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Ans- 4 (a)

$f(z)$ is entire fun^n.

$\Rightarrow f(z)$ analytic in $\mathbb{C}$

$$\Rightarrow f(z) = \sum_{n=0}^{\infty} a_n z^n, \quad z \in \mathbb{C} \quad \because \text{Taylor Series.}$$

$$\Rightarrow a_n = \frac{1}{2\pi i} \int_c \frac{f(z)}{z^{n+1}} \cdot dz$$

$\therefore$ c is positively oriented simple closed contour enclosing 0 and lying $\mathbb{C}$.

$\forall \ r > 0 \quad |z| = r$

$$\left| \frac{f(z)}{z^{n+1}} \right| \le \frac{1 + r^{1/2}}{r^{n+1}} \qquad \boxed{|f(z)| \le 1 + |z|^{1/2}}$$

$|z| = r$

2

$$= \frac{1}{r^{n+1}} + r^{-n-1/2}$$

Answer is matching word by word with other students

$$\therefore |2\pi i a_n| = \left| \int_c \frac{f(z)}{z^{n+1}} \cdot dz \right| \qquad c: |z| = r, \ r > 0$$

$$\le \left(\frac{1}{r^{n+1}} + r^{-n-1/2} \right) \times 2\pi r \qquad \text{By LM formula}$$

$$= 2\pi \left(\frac{1}{r^n} + \frac{1}{r^{n-1/2}} \right)$$

Thus, for each n.

$$|a_n| \le \frac{1}{r^n} + \frac{1}{r^{n-1/2}} \quad \forall \ r > 0$$

for $n - 1/2 > 0$, $\frac{1}{r^n} + \frac{1}{r^{n-1/2}} \to 0$ as $r \to \infty$

Hence, $a_n = 0 \ \forall \ n > K$. Thus $\boxed{f(z) = \sum_{n=0}^{\infty} a_n z^n} \Rightarrow \boxed{f(z) = a_n}$

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(b) Given C is a simple closed curve in clockwise sense enclosing '0'.

f is analytic within and on C expect at '0'.

0 is a pole of order 'm' of f.

$$\int_{c} \frac{f'(z)}{f(z)} \cdot dz = -\int_{-c} \frac{f'(z)}{f(z)} \cdot dz$$

$\hookrightarrow$ counter clockwise dirn.

0 is pole of order m of f.

$$\Rightarrow f(z) = \frac{\psi(z)}{(z-z_0)^m}, \quad z_0 = 0$$

$$\Rightarrow f(z) = \frac{\psi(z)}{z^m}, \quad \psi(z) \text{ is analytic at } z=0.$$

$$f'(z) = \frac{\psi'(z) z^m - m z^{m-1} \psi(z)}{z^{2m}} = \frac{\psi'(z)}{z^m} - \frac{m \psi(z)}{z^{m+1}}$$

$$\frac{f'(z)}{f(z)} = \frac{\psi'(z)}{\psi(z)} - \frac{m}{z}$$

$$\frac{\psi'(z)}{\psi(z)} = \phi(z) \longrightarrow \text{analytic and continous at } \boxed{z=0}$$

2 $$\operatorname{Res}\left(\frac{f'(z)}{f(z)}; 0\right) = \lim_{z \to 0} \left[\frac{\psi'(z)}{\psi(z)} - \frac{m}{z}\right]$$

$$= 0 - m = \underline{\underline{-m}}$$

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Continue 4(b)

$$\int_{-c} \frac{f'(z)}{f(z)} \cdot dz = 2\pi i \times Res\left(\frac{f'(z)}{f(z)} ; 0\right)$$

$$= 2\pi i(-m) = -2\pi i m$$

$$\therefore \int_{c} \frac{f'(z)}{f(z)} \cdot dz = 2\pi i m$$