MATRIX FACTORIZATION

&

RECOMMENDER SYSTEMS

Outline

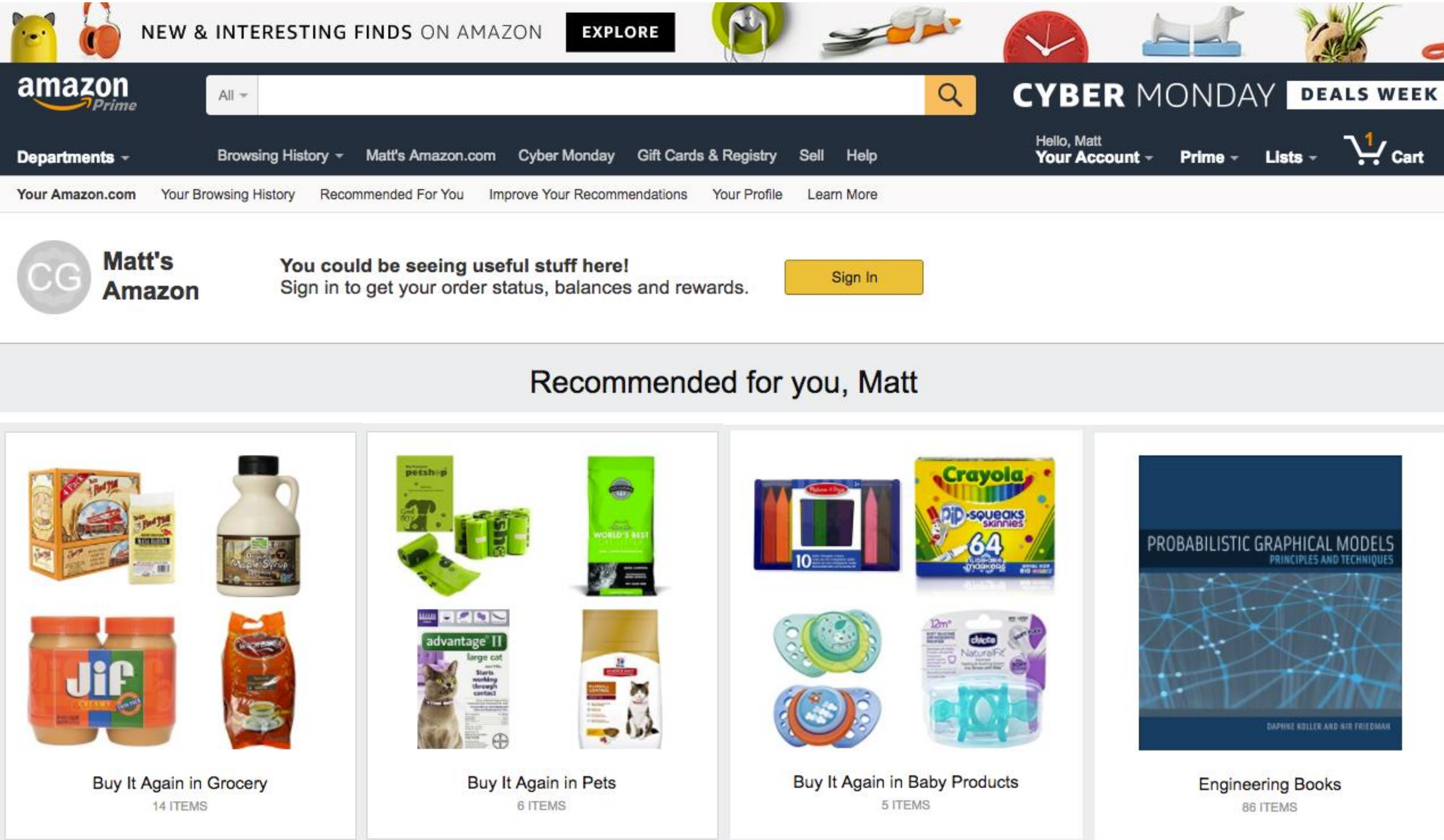
- **Recommender Systems**
  - Content Filtering
  - Collaborative Filtering
  - CF: Neighborhood Methods
  - CF: Latent Factor Methods
- **Matrix Factorization**
  - User / item vectors
  - Prediction model
  - Training by SGD
- **Extra: Matrix Multiplication in ML**
  - Matrix Factorization
  - Linear Regression
  - PCA
  - (Autoencoders)
  - K-means

Recommender Systems

A Common Challenge:

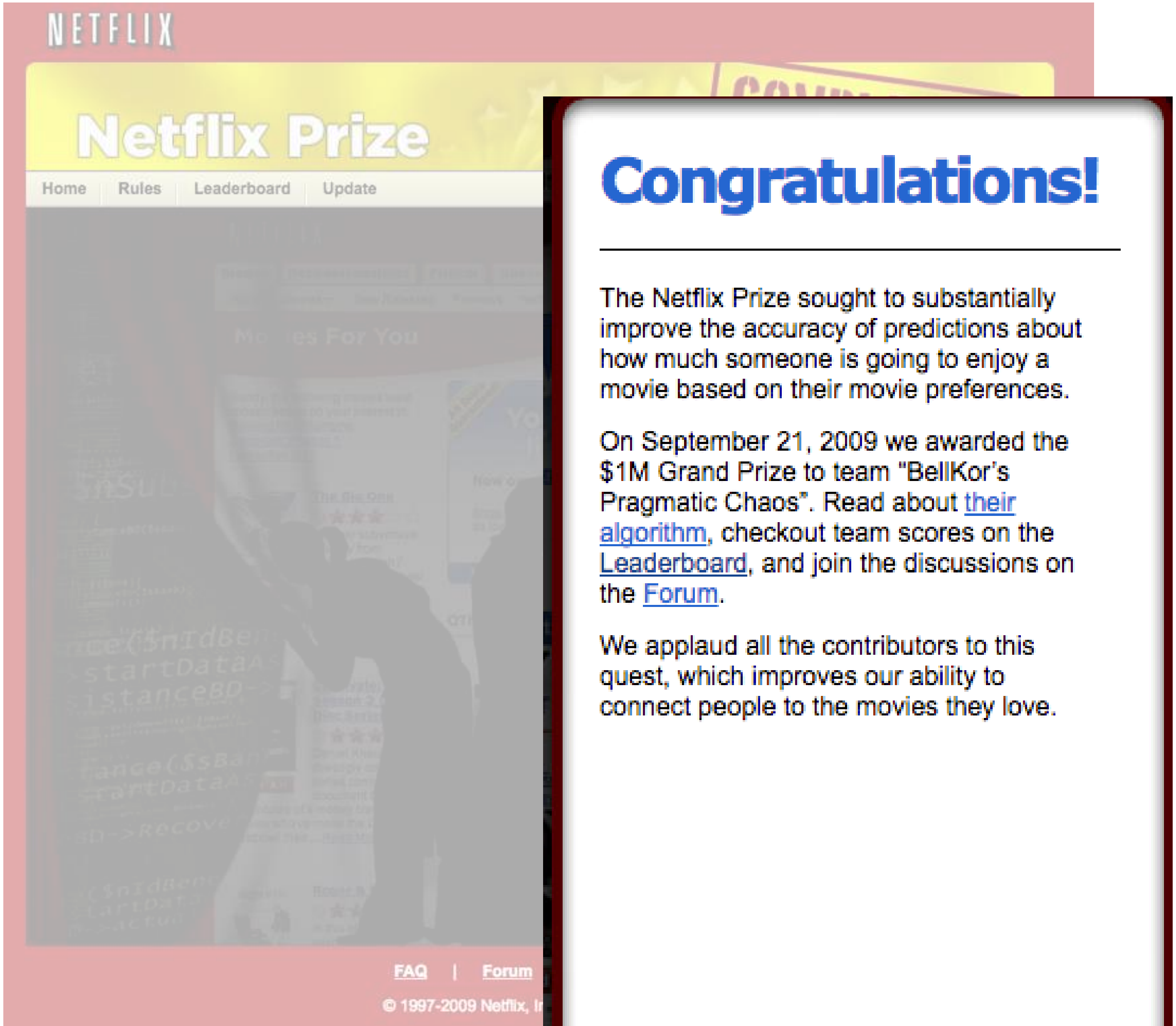
- Assume you're a company selling **items** of some sort: movies, songs, products, etc.
- Company collects millions of **ratings** from **users** of their **items**
- To maximize profit / user happiness, you want to **recommend** items that users are likely to want

Recommender Systems

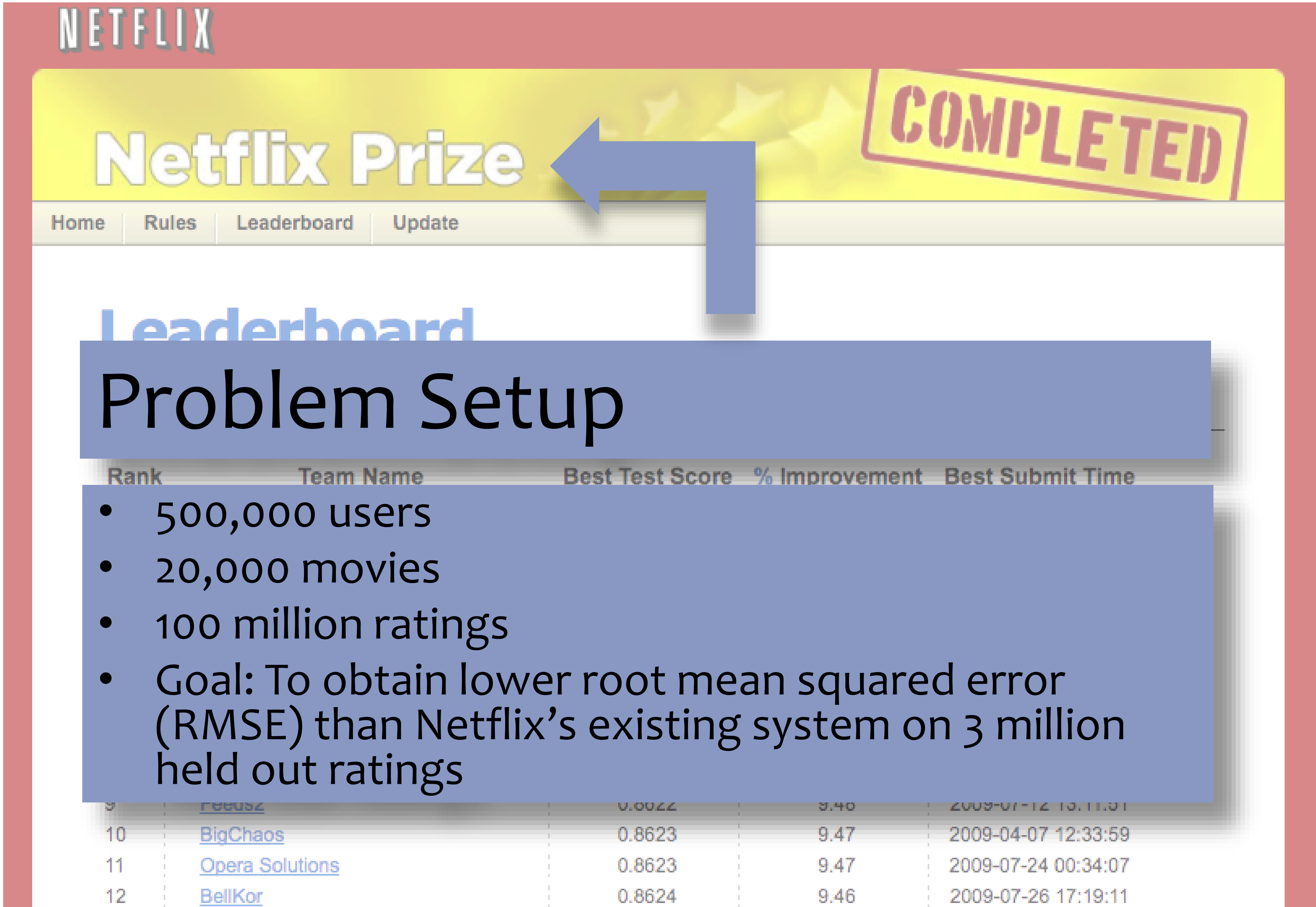


Recommender Systems

Recommender Systems



Recommender Systems



Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Recommender Systems

NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update

Leaderboard

Showing Test Score. Click here to show quiz score

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11

Recommender Systems

- **Setup:**
  - Items:
    movies, songs, products, etc.
    (often many thousands)
  - Users:
    watchers, listeners, purchasers, etc.
    (often many millions)
  - Feedback:
    5-star ratings, not-clicking 'next',
    purchases, etc.
- **Key Assumptions:**
  - Can represent ratings numerically as a user/item matrix
  - Users only rate a small number of items (the matrix is sparse)

	Doctor Strange	Star Trek: Beyond	Zootopia
Alice	1		5
Bob	3	4	
Charlie	3	5	2

Recommender Systems

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Two Types of Recommender Systems

Content Filtering

- *Example*: Pandora.com music recommendations (Music Genome Project)
- **Con:** Assumes access to side information about items (e.g. properties of a song)
- **Pro:** Got a new item to add? No problem, just be sure to include the side information

Collaborative Filtering

- *Example*: Netflix movie recommendations
- **Pro:** Does not assume access to side information about items (e.g. does not need to know about movie genres)
- **Con:** Does not work on new items that have no ratings

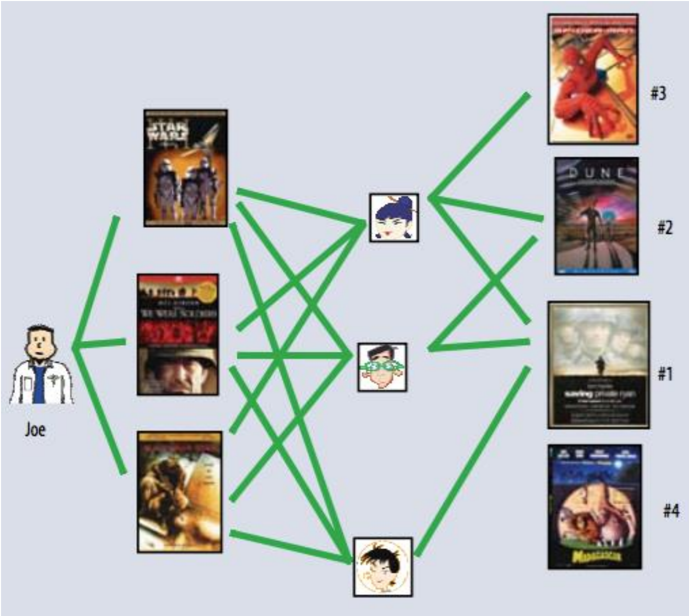
Collaborative Filtering

- **Everyday Examples of Collaborative Filtering...**
  - Bestseller lists
  - Top 40 music lists
  - The "recent returns" shelf at the library
  - Unmarked but well-used paths thru the woods
  - The printer room at work
  - "Read any good books lately?"
  - ...
- **Common insight:** personal tastes are correlated
  - If Alice and Bob both like X and Alice likes Y then Bob is more likely to like Y
  - especially (perhaps) if Bob knows Alice

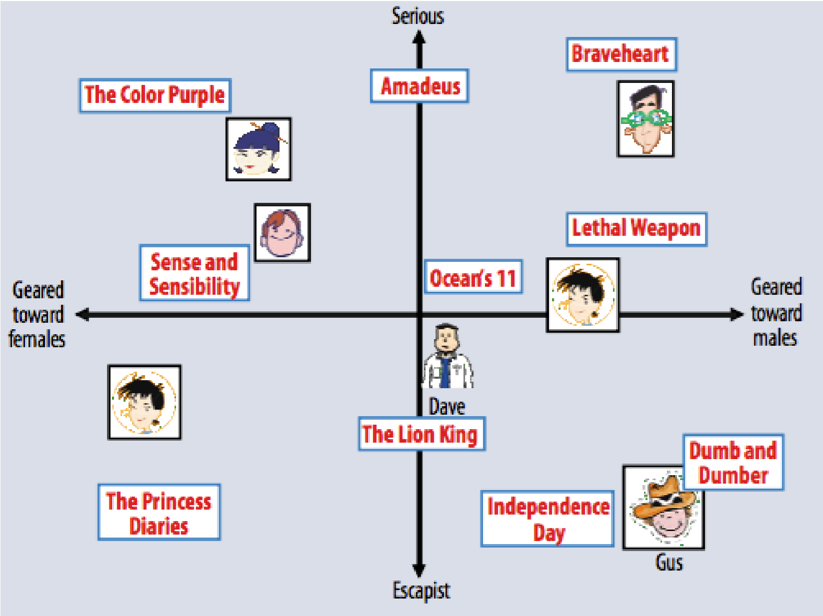
Slide from William Cohen

Two Types of Collaborative Filtering

1. Neighborhood Methods



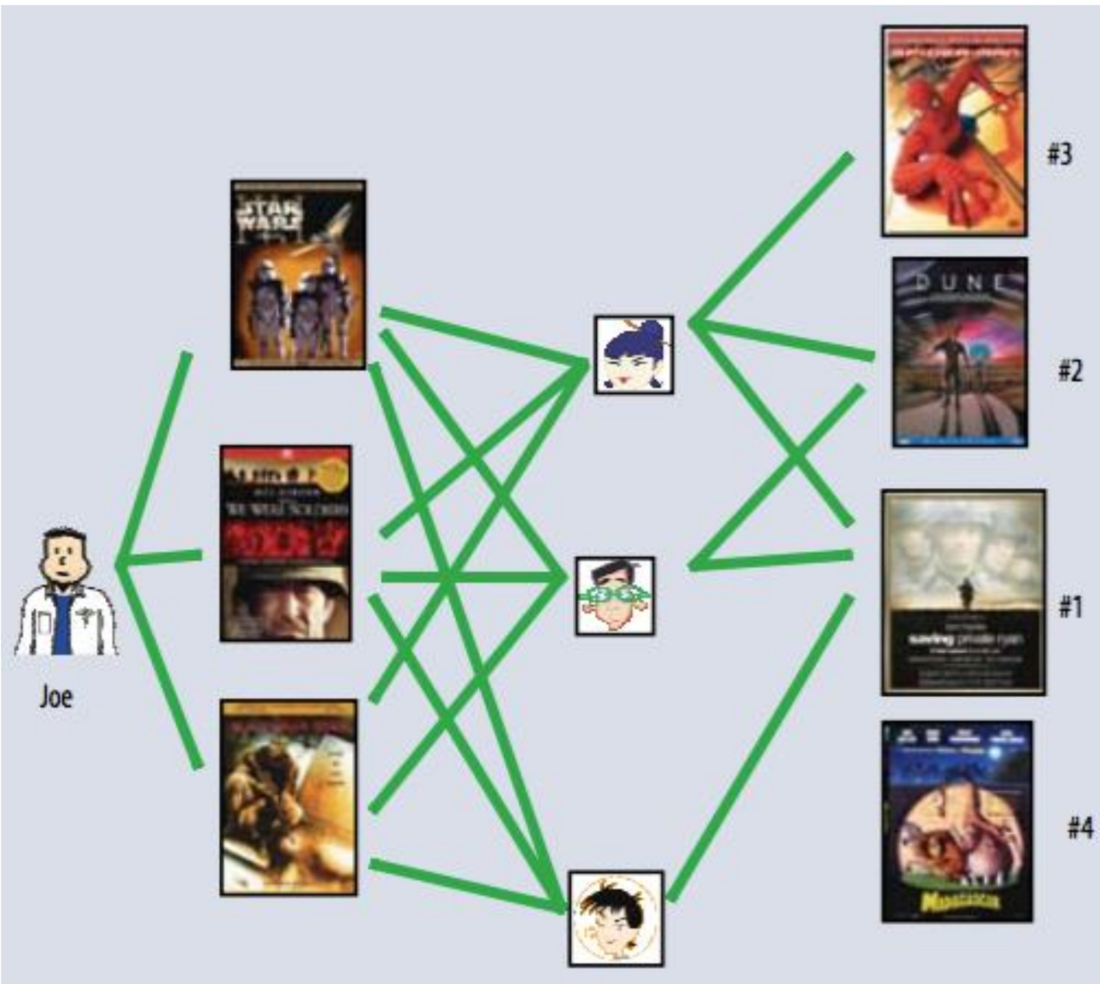
2. Latent Factor Methods



Figures from Koren et al. (2009)

Two Types of Collaborative Filtering

1. Neighborhood Methods



In the figure, assume that a green line indicates the movie was **watched**

Algorithm:

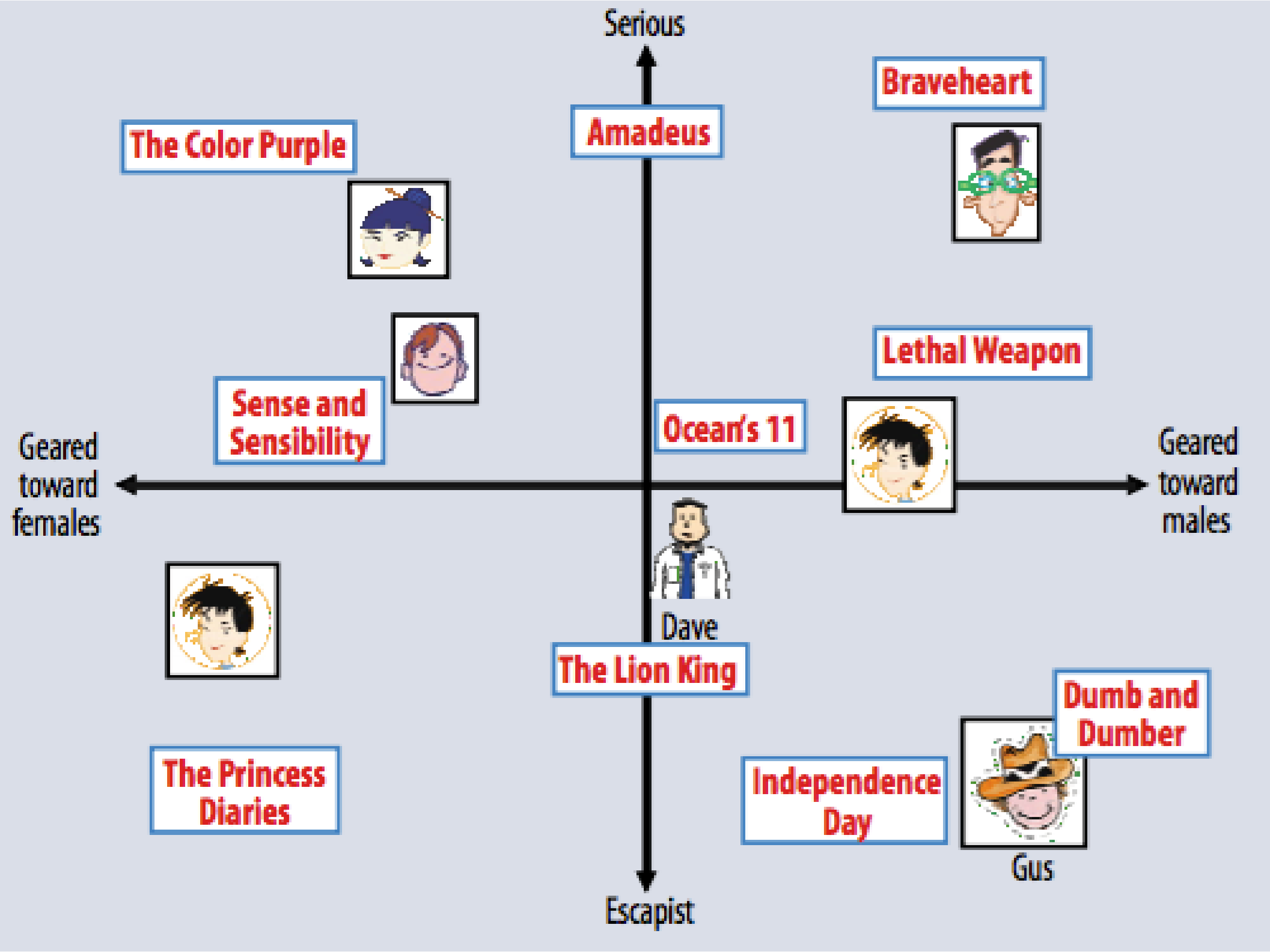
1. **Find neighbors** based on similarity of movie preferences
2. **Recommend** movies that those neighbors watched

Figures from Koren et al. (2009)

Two Types of Collaborative Filtering

2. Latent Factor Methods

- Assume that both movies and users live in some **low-dimensional space** describing their properties
- **Recommend** a movie based on its **proximity** to the user in the latent space



Figures from Koren et al. (2009)

MATRIX FACTORIZATION

Matrix Factorization
(with matrices)

- User vectors:

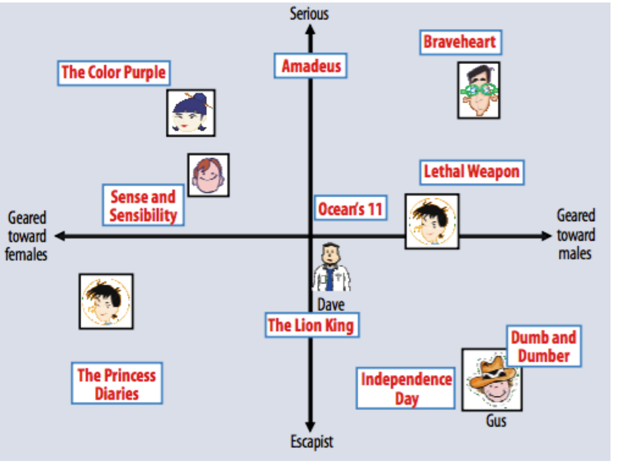
$$(W_{u*})^T \in \mathbb{R}^r$$

- Item vectors:

$$H_{*i} \in \mathbb{R}^r$$

- Rating prediction:

$$V_{ui} = W_{u*}H_{*i}$$
$$= [WH]_{ui}$$



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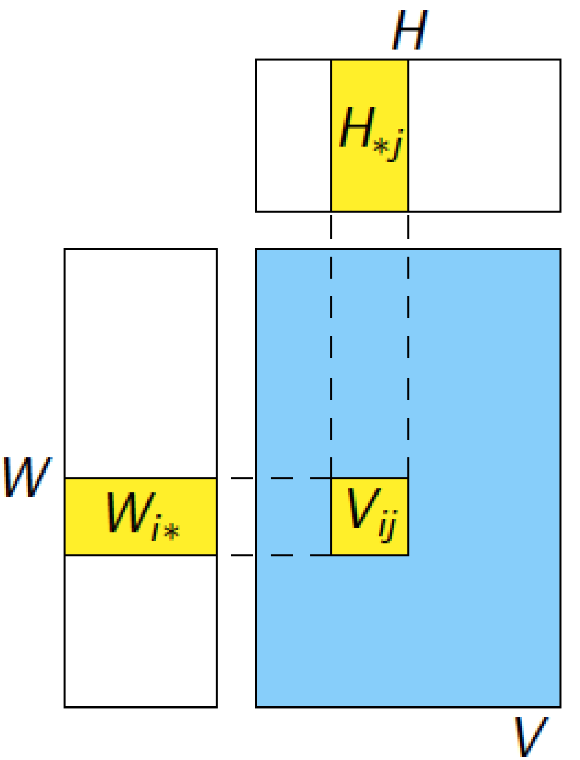
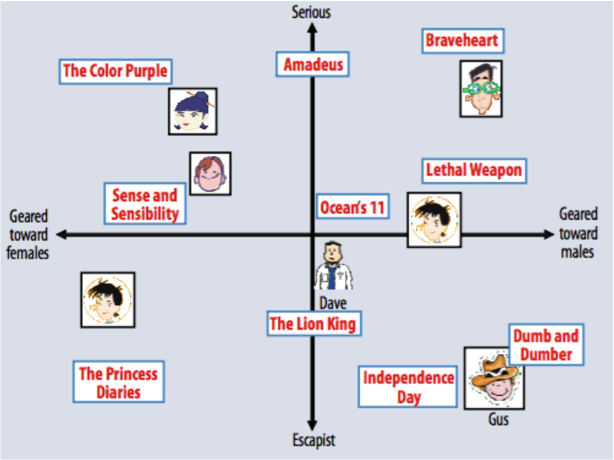


Figures from Gemulla et al. (2011)

Matrix Factorization
(with vectors)

- User vectors:
  $$\mathbf{w}_u \in \mathbb{R}^r$$
- Item vectors:
  $$\mathbf{h}_i \in \mathbb{R}^r$$
- Rating prediction:
  $$v_{ui} = \mathbf{w}_u^T \mathbf{h}_i$$



Figures from Koren et al. (2009)

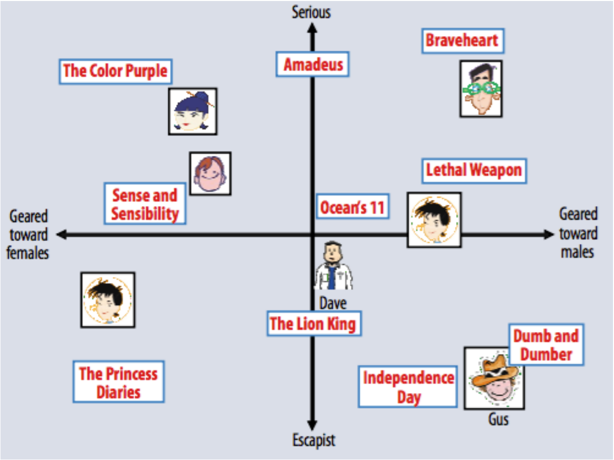
Matrix Factorization
(with vectors)

- Set of non-zero entries:

$$\mathcal{Z} = \{(u,i) : v_{ui} \neq 0\}$$

- Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\text{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$



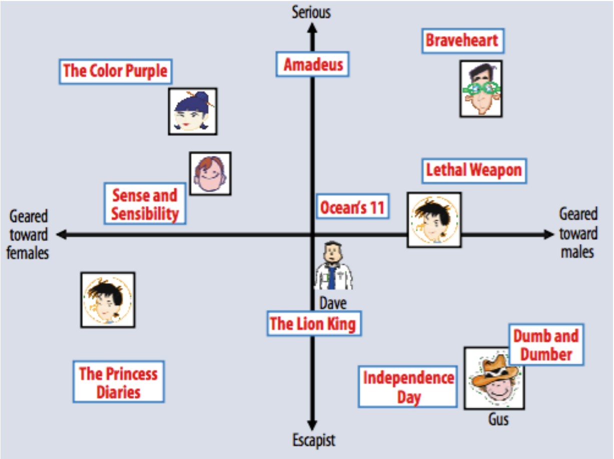
Figures from Koren et al. (2009)

Matrix Factorization
(with vectors)

- Regularized Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$



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Matrix Factorization (with vectors)

- Regularized Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

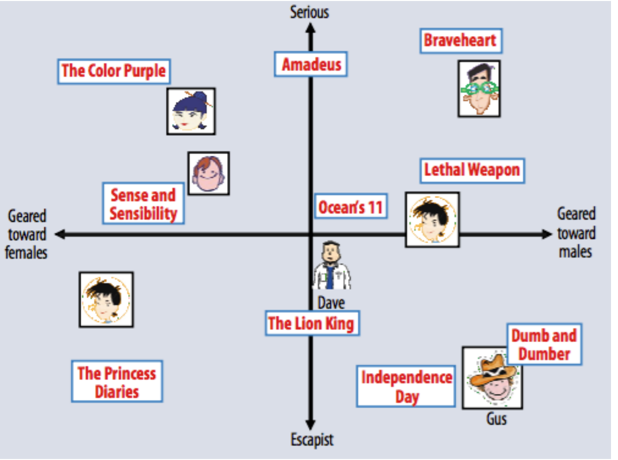
$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$

- Stochastic Gradient Descent (SGD) update for random (u,i):

$$e_{ui} \leftarrow v_{ui} - \mathbf{w}_u^T \mathbf{h}_i$$

$$\mathbf{w}_u \leftarrow \mathbf{w}_u + \gamma(e_{ui}\mathbf{h}_i - \lambda \mathbf{w}_u)$$

$$\mathbf{h}_i \leftarrow \mathbf{h}_i + \gamma(e_{ui}\mathbf{w}_u - \lambda \mathbf{h}_i)$$



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Matrix Factorization
(with matrices)

- User vectors:

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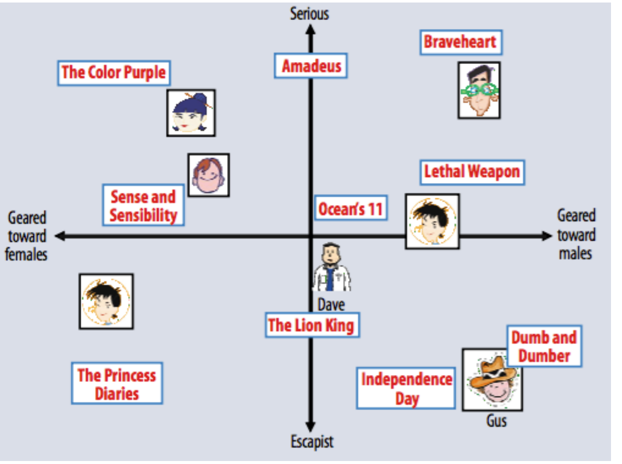
- Rating prediction:

$$\begin{aligned} V_{ui} &= W_{u*} H_{*i} \\ &= [WH]_{ui} \end{aligned}$$



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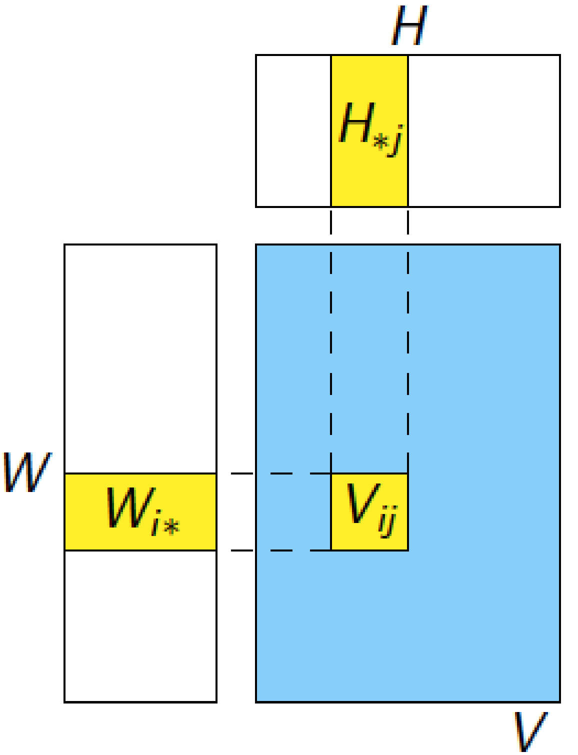


Figures from Gemulla et al. (2011)

Matrix Factorization (with matrices)

- SGD (Stochastic Gradient Descent)

require that the loss can be written as

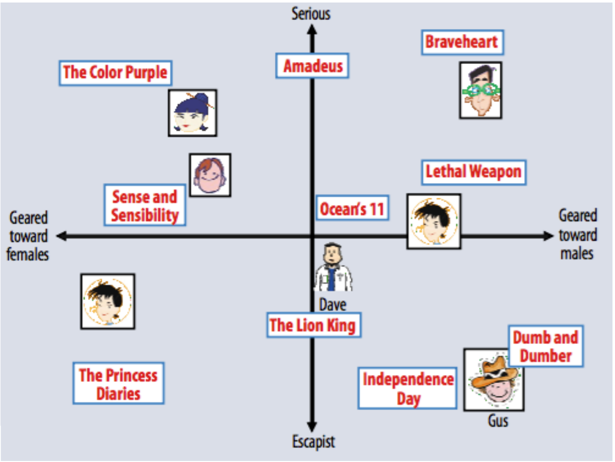
$$L = \sum_{(i,j)\in Z} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$$

Algorithm 1 SGD for Matrix Factorization

Require: A training set Z, initial values $\boldsymbol{W}_0$ and $\boldsymbol{H}_0$
while not converged **do** {step}
Select a training point $(i, j) \in Z$ uniformly at random.
$\boldsymbol{W}'_{i*} \leftarrow \boldsymbol{W}_{i*} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{W}_{i*}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
$\boldsymbol{H}_{*j} \leftarrow \boldsymbol{H}_{*j} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{H}_{*j}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
$\boldsymbol{W}_{i*} \leftarrow \boldsymbol{W}'_{i*}$
end while

step size

Figure from Gemulla et al. (2011)



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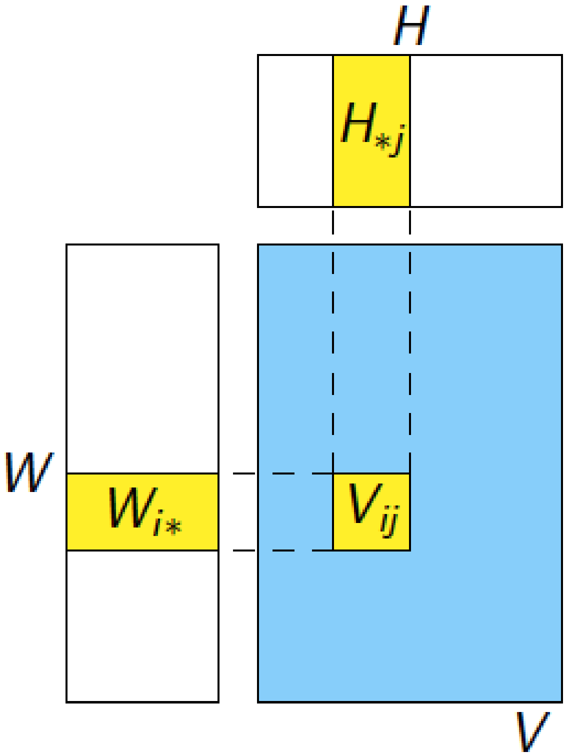


Figure from Gemulla et al. (2011)

Matrix Factorization

Example Factors

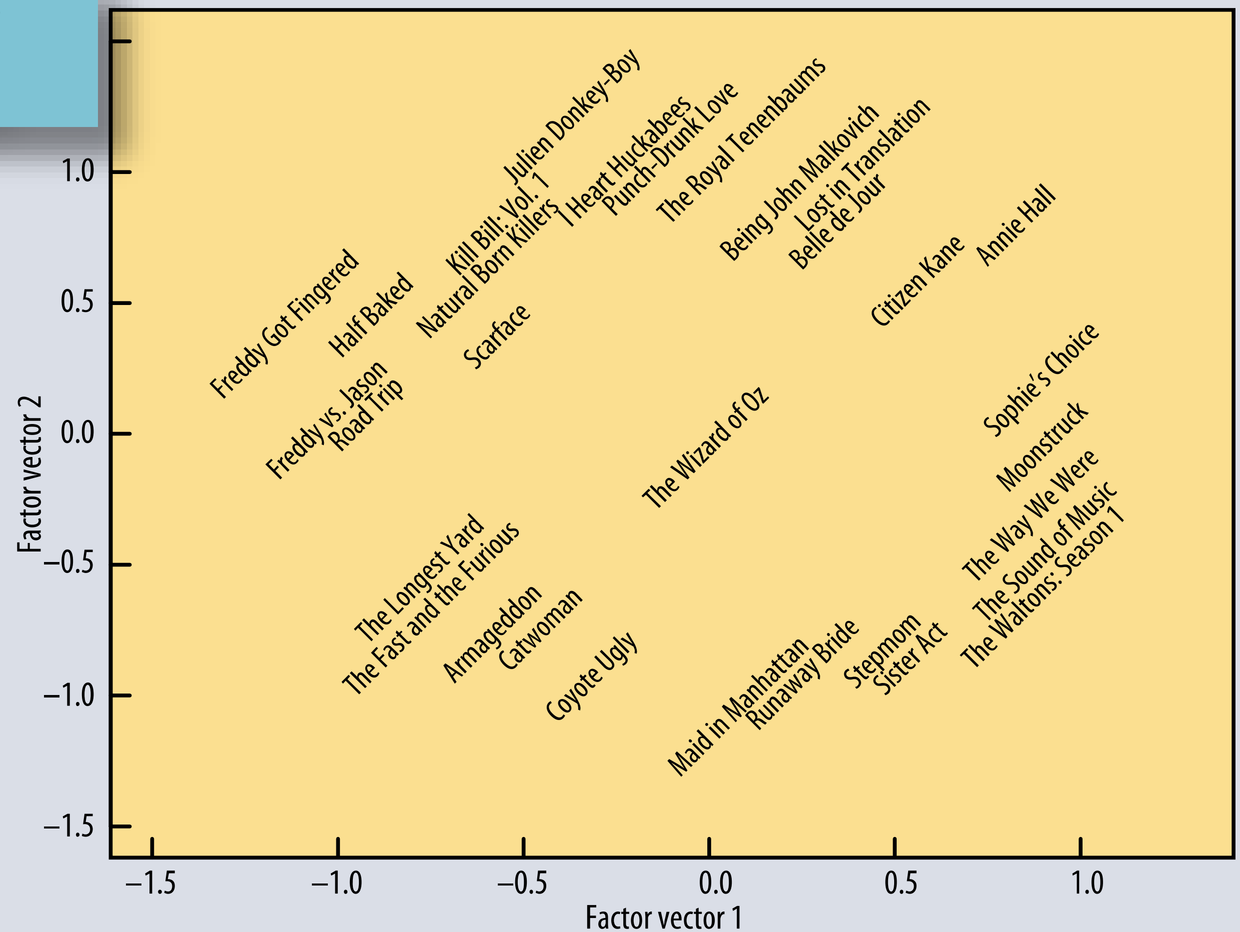


Figure 3. The first two vectors from a matrix decomposition of the Netflix Prize data. Selected movies are placed at the appropriate spot based on their factor vectors in two dimensions. The plot reveals distinct genres, including clusters of movies with strong female leads, fraternity humor, and quirky independent films.

Figure from Koren et al. (2009)

Matrix Factorization

Comparison of Optimization Algorithms

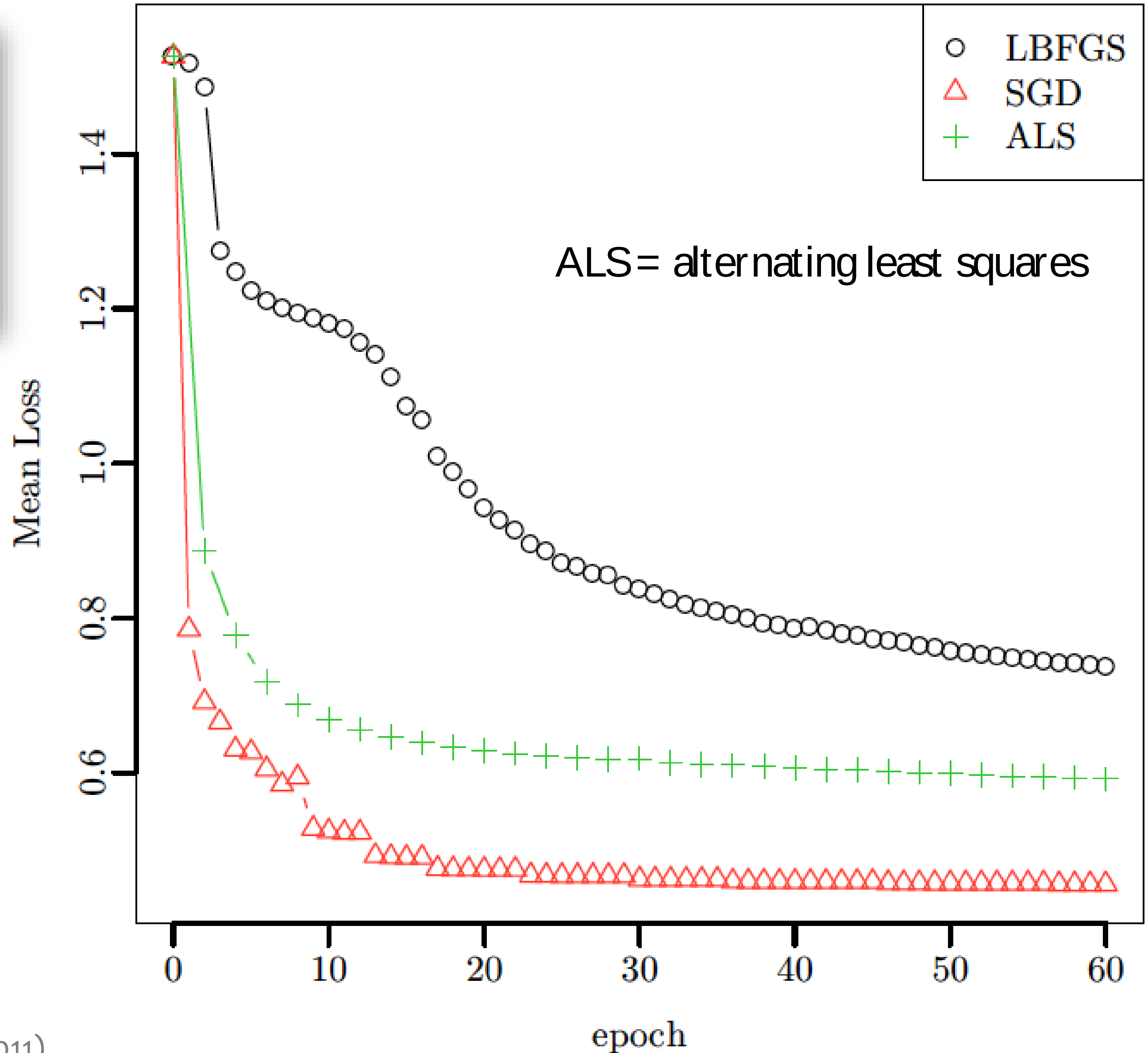
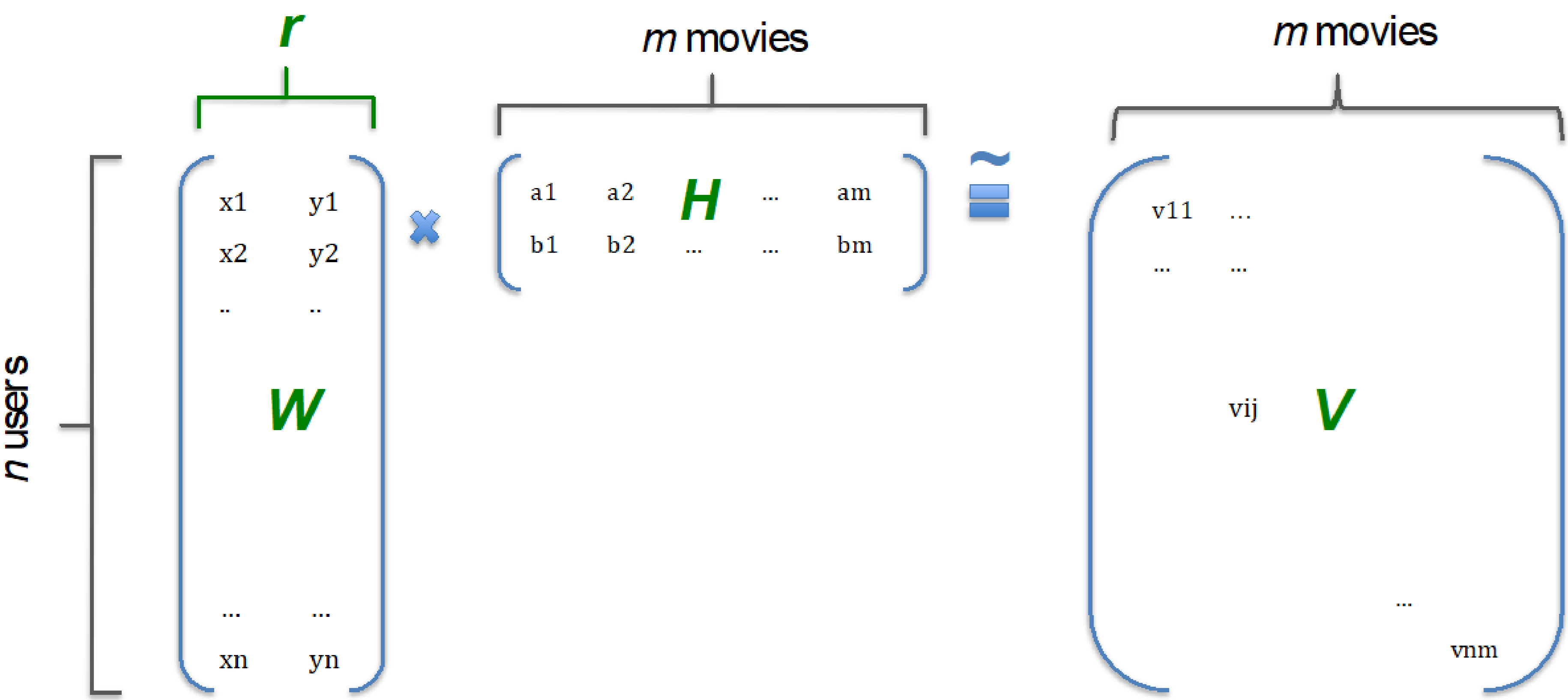


Figure from Gemulla et al. (2011)

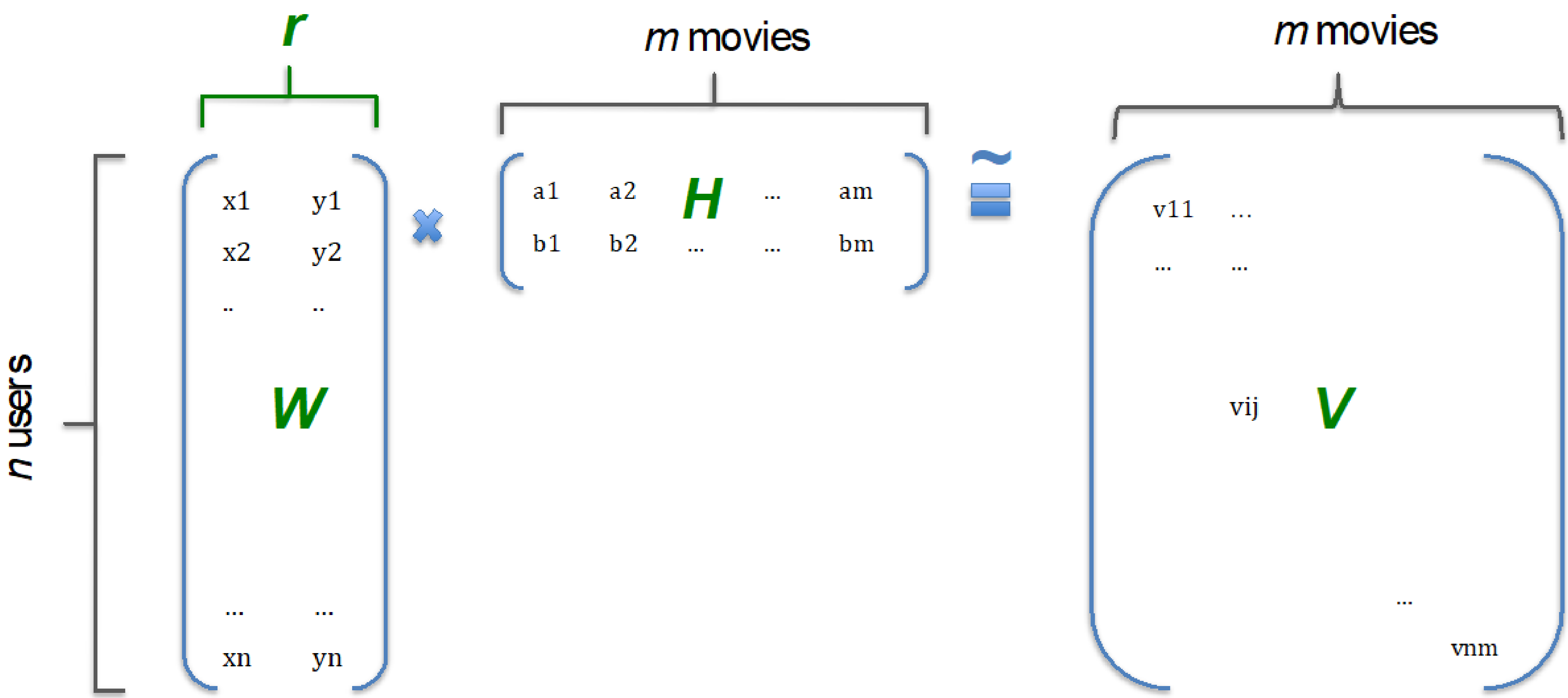
MATRIX MULTIPLICATION IN MACHINE LEARNING

Slides in this section from William Cohen



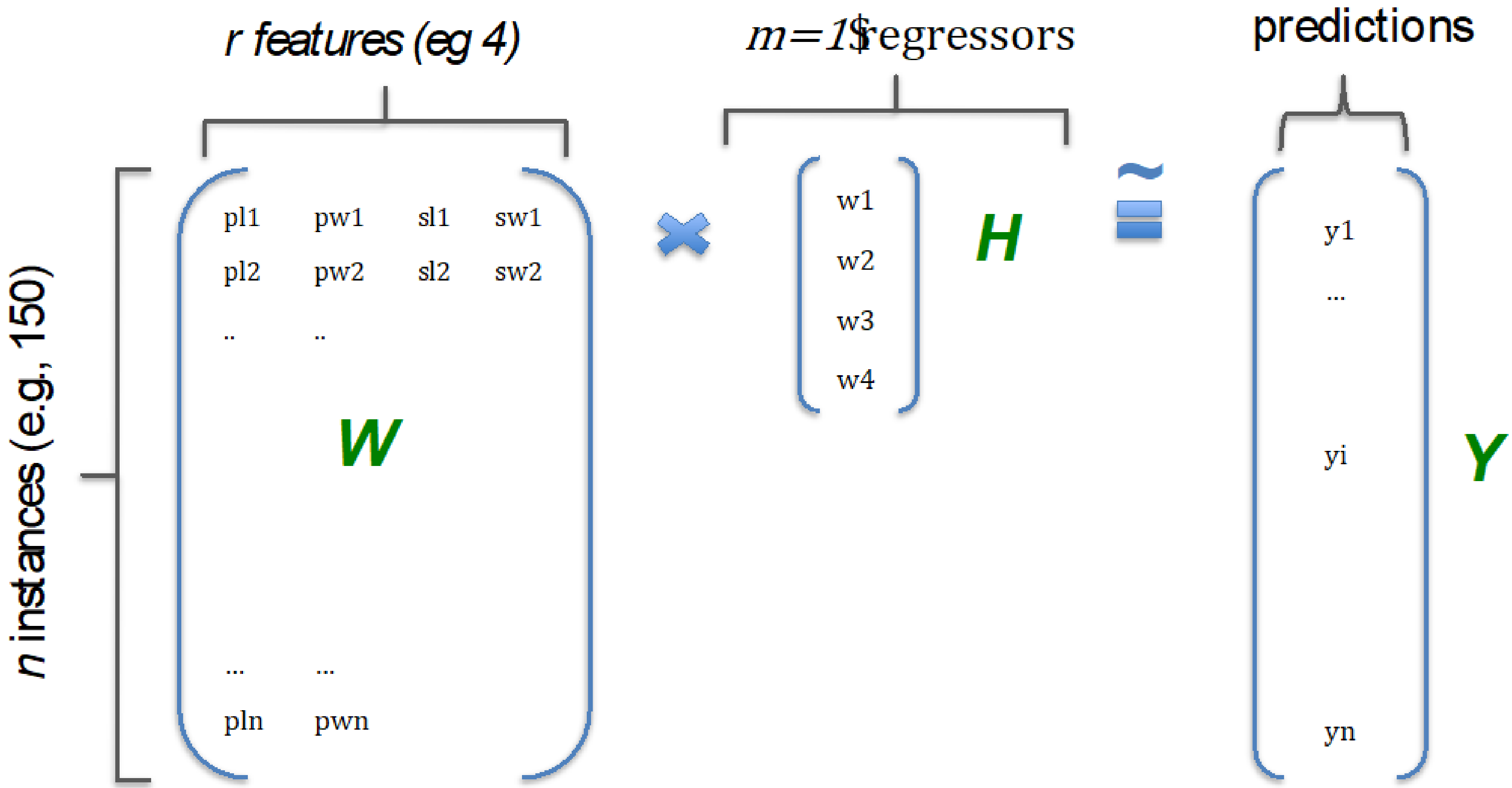
V[i,j] = user i's rating of movie j

Recovering Latent Factors

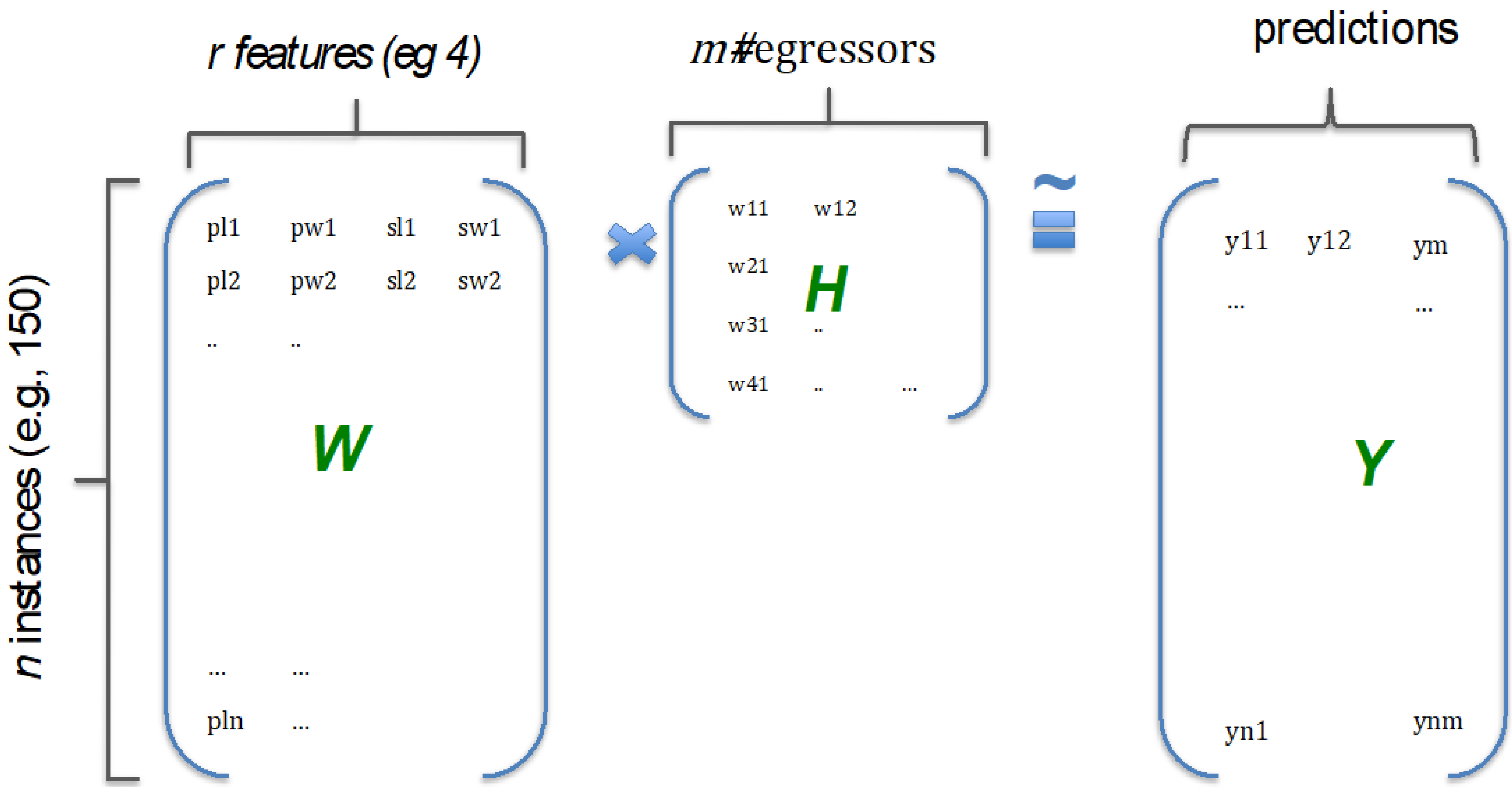


V[i,j] = user i's rating of movie j

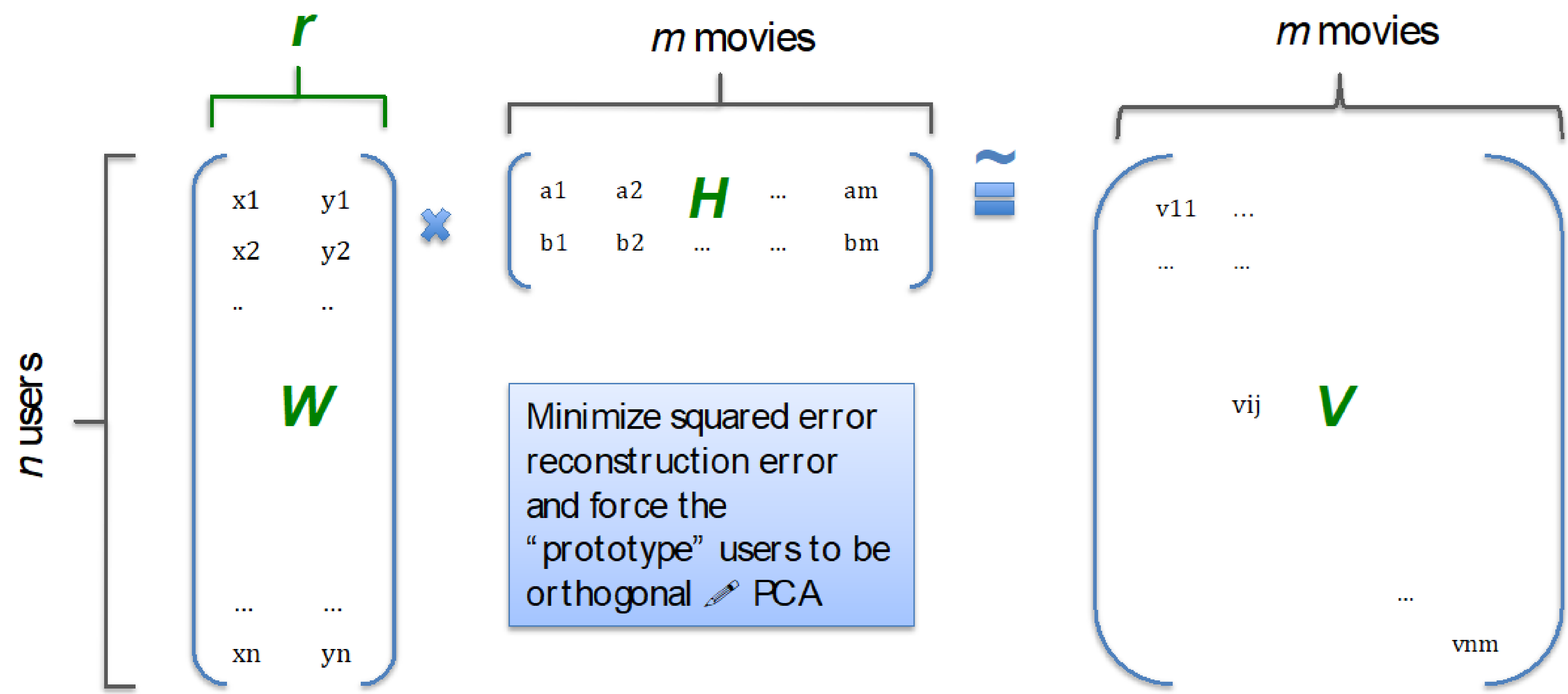
Is like Regression



Many Output at Once



Similar like PCA



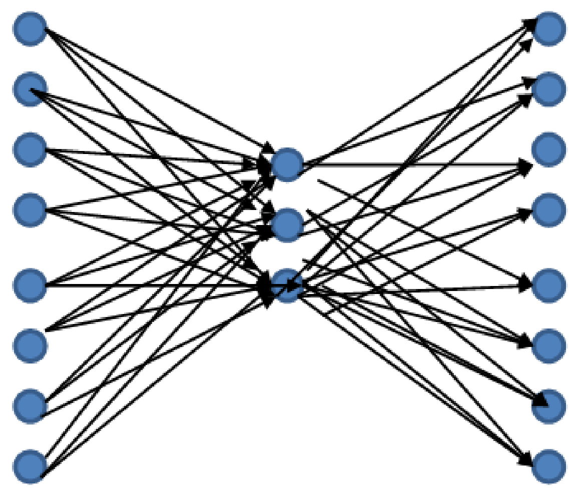
Minimize squared error reconstruction error and force the "prototype" users to be orthogonal ✎ PCA

Auto-encoder and Non-Linear PCA

- Assume we would like to learn the following (trivial?) output function:

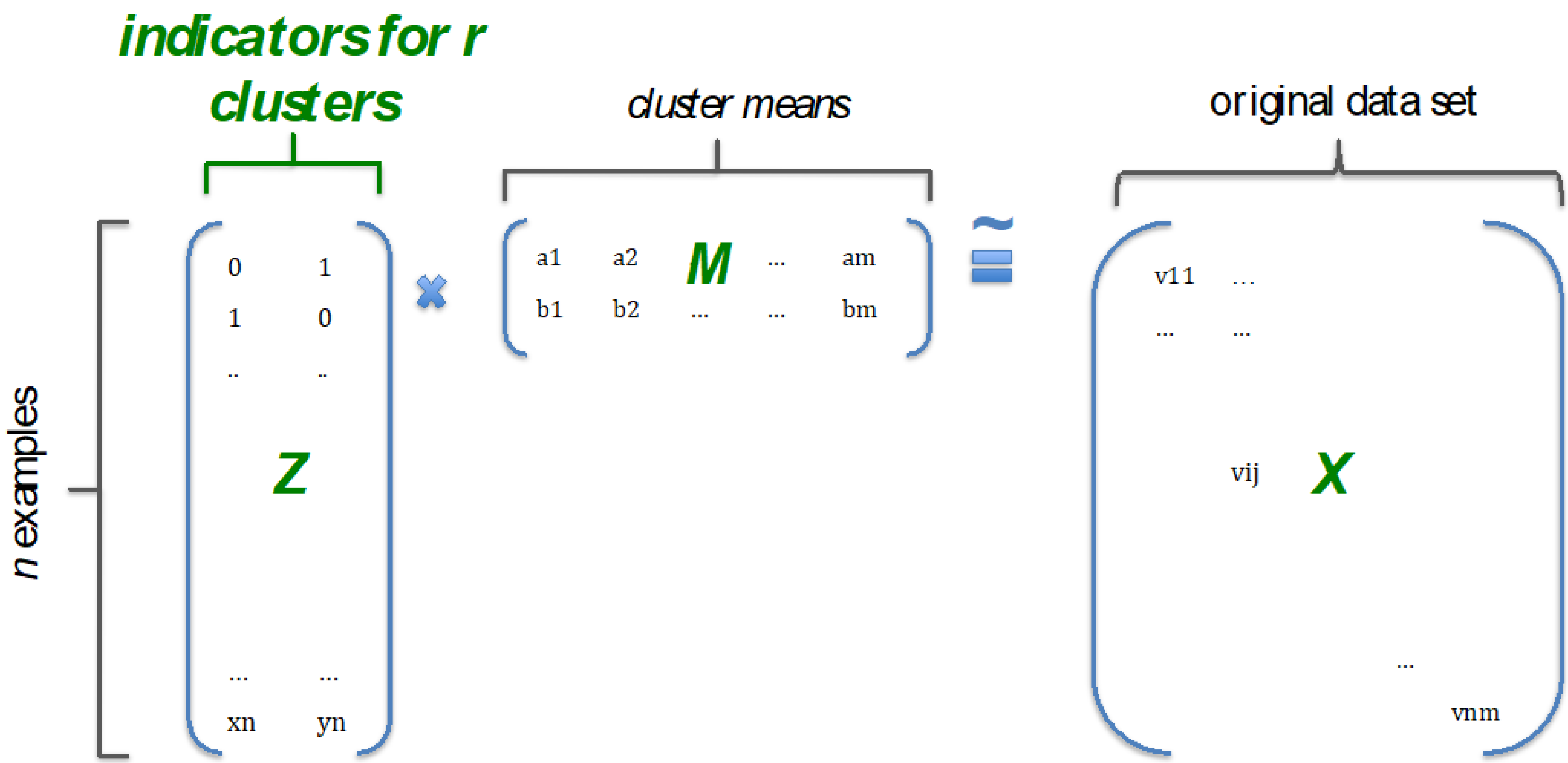
Input	Output
00000001	00000001
00000010	00000010
00000101	00000100
00001000	00001000
00010000	00010000
00100000	00100000
01000000	01000000
10000000	10000000

- Using the following network:

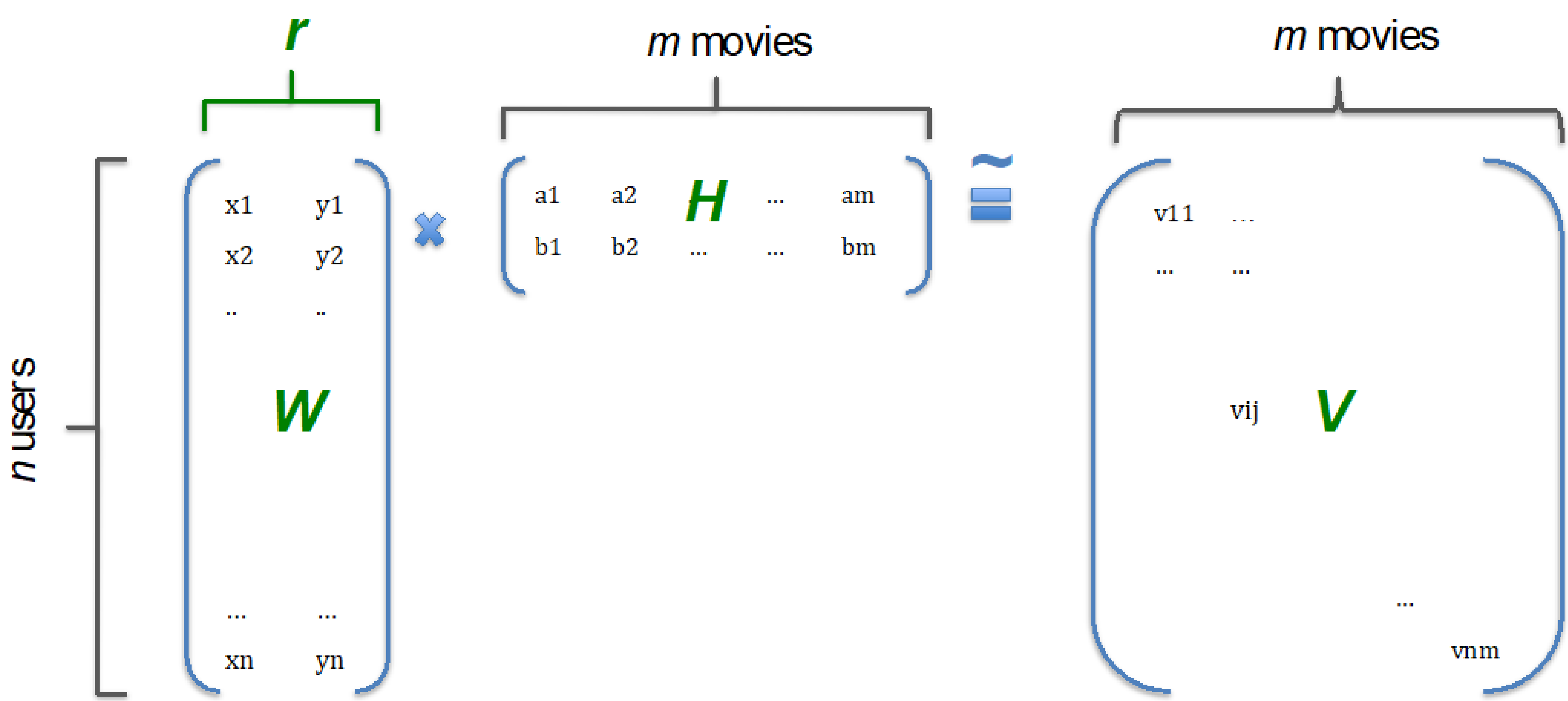


- With *linear* hidden units, how do the weights match up to W and H?

Like K-Means Clustering



Recovering Latent Factors in Matrix



V[i,j] = user i's rating of movie j

Summary

- Recommender systems solve many **real-world** (*large-scale) **problems**
- Collaborative filtering by Matrix Factorization (MF) is an **efficient** and **effective** approach
- MF is just another example of a **common recipe:**
    1. define a model
    2. define an objective function
    3. optimize with SGD