# PES University, Bengaluru
(Established under Karnataka Act 16 of 2013)

UE16MA251

### END SEMESTER ASSESSMENT (ESA)
B. TECH. 4th SEMESTER MAY 2018

### UE16MA251-LINEAR ALGEBRA

| Time: 3 Hrs | Answer All Questions | Max Marks: 100 |
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| 1. | a) | Compute L and U for the matrix $A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$ and hence find the conditions on a , b , c , d to get $A = LU$ with four pivots. | 7 |
| | b) | Let $A = \begin{bmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$ and $b = (b_1, b_2, b_3, b_4)$ Use method of Gauss elimination to find a condition on components of 'b' so that the system Ax = b is consistent .When $b = (2, 1, 0, 1)$ if $(x, 0, 0, 1)$ is a solution of the system Ax= b , find 'x' . | 7 |
| | c) | Obtain the inverse of $A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$ by Gauss Jordan method. | 6 |
| 2. | a) | If the column space of A is spanned by the vectors $(1, 3, -5)$ , $(0, 5, c)$ ,and $(-2, -1, 0)$ <br> Find the value of 'c' for which C(A) is : 1) a plane in $R^3$  2) whole of $R^3$. <br><br> Find special solutions to Ax = 0 where A has columns as given vectors. (By considering value of 'c' which results in C(A) being plane in $R^3$ ) | 7 |
| | b) | If the column space of A is spanned by the vectors $(0, 1, 1)$ , $(1, 2, 3)$ and $(4, 5, 6)$, $(7, 8, 9)$ .Find all those vectors that span left nullspace of A .Determine whether b = $(4, -2, 2)$ is in that subspace . What are the dimensions of rowspace and leftnullspace of matrix A . | 7 |
| | c) | Find the basis and dimension of these vector spaces : <br><br> 1) Space of all vectors in $R^4$ whose components add to zero . <br> 2) Space of all vectors in $R^4$ whose components are equal. <br> 3) Space of all symmetric matrices of order 2 by 2 . | 6 |
| 3. | a) | Let $T : R^4 \rightarrow R^3$ be a linear transformation defined by <br><br> $T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$ | 7 |

P.T.O.

Find the basis and dimension of : 1) Range of T  2) Kernel of T

b) Find the projection of 'b' onto the column space of A :

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix} \text{ Split 'b' into } p + q \text{ , where } p \text{ is in column space}$$

and 'q' is perpendicular to that space .Which of the four subspaces contains 'q' .

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c) Find a basis for the plane $x - 2y + 3z = 0$ in $R^3$ ,then find a basis for intersection of that plane with xy plane .Find the matrix which will project all vectors in the plane onto the line of intersection of the these two planes.

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4.

a) Determine the largest eigen value and corresponding eigen vector using power method for

matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial approximation of the eigen vector as $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ upto

seven iterations .

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b) Diagonalise the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and hence obtain $A^k$

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c) Apply Gram-Schmidt process on columns of matrix A and hence obtain QR factorization .

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}$$

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5.

a) Solve graphically (Use graph sheet available in the answer booklet only ) the LPP :

Min $Z = 20x_1 + 10x_2$

$x_1 + 2x_2 \geq 40, \quad 3x_1 + x_2 \geq 30, \quad 4x_1 + 3x_2 \geq 60$ where $x_1, x_2 \geq 0$

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b) Solve by Simplex Method :

Max $Z = 3x_1 + 2x_2 + 5x_3$

Sub to : $x_1 + 2x_2 + x_3 \leq 430, \quad 3x_1 + 2x_3 \leq 460, \quad x_1 + 4x_2 \leq 420$ where $x_1, x_2, x_3 \geq 0$

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