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PES UNIVERSITY, BANGALORE-85 (Established under Karnataka Act 16 of 2013)

ESA, B.Tech IV SEM, December- 2021

Linear Algebra and its Applications

Sub Code: UE15/16MA251

| | | 3 Hrs Answer All Questions Max Marks: 10 | 00 |
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| | | 3 Hrs Answer All Questions Max Marks: 10 | 7 |
| 1. | | Use Gaussian elimination to solve the equations $2x_1 + 3x_2 + x_3 = 3$, $4x_1 + 7x_2 + 5x_3 = 2$, and $2x_1 + 4x_2 + x_3 = 1$. Which 3 matrices E_{21} , E_{31} and E_{32} put the coefficient matrix A into triangular form U. | |
| | <i>b)</i> | Invert the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by Gauss-Jordan method. Given $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ -1 & 1 & 4 \end{pmatrix}$, find the LU decomposition of A. | 7 |
| | c) | | 6 |
| 2. | a) | For every C, find the special solution Ax=0 where $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & C & 2 & 2 \end{bmatrix}$. | 7 |
| | | Find the basis and dimension for the column space and row space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 7 & 10 & 11 \\ 2 & 4 & 7 & 7 \end{bmatrix}.$ | 7 |
| | ĺ | Decide the dependence or independence of the vectors $(1,-3,2)$, $(2,1,-3)$ and $(-3,2,1)$. | 6 |
| 3. | a) | Solve $Ax = b$ by least squares and find $p = A\hat{x}$ if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. | 7 |
| | 1 ^ | On the space P_3 of cubic polynomials, what matrix represents the second derivatives of cubics? What is the column space of this matrix? | 7 |
| | c) | Determine the new point by projecting $x = (4,2)$ on the x-axis and then rotate | 6 |

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| | | by 45° in counter clockwise. | |
| 4. | | From the given vectors find orthonormal vectors using Gram-Schmidt process where = $(1, 1, 0)$, $b = (1, 0, 1)$ and $c = (0, 1, 1)$. | 7 |
| | b) | Find Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. | 7 |
| | c) | Project $b = (0,3,0)$ onto each of the orthonormal vectors $a_1 = (2/3,2/3,-1/3)$, $a_2 = (-1/3,2/3,2/3)$ and find its projection p onto the plane of a_1 and a_2 . | 6 |
| 5. | a) | A dealer deals in only two items — computer keyboards and computer mouse. He has Rs 50,000 to invest and has storage space of at most 60 pieces. A computer keyboard costs Rs 2500 and a computer mouse Rs 500. He estimates that from the sale of one keyboard, he can make a profit of Rs 250 and that from the sale of one computer mouse a profit of Rs 75. He wants to know how many computer keyboards and mouse he should buy from the available money so as to maximize his total profit, assuming that he can sell all the items which he buys. Formulate the above situation into a Linear programming model and also provide a graphical solution. (Draw a neat sketch on the graph sheet). Also find the maximum profit. | 10 |
| | <i>b)</i> | Use the simplex method to solve the following LP problem: Maximize $z = 3x_1 + 5x_2 + 4x_3$ subject to the constraints $2x_1 + 3x_2 \le 8$, $2x_2 + 5x_3 \le 10$, $3x_1 + 2x_2 + 4x_3 \le 15$ and $x_1, x_2, x_3 \ge 0$. | 10 |