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PES University, Bengaluru
(Established under Karnataka Act No. 16 of 2013)

UE15MA251

DECEMBER 2019 END SEMESTER ASSESSMENT B.Tech. IV SEMESTER

UE15MA251 - LINEAR ALGEBRA

Time: 3 Hrs | Answer All Questions | Max Marks: 100

1a	Apply Gaussian Elimination to the system of equations $u + v + w = -2$, $3u + 3v - w = 6$ and $u - v + w = -1$ and find the solution. What coefficient of v in the third equation, in place of the present -1 would make the system singular ?	7
1b	Invert the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ by the Gauss – Jordan method.	7
1c	If the elementary row transformations $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 + 2R_2$ put matrix A into upper triangular form $U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, write down the associated elementary matrices and the matrix A. Find also the lower triangular factor L of A.	6
2a	Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$ to its echelon form and find its rank. What are the free variables and special solutions to $Ax = 0$?	7
2b	Find a basis and the dimension of the column space and left null space of $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 4 & 3 \\ 2 & 7 & -8 & 6 \end{bmatrix}$	7
2c	Let $v_1 = (1, 2, 3)$, $v_2 = (4, 5, 6)$ and $v_3 = (2, 1, 0)$. Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. If possible, find a linear dependence relation among these three vectors.	6
3a	Let $b = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $a = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of b onto the line through a. Then write b as the sum of two orthogonal vectors, one in span { a } and one orthogonal to a. Find the two associated projection matrices .	7
3b	Find the least squares solution of the inconsistent system $Ax = b$ where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. Also find the projection of b onto C (A) and the error vector e.	7
3c	If $H = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ is the reflection matrix about the θ line find the least positive value of θ. For this value of θ find the rotation matrix and the projection matrix.	6

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4a	Find the QR factorization of the matrix A whose column space is spanned by the vectors (3, 6, 0) and (1, 2, 2).	7
4b	Find the eigenvalues and the corresponding eigenvectors of $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$.	7
4c	Find the matrix S that diadonalizes $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Hence compute A^k for some k.	6
5a	A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, not more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society? Formulate a Linear Programming Model and solve by graphical method. (**Use only the graph sheet for sketch**)	8
5b	Solve using the Simplex method of LPP: Maximize $Z = x_1 + 2x_2 - x_3$ subject to the constraints $2x_1 + x_2 + x_3 \leq 14$, $4x_1 + 2x_2 + 3x_3 \leq 28$, $2x_1 + 5x_2 + 5x_3 \leq 30$ where $x_1, x_2, x_3 \geq 0$. Write down the optimal solution and the maximum value of Z.	12