

Joint density of states in Semiconductor:-

density of state for a semiconductor is defined the same way as for any material: (no. of energy states per unit volume in energy range E to $E + dE$). For a semiconductor, however two or more bands can play role, electrons in the conduction band and holes in the valence band. So, for a semiconductor, we used to calculate the joint density of states, i.e. the density of states in conduction band and the density of states in valence band.

Density of states is calculated for electron-hole pair having same wave vector ($\vec{r}e\vec{K}$) which means only allowed transitions are considered. (not oblique.)

By using parabolic approximation,

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c^*} \quad - (1)$$

$$E_1 = E_v - \frac{\hbar^2 k^2}{2m_v^*} \quad - (2)$$

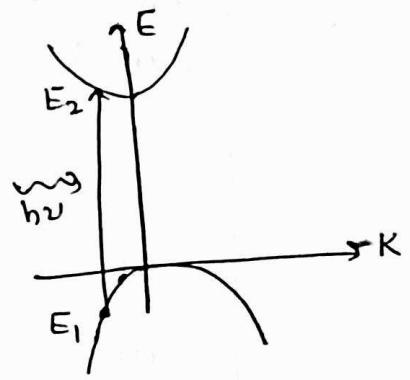
where, $E_c \rightarrow$ bottom edge of C.B. (Note \rightarrow free e^- energy $E = P^2/2m = \frac{\hbar^2 k^2}{2m}$)

$E_v \rightarrow$ upper edge of V.B.

m_c^* and m_v^* \rightarrow effective mass of e^- and hole in C.B and V.B respectively.

so from equ. (1) and (2)

$$\begin{aligned} h\nu &= E_2 - E_1 = E_c + \frac{\hbar^2 k^2}{2m_c^*} - E_v + \frac{\hbar^2 k^2}{2m_v^*} \\ &= (E_c - E_v) + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_c^*} + \frac{1}{m_v^*} \right) \end{aligned} \quad - (3)$$



$$\therefore E_0 = E_V + E_g$$

and let $\frac{1}{m_e^*} + \frac{1}{m_V} = \frac{1}{m_r^*} \Rightarrow$ reduced mass

So, eqn (3) \Rightarrow

$$hv = E_g + \frac{\hbar^2 k^2}{2 m_r^*}$$

$$\text{or } (hv - E_g) = \frac{\hbar^2 k^2}{2 m_r^*}$$

$$\text{or } k^2 = \frac{2 m_r^* (hv - E_g)}{\hbar^2}$$

Putting the value of k^2 in eqn (1), we have

$$E_b = E_0 + \frac{W^2}{2 m_e^*} \cdot \frac{2 m_r^* (hv - E_g)}{k^2}$$

$$E_b = E_0 + \frac{m_r^*}{m_e^*} (hv - E_g) \quad \text{--- (4)}$$

So, no. of states (in a range E_2 to $E_2 + dE_2$) per unit volume can be written as (in conduction band)

$$g_c(E_2) dE_2 = g(v) dv \Rightarrow \text{no. of states for photons of energy between } h\nu \text{ & } h(\nu + dv) \text{ to interact with}$$

$$\text{So, } g(v) = g_c(E_2) \frac{dE_2}{dv} \quad \text{--- (5)}$$

$$\text{Now } g_c(E_2) = \frac{1}{2\lambda^2} \left(\frac{2 m_e^*}{\hbar^2} \right) (E_b - E_c)^{y_2} \quad \text{--- (6)}$$

$$\text{So, eqn (5) } \Rightarrow g(v) = \frac{1}{2\lambda^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{y_2/2} (E_b - E_c)^{y_2} \cdot \frac{dE_2}{dv}$$

$$\text{from eqn (4) } \Rightarrow \frac{dE_2}{dv} = \Omega + \frac{m_r^*}{m_e^*} \cdot h$$

$$\text{So, } g(v) = \frac{1}{2\lambda^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{y_2/2} (E_b - E_c)^{y_2} \cdot \frac{h \cdot m_r^*}{m_e^*}$$

$$= \frac{1}{\lambda \hbar^2} \left(\frac{2 m_e^*}{\hbar^2} \right)^{y_2/2} \cdot \left(\frac{m_r^*}{m_e^*} \right)^{y_2} (hv - E_g)^{y_2} \left(\frac{m_r^*}{m_e^*} \right)$$

$$\boxed{g(v) = \frac{1}{\lambda \hbar^2} (2 m_r^*)^{y_2/2} (hv - E_g)^{y_2}}$$

Optical gain in a Semiconductor :-

The optical joint density of states takes into account the no. of states available in both the C.B and the valance bands with which a photon of energy $h\nu$ can interact and is given by

$$\rho(v) = \frac{(2m_r)^{3/2}}{\pi h^2} (h\nu - E_g)^{1/2}; \quad h\nu > E_g \quad (1)$$

Note that only photons with energy $h\nu > E_g$ can participate in the emission and absorption process.

If $h\nu = E_2 - E_1$ is the energy of an incident photon then for absorption of the photon an electron should be present in the state with energy E_1 in the V.B. and there should be an unoccupied state at energy E_2 in the C.B.. Thus, the probability of absorption is given by (in quasi-equilibrium),

$$P_a(v) = f(E_1) [1 - f_c(E_2)] \rightarrow \text{Probability that a state with energy } E \text{ remains unoccupied.}$$

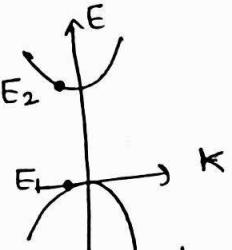
↓
Prob. of
occupation of a
state E

Similarly for emission of photon, it requires that an electron exists at energy level E_2 in the C.B and there exists a hole at energy level E_1 in the V.B. Therefore, the probability of emission -

$$P_e(v) = f_c(E_2) [1 - f(E_1)] \quad (3)$$

for net emission, we must have, $P_e(v) > P_a(v)$
or $f_c(E_2) [1 - f_v(E_1)] > f_v(E_1) [1 - f_c(E_2)] \quad (4)$

$$P_e(v) = \left[\frac{1}{e^{(E_2 - E_{Fc})/kT} + 1} \right] \left[\frac{e^{(E_1 - E_{Fv})/kT}}{e^{(E_1 - E_{Fv})/kT} + 1} \right] \quad (5)$$



$$\text{and } P_a(\nu) = \left[\frac{1}{1 + e^{\frac{(E_1 - E_{fv})}{kT}}} \right] \left[\frac{e^{\frac{(E_2 - E_{fc})}{kT}}}{1 + e^{\frac{(E_2 - E_{fc})}{kT}}} \right] \quad (6)$$

from equ. (5) and (6) =)

$$\Rightarrow e^{\frac{(E_1 - E_{fv})}{kT}} > e^{\frac{(E_2 - E_{fc})}{kT}}$$

$$\Rightarrow \text{or } E_1 - E_{fv} > E_2 - E_{fc}$$

$$\text{or } E_{fc} - E_{fv} > E_2 - E_1 = h\nu$$

$$\boxed{E_{fc} - E_{fv} > E_g} \quad (7)$$

when the separation between the quasi-fermi levels in a sc exceeds the bandgap, then for all frequencies ν that satisfy equ. (7), it is possible to have light amplification or overall optical gain.

The gain coefficient for amplification of radiation of frequency ν by stimulated emission in a s.c. is given by

$$\gamma(\nu) = \frac{(c/n)^2}{\Theta \tau_r} \frac{P(\nu)}{\nu^2} \Delta P \quad (8)$$

where $\tau_r \rightarrow$ recombination time

$P(\nu) \rightarrow$ joint density of stat

$$\Delta P = P_e(\nu) - P_a(\nu)$$

for amplification, $\gamma(\nu) > 0$ which requires $\Delta P > 0$
i.e. the probability of emission has to be greater than
the probability of absorption.

Optical loss or Absorption coefficient in thermal equilibrium:

A semiconductor in thermal equilibrium has only a single fermi level i.e. $E_F = E_{fc} = E_{fv}$

and the carrier distribution is given by the fermi function and therefore both $f(E_1)$ and $[1 - f(E_2)]$ are

Energy is most stable. Therefore the electrons in S.

much larger than $f(E_2)$ and $[1 - f(E_1)]$. Thus,

$$P_e(v) < P_a(v)$$

also $\Delta P = P_e(v) - P_a(v) = -ve$

Therefore, the gain coefficient $\gamma(v)$ in eqn (8) is always negative. (since $E_2 > E_1$).

Thus, a semiconductor in thermal equilibrium, whether it is intrinsic or doped, always attenuates light and we have the overall optical loss in sc.

Energy density States of photon :-

The behaviour of photons depends upon the distribution of energy among the photons. The distribution of a fixed amount of energy among a number of identical particles depends upon the density of available energy states and the probability that a given state will be occupied.

The probability that a given energy state will be occupied is given by the distribution function, but if there are more available energy states in a given energy interval than that will give a greater weight to the probability for that energy interval.

$$N(E)dE = g(E)dE \cdot F(E) \rightarrow \text{Distribution function}$$

↓
 no. of Particles per unit volume with energy between E and $E+dE$.
 Density of states

— (1)

Since, photons are bosons, the distribution function is.

Bose-Einstein distribution function

$$F(E) = \frac{1}{A(e^{h\nu/RT} - 1)} \quad (2)$$

where the normalization constant $A=1$ for photons.

The determination of how many ways there are to obtain an energy in an incremental energy range dE can be approached as the no. of possible standing waves in a cubical box.

Electromagnetic wave equation in 3D can be written as

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \quad (3)$$

The solution of equation (3) is

$$E = E_0 \sin \frac{n_1 \pi x}{L} \sin \frac{n_2 \pi y}{L} \sin \frac{n_3 \pi z}{L} \sin \frac{2 \pi c t}{\lambda}$$

Substituting this solution in eqn (3), we get

$$\left(\frac{n_1 \pi}{L}\right)^2 + \left(\frac{n_2 \pi}{L}\right)^2 + \left(\frac{n_3 \pi}{L}\right)^2 = \left(\frac{2L}{\lambda}\right)^2$$

$$\text{or } n_x^2 + n_y^2 + n_z^2 = \frac{4L^2}{\lambda^2} \quad (4)$$

Treating the 'n' as a space such that a given set of n values determines a point in that space or we can say that the no. of possible states is proportional to the volume in n-space.

It is convenient to define a radius 'R' in n-space:

$$\text{re } R^2 = n_x^2 + n_y^2 + n_z^2$$

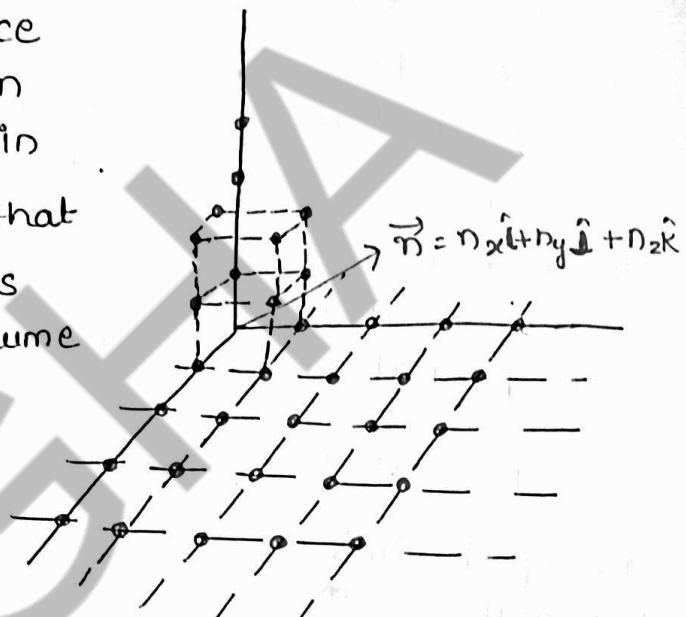
Using the photon energy, $E = pc = h\nu = \frac{hc}{\lambda}$
the energy can be expressed in terms of R,

$$R = \frac{2L}{\lambda} = \frac{2LE}{hc}$$

$$\text{or } E = \frac{hcR}{2L}$$

The n-space associated with the standing wave solutions involve only positive values of n. so, the volume must be divided by 8. It then must be multiplied by '2' to account for the two planes of polarization of photons. The no. of values is then,

$$N = 2 \times \frac{1}{8} \times \frac{4}{3} \pi R^3 = \frac{\pi}{3} \left(\frac{2LE}{hc}\right)^3$$



8.

The no. of states per unit volume is

$$n_s = \frac{N}{L^3} = \left(\frac{8\pi}{3}\right) \frac{E^3}{(hc)^3}$$

The final density of states, as a function of energy is then the derivative of this population with respect to energy,

$$\rho(E) = \frac{dns}{dE} = \frac{d}{dE} \left[\left(\frac{8\pi}{3}\right) \frac{E^3}{(hc)^3} \right]$$

$$\boxed{\rho(E) = \frac{8 \cdot 8\pi E^2}{3 \cdot (hc)^3} = \frac{8\pi}{(hc)^3} \cdot E^2}$$

This represents the no. of photons per unit volume per unit energy at energy E .

In terms of frequency,

$$\rho(v) = \frac{dns}{dv} = \frac{d}{dv} \left[\frac{8\pi}{3} \cdot \frac{h^3 v^3}{c^3} \right]$$

$$\rho(v) = \frac{8\pi}{3} \cdot \frac{h^3}{c^3} \cdot \frac{v^2}{v}$$

$$\boxed{\rho(v) = \frac{8\pi v^2}{c^3}}$$

In terms of wavelength :-

$$\rho(\lambda) = \frac{dns}{d\lambda} = \frac{d}{d\lambda} \left[\frac{8\pi}{3} \cdot \frac{(hc)^3}{\lambda^3 (hc)^3} \right]$$

$$\rho(\lambda) = \frac{d}{d\lambda} \left(\frac{8\pi}{3} \cdot \frac{1}{\lambda^3} \right)$$

$$\rho(\lambda) = -\frac{8\pi}{3} \cdot \frac{1}{\lambda^4}$$

$$\boxed{\rho(\lambda) = \frac{8\pi}{\lambda^4}}$$

(neglecting '-' sign)

Fermi Golden Rule :-

Fermi golden Rule provides one way to calculate the transition rate (Probability of transition per unit time) between two certain quantum mechanically defined states.

It gives the rate of any decay process. It can be given as

$$W_{if} = \frac{4\pi^2}{h} |M|^2 \frac{dn_f}{dE} \quad \text{--- (i)}$$

where, $W_{if} \rightarrow$ rate of transition from initial state ψ_i to final state ψ_f .

$M \rightarrow$ the Matrix element

$$M = \int \psi_f^* H_{int} \psi_i dv \text{ for all the transition and}$$

H_{int} is the interaction responsible for it.

$$\text{also, } M = \langle f | H_{int} | i \rangle$$

$$\text{so equ. (i)} \Rightarrow W_{if} = \frac{4\pi^2}{h} |\langle f | H_{int} | i \rangle|^2 \frac{dn_f}{dE}$$

Here, $\frac{dn_f}{dE} \rightarrow$ density of states.

Photovoltaic Effect: Photovoltaic effect is the phenomenon in which the incidence of light upon a junction, induces the generation of an electromotive force.

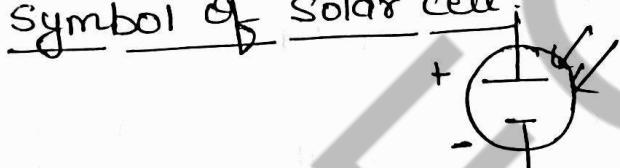
The photovoltaic effect produces voltage (or corresponding electric current) in a material upon exposure to electromagnetic radiation.

In most photovoltaic applications, the radiation is sun light; and for this reason, the devices making use of photovoltaic effect to convert solar energy into electrical energy are known as Solar cells.

Solar cell: The solar cell generates electric potential when it is illuminated by optical irradiation.

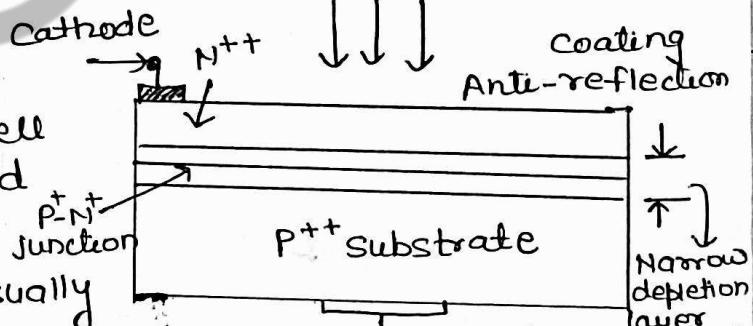
It is used to generate electricity from sun light, therefore, it is known as Solar cell.

Symbol of Solar cell:



Solar radiation

Cathode



Construction: The solar cell consists of a heavily doped P-N Junction. The top layer junction forming the N^{++} region is usually made very thin to allow solar radiation to fall into the junction. The resulting junction will be extremely low ($\approx 20\text{nm}$) becoz the doping level is extremely high. Solar cells are designed to have large surface area to allow a large amount of light into the device. Usually, the anode connection will be made from the bottom (P^{++} substrate) of the device and the cathode from the top (N^{++} layer). An anti-reflection coating on top of the top N^{++} layer is made to prevent light losses due to

reflection from the surface of the solar cell.

This is very essential becoz Si is a highly reflective material. Each solar cell typically produces an output of about 0.6 V. So, the total output voltage can be increased by connecting them in series, and to increase the current, the total resistance should be reduced which can be obtained by connecting several series of solar cells in parallel combination to get higher output power;

working :- when sunlight falls on the top surface, it is absorbed mostly within a small distance below the surface where the junction is located.

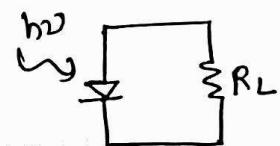
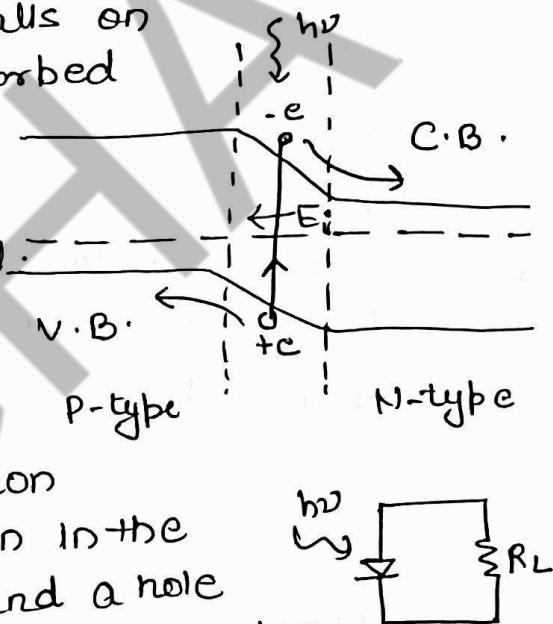
Each absorbed light photon creates an electron and a hole. As shown in energy band diagram, the absorption

of photon causes an electron in the V.B to move to the C.B, and a hole is left in the V.B. The built-in electric

field E_i in the depletion region forces the hole towards the p-type region and the electron towards the n-type region. The two sides of the junction thus become oppositely charged so that a forward voltage is generated across the junction. This voltage drives a current around an external circuit, and the junction acts as a cell.

The total current passing through solar cell is given by

$$I = I_s (e^{V/kT} - 1) - I_L$$



where, $V \rightarrow$ bias voltage applied to collect charge generated by light.

$I_s \rightarrow$ reverse saturation current

$I_L \rightarrow$ photocurrent passes through load

$$I_L = eAG_1(L_p + L_n)$$

$A \rightarrow$ cross sectional area of junction.

L_p and $L_n \rightarrow$ are diffusion lengths of minority carriers on the p and n side respectively.

I-V characteristics of Solar cell:

The I-V characteristic curve is shown in figure. By properly choosing a load, the product ($I_{sc}V_{oc}$) can be extracted; where I_{sc} is known as the short-circuit current equal to photo current I_L and V_{oc} is the open-circuit voltage of the cell.

The shaded area in the fig. represent the maximum power rectangle. The quantities I_m and V_m correspond to the current and voltage respectively, for the maximum power output.

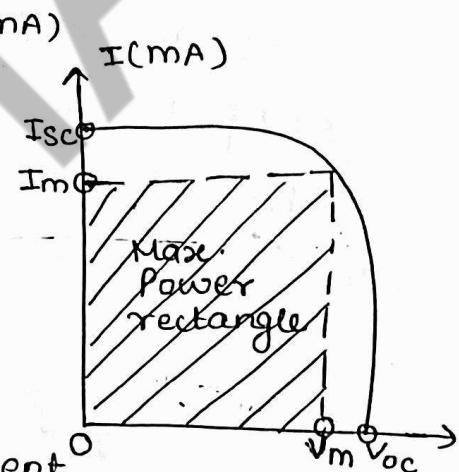
$$P_m = I_m \times V_m$$

And, the conversion efficiency of a solar cell is given by, $\eta = \frac{\text{Maximum power output}}{\text{Input power}} = \frac{I_m V_m}{P_{in}}$

$$\text{or } \eta = \frac{f.f. \cdot I_{sc} \cdot V_{oc}}{P_{in}}$$

where, fill factor (F.F.) is defined as the maximum Power rectangle of $(I_{sc} \times V_{oc})$

$$F.F. = \frac{P_m}{I_{sc} \cdot V_{oc}} = \frac{I_m V_m}{I_{sc} \cdot V_{oc}}$$



Solar cell efficiency :- solar cell efficiency is the ratio of the electrical output of a solar cell to the incident energy in the form of sun light. The energy conversion efficiency (n) of a solar cell is the percentage of the solar energy to which the cell is exposed that is converted into electrical energy.

This is calculated by dividing a cell's power output (in watt) at its maximum power point (P_m) by the input light (E in W/m^2) and the surface area of solar cell (A in m^2)

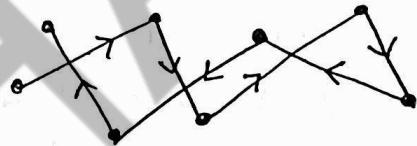
$$n = \frac{P_m}{E \times A}$$

Theory of Drude Model :- The Drude model was the first attempt to use the idea of a gas of electrons, free to move between positively charged ionic cores, in order to explain the properties of metal. The classical free electron theory reveals that the free electrons are fully responsible for electrical conduction.

The assumptions of the drude model are:

- The free electrons or electron gas, available in a metal move freely here and there during the absence of an electronic field similar to the gas molecules moving in a vessel.
- These free electrons collide with other free electron or positive ion core or the walls of the container. Collisions of this type are known as elastic collisions. The total energy of an electron is assumed to be purely K.E.
- Between collisions, electron do not interact with each other or with ions.
- Collisions are instantaneous and result in a change in electron velocity.
- An electron suffers a collision with probability per unit time τ^{-1} (relaxation time approximation) i.e τ^{-1} is the scattering rate.
- Electrons achieve thermal equilibrium with their surroundings only through collisions.

On the other hand, suppose an electric field is applied to the material through an external arrangement. The free electrons available in the metal gain some amount of energy and are directed to



move towards a higher potential towards a lower potential. These electrons acquire a constant velocity known as drift velocity.

Determination of electrical conductivity:

With the help of Drude Model, we can derive an expression for electrical conductivity. An external static electric field can affect electron velocity during the time interval between two successive collisions. But the gain in velocity is destroyed each time a collision occurs since the average velocity immediately after the collision is zero.

A larger influence of electric field is reflected in a larger mean free time or relaxation time τ . The probability of collision per unit time is defined as $\gamma\tau$. Therefore, the probability that a collision occurs in a small time interval $d\tau$ is simply $d\tau/\tau$.

Taking the acceleration of an electron of mass 'm' as eE/m , the mean drift velocity can be written as

$$v_d = \left(-\frac{eE}{m}\right)\tau \quad -(1)$$

If, there are a total of n electrons per m^3 in the metal, all with constant drift velocity v_d , we have, the net electric current density

$$J = -nev_d = \left(\frac{n e^2 \tau}{m}\right) E \quad -(2)$$

$$J = \sigma E$$

where,
$$\boxed{\sigma = \frac{n e^2 \tau}{m}} \quad -(3)$$

is defined as the electrical conductivity.

The form of equ.(3) remain the same in all models including those based on quantum physics.

It must be, however, stated that the electrical conductivity σ is not universally scalar since in some complicated situation J becomes non-linear with respect to E .

The electrical conductivity is often expressed in terms of the drift mobility of electrons μ as

$$\sigma = ne\mu \quad \text{--- (4)}$$

where, $\mu = \frac{|V_d|}{E} \quad \text{--- (5)}$

Drude put equ.(3) in a further useful form by exploiting the ideas of K.E. theory such as to define τ in terms of thermal velocity v_{th} expressed by the following relations.

$$\tau = \frac{A}{v_{th}} \quad \text{--- (6)}$$

where, $A \rightarrow$ electron mean free path.

$$\frac{3}{2} K_B T = \frac{1}{2} m v_{rms}^2 = \frac{1}{2} m v_{th}^2 \quad \text{--- (7)}$$

Using these relation equ.(3) =

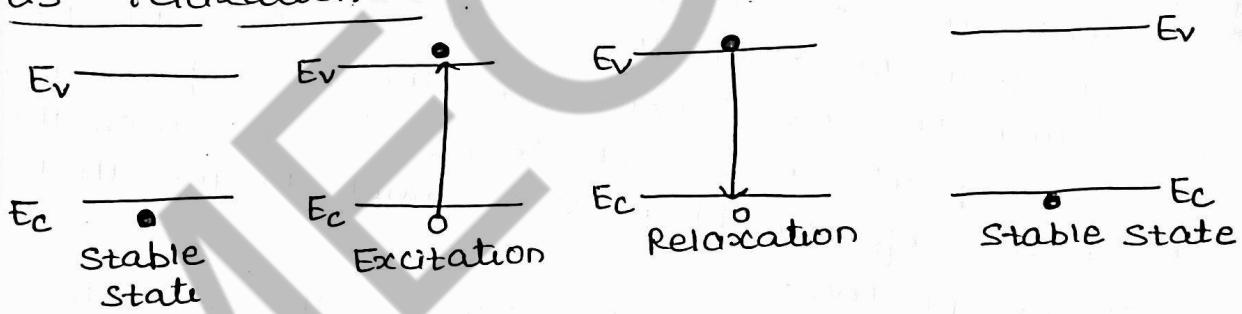
$$\boxed{\sigma = \frac{ne^2 A}{(3mK_B T)^{1/2}}} \quad \text{--- (8)}$$

The relation (8) is the achievement of Drude theory since it gives the right magnitude of electrical conductivity and correctly describes that conductivity increases with decrease in temperature.

UNIT-3

Concept of optical transitions in bulk S.C. :-

Among energy states, the state with the lowest energy is most stable. Therefore, the electrons in S.C. tend to stay in low energy states. If they are excited by thermal energy, light or electron beams, the electrons absorb these energies and transit to higher energy states. These transitions of the electrons from low energy states to high energy states are called excitations. High energy states, however, are unstable. As a result, to take stable states, the electrons in high energy states transit to low energy states in certain lifetimes. These transitions of the excited electrons from high energy states to low energy states are referred as relaxations.



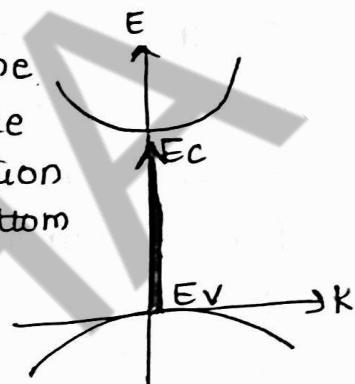
2. Optical absorption process :- If a wide range of electromagnetic energy is consumed by the valence electron, the excited electrons go to various energy levels of the conduction band. Besides this, there are several mechanisms of absorption by which electrons (and holes) absorb optical energy. But for Bulk semiconductor, we generally discuss band to band transitions.

In band to band transition, an electron in the V.B. absorbs a photon with enough energy to be excited

to the conduction band², leaving a hole behind. These band to band transitions can further categorized as direct and indirect band gap transitions. In direct band gap materials, the energy transition is more direct, means no change occurs in momentum whereas in indirect band gap material, large change in momentum occurs.

(a) Direct band gap absorption transition:-

Let us consider the transitions of the electrons from the top of the valence band to the bottom of the conduction band. A semiconductor, in which bottom of the conduction band and the top of the valence band are placed at a common wave vector



\mathbf{k} , is the direct transition S.C.

In transitions of the electrons, the energy and momentum are conserved. Therefore phonon do not take part in direct transitions. Because the wave vector \mathbf{k} of the phonons is much larger than that of the photon.

Hence, in the direct transitions, the transition probabilities are determined by only the electron transition probabilities.

The absorption coefficient for direct-gap absorption can be written as

$$\alpha(\hbar\omega) = A(\hbar\omega - E_g)$$

↓ ↓
 absorption magnetic
 coeff. vector pot.

(b) Indirect band gap absorption transition :-

A semiconductor, in which the bottom of the conduction band and the top of the valence band have different k -values is the indirect transition semiconductor. The phonon transitions accompany the indirect transitions to satisfy the momentum

conservation law. So, In indirect transitions, the transition probabilities are given by a product of the electron transition probabilities and the phonon transition probabilities.

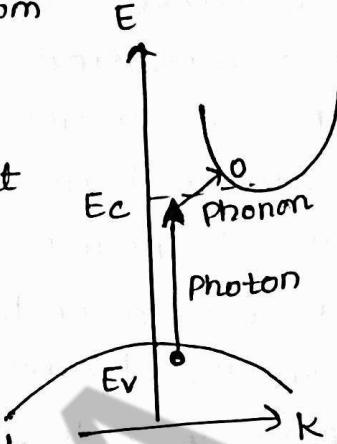
The absorption coefficient of transition with phonon involvement

$$\alpha(\hbar\omega) = A^2 (\hbar\omega - E_{th})^2 \rightarrow \text{energy involving phonons.}$$

3. Concept of Recombination process :- In semiconductors, the transitions of electrons from high energy states to low energy states are designated recombinations of the electrons and the holes. In the recombinations of the electrons and the holes, there are radiative recombinations and non-radiative recombinations.

The radiative recombinations emit photons, and the energy of photons correspond to a difference in the energies between the initial and final energy states related to transitions.

In contrast, in the non radiative recombinations, the phonons are emitted to crystal lattice or the electrons are trapped in the defects, and the transition energy is transformed into forms other than light.

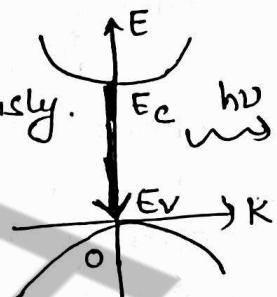


Radiative or direct Recombination process:

In direct recombination, an excess population of electrons and holes decays by electrons falling from the conduction band to empty states (holes) in the valence band. Energy lost by an electron in making the transition is given up a photon.

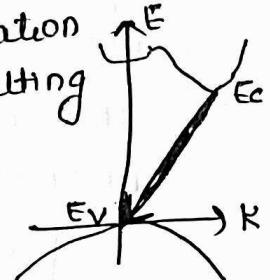
Direct recombination occurs spontaneously.

The probability that an electron and a hole will recombine is constant in time.



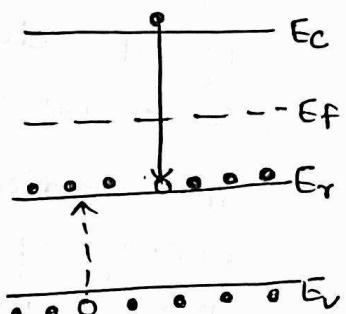
Non-Radiative or indirect recombination:

The vast majority of the recombination events in indirect materials occur via recombination levels within the band gap, and the resulting energy loss by recombining electrons is usually given up to the lattice as heat rather than by the emission of photons.



Any impurity or lattice defect can serve as recombination center if it is capable of receiving a carrier of one type and subsequently capturing the opposite type of carrier, thereby annihilating the pair.

Fig. illustrates a recombination level E_r which is below E_F and therefore is substantially filled with electrons. When excess holes and electrons are created in material, each electron hole pair recombine at E_r in the two steps. (a) hole capture (b) electron capture.



Since the recombination centers in fig. are filled at equilibrium, the first event in the recombination

process is hole capture. It is important to note that the event is equivalent to an electron at Er falling to the valence band, leaving behind an empty state in the recombination level. Thus, in hole capture, energy is given up as heat to the lattice. Similarly, energy is given up when a conduction band electron subsequently falls to the empty state in Er. When both of these events have occurred, the recombination center is back to its original state (filled with an electron), but an EHP is missing. Thus, one EHP recombination has taken place, and the center is ready to participate in another recombination event by capturing a hole.

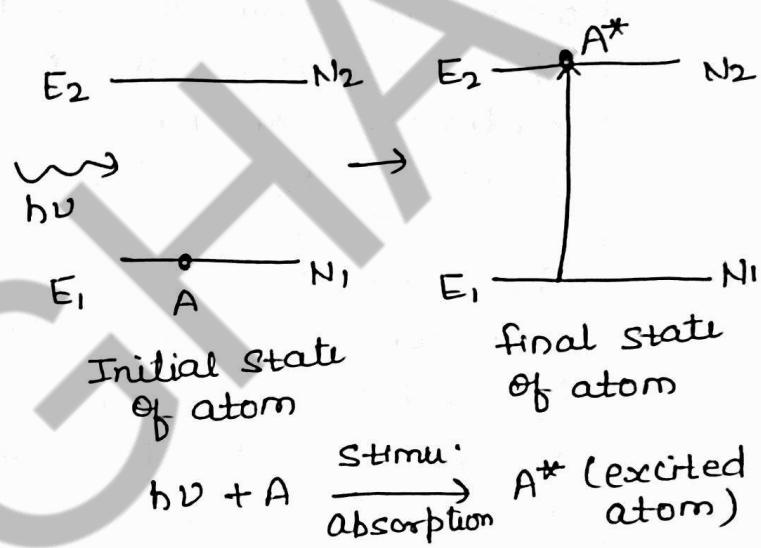
Stimulated Absorption :-

An atom has a no. of quantized energy states. Initially an atom is in the ground state i.e all of its electrons possess the lowest possible energy states. When energy is given in the form of electromagnetic radiation, the atom goes to the excited state i.e its electrons jump to a higher energy state by absorbing a quanta or photon. This process is called stimulated absorption or induced absorption.

If E_1 and E_2 are the energies of an electron in the initial and final states respectively and ν is the frequency of absorbed radiation then

$$E_2 - E_1 = h\nu$$

$$\text{or } \nu = \frac{E_2 - E_1}{h}$$



The probability of absorption process depends upon the energy density $u(\nu)$ of the radiation and the no. of atoms or electrons N_1 in the ground state.

$$\text{Thus, } (P_{12})_{\text{sti. Abs.}} \propto N_1 u(\nu)$$

$$\text{or } (P_{12})_{\text{sti. Abs.}} = B_{12} N_1 u(\nu) \quad \leftarrow \begin{matrix} \text{(Stimulated} \\ \text{Absorption} \\ \text{Rate)} \end{matrix}$$

where,

$B_{12} \rightarrow$ is a proportionality constant called "Einstein's coefficient for absorption process" and depends upon the properties of state or energy levels E_1 and E_2 .

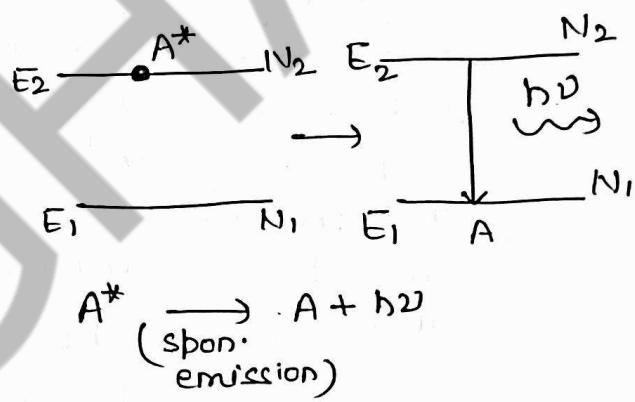
Emission process :- when the atom is in the excited state it can make a transition to a lower energy state through the emission of electromagnetic radiation. The emission process can occur in two different ways.

1. Spontaneous emission
2. Stimulated emission

Spontaneous emission :-

when an atom in excited state emits radiation spontaneously, in the absence of any incident radiation. The process is known as spontaneous emission of radiation.

The rate of spontaneous emission or probability of emission is proportional to the no. of atoms in the excited state i.e N_2 .



$$(P_{21})_{\text{spont. emi.}} \propto N_2$$

$$\text{or } (P_{21}) = A_{21} N_2 \quad (\text{spontaneous emission rate})$$

where, A_{21} → "Einstein's coefficient of spontaneous emission of radiation".

Stimulated emission :- when an atom is in the excited state, then an incident photon of correct energy ($h\nu = E_2 - E_1$) may cause the atom to jump to lower energy state, emitting an additional photon of same frequency. Thus, now two photons of same frequency are present.

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This phenomenon is called stimulated emission of radiation.

The rate of stimulated emission

or probability of emission depends both on the intensity of the external radiation and on the no. of atoms in the upper level i.e N_2

The rate of stimulated emission or probability of emission depends both on the intensity of the external radiation and on the no. of atoms in the upper level i.e N_2

$$(P_{21})_{\text{stim. emis.}} \propto N_2 u(v)$$

$$(P_{21})_{\text{stim}} = B_{21} N_2 u(v) \quad \begin{matrix} \leftarrow \\ \text{(stimulated emission rate)} \end{matrix}$$

where, $B_{21} \rightarrow \text{"Einstein's coefficient of stimulated emission of radiation"}$.

Hence, total probability of emission,

$$(P_{21})_{\text{emission}} = (P_{21})_{\text{spont.}} + (P_{21})_{\text{stim.}}$$

$$(P_{21})_{\text{emission}} = [A_{21} + B_{21} u(v)] N_2$$

\rightarrow Optical transition in thermal equilibrium :-
Relation Between Einstein's coefficients :-
 Consider an assembly of atoms in thermal equilibrium at temperature T with radiation of frequency v and energy density $u(v)$. Let N_1 and N_2 be the no. of atoms in energy states 1 and 2 respectively at any instant.

The Probability of absorption is given by

$$P_{12} = B_{12} N_1 u(v) \quad (1)$$

and the total probability of emission is given by

$$P_{21} = [A_{21} + B_{21} u(v)] N_2 \quad (2)$$

At equilibrium, the absorption and emission rates must be equal i.e.

$$B_{12} N_1 u(v) = A_{21} N_2 + B_{21} N_2 u(v)$$

$$u(v) [N_1 B_{12} - N_2 B_{21}] = A_{21} N_2$$

$$u(v) = \frac{A_{21} N_2}{N_1 B_{12} - N_2 B_{21}}$$

$$\text{or } u(v) = \frac{A_{21} N_2}{N_2 B_{21} \left[\left(\frac{N_1}{N_2} \right) \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]}$$

$$u(v) = \frac{A_{21}}{B_{21} \left[\left(\frac{N_1}{N_2} \right) \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \quad \text{--- (3)}$$

As, the B_{12} and B_{21} depends upon the properties of energy states involved in transitions

$$\text{so, } \boxed{B_{12} = B_{21}} \quad \text{--- (A)}$$

So equ. (3) =

$$u(v) = \frac{A_{21}}{B_{21} \left(\frac{N_1}{N_2} - 1 \right)} \quad \text{--- (4)}$$

According to Boltzmann distribution law, the no. of atoms N_1 and N_2 in energy states E_1 and E_2 in thermal equilibrium at temp. T are given by

$$N_1 = N_0 e^{-E_1/RT} \quad \text{and} \quad N_2 = N_0 e^{-E_2/RT}$$

$$\text{so } \frac{N_1}{N_2} = e^{(E_2 - E_1)/RT} = e^{hv/RT} \quad (\because E_2 - E_1 = hv)$$

Put the value of N_1/N_2 in equ. (4)

$$u(v) = \frac{A_{21}}{B_{21} \left(e^{hv/RT} - 1 \right)} \quad \text{--- (5)}$$

Comparing eqn (4) with the Planck's radiation formula

$$u(v) = \frac{8\pi h v^3}{c^3} \left(\frac{1}{e^{hv/kt} - 1} \right)$$

we have

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}} - \textcircled{B}$$

Equations \textcircled{A} and \textcircled{B} are called Einstein's relations.