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MA-203

Ans-1

Given,

$$\begin{bmatrix} u \\ v \end{bmatrix}_t = \begin{bmatrix} 1 & 1/2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}_{xx}, \quad t>0, \quad 0<x<\pi$$

$$u_t = u_{xx} + \frac{v_{xx}}{2}, \quad v_t = 2v_{xx}$$

Solving, $v_t = 2v_{xx}$

This is similar to heat Eq^n., $\alpha^2 = 2$

Let $v(x,t) = X(x)\,T(t)$

$\Rightarrow XT' = 2X''T$

$\Rightarrow \dfrac{X}{X''} = \dfrac{2T}{T'}$

$\therefore$ As RHS and LHS dependent on 2 diff. variables.

$\Rightarrow X'' = KX$

$\Rightarrow T' = 2KT$

$\therefore$ 3 Cases arise :

$K=0$

$K=\lambda^2$ i.e. $K>0$

$K=-\lambda^2$ i.e. $K<0$

Kuldeep Singh
190001030
Singh.

Date
Page 2

continue-①

➤ $K=0$

$V(x,t) = A_1 x + B_1$

$V(x,0) = \sin(x)$ and $V(0,t) = V(\pi,t) = 0$

$V(0,t) = B_1 = 0$

$V(\pi,t) = A_1 \pi = 0 \Rightarrow \boxed{A_1 = 0}$

$V(x,t) = 0$

$\therefore$ $K=0$, Sol^n does not exist.

➤ $K>0$ i.e $K = \lambda^2$

$V(x,t) = (A_1 e^{\lambda x} + A_2 e^{-\lambda x}) e^{\lambda^2 2t}$

$V(0,t) = (A_1 + A_2) e^{\lambda^2 2t} = 0 \Rightarrow \boxed{A_1 = -A_2}$

$V(x,t) = A_1 (e^{\lambda x} - e^{-\lambda x}) e^{2\lambda^2 t}$

$V(\pi,t) = A_1 (e^{\lambda\pi} - e^{-\lambda\pi}) e^{2\lambda^2 t} = 0 \Rightarrow \boxed{A_1 = 0}$

$V(x,t) = 0$

$\therefore$ $K>0$, Sol^n does not exist.

➤ $K<0$

$V(x,t) = A_1 \cos\lambda x + A_2 \sin\lambda x) e^{-\lambda^2 2t}$

$V(0,t) = 0$

$= A_1 e^{-\lambda^2 2t} = 0 \Rightarrow \boxed{A_1 = 0}$

$V(\pi,t) = A_2 \sin \pi\lambda\, e^{-\lambda^2 2t} = 0$

$\Rightarrow A_2 = 0$ or $\pi\lambda = n\pi \Rightarrow \boxed{\lambda = n}$

$\Longrightarrow A_2$ can't be 0.

$\therefore V(x,t) = A_2 (\sin nx)\, e^{-n^2 2t}$

$V(x,t) = \sum_{n=0}^{\infty} a_n \sin(nx)\, e^{-n^2 2t}$

$V(x,0) = \sum a_n \sin(nx) = \sin(x)$

Kuldeep Singh
190001030

Date
Page (3)

continue-①

$\Rightarrow a_n = 0 \quad \forall \; n \neq 1$

$\Rightarrow \boxed{a_1 = 1}$

$$\boxed{V(x,t) = \sin(x)\, e^{-2t}}$$

$\rightarrow \quad u_t = u_{xx} + \frac{V_{xx}}{2}$

$u_t = u_{xx} + \frac{V_t}{4}$

$u_t = u_{xx} + \sin(x)(-2)\, e^{-2t}$

$u_t = u_{xx} - 2\sin(x)\, e^{-2t}$

Let $\quad u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin(nx)$

$\rightarrow u(0,t) = u(\pi,t) = 0$ is satisfied.

$\rightarrow \sum u_n'(t)\sin(nx) = \sum u_n(t)(-n^2)\sin(nx) - 2\sin(x)e^{-2t}$

$\Rightarrow \sum \sin(nx)\left[u_n'(t) + u_n(t)\, n^2\right] = -2\sin(x)\, e^{-2t}$

Let, $\quad u_n(t) = 0 \quad \forall \; n \neq 1$

$\rightarrow \sin(x)\left[u_1'(t) + u_1(t)\right] = -2\sin(x)\, e^{-2t}$

$\Rightarrow \frac{\partial u_1}{\partial t} + u_1 = -2e^{-2t}$

$\Rightarrow e^{t}\frac{\partial u_1}{\partial t} + e^{t}u_1 = -2e^{-t}$

$\Rightarrow \int \partial(u_1 e^{t}) = \int -2e^{-t}\, dt$

Kuldeep Singh
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Date:
Page: 4

$$u_1 e^t = 2e^{-t} + c$$
$$u_1 = 2e^{-2t} + c\, e^{-t}$$

$$u(x,t) = \sum u_n(t) \sin(nx)$$
$$= (2e^{-2t} + ce^{-t}) \sin(x)$$

$$u(x,0) = \sin x$$
$$(2+c) \sin(x) = \sin(x) \Rightarrow 2+c=1 \Rightarrow \boxed{c=-1}$$

$$\therefore u(x,t) = (2e^{-2t} - e^{-t}) \sin(x)$$

4.5