UE20MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

.**Unit 3: Orthogonality** Linear Transformations , Orthogonal Vectors and Subspaces, Orthogonal Bases, Cosines and Projections onto Lines, Projections and Least Squares.

Class No.	Portions to be covered
29-30	Linear Transformations , Examples
31	Transformations Represented by Matrices
32-33	Rotations, Reflections and Projections
34	**Matlab Class Number 5 –Span of Column space of A**
35-36	Orthogonal Vectors and Subspaces, Orthogonal Bases
37-38	Cosines and Projections onto Lines
39	Projections and Least Squares
40	**Applications**
41	**Matlab Class Number 6 –Four fundamental Subspaces of A**

Classwork problems:

1.	Find the image of these points after applying the transformation given: (i)Reflect (-3,2) across 90° line and then project on x-axis. (ii)Project (3,4) on y-axis and then rotate by 45° in clockwise direction. **Answer:** (i)(3,0) (ii) $2\sqrt{2}(-1,1)$
2.	Which of these transformations are not linear? Give reasons. $(i)T(x,y,z)=(x,y,0)$ $(ii)T(x,y)=(x+1,y+2)$ $(iii)T(x,y,z)=(\lvert x\rvert,y+z)$ $(iv)T(x,y)=(x+3,\ 2y,\ x+y)$ **Answer:** (ii)T(0) is not equal to 0 (iv)T(kv) $\neq$ KT(v)
3.	Find a 2x2 matrix A that maps (i)(1,3) and (1,4) into (-2,5) and (3,-1) respectively. (ii)Find the image of (2,-4) and (-1,2) **Answer:** $(i)A=\begin{pmatrix}-17 & 5\\ 23 & -6\end{pmatrix}$ (ii) (-54,70), (27,-35).
4.	For each of the following linear transformations T, find a basis and the dimension of the range and kernel of T: (i) $T:\mathbb{R}^3\to\mathbb{R}^2$ defined by T(x, y, z)=(x+2y-z, x+y-2z) (ii) $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by T(x, y)=(x+y, x-2y, 3x+y) **Answer:** (i) {(1,1),(2,1)} 2; {(3,-1,1)} 1 (ii) {(1,1,3),(1,-2,1)} 2; {(0,0)} 0

5. Find the matrix of the linear transformation T on $\mathbb{R}^3$ defined by T(x,y,z)=(x+2y-z, y+z, x+y-2z) with respect to
(i)the standard basis (1,0,0),(0,1,0), ((0,0,1)
(ii)the basis (1,1,1),(1,1,0), ((1,0,0)

Answer: $(i)\begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \quad (ii)\begin{pmatrix} 0 & 2 & 1 \\ 2 & -1 & -1 \\ 0 & 2 & 1 \end{pmatrix}$

6. From the cubics P_3 to P_2 what matrix represents d/dt ($3t^3-5t^2+2t+3$)?

7. Find a linear mapping $T:\mathbb{R}^4 \to \mathbb{R}^3$ whose Kernel is spanned by (1,2,3,4) and (0,1,1,1).

Answer: T(x,y,z,t)=(x+y-z, 2x+y-t, 0)

8. Find the projection of w=(-1,1,4,3) onto the column space of V spanned by v_1=(1,1,0,1), v_2=(0,-1,1,1) .Which space does p belong to? Find a basis of the orthogonal complement $V^{\perp}$ of V. Split the vector w=v+u such that v is in C(V) and u is a vector in N(V^T).

Answer: p=(1,-1,2,3), {(-,1,1,0), (-2,1,0,1)}; p=v, u=(-2,2,2,0)

9. Let P be the plane in $\mathbb{R}^4$ with equation x+y+z+t=0. What is the basis for $P^{\perp}$? what matrix has the plane P as its null space?

Answer: Basis for $P^{\perp}$={(1,1,1,1)}, A.

10. Find the matrix P that projects every point in $\mathbb{R}^3$ onto the line of intersection of the planes x-2y+3z=0 and y-z=0. What are the column space and row space of this matrix.

Answer: C(P) and C(P^T) is a line in R^3.

11. Let A=[2 -2 1] and let V be the nullspace of A. Find (i)a basis for V and a basis for $V^{\perp}$ (ii)a projection matrix P_1 onto $V^{\perp}$ (iii)the projection matrix P_2 onto V.

Answer: {(2,1,0), (1,0,1)}; {(2,-2,1)}

12. A sales organization obtains the following data relating the number of salespersons to annual sales:

Number of salespersons: x	5	6	7	8	9	10
Annual sales(in millions of rupees):y	2	3	4	5	6	7

(a)Find the least squares line relating x and y.
(b)Use the equation obtained in (a) to estimate the annual sales when there are 14 salespersons.

Answer: (a)y=x-3 (b)11millions of rupees.

13.	If S is the subspace of R^3 containing only the zero vector, what is $S^{\perp}$? If S is spanned by (1,1,1), what is $S^{\perp}$? If S is spanned by (2,0,0)and (0,0,3), what is $S^{\perp}$? **Answer:** R^3; {(-1,1,0), (-1,0,1)}; {(0,1,0)}
14.	What multiple of a=(1,1,1) is closest to b=(2,4,4)? Find also the point closest to a on the line through b. **Answer:** (10/3)a ; (5/9,10/9,10/9)
15.	Find $\|E\|^2 = \|Ax-b\|^2$ and solve the normal equations $A^T A\hat{x} = A^T b$.Find the solution $\hat{x}$and the projection p=A$\hat{x}$. (Use Least squares method) Given $A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{pmatrix}, x = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, b = \begin{pmatrix} 3 \\ 1 \\ 2 \\ -1 \end{pmatrix}$ **Answer:** (24/17,-8/17); p=(8/17(5,3,1,-4)