PES University

PES University, Bengaluru
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UE20MA251

May 2022: END SEMESTER ASSESSMENT (ESA) B TECH IV SEMESTER

UE20MA251 - Linear Algebra and Its Applications

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1	a)	Determine the values of 'a' and 'b' for which the linear system $x + z = 4$, $2x + y + 3z = 5$ and $-3x - 3y + (a^2-5a)z = b-8$ has (a) no solution (b) a unique solution (c) infinitely many solutions.	7
	b)	Which elementary matrices E_{21} and E_{32} put A into upper triangular form $E_{32}E_{21}A = U$? Multiply by E_{32}^{-1} and E_{21}^{-1} to factor A into $LU = E_{21}^{-1}E_{32}^{-1}U$ given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$.	6
	c)	Use Gauss Jordan method to find the third column of B given $A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$ such that AB = I.	7
2	a)	For every c find Row reduced echelon form R and the special solutions to Ax=0, given $A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{pmatrix}$	7
	b)	Find a basis for the space spanned by the vectors $v_1 = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ -3 \end{pmatrix}$, $v_3 = \begin{pmatrix} -3 \\ -4 \\ 1 \\ 6 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ -3 \\ -8 \\ 7 \end{pmatrix}$, $v_5 = \begin{pmatrix} 2 \\ 1 \\ -6 \\ 9 \end{pmatrix}$. Which space do the basis vectors span?	6
	c)	Given $A = \begin{pmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{pmatrix}$, find a non-zero vector in Null space of A. Determine if u=(3, -1,3) is in column space of A? Does U belong to N(A)? Give reason.	7
3	a)	Find the projection p of w=(-1,1,4,3) onto the column space of V spanned by the vectors v_1=(1,1,0,1), v_2=(0,-1,1,1) .Which space does p belong to? Find a basis of the orthogonal complement $V^\perp$ of V. Split the vector w=v+u such that v is in C(V) and u is a vector in $N(V^T)$.	7
	b)	Let T be a linear mapping $T: R^4 \to R^3$ defined by $T(x, y, z, t)=(x - y + z + t,\ x + 2z - t,\ x + y + 3z - 3t)$. Write the matrix of transformation T and find the basis and dimension of Range of T and kernel of T.	6

c) Project the vector $b=(1,0,0)$ onto the vectors $a_1=(-1,2,2)$, $a_2=(2,2,-1)$ and $a_3=(2,-1,2)$. Add the projections $p_1+p_2+p_3$. Why is $p_1+p_2+p_3=b$? What is the product of the projection matrices P_1P_2 where P_1 and P_2 are the projection matrices on a_1 and a_2. 7

4 a) Apply Gram – Schmidt process to produce an orthonormal basis for $a_1=(2, 0, 1)$, $a_2=(4, -1, 2)$. Factor $A=QR$ where $A=(a_1, a_2, a_3)$. 7

b) The eigen vectors of a 3x3 matrix A corresponding to the eigen values -2. 3. 6 are (1, 0, -1), (1, -1, 1) and (1, 2, 1). Find the matrix A. 6

c) Find all eigen values and eigen vectors of $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$ and write two different diagonalising matrices S_1 and S_2. 7

5 a) The sum of squares of the quadratic form of a symmetric matrix is $x^T Ax = 4(x-\frac{1}{2}y)^2 + 2(y+\frac{1}{2}z)^2 + \frac{1}{2}z^2$. Determine A, its multipliers, its pivots and LDU? 6

b) The following table lists the weights and heights of 5 boys: 5

Boy	# 1	# 2	# 3	# 4	# 5
Weight(lbs)	120	125	125	135	145
Height(in)	61	60	64	68	72

Find the covariance matrix for the above data.

c) Find SVD of the matrix $A=\begin{pmatrix}1 & 1\\ -1 & 0\\ 0 & 1\end{pmatrix}$ 9

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