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**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: P(X > 50) = 1 - P(X ≤ 50).

Z = (X - 45)/8.0=(50-45)/8.0

Thus the question can be answered by using the normal table to find

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0) = P(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer: False. Since the *μ* is given 38.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer: As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). And 2X1 will just scale the normal distribution by 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

Answer: 48.5, 151.5

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer:

Mean=5+7=12

Mean Profit=12\*45=540 Million

SD=32+42=25=25\*45=315 Million

stats.norm.interval(0.95,540,315)

Range is (-77.38865513011706, 1157.388655130117) Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer:

5th percentile=-1.64

X=μ + Zσ =540+(-1.64)\*315

npround(X,2)

=23.4 Million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: Division 2 with distribution N(7, 42)