## Homework 4

Anushka Menon and Yashavee Goel

```
In [229... %matplotlib inline
         import numpy as np
         import matplotlib.pyplot as plt
         import scipy.special as sp
         import math
In [230... m pi = np.array([0.176, 0.234, 0.260, 0.284, 0.324])
         f = np.array([960, 1025, 1055, 1085, 1130])
         delta f = np.array([25, 20, 15, 10, 8])
         def func(m pi, Phi):
             return Phi[0]+Phi[1]*m pi+Phi[2]*m pi**2
         def chi sq(f, delta f):
             for i in range(5):
                  return np.sum(0.5*(f[i]-func(m pi, Phi))/delta f[i])
In [235...
          def P_dot(Phi, m_pi, f, delta_f):
             p dot = []
             for i in range(3):
                 sum = 0
                 for j in range(5):
                      sum += ((beta*(f[j]-func(m pi, Phi))*m pi[j]**i)/delta f[j])
                 p dot.append(sum)
              return np.array(p dot)
         def leapfrog(N_md, P_0, Phi_0, J, beta, h, N):
             eps = 1/N md
             P = P 0.astype("float")
             Phi = Phi_0.astype("float")
             Phi += eps/2*Phi
             for i in range(N md-1):
                 P += eps*P_dot(Phi, m_pi, f, delta_f)
                 Phi += eps*P
             P -= eps*P_dot(Phi, m_pi, f, delta_f)
             Phi += eps/2*Phi
             return P, Phi
         def Hamiltonian(P, Phi, J, beta, h, N):
              return P[0]**2/2+P[1]**2/2+P[2]**2/2+chi_sq(f, delta_f)
In [236... P 0 = np.array([1,2,3])
         Phi 0 = np.array([1,2,3])
         beta = 1000
         N md = 20
         N_cfg = 10000
         h = 0.5
         N = 20
         J = 1/N
         Phi = np.array([800,800,600])
In [237...diff = []
```

```
range_1 = range(1,100)
         for N md in range 1:
             arr = leapfrog(N_md, P_0, Phi_0, J, beta, h, N)
             P_prime = arr[0]
                                     # We need final values of p f and phi f to ca
             Phi prime = arr[1]
             H f = Hamiltonian(P 0, Phi 0, J, beta, h, N)
             H_0 = Hamiltonian(P_prime, Phi_prime, J, beta, h, N)
               The following is the quantity we want on y-axis
             diff.append(abs((H f-H 0)/H 0))
         ValueError
                                                 Traceback (most recent call last)
         Cell In [237], line 5
              2 range 1 = range(1, 100)
              4 for N md in range 1:
                    arr = leapfrog(N_md, P_0, Phi_0, J, beta, h, N)
                    P_prime = arr[0]  # We need final values of p_f and phi
               6
         _f to calculate H_f
                    Phi prime = arr[1]
              7
         Cell In [235], line 19, in leapfrog(N md, P 0, Phi 0, J, beta, h, N)
              P += eps*P dot(Phi)
              18
                    Phi += eps*P
         ---> 19 P -= eps*P dot(Phi)
              20 Phi += eps/2*Phi
              21 return P, Phi
         ValueError: operands could not be broadcast together with shapes (3,) (3,5)
        (3,)
In [234... plt.figure(figsize=(12,10))
         plt.yscale('log')
         plt.grid()
         plt.xlabel("$N {\md}$")
         plt.ylabel("$H_{\f}$ - $H_{\o}$ / $H_{\o}$")
         plt.title("Convergence of leap frog integrator", size =12)
         plt.plot(range 1, diff, 'o')
         plt.show()
```

```
Traceback (most recent call last)
ValueError
Cell In [234], line 7
      5 plt.ylabel("$H_{\f}$ - $H_{\o}$ / $H_{\o}$")
      6 plt.title("Convergence of leap frog integrator", size =12)
----> 7 plt.plot(range 1, diff, 'o')
      8 plt.show()
File ~/.local/lib/python3.8/site-packages/matplotlib/pyplot.py:2730, in plo
t(scalex, scaley, data, *args, **kwargs)
   2728 @ copy docstring and deprecators(Axes.plot)
   2729 def plot(*args, scalex=True, scaley=True, data=None, **kwargs):
-> 2730
            return gca().plot(
  2731
                *args, scalex=scalex, scaley=scaley,
                **({"data": data} if data is not None else {}), **kwargs)
   2732
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/ axes.py:1662, in
Axes.plot(self, scalex, scaley, data, *args, **kwargs)
   1419 """
   1420 Plot y versus x as lines and/or markers.
  1421
   (\ldots)
   1659 (``'green'``) or hex strings (``'#008000'``).
   1660 """
   1661 kwargs = cbook.normalize kwargs(kwargs, mlines.Line2D)
-> 1662 lines = [*self. get lines(*args, data=data, **kwargs)]
   1663 for line in lines:
   1664
            self.add line(line)
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/ base.py:311, in
process plot var args. call (self, data, *args, **kwargs)
    309
            this += args[0],
            args = args[1:]
    310
--> 311 yield from self. plot args(
            this, kwargs, ambiguous fmt datakey=ambiguous fmt datakey)
    312
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/ base.py:504, in
process plot var args. plot args(self, tup, kwargs, return kwargs, ambiguo
us fmt datakey)
            self.axes.yaxis.update_units(y)
    503 if x.shape[0] != y.shape[0]:
--> 504
            raise ValueError(f"x and y must have same first dimension, but
                             f"have shapes {x.shape} and {y.shape}")
    505
    506 if x.ndim > 2 or y.ndim > 2:
            raise ValueError(f"x and y can be no greater than 2D, but have
    507
    508
                             f"shapes {x.shape} and {y.shape}")
ValueError: x and y must have same first dimension, but have shapes (99,) a
nd(0,)
Error in callback <function draw all if interactive at 0x7f3738102670> (fo
r post execute):
```

```
Traceback (most recent call last)
ParseException
ParseException: Expected {accent | symbol | function | operatorname | group
| frac | dfrac | binom | genfrac | overset | underset | sqrt | overline}, f
ound '\x0c' (at char 3), (line:1, col:4)
During handling of the above exception, another exception occurred:
                                          Traceback (most recent call last)
ParseSyntaxException
File ~/.local/lib/python3.8/site-packages/matplotlib/ mathtext.py:1983, in
Parser.parse(self, s, fonts object, fontsize, dpi)
  1982 try:
-> 1983
           result = self. expression.parseString(s)
  1984 except ParseBaseException as err:
File ~/.local/lib/python3.8/site-packages/pyparsing/core.py:1141, in Parser
Element.parse string(self, instring, parse all, parseAll)
  1139
           else:
  1140
               # catch and re-raise exception from here, clearing out pypa
rsing internal stack trace
-> 1141
               raise exc.with traceback(None)
  1142 else:
ParseSyntaxException: Expected {accent | symbol | function | operatorname |
group | frac | dfrac | binom | genfrac | overset | underset | sqrt | overli
ne\}, found '\x0c' (at char 3), (line:1, col:4)
The above exception was the direct cause of the following exception:
ValueError
                                          Traceback (most recent call last)
File ~/.local/lib/python3.8/site-packages/matplotlib/pyplot.py:119, in dra
w all if interactive()
   117 def draw all if interactive():
           if matplotlib.is interactive():
    118
--> 119
                draw all()
File ~/.local/lib/python3.8/site-packages/matplotlib/ pylab helpers.py:132,
in Gcf.draw all(cls, force)
    130 for manager in cls.get all fig managers():
   131
           if force or manager.canvas.figure.stale:
--> 132
                manager.canvas.draw_idle()
File ~/.local/lib/python3.8/site-packages/matplotlib/backend bases.py:2054,
in FigureCanvasBase.draw idle(self, *args, **kwargs)
  2052 if not self._is_idle_drawing:
  2053
           with self. idle draw cntx():
-> 2054
                self.draw(*args, **kwargs)
File ~/.local/lib/python3.8/site-packages/matplotlib/backends/backend agg.p
y:408, in FigureCanvasAgg.draw(self)
    404 # Acquire a lock on the shared font cache.
    405 with RendererAgg.lock, \
   406
             (self.toolbar._wait_cursor_for_draw_cm() if self.toolbar
    407
             else nullcontext()):
--> 408
            self.figure.draw(self.renderer)
    409
           # A GUI class may be need to update a window using this draw, s
0
            # don't forget to call the superclass.
    410
    411
           super().draw()
File ~/.local/lib/python3.8/site-packages/matplotlib/artist.py:74, in fina
```

```
lize rasterization.<locals>.draw wrapper(artist, renderer, *args, **kwargs)
     72 @wraps(draw)
     73 def draw wrapper(artist, renderer, *args, **kwargs):
            result = draw(artist, renderer, *args, **kwargs)
     75
            if renderer. rasterizing:
     76
                renderer.stop rasterizing()
File ~/.local/lib/python3.8/site-packages/matplotlib/artist.py:51, in allow
rasterization.<locals>.draw wrapper(artist, renderer)
     48
            if artist.get agg filter() is not None:
     49
                renderer.start filter()
---> 51
            return draw(artist, renderer)
     52 finally:
            if artist.get agg filter() is not None:
File ~/.local/lib/python3.8/site-packages/matplotlib/figure.py:3074, in Fig
ure.draw(self, renderer)
   3071
                # ValueError can occur when resizing a window.
   3073 self.patch.draw(renderer)
-> 3074 mimage. draw list compositing images(
            renderer, self, artists, self.suppressComposite)
   3077 for sfig in self.subfigs:
   3078
            sfig.draw(renderer)
File ~/.local/lib/python3.8/site-packages/matplotlib/image.py:131, in draw
_list_compositing_images(renderer, parent, artists, suppress_composite)
    129 if not composite or not has images:
    130
            for a in artists:
--> 131
                a.draw(renderer)
    132 else:
           # Composite any adjacent images together
    133
    134
            image group = []
File ~/.local/lib/python3.8/site-packages/matplotlib/artist.py:51, in allow
rasterization.<locals>.draw wrapper(artist, renderer)
            if artist.get agg filter() is not None:
     48
     49
                renderer.start_filter()
---> 51
            return draw(artist, renderer)
     52 finally:
            if artist.get agg filter() is not None:
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/ base.py:3071, in
AxesBase.draw(self, renderer)
   3068
            for spine in self.spines.values():
   3069
               artists.remove(spine)
-> 3071 self. update title position(renderer)
   3073 if not self.axison:
            for _axis in self._axis_map.values():
   3074
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/_base.py:3015, in
AxesBase. update title position(self, renderer)
   3013 top = max(top, bb.ymax)
   3014 if title.get text():
            ax.yaxis.get tightbbox(renderer) # update offsetText
-> 3015
   3016
            if ax.yaxis.offsetText.get text():
   3017
                bb = ax.yaxis.offsetText.get_tightbbox(renderer)
File ~/.local/lib/python3.8/site-packages/matplotlib/axis.py:1269, in Axis.
get_tightbbox(self, renderer, for_layout_only)
   1267 # take care of label
   1268 if self.label.get_visible():
```

```
bb = self.label.get window extent(renderer)
-> 1269
  1270
           # for constrained/tight layout, we want to ignore the label's
           # width/height because the adjustments they make can't be impro
  1271
ved.
           # this code collapses the relevant direction
  1272
  1273
           if for layout only:
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:915, in Text.g
et window extent(self, renderer, dpi)
   910
           raise RuntimeError(
   911
                "Cannot get window extent of text w/o renderer. You likely
                "want to call 'figure.draw without rendering()' first.")
   912
   914 with cbook. setattr cm(self.figure, dpi=dpi):
--> 915
           bbox, info, descent = self._get_layout(self._renderer)
           x, y = self.get_unitless_position()
   916
   917
           x, y = self.get transform().transform((x, y))
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:321, in Text.
get layout(self, renderer)
   319 clean line, ismath = self. preprocess math(line)
   320 if clean line:
          w, h, d = get text metrics with cache(
--> 321
                renderer, clean line, self. fontproperties,
   322
                ismath=ismath, dpi=self.figure.dpi)
   323
   324 else:
   325
        w = h = d = 0
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:97, in get te
xt metrics with cache(renderer, text, fontprop, ismath, dpi)
     94 """Call ``renderer.get_text_width_height_descent``, caching the res
ults."""
     95 # Cached based on a copy of fontprop so that later in-place mutatio
ns of
     96 # the passed-in argument do not mess up the cache.
---> 97 return get text metrics with cache impl(
     98
           weakref.ref(renderer), text, fontprop.copy(), ismath, dpi)
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:105, in get t
ext metrics with cache impl(renderer ref, text, fontprop, ismath, dpi)
    101 @functools.lru cache(4096)
   102 def get text metrics with cache impl(
                renderer_ref, text, fontprop, ismath, dpi):
   103
           # dpi is unused, but participates in cache invalidation (via th
   104
e renderer).
           return renderer ref().get text width height descent(text, fontp
--> 105
rop, ismath)
File ~/.local/lib/python3.8/site-packages/matplotlib/backends/backend agg.p
y:238, in RendererAgg.get_text_width_height_descent(self, s, prop, ismath)
   234
           return w, h, d
   236 if ismath:
           ox, oy, width, height, descent, font image = \
--> 238
                self.mathtext_parser.parse(s, self.dpi, prop)
            return width, height, descent
   239
   241 font = self._prepare_font(prop)
File ~/.local/lib/python3.8/site-packages/matplotlib/mathtext.py:226, in Ma
thTextParser.parse(self, s, dpi, prop)
   222 # lru cache can't decorate parse() directly because prop
   223 # is mutable; key the cache using an internal copy (see
```

```
224 # text._get_text_metrics_with_cache for a similar case).
   225 prop = prop.copy() if prop is not None else None
--> 226 return self. parse cached(s, dpi, prop)
File ~/.local/lib/python3.8/site-packages/matplotlib/mathtext.py:247, in Ma
thTextParser. parse cached(self, s, dpi, prop)
    244 if self. parser is None: # Cache the parser globally.
           self. class . parser = mathtext.Parser()
--> 247 box = self. parser.parse(s, fontset, fontsize, dpi)
   248 output = mathtext.ship(box)
    249 if self. output type == "vector":
File ~/.local/lib/python3.8/site-packages/matplotlib/ mathtext.py:1985, in
Parser.parse(self, s, fonts object, fontsize, dpi)
           result = self. expression.parseString(s)
  1984 except ParseBaseException as err:
-> 1985
           raise ValueError("\n".join(["",
   1986
                                        err.line,
                                        " " * (err.column - 1) + "^",
  1987
  1988
                                        str(err)])) from err
  1989 self._state_stack = None
  1990 self. in subscript or superscript = False
ValueError:
H {}
Expected {accent | symbol | function | operatorname | group | frac | dfrac
| binom | genfrac | overset | underset | sqrt | overline}, found '\x0c'
(at char 3), (line:1, col:4)
```

```
ParseException
                                          Traceback (most recent call last)
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ound '\x0c' (at char 3), (line:1, col:4)
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ParseSyntaxException
File ~/.local/lib/python3.8/site-packages/matplotlib/ mathtext.py:1983, in
Parser.parse(self, s, fonts object, fontsize, dpi)
  1982 try:
           result = self. expression.parseString(s)
-> 1983
  1984 except ParseBaseException as err:
File ~/.local/lib/python3.8/site-packages/pyparsing/core.py:1141, in Parser
Element.parse string(self, instring, parse all, parseAll)
  1139
           else:
               # catch and re-raise exception from here, clearing out pypa
  1140
rsing internal stack trace
-> 1141
               raise exc.with traceback(None)
  1142 else:
ParseSyntaxException: Expected {accent | symbol | function | operatorname |
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ne\}, found '\x0c' (at char 3), (line:1, col:4)
The above exception was the direct cause of the following exception:
ValueError
                                          Traceback (most recent call last)
File ~/.local/lib/python3.8/site-packages/IPython/core/formatters.py:339, i
n BaseFormatter. call (self, obj)
   337
           pass
    338 else:
           return printer(obj)
--> 339
   340 # Finally look for special method names
   341 method = get real method(obj, self.print method)
File ~/.local/lib/python3.8/site-packages/IPython/core/pylabtools.py:151, i
n print_figure(fig, fmt, bbox_inches, base64, **kwargs)
            from matplotlib.backend bases import FigureCanvasBase
    148
   149
           FigureCanvasBase(fig)
--> 151 fig.canvas.print figure(bytes io, **kw)
    152 data = bytes io.getvalue()
    153 if fmt == 'svg':
File ~/.local/lib/python3.8/site-packages/matplotlib/backend bases.py:2314,
in FigureCanvasBase.print_figure(self, filename, dpi, facecolor, edgecolor,
orientation, format, bbox inches, pad inches, bbox extra artists, backend,
**kwarqs)
  2308
            renderer = _get_renderer(
  2309
                self.figure,
  2310
                functools.partial(
  2311
                    print_method, orientation=orientation)
  2312
           with getattr(renderer, "_draw_disabled", nullcontext)():
  2313
-> 2314
               self.figure.draw(renderer)
  2316 if bbox inches:
  2317
           if bbox inches == "tight":
```

File ~/.local/lib/python3.8/site-packages/matplotlib/artist.py:74, in fina

```
lize rasterization.<locals>.draw wrapper(artist, renderer, *args, **kwargs)
     72 @wraps(draw)
     73 def draw wrapper(artist, renderer, *args, **kwargs):
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            if renderer. rasterizing:
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File ~/.local/lib/python3.8/site-packages/matplotlib/artist.py:51, in allow
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            if artist.get agg filter() is not None:
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   3071
                # ValueError can occur when resizing a window.
   3073 self.patch.draw(renderer)
-> 3074 mimage. draw list compositing images(
            renderer, self, artists, self.suppressComposite)
   3077 for sfig in self.subfigs:
   3078
            sfig.draw(renderer)
File ~/.local/lib/python3.8/site-packages/matplotlib/image.py:131, in draw
_list_compositing_images(renderer, parent, artists, suppress_composite)
    129 if not composite or not has images:
    130
            for a in artists:
--> 131
                a.draw(renderer)
    132 else:
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    133
    134
            image group = []
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File ~/.local/lib/python3.8/site-packages/matplotlib/axes/ base.py:3071, in
AxesBase.draw(self, renderer)
   3068
            for spine in self.spines.values():
   3069
               artists.remove(spine)
-> 3071 self. update title position(renderer)
   3073 if not self.axison:
            for _axis in self._axis_map.values():
   3074
File ~/.local/lib/python3.8/site-packages/matplotlib/axes/_base.py:3015, in
AxesBase. update title position(self, renderer)
   3013 top = max(top, bb.ymax)
   3014 if title.get text():
            ax.yaxis.get tightbbox(renderer) # update offsetText
-> 3015
   3016
            if ax.yaxis.offsetText.get text():
   3017
                bb = ax.yaxis.offsetText.get_tightbbox(renderer)
File ~/.local/lib/python3.8/site-packages/matplotlib/axis.py:1269, in Axis.
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   1267 # take care of label
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```
bb = self.label.get window extent(renderer)
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  1270
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           # width/height because the adjustments they make can't be impro
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  1272
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File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:915, in Text.g
et window extent(self, renderer, dpi)
   910
           raise RuntimeError(
   911
                "Cannot get window extent of text w/o renderer. You likely
                "want to call 'figure.draw without rendering()' first.")
   912
   914 with cbook. setattr cm(self.figure, dpi=dpi):
--> 915
           bbox, info, descent = self._get_layout(self._renderer)
           x, y = self.get_unitless_position()
   916
   917
           x, y = self.get transform().transform((x, y))
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:321, in Text.
get layout(self, renderer)
   319 clean line, ismath = self. preprocess math(line)
   320 if clean line:
          w, h, d = get text metrics with cache(
--> 321
                renderer, clean line, self. fontproperties,
   322
                ismath=ismath, dpi=self.figure.dpi)
   323
   324 else:
   325
        w = h = d = 0
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:97, in get te
xt metrics with cache(renderer, text, fontprop, ismath, dpi)
     94 """Call ``renderer.get_text_width_height_descent``, caching the res
ults."""
     95 # Cached based on a copy of fontprop so that later in-place mutatio
ns of
     96 # the passed-in argument do not mess up the cache.
---> 97 return get text metrics with cache impl(
     98
           weakref.ref(renderer), text, fontprop.copy(), ismath, dpi)
File ~/.local/lib/python3.8/site-packages/matplotlib/text.py:105, in get t
ext metrics with cache impl(renderer ref, text, fontprop, ismath, dpi)
    101 @functools.lru cache(4096)
   102 def get text metrics with cache impl(
                renderer_ref, text, fontprop, ismath, dpi):
   103
           # dpi is unused, but participates in cache invalidation (via th
   104
e renderer).
           return renderer ref().get text width height descent(text, fontp
--> 105
rop, ismath)
File ~/.local/lib/python3.8/site-packages/matplotlib/backends/backend agg.p
y:238, in RendererAgg.get_text_width_height_descent(self, s, prop, ismath)
   234
           return w, h, d
   236 if ismath:
           ox, oy, width, height, descent, font image = \
--> 238
                self.mathtext_parser.parse(s, self.dpi, prop)
            return width, height, descent
   239
   241 font = self._prepare_font(prop)
File ~/.local/lib/python3.8/site-packages/matplotlib/mathtext.py:226, in Ma
thTextParser.parse(self, s, dpi, prop)
   222 # lru cache can't decorate parse() directly because prop
   223 # is mutable; key the cache using an internal copy (see
```

```
224 # text. get text metrics with cache for a similar case).
             225 prop = prop.copy() if prop is not None else None
         --> 226 return self. parse cached(s, dpi, prop)
         File ~/.local/lib/python3.8/site-packages/matplotlib/mathtext.py:247, in Ma
         thTextParser. parse cached(self, s, dpi, prop)
             244 if self. parser is None: # Cache the parser globally.
                     self. class . parser = mathtext.Parser()
         --> 247 box = self. parser.parse(s, fontset, fontsize, dpi)
             248 output = mathtext.ship(box)
             249 if self. output type == "vector":
         File ~/.local/lib/python3.8/site-packages/matplotlib/ mathtext.py:1985, in
         Parser.parse(self, s, fonts object, fontsize, dpi)
            1983
                     result = self. expression.parseString(s)
            1984 except ParseBaseException as err:
         -> 1985
                     raise ValueError("\n".join(["",
            1986
                                                  err.line,
                                                  " " * (err.column - 1) + "^",
            1987
            1988
                                                 str(err)])) from err
            1989 self. state stack = None
            1990 self._in_subscript_or_superscript = False
         ValueError:
         H {}
         Expected {accent | symbol | function | operatorname | group | frac | dfrac
          | binom | genfrac | overset | underset | sqrt | overline}, found '\x0c'
          (at char 3), (line:1, col:4)
         <Figure size 1200x1000 with 1 Axes>
         Answer 3: HMC Algorithm
In [223... def markov chain(Phi i,N cfg,N md,beta,J,h,N):
             chain = []
             chain.append(Phi i)
             for i in range(N cfg):
                 p = np.random.normal(0,1) # Random sample
                 new = leapfrog(N md, P, chain[-1], J, beta, h, N) # Integrating lea
                 prob = np.exp(Hamiltonian(P,chain[-1], J, beta, h, N)-Hamiltonian(n∈
         # Given probability distribution function defined above
                 if prob > np.random.uniform(0,1): # Accept/reject
                     chain.append(new[1])
```

```
else:
                      chain.append(chain[-1])
             return chain
In [ ]:
```

## Computational Physics Exercise 4

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1. 
$$\mathcal{H}[p, \phi] = \frac{p_0^2 + p_1^2 + p_2^2 + \beta \chi^2(\phi)}{2}$$

$$\phi_0 = \frac{\partial}{\partial \rho_0} \mathcal{H} = \rho_0, \phi_1 = \rho_1, \phi_2 = \rho_2$$

$$\chi^{2}(\phi) = \frac{1}{2} \underbrace{\frac{5}{i=1}}_{s_{f_{i}}} (f_{i} - f(\chi_{i}, \phi))^{2}$$

$$\dot{p}_{o} = -\frac{\partial \mathcal{H}}{\partial \phi_{o}} = +\frac{\beta}{2} \underbrace{\frac{5}{8}}_{i=1}^{5} \underbrace{\frac{f_{i} - f(x_{i}, \phi)}{8f_{i}^{2}}}_{f_{i} - f(x_{i}, \phi)}$$

$$= \frac{\beta}{8f_{i}^{2}} \underbrace{\frac{5}{i=1}}_{i=1}^{5} \underbrace{\left(f_{i} - f(x_{i}, \phi)\right)}_{f_{i} - f(x_{i}, \phi)}$$

$$\dot{\rho}_1 = \frac{\beta}{8f_1^2} \chi_i \stackrel{\text{Z}}{=} \left( f_i - f(\chi_i, \phi) \right)$$

$$\hat{\rho}_{2} = \frac{\beta}{8f_{i}^{2}} \chi_{i}^{2} \stackrel{5}{\underset{i=1}{\text{i}}} \left( f_{i} - f(\chi_{i}, \phi) \right)$$