Fuzzy logic = Computing with words

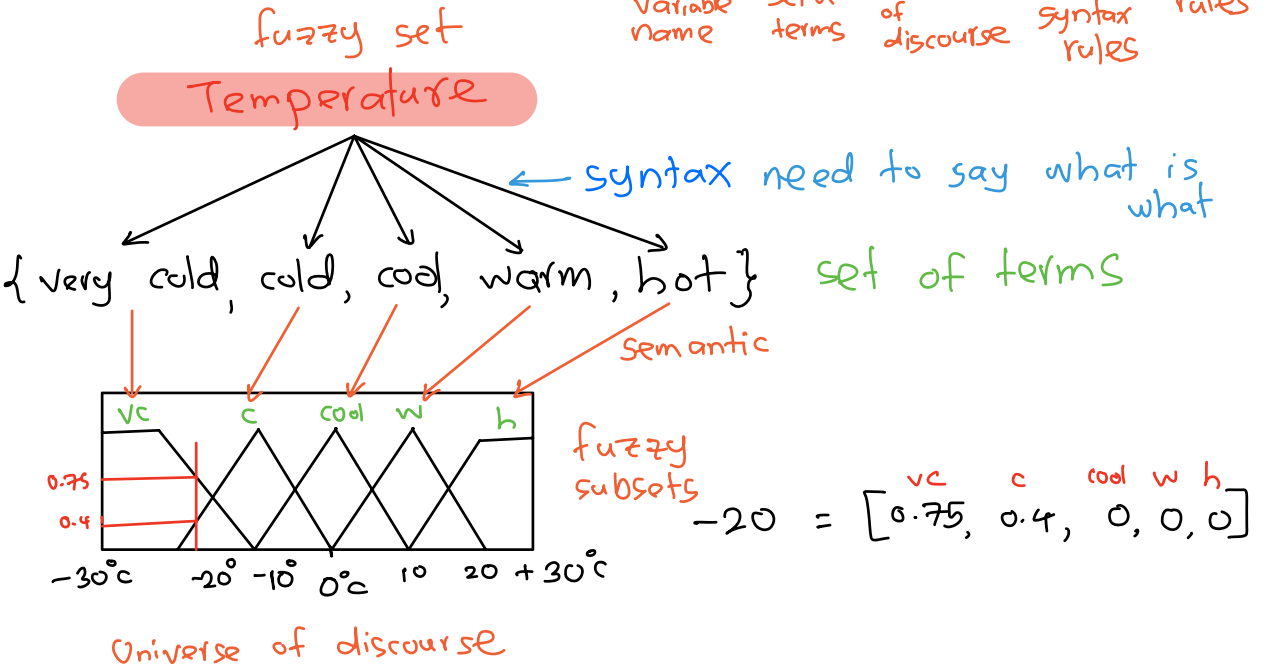
how can we model human thinking

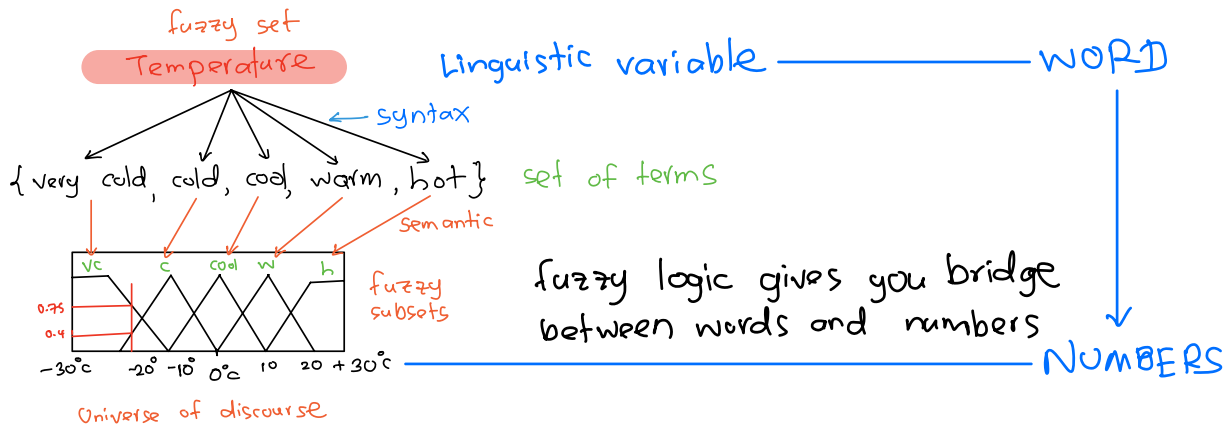
"linguistic variables that is variables whose values are not number but words or sentances in a natural or artificial language."

— L.A Zadeh

linguistic variable = $\langle x, T(x), U, G, M \rangle$ collection

- x — Variable name
- $T(x)$ — Set of terms
- U — Universe of discourse
- G — Set of syntax rules
- M — Set of semantic rules





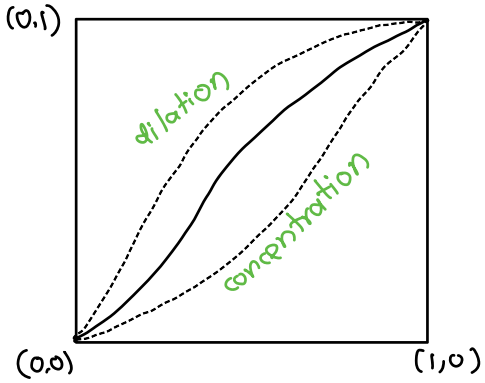
$A = \{x \mid x \in X \text{ and } x \text{ is } \Delta\}$ classical set

* x come from temperature and x is very hot temperature

$A_{fuzzy} = \{x, \mu_A(x) \mid x \in X,\ \mu_A(x) \in [0,1]\}$

Membership function μ_A quantifies the degree of belongingness of x to A.

Since A is now a linguistic variable, we can modify its meaning.



$$\mu_A^{CON}(x) = [\mu_A(x)]^2$$
$$\mu_A^{DIL}(x) = [\mu_A(x)]^{0.5}$$

μ_A is known (eg. bright, cold, old....)

$\mu_{very\ A} = CON(A)$

$\mu_{more\_or\_less\ A} = DIL(A)$

$\mu_{very\_very\ A} = CON(CON(A))$

$\mu_{not\_very\ A} = 1 - CON(A)$

$\mu_{more\ A} = A^{1.25}$

$\mu_{less\ A} = A^{0.75}$

Modus Ponens

Rule: If A is true, then B is true

Observation: A is true

Conclusion: B is true

Modus Tollens

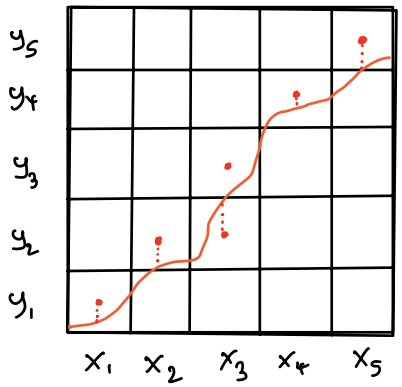
Rule: If A is true, then B is true

Observation: B is false

Conclusion: A is false

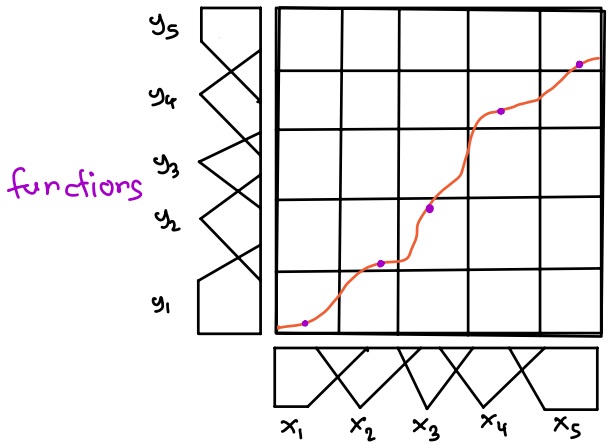
AI is function approximation.

(mis) Use DTs for function approximation.



scalors

Decision trees are not good for function approximation, because they are discreate.



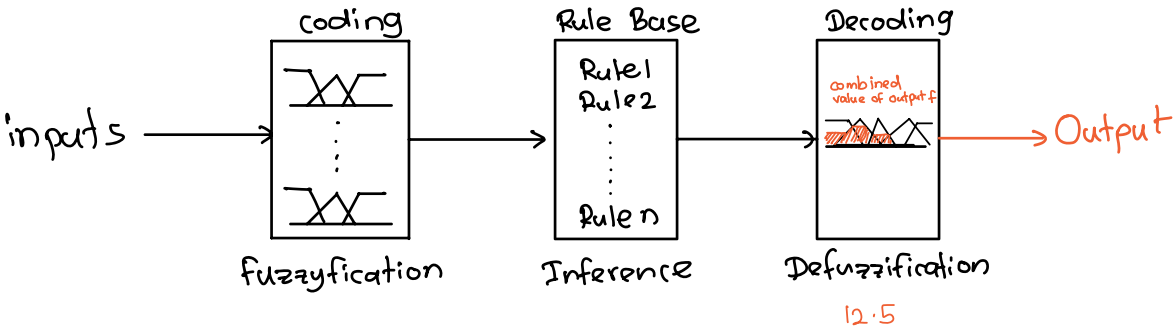
functions

If x_1 then y_1

If x_2, then y_2

⋮ ⋮

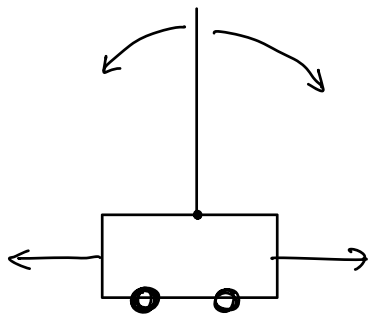
} A set of rules



12.5

$-22^{\circ}c = [0.75 \quad 0.25 \quad 0 \quad 0 \quad 0]$

Example : Inverted Pendulum

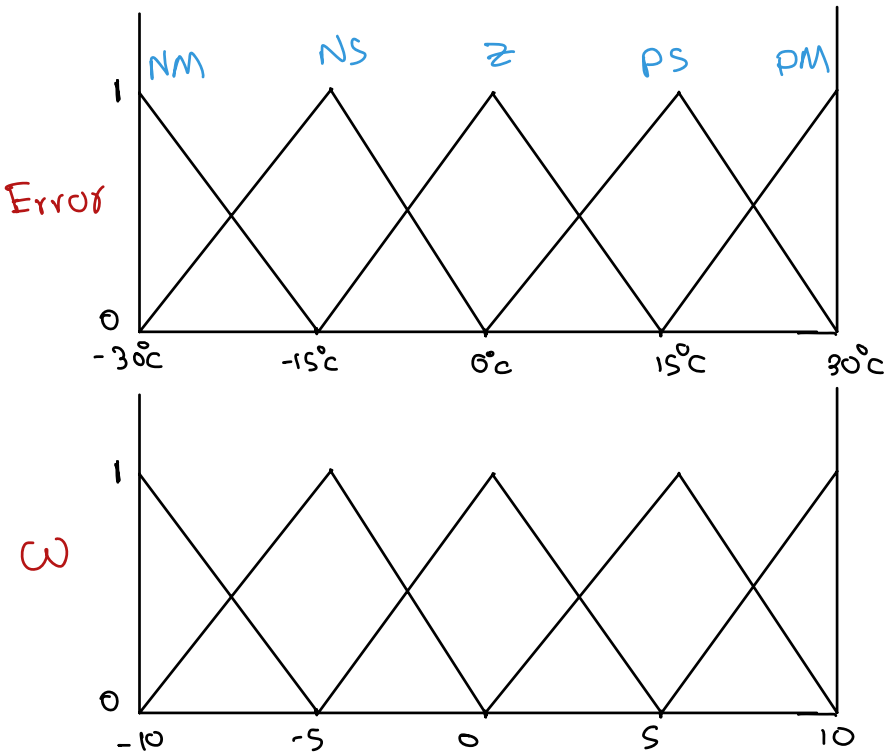


inputs : ① error (difference) in radian

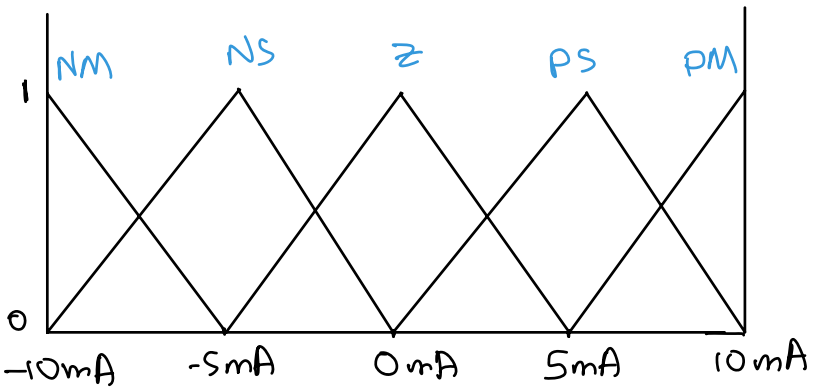
② ω

output : current (+/- for direction)

1. Fuzzification



Current



Rules

Error	ω	I
Z	Z	Z
Z	NM	PM
Z	PS	NS
PM	Z	NM

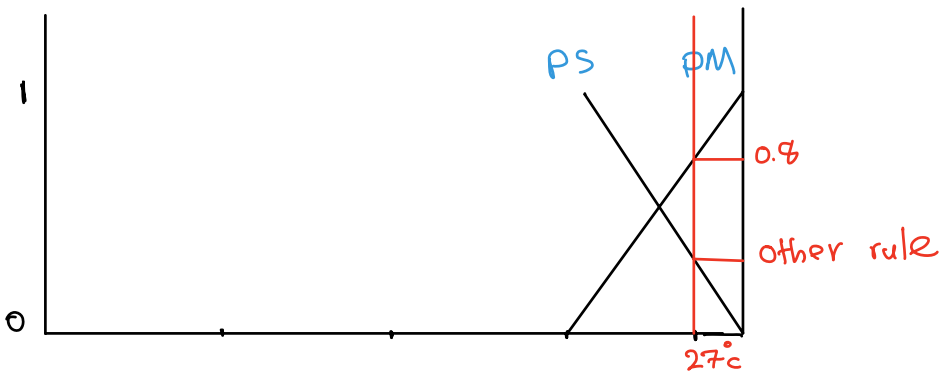
total rules ⟶ 5^3

input : error = 27°c

ω = −1.5 rads^{-1}

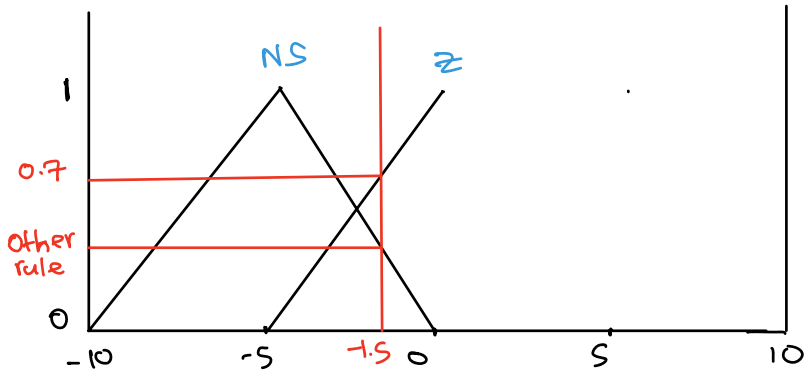
I = ?

Some rules will fire

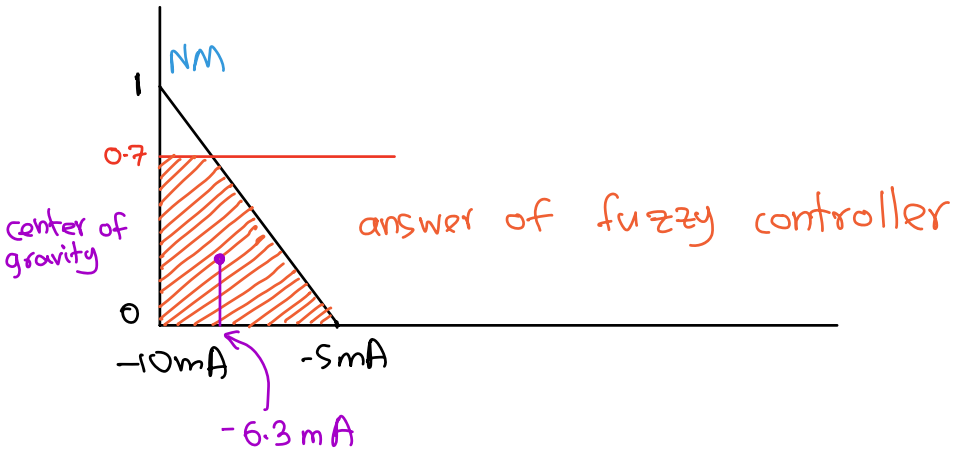


If error is PM and ω is Z

Then current is NM



$$AND = \min(0.8, 0.7) = 0.7$$



Fuzzy controllers can approximate the relationship between inputs and outputs via interpolation in a vague environment.

How do we get rules? from experts

from data via clustering / GAs

Evolving fuzzy rules