Tutorial-4

1). $T(n) = 3T(n/2) + n^2$

Answer) $a = 3, \ b = 2, \ f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ according to master Theorem

$T(n) = \theta(n^2)$

2). $T(n) = 4T(n/2) + n^2$

$a = 4, \ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (Case 2)

$\therefore$ according to master Theorem

$T(n) = O(n^2 \log n)$

3.) $T(n) = T(n/2) + 2^n$

$a = 1, \ b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore$ according to master Theorem

$T(n) = \theta(2^n)$.

4.) $T(n) = 2^n T(n/2) + n^n$

$\therefore$ Master's Theorem is Not applicable as a is function.

5.) $T(n) = 16T(n/4) + n$

$a = 16, \ b = 4, \ f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$\therefore T(n) = \theta(n^2)$

6.) $T(n) = 2T(n/2) + n \log n$

$a = 2,\ b = 2,\ F(n) = n \log n$

$n^{\log_b^a} = n^{\log_2^2} = n$

Now, $f(n) > n$

$\therefore$ According to masters Theorem $T(n) = \theta(n \log n)$

7.) $T(n) = aT\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2^2} = n$

$n > f(n)$

$\therefore$ According to masters Theorem $T(n) = \theta(n)$

8.) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4^2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to masters Theorem $T(n) = \theta(n^{0.5})$

9.) $T(n) = 0.5\,T(n/2) + \frac{1}{n}$.

$\therefore$ Masters Not applicable as $a < 1$.

10.) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b^a} = n^{\log^{16}} = n^2$

$n^2 < n!$

$\therefore$ According to masters Theorem $T(n) = \theta(n!)$

11) $T(n) = 4T\left(\frac{n}{4}\right) + \log n$

$a=4,\ b=2,\ f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^2)$

12.) $T(n) = \text{sqrt}(n) + (n/2) + \log n$

$\therefore$ Master's not applicable as a is Not constant.

13.) $T(n) = 3T(n/2) + n$

$a=3,\ b=2 \qquad f(n)=n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to Master's theorem, $T(n) = O(n^{\log_2 3})$

14.) $T(n) = 3T(n/3) + \sqrt{n}$.

$a=3,\ b=3,\ f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to Master's Theorem, $T(n) = \Theta(n)$

15.) $T(n) = 4T(n/2) + cn$

$a=4,\ b=a,\ f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

$\therefore$ According to Master's Theorem, $T(n) = \Theta(n^2)$

16.) $T(n) = 3T(n/4) + n\log n$

$\Rightarrow$ $a=3,\ b=4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$$n^{0.79} < n \log n.$$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n \log n)$

17) $$T(n) = 3T(n/3) + n/2$$

$$a = 3, \quad b = 3, \quad f(n) = \frac{n}{2}$$

$$n^{\log_b a} = n^{\log_3 3} = n$$

$$\Theta(n) = \Theta\left(\frac{n}{2}\right)$$

$\therefore$ According to Master's theorem $T(n) = \Theta(n \log n)$.

18) $$T(n) = 6T(n/3) + n^2 \log n.$$

$\rightarrow$ $$a = 6, \quad b = 3, \quad f(n) = n^2 \log n$$

$$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$$

$$n^{1.63} < n^2 \log n$$

$\therefore$ According to Master's theorem $T(n) = \Theta(n^2 \log n)$

19.) $$T(n) = 4T(n/2) + n/\log n.$$

$$a = 4, \quad b = 2, \quad f(n) = n/\log n.$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$n^2 > n/\log n.$$

$\therefore$ According to Master's theorem $T(n) = \Theta(n^2)$.

20). $$T(n) = 64T(n/8) - n^2 \log n.$$

Master's theorem is not applicable as $f(n)$ is Not increasing function.

21.) $$T(n) = 7T(n/3) + n^2$$

$\Rightarrow$ $$a = 7, \quad b = 3, \quad f(n) = n^2$$

$$n^{\log_b a} = n^{\log_3 7} = n^7$$

$$n^7 < n^2$$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's Theorem isn't applicable since regularity condition is violated in case 3.