ENGINEERING MATHEMATICS-IV
LINEAR ALGEBRA
MATLAB

Department of Science and Humanities

Projection matrices and least squares:

Projections:

We know that $P = A(A^TA)^{-1}A^T$ is a matrix that projects a vector b onto the space spanned by the columns of A. If b is perpendicular to the column space, then it's in the left nullspace $N(A^T)$ of A and Pb = 0. If b is in the column space then b = Ax for some x, and Pb = b.

A typical vector will have a component p in the column space and a component e perpendicular to the column space (in the left nullspace); its projection is just the component in the column space. The matrix projecting b onto $N(A^T)$ is I − P:

e = b − p

e = (I − P)b.

Naturally, I − P has all the properties of a projection matrix.

Projection matrices and least squares:

Least squares:

We want to find the closest line b = C + Dt to the points (1, 1), (2, 2), and (3, 2). The process we're going to use is called linear regression; this technique is most useful if none of the data points are outliers. By "closest" line we mean one that minimizes the error represented by the distance from the points to the line. We measure that error by adding up the squares of these distances. In other words, we want to minimize

$$\|Ax-b\|^2 = \|e\|^2 .$$

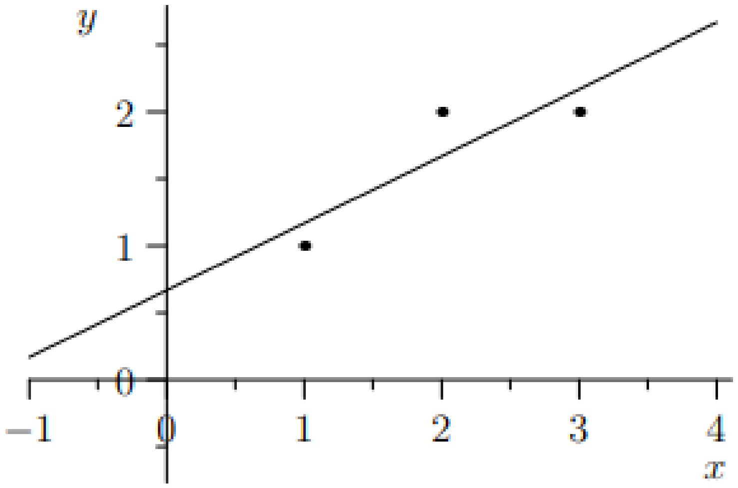


Figure 1: Three points and a line close to them.

Projection matrices and least squares:

Find the projection for the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$ **;** $x = \begin{pmatrix} u \\ v \end{pmatrix}$ **and**

$$b = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}.$$

Code:

```
A=[1,0;0,1;1,1]
b=[1;3;4]
x = lsqr(A,b)
```

Projection matrices and least squares:

Output:

```
A =
        1   0
        0   1
        1   1
b =
        1
        3
        4
lsqr converged at iteration 2 to a solution with relative residual 6.7e-17.
x =
        1.0000
        3.0000
```

Projection matrices and least squares:

Find the projection for the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 1 \end{pmatrix}$; $x = \begin{pmatrix} u \\ v \end{pmatrix}$ and

$$b = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}.$$

Code:

```
A=[1,0;0,2;3,1]
b=[1;0;4]
x = lsqr(A,b)
```

Projection matrices and least squares:

OUTPUT:

```
A =
        1       0
        0       2
        3       1
b =
        1
        0
        4
lsqr converged at iteration 2 to a solution with relative residual 0.076.
x =
        1.2927
        0.0244
```

Projection matrices and least squares:

Find the point on a plane x+y-z=0 that is closest to (2,1,0)

Code:

```
syms   c
P=[2,1,0]+c*[1,1,-1]
s=1*(c+2)+1*(c+1)-1(-c)==0
s1=solve(s,c)
p=[2,1,0]+s1*[1,1,-1]
```

Projection matrices and least squares:

Output

P =

[3*c + 1, 4*c, c + 1]

s =

26*c + 4 == 1

s1 =

-3/26

p =

[17/26, -6/13, 23/26]

Projection matrices and least squares:

Find the point on a plane 3x+4y+z=1 that is closest to (1,0,1)

Code:

```
syms   c
P=[1,0,1]+c*[3,4,1]
s=3*(1+3*c)+4*(4*c)+(1+c)==1
s1=solve(s,c)
p=[1,0,1]+s1*[3,4,1]
```

Projection matrices and least squares:

Output:

```
P =
        [c + 2, c + 1, -c]

s =
        3*c + 3 == 0

s1 =
        -1

p =
        [1, 0, 1]
```

Projection matrices and least squares:

Let $u = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ **onto** $v = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$ **and find P, the matrix that will project any matrix onto the vector v. Use the result to find proj$_v$ u.**

Code:

```
u=[1;7]
u =
        1
        7
v=[-4;2]
v =     -4
        2
```

Projection matrices and least squares:

```
P=(v*transpose(v))/(transpose(v)*v)

P =
        0.8000      -0.4000
       -0.4000       0.2000

P*u

ans =
       -2
        1
```

Projection matrices and least squares:

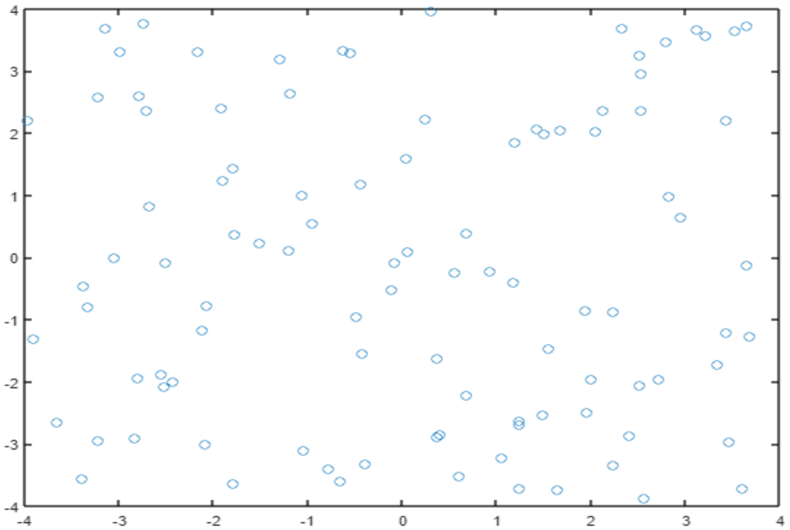
Projecting a lot of vector on a single vector:

Code:

```
u=8*rand(2,100)-4;
x=u(1,: )
y=u(2,: )
plot(x,y,'o')
```

In the below figure I have generated a 100 random vectors.

Projection matrices and least squares:



In this figure each circle represents the tip of a vector whose tail begins at the origin.

Next , I will take the projection matrix P to project each of the 100 2 by 1 vectors in matrix U onto the vector v,

Projection matrices and least squares:

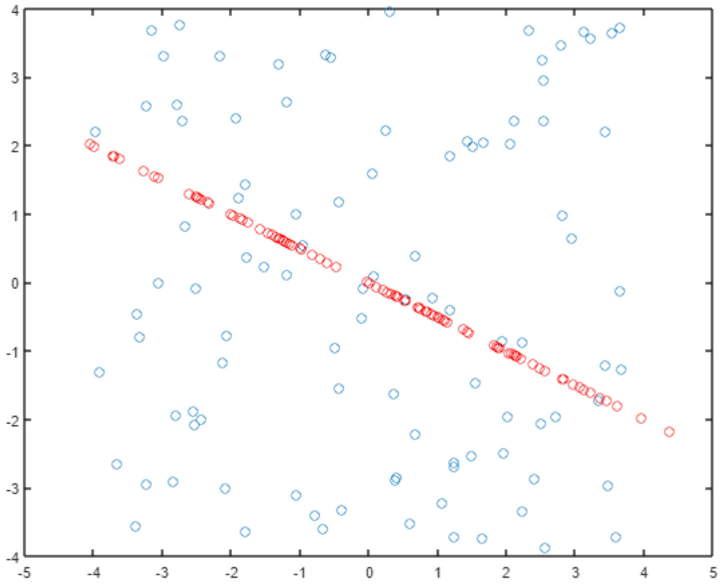
Code:

```
>> P=[0.8,-0.4;-0.4,0.2]

P =
        0.8000        -0.4000
       -0.4000         0.2000

>> Pu=P*u;
x=Pu(1,:)
y=Pu(2,:)
hold on
plot(x,y,'ro‘)
```

Projection matrices and least squares:



Here each vector in the matrix u is projected onto a line in the direction of the vector v=[-1;2].

Projection matrices and least squares:

Example Problems:

1. Find the least square fit for this system

$$x + 2y = 3$$
$$3x + 2y = 5$$
$$x + y = 2.09$$

2. <u>Find the point on a plane 13x+4y+z=1 that is closest to (1,-1,1)</u>

Projection matrices and least squares:

Example Problems:

3. Find the least square fit for this system

$$x + 2y + z = 3$$
$$3x + 2y - 2z = 5$$
$$x + y + 7z = 21.09$$

THANK YOU