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RANKA

(9) Let, f be analytic function. S.t.

$$f(z_n) = (-1)^n z_n$$

$\{z_n\} \subset N(0,1)$, $z_n \neq 0$ $z_n \to 0$ as $n \to \infty$

Let, $z_n \to 0$
$n \to \infty$
$\Rightarrow$ $z = 0$ is limit point of $\{z_{2n}\}$
$0 \in N(0,1)$

Let, $g(z) = z$, $f(z) = (-1)^n z_n$

$S = \{z_{2n}\}$ $n \geq 0$ $D = N(0,1)$

$\Rightarrow$ 0 is limit point of S and 0 belongs to $N(0,1)$.

By Uniqueness theorem.
$\Rightarrow$ $f(z) = g(z) = z$ $\Rightarrow$ $f(z) = z$

Answer matches word by word with the answer of other students, even the mistakes. Marks deducted.

But for $n = 2n+1$
$f(z_n) = -z_n = z_n$

1.25

$\Rightarrow z_n = 0$
But, 0 is not in $\{z_n\}$

So, Contradiction!
$\Rightarrow$ There does not exist any analytic function.

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In other, $p(z)e^{1/z}$ must have singularities Since, its Laurent Expansion contains terms of negative power.

So, $p(z) = 0$, their no Polynomial S.t. $p(z)e^{1/z}$ is entire.

(C) Let D: domain $|z| \le 1$ and contour C enclosing D be $C : |z| = 1$ as well.

Now
$|1-z|$ is analytic $\forall\ z \in \mathbb{C}$
$\Rightarrow$ $|1-z|$ analytic in D.

- $|1-z|$ is continous $\forall\ z \in \mathbb{C}$ and hence, at $|z| = 1$ to D.
- $|1-z|$ is a non-zero.
- Domain $|z| \le 1$ is bounded with $M = 1$.

All the condition for maximum modulous theorem are satisfied and Hence, maximum of $|1-z|$ lies in $|z| = 1$

$|1 + (-z)| \le |1| + |-z|$,
$\le |1| + |z|$
$= 2$

0

$\boxed{a = 1}$, $\boxed{b = -2}$

$|a+b| \le |a| + |b|$

why??

Hence, Maximum value is $\boxed{2}$. The Point is $\boxed{z = -1}$

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In other, $p(z)e^{1/z}$ must have singularities Since, its Laurent Expansion contains terms of negative power.

0.5

So, $p(z) = 0$, their no Polynomial S.t. $p(z)e^{1/z}$ is entire.

(c) Let D: domain $|z| \le 1$ and contour C enclosing D be $C : |z| = 1$ as well.

Now

$|1-z|$ is analytic $\forall$ $z \in \mathbb{C}$

$\Rightarrow$ $|1-z|$ analytic in D.

- $|1-z|$ is continous $\forall$ $z \in \mathbb{C}$ and hence, at $|z| = 1$ to D.
- $|1-z|$ is a non-zero.
- Domain $|z| \le 1$ is bounded with $M = 1$.

All the condition for maximum modulous theorem are satisfied and Hence, maximum of $|1-z|$ lies in $|z| = 1$

$$|1+(-z)| \le |1| + |-z|$$
$$\le |1| + |z|$$
$$= 2$$

$\boxed{a = 1}$, $\boxed{b = -2}$

$|a+b| \le |a| + |b|$

Hence, Maximum value is $\underline{2}$.

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