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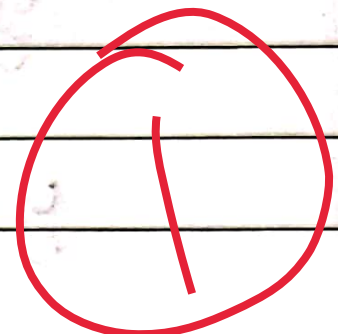
Ans-2

(a) The boundary conditions are:

? $y_{tt}(x,t) = a^2 y_{xx}(x,t) ; x > 0, t > 0$

$$y(x,0) = 0, \quad y_t(x,0) = 0 ; x \geq 0$$

$$y(0,t) = f(t) ; t \geq 0$$



$$(b) \quad y(x,t) = \phi(x+at) + \psi(x-at)$$

Using boundary conditions;

$$y(x,0) = \phi(x) + \psi(x) = 0 \Rightarrow \boxed{\phi(x) = -\psi(x)} \quad \text{--- (1)}$$

$$y_t(x,0) = a\phi'(x+at) - a\psi'(x-at) = 0$$

$$\therefore \phi'(x) - \psi'(x) = 0$$

integrate w.r.t. x ,

$$\boxed{\phi(x) - \psi(x) = C_1}$$

\therefore where C_1 is an integration const.

from (1),

$$\phi(x) + \phi(x) = C_1 \quad ?$$

$$\boxed{\phi(x) = \frac{C_1}{2} = C} \quad X$$

$$\boxed{\psi(x) = -\phi(x) = -C} \quad X$$

Hence,

$$\exists \text{ a } C \text{ s.t. } \underline{\phi(x) = C} \text{ \& \& } \underline{\psi(x) = -C}$$

0.5

(c) Now,

$$y(x,t) = \begin{cases} \int_0^0 f(t - x/a) & x \geq at \\ f(t - x/a) & x \leq at \end{cases}$$

The part of String to the right of the point $x=at$ on the x -axis is unaffected by the movement of the string prior to time t .

$$y(x,t) = \phi(x+at) + \psi(x-at), x \geq 0$$

from prev. result, $\psi(x) = f(x/a) - c, x \geq 0$

$$\psi(x) = f(x/a) - c$$

$$\Rightarrow y(x) = -c, x \geq 0$$

$$\Rightarrow y(x-at) = -c, x \geq at \quad \text{--- (1)}$$

$$\text{Also } \phi(x) = c, x \geq 0$$

$$\Rightarrow \phi(x+at) = c$$

domain?

$$\Rightarrow \psi(x-at) + \phi(x+at) = 0, x \geq at$$

$$\Rightarrow y(x,t) = 0, x \geq at$$

$$\text{Now } \phi(x) = c, x \geq 0$$

$$\Rightarrow \phi(x+at) = c, x \geq -at$$

$$\text{and } \psi(-x) = f(x/a) - c, x \geq 0$$

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$$\Rightarrow \psi(x) = f(-x/a) - c, \quad x \leq 0$$

$$\psi(x-at) = f(t-x/a) - c, \quad x \leq at$$

$$\Rightarrow \cancel{\psi(x-at) = f(t-x/a)}$$

$$\Rightarrow \boxed{\psi(x-at) + \phi(x+at) = f(t-x/a)} \quad x \leq at$$

