_2 \geq 0$ H_1: $\mu_1 - \mu_2 < 0$	H_0: $\mu_1 - \mu_2 \leq 0$ H_1: $\mu_1 - \mu_2 > 0$	H_0: $\mu_1 - \mu_2 = 0$ H_1: $\mu_1 - \mu_2 \neq 0$

Hypothesis tests for $\mu_1 - \mu_2$

Two Population Means, Independent Samples

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \mu_1 - \mu_2 \geq 0$	$H_0: \mu_1 - \mu_2 \leq 0$	$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 < 0$	$H_1: \mu_1 - \mu_2 > 0$	$H_1: \mu_1 - \mu_2 \neq 0$



Reject H_0 if $Z < -Z_\alpha$

Reject H_0 if $Z > Z_\alpha$

Reject H_0 if $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$

Example

- You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
•Sample mean	**3.27**	**2.53**
•Sample std dev	**1.30**	**1.16**

Assuming both populations are approximately normal with equal variances, is there a difference in average yield (α = 0.05)?

Example (cont.)

The test statistic is:

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(3.27 - 2.53\right) - 0}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$

$$S_p^2 = \frac{(n_1 - 1)S_1{}^2 + (n_2 - 1)S_2{}^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(21 - 1)1.30^2 + (25 - 1)1.16^2}{(21\text{-}1) + (25 - 1)} = 1.5021$$

Example (cont.)

H0: μ1 - μ2 = 0 i.e. (μ1 = μ2)

H1: μ1 - μ2 ≠ 0 i.e. (μ1 ≠ μ2)

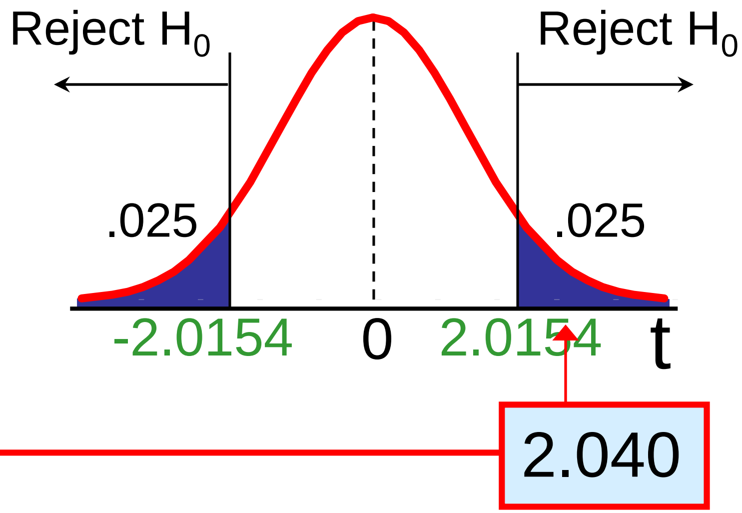
α = 0.05

df = 21 + 25 - 2 = 44

Critical Values: t = ± 2.0154

Test Statistic:

$$t = \frac{3.27 - 2.53}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$



Decision:

Reject H_0 at α = 0.05

Conclusion:

There is evidence of a difference in means.

Testing the Difference Between Two Population Means: Matched Pairs

Large Sample,

$$Z = \frac{\bar{d} - \mu_D}{\frac{s_D}{\sqrt{n}}}$$

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_a : \mu_1 - \mu_2 \neq 0$$

Small Sample

$$t = \frac{\bar{d} - \mu_D}{\frac{s_D}{\sqrt{n}}}$$

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_a : \mu_1 - \mu_2 > 0$$
$$\mu_1 - \mu_2 < 0$$

Sample Mean

$$\bar{d} = \frac{\sum_{i=1}^{n} D_i}{n}$$

Sample Standard Deviation

$$S_D = \sqrt{\frac{\sum_{i=1}^{n} (D_i - \bar{d})^2}{n - 1}}$$

Example

Assume you send your salespeople to a "customer service" training workshop. Has the training made a difference in the number of complaints? You collect the following data:

Salesperson	Number of Complaints: Before (1)	After (2)	(2) - (1) Difference, D_i
C.B.	6	4	- 2
T.F.	20	6	-14
M.H.	3	2	- 1
R.K.	0	0	0
M.O.	4	0	- 4
			-21

$$\bar{d} = \frac{\sum D_i}{n}$$

$$= -4.2$$

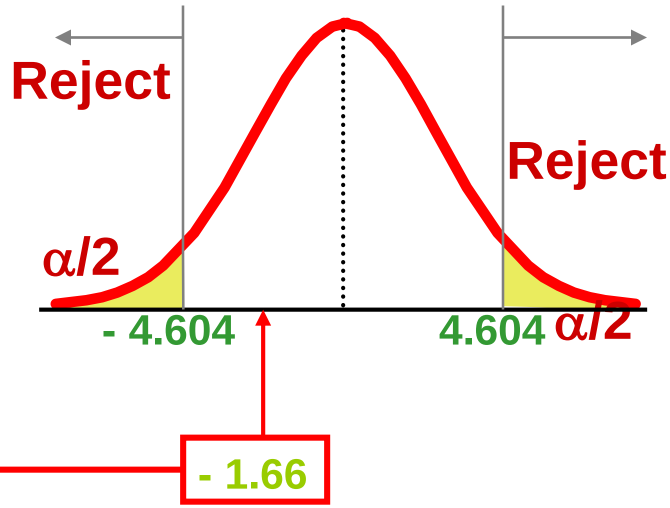
$$S_D = \sqrt{\frac{\sum (D_i - \bar{D})^2}{n-1}}$$

$$= 5.67$$

Example (cont.)

Has the training made a difference in the number of complaints (at the 0.01 level)?

H_0: $\mu_D = 0$
H_1: $\mu_D \neq 0$



$\alpha = .01$ $\quad \overline{D} = -4.2$

Critical Value = ± 4.604

d.f. = n - 1 = 4

Test Statistic:

$$t = \frac{\overline{D} - \mu_D}{S_D/\sqrt{n}} = \frac{-4.2 - 0}{5.67/\sqrt{5}} = \boxed{-1.66}$$

Decision: Do not reject H_0

(t stat is not in the reject region)

Conclusion: There is not a significant change in the number of complaints.

Example

- You're a marketing research analyst. You want to compare a client's calculator to a competitor's. You sample **8** retail stores.

At the **.01** level, does your client's calculator sell for ***less than*** their competitor's

Store	(1) Client	(2) Competitor
1	$ 10	$ 11
2	8	11
3	7	10
4	9	12
5	11	11
6	10	13
7	9	12
8	8	10

Example

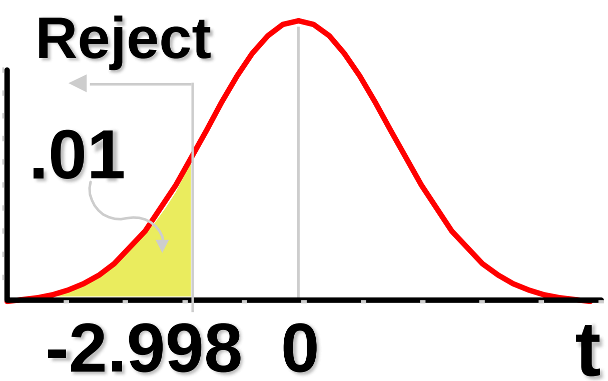
H_0: $\mu_D = 0$ ($\mu_D = \mu_1 - \mu_2$)

H_a: $\mu_D < 0$

$\alpha = .01$

df = 8 - 1 = 7

Critical Value(s):



Test Statistic:

$$t = \frac{\bar{x}_D - D_0}{\frac{s_D}{\sqrt{n_D}}} = \frac{-2.25 - 0}{\frac{1.16}{\sqrt{8}}} = -5.486$$

Decision:

Reject at $\alpha = .01$

Conclusion:

There Is Evidence Client's Brand (1) Sells for Less

Hypothesis Tests for Two Population Proportions

$$Z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}, \quad \hat{p}_1 = \frac{X_1}{n_1}, \quad \hat{p}_2 = \frac{X_2}{n_2}$$

Lower-tail test:

H_0: $p_1 \geq p_2$
H_1: $p_1 < p_2$

i.e.,

H_0: $p_1 - p_2 \geq 0$
H_1: $p_1 - p_2 < 0$

Upper-tail test:

H_0: $p_1 \leq p_2$
H_1: $p_1 > p_2$

i.e.,

H_0: $p_1 - p_2 \leq 0$
H_1: $p_1 - p_2 > 0$

Two-tail test:

H_0: $p_1 = p_2$
H_1: $p_1 \neq p_2$

i.e.,

H_0: $p_1 - p_2 = 0$
H_1: $p_1 - p_2 \neq 0$

Hypothesis Tests for Two Population Proportions

Population proportions

Lower-tail test:

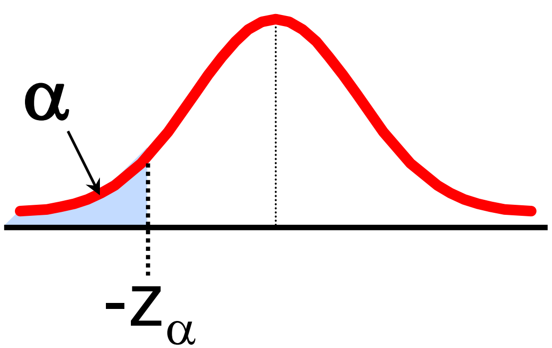
H_0: $p_1 - p_2 \geq 0$
H_1: $p_1 - p_2 < 0$

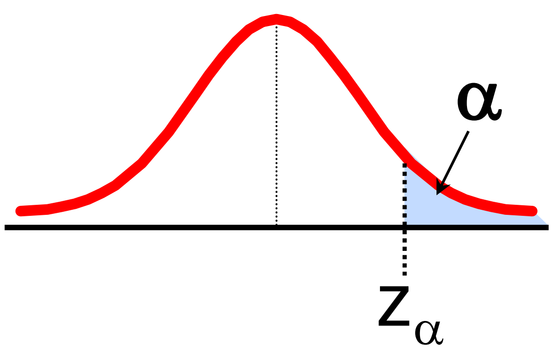
Upper-tail test:

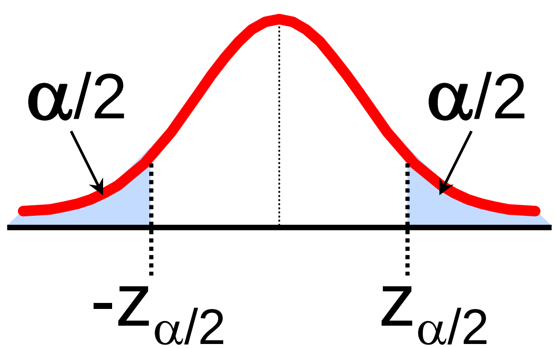
H_0: $p_1 - p_2 \leq 0$
H_1: $p_1 - p_2 > 0$

Two-tail test:

H_0: $p_1 - p_2 = 0$
H_1: $p_1 - p_2 \neq 0$







Reject H_0 if $Z < -Z_{\alpha}$

Reject H_0 if $Z > Z_{\alpha}$

Reject H_0 if $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$

Example

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?

- In a random sample, 36 of 72 men and 31 of 50 women indicated they would vote Yes

- Test at the .05 level of significance

Example (cont.)

- The hypothesis test is:

H_0: $p_1 - p_2 = 0$ (the two proportions are equal)
H_1: $p_1 - p_2 \neq 0$ (there is a significant difference between proportions)

- The sample proportions are:
  - Men: $\hat{p}_1 = 36/72 = .50$
  - Women: $\hat{p}_2 = 31/50 = .62$

- The pooled estimate for the overall proportion is:

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{36 + 31}{72 + 50} = \frac{67}{122} = .549$$

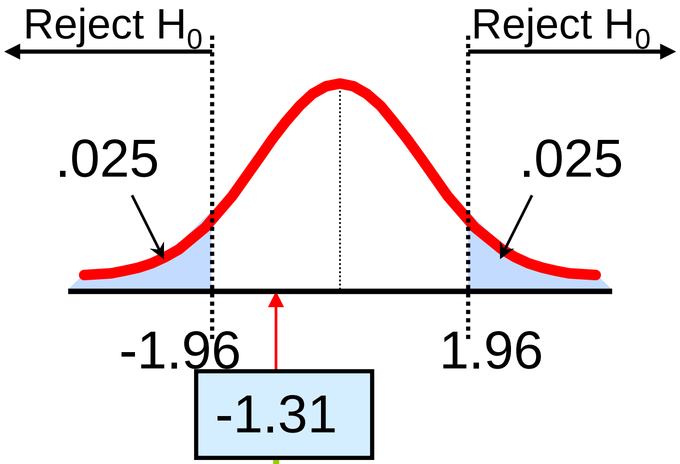
Example (cont.)

The test statistic for $p_1 - p_2$ is:

$$z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\hat{p}\left(1 - \hat{p}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{\left(.50 - .62\right) - \left(0\right)}{\sqrt{.549\left(1 - .549\right)\left(\frac{1}{72} + \frac{1}{50}\right)}} = \boxed{-1.31}$$

Critical Values = ±1.96
For α = .05



Decision: Do not reject H_0

Conclusion: There is not significant evidence of a difference in proportions who will vote yes between men and women.

Testing a Population Variance

The test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$H_0 : \sigma^2 = \sigma_0^2$$

$$H_a : \sigma^2 \neq \sigma_0^2$$

$$H_0 : \sigma^2 = \sigma_0^2$$

$$H_a : \sigma^2 > \sigma_0^2$$

$$\sigma^2 < \sigma_0^2$$

Example

- Consider a container filling machine. Management wants a machine to fill 1 liter (1,000 cc's) so that that variance of the fills is less than 1 cc^2. A random sample of n=25 1 liter fills were taken. Does the machine perform as it should at the 5% significance level?

- We want to show that:

  $H_1: \sigma^2 < 1$

  Variance is less than 1 cc^2

  (so our null hypothesis becomes: $H_0: \sigma^2 = 1$). We will use this test statistic:

$$\boxed{\chi^2 = \frac{(n-1)s^2}{\sigma^2}}$$

Example (cont.)

- Since our alternative hypothesis is phrased as:

  H_a: $\sigma^2 < 1$

- We will reject H0 in favor of H1 if our test statistic falls into this rejection region:

$$\chi^2 < \chi^2_{1-\alpha,n-1} = \chi^2_{1-.05,25-1} = \chi^2_{.95,24} = 13.8484$$

- We compute the sample variance to be: s^2=.8088
- And thus our test statistic takes on this value…

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(25-1)(.8088)}{1} = 19.41$$

compare

There is not enough evidence to infer that the claim is true.

$$\chi^2 = 19.41 \quad < \quad \chi^2_{1-\alpha,n-1} = 13.8484$$

Testing the Ratio of Two Populations Variances

$H_0: \sigma_1^2 / \sigma_2^2 = 1$
$H_1: \sigma_1^2 / \sigma_2^2 \neq 1$

Two-tail test

Lower-tail test

$H_0: \sigma_1^2 \geq \sigma_2^2$
$H_1: \sigma_1^2 < \sigma_2^2$

One-tail test

$H_0: \sigma_1^2 / \sigma_2^2 = 1$
$H_1: \sigma_1^2 / \sigma_2^2 > 1$

Covers both

Read book pages 379 - 382

Upper-tail test

$H_0: \sigma_1^2 \leq \sigma_2^2$
$H_1: \sigma_1^2 > \sigma_2^2$

The F test statistic is (two-tail) :

$$\boxed{F = \frac{s_1^2}{s_2^2}} \quad \boxed{if\ s_1^2 > s_2^2}$$

$$\boxed{F = \frac{s_2^2}{s_1^2}} \quad \boxed{if\ s_2^2 > s_1^2}$$

For one-tail test, always perform Upper tail test with adjustment on making proper H_1 :

e.g. H1: $\sigma_1^2 < \sigma_2^2$ should be changed to H1: $\sigma_2^2 / \sigma_1^2 > 1$

Example

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
Mean	**3.27**	**2.53**
Std dev	**1.30**	**1.16**

Is there a difference in the variances between the NYSE & NASDAQ at the $\alpha = 0.05$ level?

F Test: Example Solution

- Form the hypothesis test:

H_0: $\sigma^2{}_1/\sigma^2{}_2 = 1$	(there is no difference between variances)
H_1: $\sigma^2{}_1/\sigma^2{}_2 \neq 1$	(there is a difference between variances)

Find the F critical values for $\alpha = 0.05$:

Here, NYSE s > NASDAQ s ,

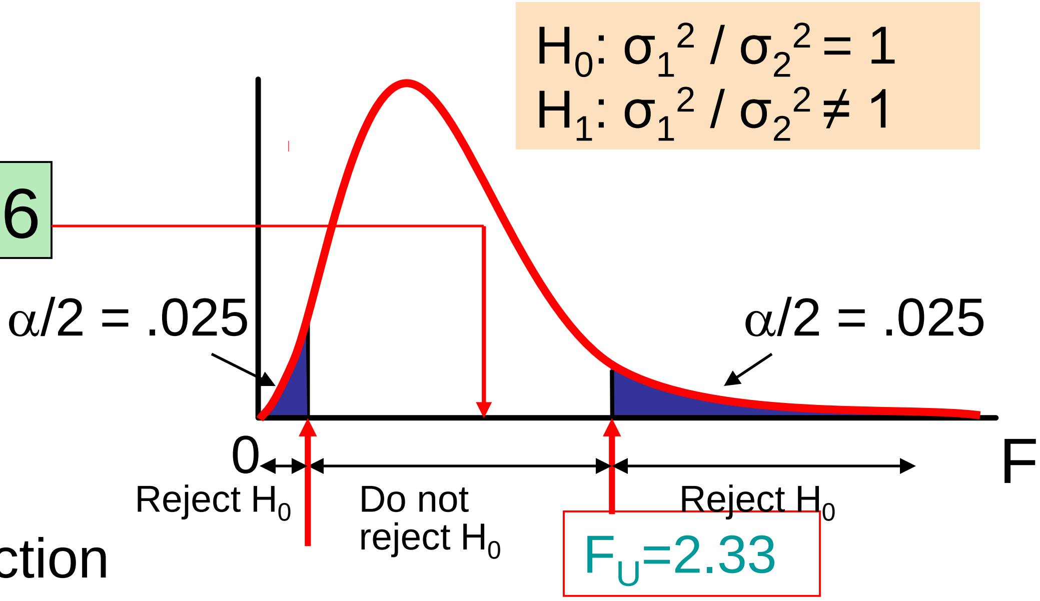
- Numerator:
  - $n_1 - 1 = 21 - 1 = 20$ d.f.
- Denominator:
  - $n_2 - 1 = 25 - 1 = 24$ d.f.

$$\boxed{F_U = F_{.025,\ 20,\ 24} = 2.33}$$

Example (cont.)

The test statistic is:

$$F = \frac{S_1^2}{S_2^2} = \frac{1.30^2}{1.16^2} = \boxed{1.256}$$



- F = 1.256 is not in the rejection region, so we do not reject H_0

- Conclusion: There is not sufficient evidence of a difference in variances at α = .05