Part 5

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0.0.2 Answer 5 : Autocorrelation time for the absolute magnetization

This is only for part 5. Sorry for the confusion.

[55]:
```
# Necessary modules

%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import random as r
from random import choice
import sympy

# We now work to generalize the Ising model to 2-Dimensions and or that the␣
  ↪lattice size becomes (N_x) X (N_y) where we
# assume periodic boundary conditions in both, x and y, directions. It is given␣
  ↪that we use the same coupling constant in
# both directions.

# Number of sweeps for thermalization for every value of J,h
n_therm = 20000

# Number of measurements for each J,h
n_meas = 100

# External magnetic field
h = 0
```

[56]:
```
#  Beginning by defining the two point correlation function

# Taken from the discussion in the tutorial

# This function returns the ft values and takes care of k and -k indices in the␣
  ↪summation

def C(s):
    skp = np.fft.fft2(s, norm = 'backward')
    # print(skp)
    skm = np.fft.ifft2(s, norm = 'forward')
    return np.fft.fft2(skm*skp, norm = 'backward')/s.size**2
```

[57]:
```
J_c = 0.44068 # Critical J
```

[58]:
```
# To calculate change in energy after one spin flips at position (x,y)

# This is the general formula and we have already defined h=0 above
def energy_flip(s,J,h,x,y,n):
#     Here, the s is the spin, x and y are th positions of the 2 D lattice,␣
 ↪J,n, and h are the same variables as defined
#     before.
#     The function calculates the energy after the spin at one site is flipped.
    return 2 * s[x][y]*(J*(s[((x+1)%n)][y] + s[((x-1)%n)][y] + s[x][((y+1)%n)]␣
 ↪+ s[x][((y-1)%n)]) + h)


def energy(s,J,h,x,y,n):
    return -J*((s[((x+1)%n)][y] + s[((x-1)%n)][y] + s[x][((y+1)%n)] +␣
 ↪s[x][((y-1)%n)])*s[x][y])-h*s[x][y]
```

[59]:
```
# fft of sx and sy

def variables(n,J,h,obs):

# Defining local arrays

    m = np.array([])  # Magnetization
    E = np.array([])  # Energy
    m_absolute = np.array([]) # Absolute value of magnetization
    prob = np.array([]) # Probability information

#   Now we need to assign spins to the site and because this is a 2D lattice,␣
 ↪we have to keep in mind the dimensions
#   x and y and do that. This has been done as follows:
#   The variable (s) is the spin here and n is some random integer

    s = [[choice((+1,-1)) for x in range(n)] for y in range(n)]

    for j in range(n_therm):
#           Assigning a random integer value to x and y
        x = np.random.randint(n)
        y = np.random.randint(n)

#   We need to define the change of energy after flipping the lattice site␣
 ↪(x,y) picked randomly. We call the defined
```

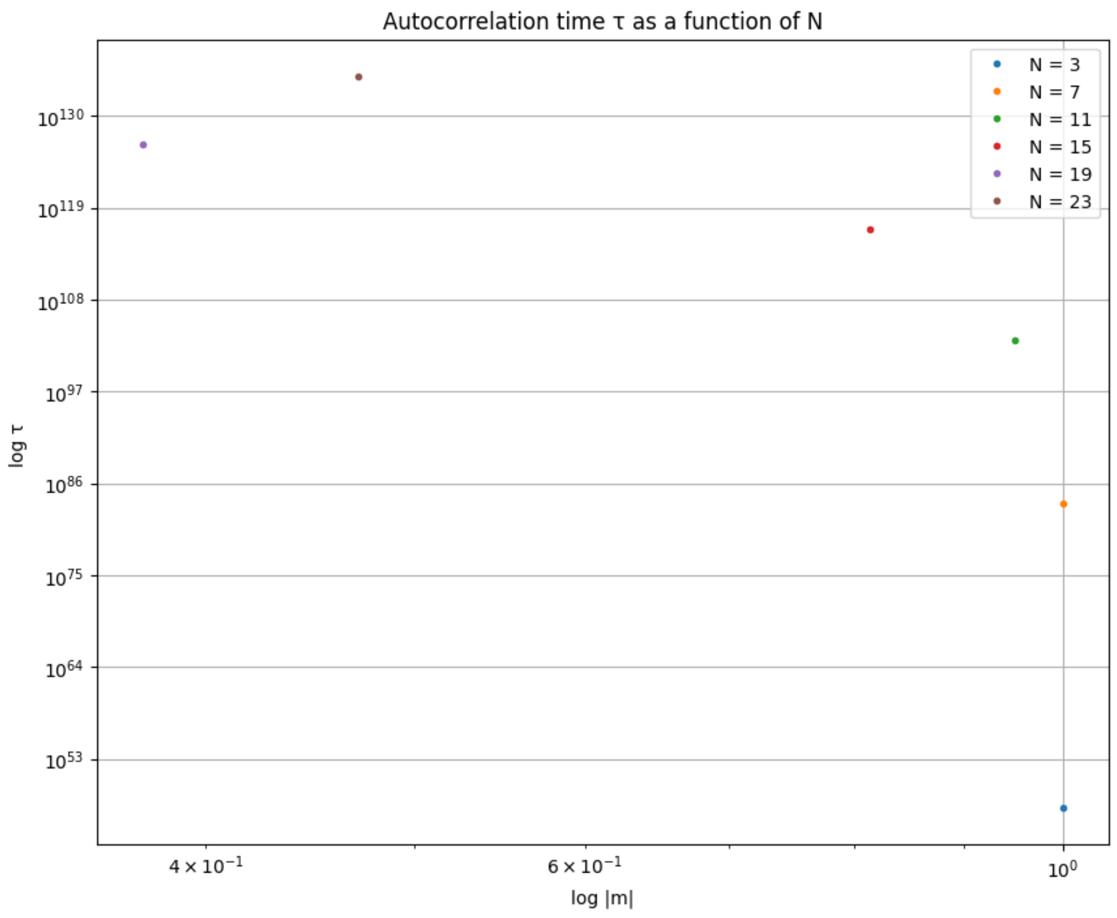
```
#   function energy_flip to calculate that.

        delta_energy = energy_flip(s,J,h,x,y,n)
        if delta_energy < 0:
            s[x][y] *= -1         # Condition to accept the spin flip
        else:
            if np.random.uniform(0,1) <= np.exp(-delta_energy):
                s[x][y] *= -1

    for i in range(n_meas):
        for j in range(n**2):  #sweeping the lattice
            x = j % n
            y = j // n
            delta_energy = energy_flip(s,J,h,x,y,n)
            if delta_energy < 0:
                s[x][y] *= -1         # Condition to accept the spin flip
            else:
                if np.random.uniform(0,1) <= np.exp(-delta_energy):
                    s[x][y] *= -1
                    prob = np.append(prob,1.)    # Accept.
                else:
                    prob = np.append(prob,0.)     # We reject the other values.

        #         Measurements

        if obs == "m":
            m=np.append(m,np.mean(s))
            obs=np.mean(m)
        if obs == "E":
            temp = np.array([])
            for x in range(n):
                for y in range(n):
                    temp = np.append(temp,energy(s,J,h,x,y,n))
            E=np.append(E,np.mean(temp))
            obs=np.mean(E)
        if obs == "m_absolute":
            m_absolute=np.append(m_absolute,np.absolute(np.mean(s)))
            obs=np.mean(m_absolute)

    return obs
```

[60]:
```
# plt.figure(figsize=(12,10))
# plt.title("C(s)",fontsize=12)
# plt.grid()
# plt.plot(C,r)
# plt.legend()
```

```
[61]: def int_autocorrelation(N):
          for z in range(1,100):
              tau_int = N**z
          return tau_int

      tau_int3 = int_autocorrelation(3)
      tau_int7 = int_autocorrelation(7)
      tau_int11 = int_autocorrelation(11)
      tau_int15 = int_autocorrelation(15)
      tau_int19 = int_autocorrelation(19)
      tau_int23 = int_autocorrelation(23)
```

Autocorrelation time   as a function of N

```
[63]: # Now plotting the required variables

      J = 0.5     # Keeping J fixed at a certain value > J_c
      h = 0

      # We are supposed to take different values of N (N_x = N_y), so we define as␣
       ↪follows:

      mag_3  = [variables(3,J,h,"m_absolute")]
      mag_7  = [variables(7,J,h,"m_absolute")]
      mag_11 = [variables(11,J,h,"m_absolute")]
      mag_15 = [variables(15,J,h,"m_absolute")]
      mag_19 = [variables(19,J,h,"m_absolute")]
      mag_23 = [variables(23,J,h,"m_absolute")]

      # For different values of N:

      plt.figure(figsize=(10,8))
      plt.title("Autocorrelation time   as a function of N",fontsize=12)
      plt.grid()
      plt.ylabel("log   ")
      plt.xlabel("log |m|")
      plt.loglog(mag_3, tau_int3,'.', mag_7, tau_int7,'.', mag_11,tau_int11,'.',␣
       ↪mag_15, tau_int15,'.', mag_19, tau_int19,'.', mag_23, tau_int23,'.')
      plt.legend(("N = 3", "N = 7","N = 11","N = 15","N = 19","N = 23"))
      plt.show()
```



0.0.3 Answer 5 : Dynamical variable

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[ ]:
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