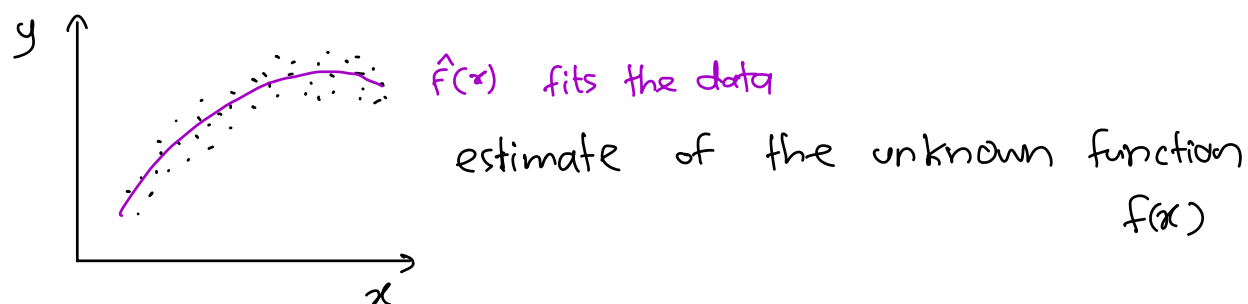


Problem: Understanding the input output relationship
 $f(x)$ is unknown

Given: Data from the past



simple solution: linear regression

to regress = moving backward

reasoning backward

→ learning from the past

Fitting the data: $y = \alpha + \beta x + \epsilon$
noise
linear combination

simplified → $\epsilon = 0$

$$y = \alpha + \beta x$$

$$E(y|x) = \alpha + \beta x = w_0 + w_1 x$$

weights

Intelligence is to find w_0, w_1 !
model ← | → model free

$$E(w_0, w_1 | x) = \frac{1}{2} \sum_{i=1}^N [y_i - (w_1 x_i + w_0)]^2$$

desired output estimate by guessing w_0, w_1

Build partial derivative wrt w_0, w_1 and simplify.

$$\frac{\partial F}{\partial w_0} = \frac{1}{2} \sum_{i=1}^N 2[y_i - (w_1 x_i + w_0)] (0 - (0+1)) = 0$$

$$\sum_{i=1}^N y_i = \sum_{i=1}^N w_1 x_i + w_0 = N w_0 + w_1 \sum_{i=1}^N x_i$$

$$\frac{\partial F}{\partial w_1} = \frac{1}{2} \sum_{i=1}^N 2[y_i - (w_1 x_i + w_0)] (0 - x_i - 0) = 0$$

$$\sum_{i=1}^N y_i x_i = w_0 \sum_{i=1}^N x_i + w_1 \sum_{i=1}^N x_i^2$$

Matrix form: $AW = y$

$$A = \begin{bmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \quad W = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$y = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \end{bmatrix} \quad W = A^{-1} y$$

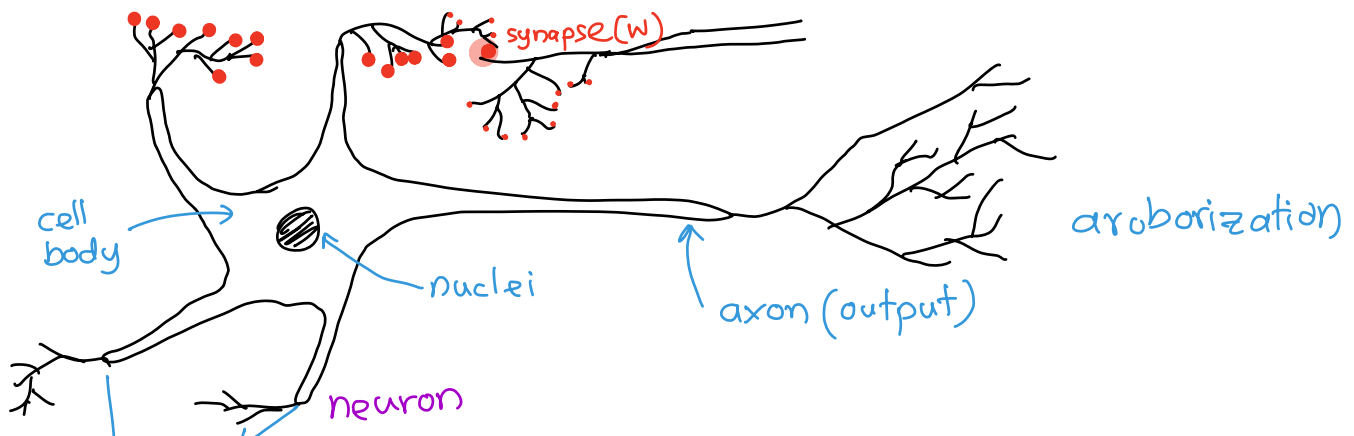
What about $y(x) = w_0 + w_1 x + w_2 x^2$

this is linear; linear combination of weights

forget about x^2

If this is still a linear regression, then how do we do non linear regression?

→ Neural networks



↓
dendrites (inputs)

Intelligence is changing synapse

How many neurons?

zebra fish $\approx 250k$ neurons

Adult human $\approx 10^{10} - 10^{12}$ neurons

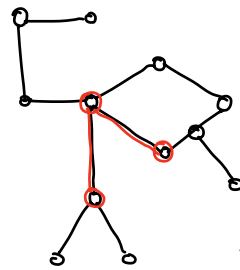
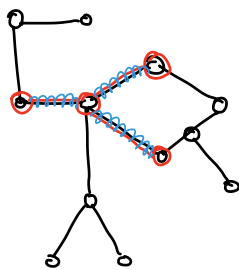
$\approx 10^{14}$ synapses

Synapses that increase the potential are in the excitatory mode.

Synapses that decrease the potential are in the inhibitory mode.

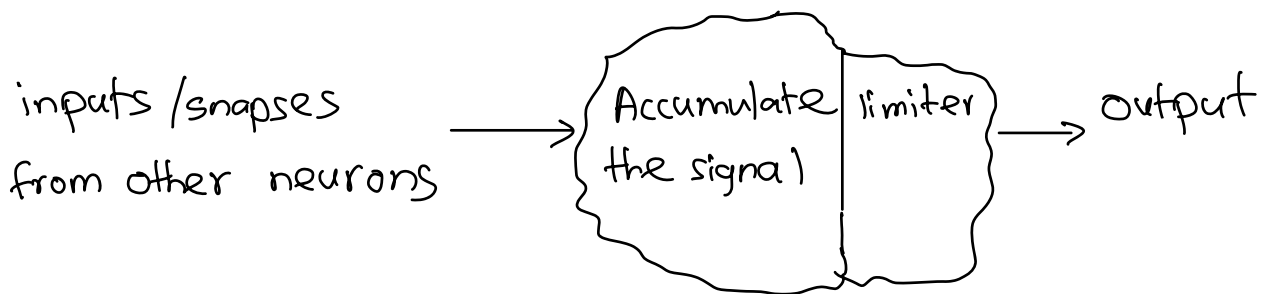
The synaptic networks are plastic.

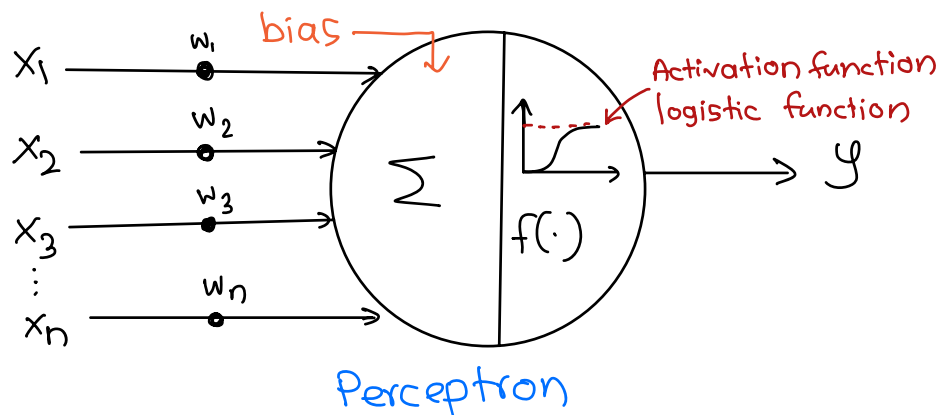
The plasticity is the most obvious manifestation of intelligence.



same network can
exhibit different
sub-graphs of connections

Abstraction of Neurons

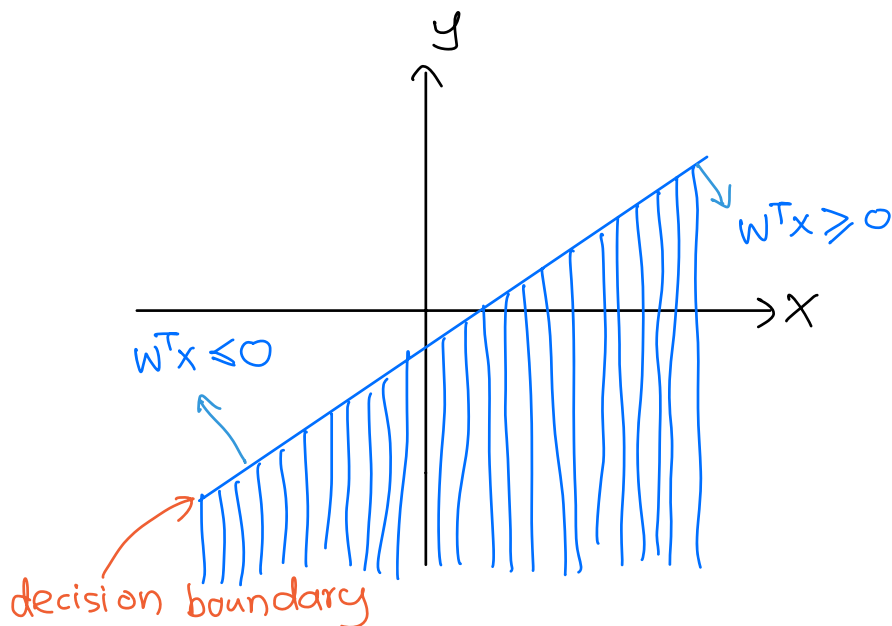




In perceptron, the logistic function is a "hard limiter" i.e. (threshold)

$$S = \sum_{i=1}^n w_i x_i + \text{bias}$$

This is a line (i.e. hyperplane). Hence the perceptron can separate 2 classes.



$$x_0 = +1, \quad w_0 = b$$

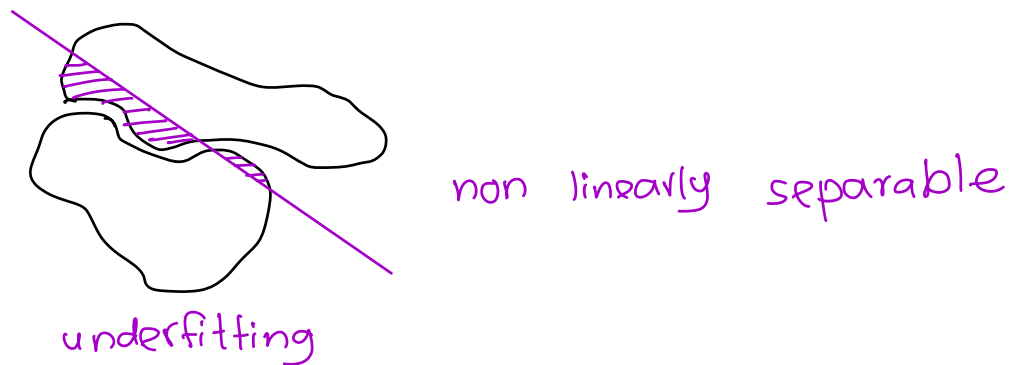
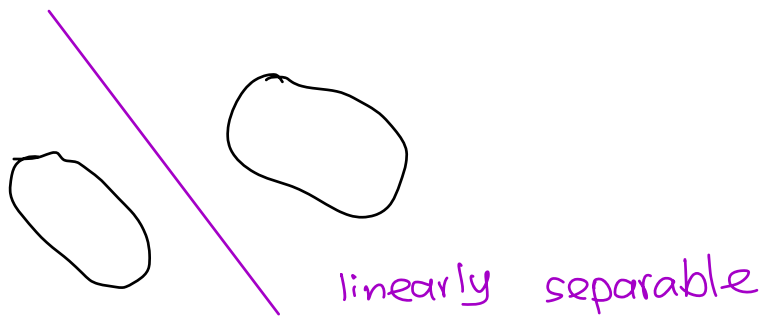
$$S = \sum_{i=0}^m x_i w_i$$

Now we have to iterate over data

$$S(n) = \sum_{i=0}^m w_i(n) \cdot x_i(n)$$

← number of instances
← number of iterations

$$= w^T(n) \cdot x(n)$$



Weight adjustment

$x(n)$ correctly classified by $w(n)$

$$w(n+1) = w(n) \quad \text{if} \quad w^T \cdot x(n) \underset{\text{boundary decision}}{\geq} 0 \quad x(n) \in c_{i,j}$$

no change since classification is correct

If misclassified,

$$w(n+1) = w(n) - \eta(n) \cdot \underbrace{x(n)}_{\text{contribution of input}}$$

If $\eta(n) = \eta > 0$ fixed increment adaptation

How η is not very important as long as positive
(Its just scales the contribution)

Perceptron can be proven to converge $\eta = 1$

