UE21MA241B: Linear Algebra & its Applications (4-0-0-4-4)

Preamble:
This course familiarises students with some basic techniques in matrix theory which are essential for analysing linear systems and their inter relations and applications to Engineering. The course provides hands-on experience in basic programming concepts using MATLAB for solving problems relevant to these areas.

Course Objectives:
- To solve systems of Linear Equations using Matrices.
- To analyze problems using the notion of Vector spaces and subspaces.
- To apply Orthogonality of Vectors & Subspaces and Gram Schmidt Orthogonalization to produce Orthonormal Vectors.
- To find Eigenvalues, Eigenvectors for Diagonalization, Singular Value Decomposition, which is used to compute Pseudo Inverse, Least Squares fitting of data, Multivariable control, and Matrix Approximation.
- To visualize solution to linear system of equations with different approaches using MATLAB.

Course Outcomes:
After completing this course, students will be able to:
- Solve systems of Linear Equations using Matrix Transformations, Interpret the nature of Solutions, Visualize Consistency of Linear system of Equations and also compute inverse of a Matrix.
- Demonstrate the ability to work within Vector Spaces, distil Vector Space properties and understand the concepts of the four fundamental Subspaces, Linear Span, Linear Independence, Dimension and Basis
- Analyze Matrix Representation with respect to Standard and Nonstandard Bases.
- Learn the concepts of Orthogonal Vectors and Orthogonal Subspaces and apply the Gram-Schmidt process to find an Orthonormal Basis in a Subspace
- Analyze Eigenvalues, Eigenvectors and Diagonalization of a Matrix.
- Apply the concept of Positive Definite Matrices, Singular Value Decomposition into Application problems.

Course Content:
Unit 1: Matrices and Gaussian Elimination
Introduction, The Geometry of Linear Equations, Gaussian Elimination, Singular Cases, Elimination Matrices, Triangular Factors -LU decomposition & Cholesky's method and Row Exchanges, Inverses and Transposes, Inverse by Gauss -Jordan method
Self-Learning Component: Algebra of Matrices. **12 Hours**

Unit 2: Vector Spaces
Vector Spaces and Subspaces (definitions only), Linear Independence, Basis and Dimensions, Row reduced Echelon form, Sum of subspaces, Direct Sums,The Four Fundamental Subspaces, Rank-Nullity theorem.
Self-Learning Component: Examples of Vector Spaces and Subspaces. **12 Hours**

Unit 3: Orthogonality
Linear Transformations, Algebra of Linear transformations, Invertible maps, Isomorphisms, Orthogonal Vectors and Subspaces, Orthogonal Bases, Cosines and Projections onto Lines, Projections and Least Squares. **11 Hours**

Unit 4: Orthogonalization, Eigenvalues and Eigenvectors

Orthogonalization, The Gram-Schmidt Orthogonalization process, Introduction to Eigenvalues and Eigenvectors, Properties of Eigenvalues and Eigenvectors, Spectral theorem, Symmetric Matrices, Diagonalization of a Matrix, Cayley-Hamilton theorem (statement only).

11 Hours

Unit 5: Singular Value Decomposition

Quadratic Forms, Definitions of Positive definite, negative definite, positive semi-definite negative semi-definite, indefinite forms and matrices, Tests for Positive Definiteness, problems on Positive Definite Matrices and Least Squares, problems on Semi Definite Matrices, Singular Value Decomposition and Pseudoinverse. **10 Hours**

Applications to Biotechnology Engineers:

Unit-1: Robust finger print image enhancement using Gauss elimination/Gaussian filters for MRI Images' noise reduction

Unit-2. DNA sequence analysis problems.

Unit-3: Problems on orthogonal transcription systems which are essential for building synthetic gene regulation and custom genetic circuits.

Unit-4: Problems involving making of multiple copies of DNA, called DNA cloning and problems on making large amount of specific human protein, Modelling Epidemic spread.

Unit-5: DNA gene expression data via SVD.

Applications to Civil Engineers:

Unit-1:Structural Analysis and Finite element methods, Structural Dynamics, Traffic Studies and Analysis

Unit-2: Analytical solutions and applications in Civil Engineering.

Unit-3:Earthquake Engineering, Wind Analysis of Circular Tanks, Structural Analysis

Unit-4: Determination of frequency and mode shapes in Earthquake Engineering, Structural Dynamics and Theory of Elasticity. Finite Element Methods

Unit-5:Risk Assessment of Floods, best fit curve and optimal curve in geotechnical engineering, and Estimation and Costing.

Applications to Mechanical Engineers:

Unit-1: Finite difference formulations for heat transfer by conduction in slabs & fins. Finding natural frequencies and mode shapes of Multi Degrees-of-Freedom systems in Mechanical Vibrations, Solving equations of equilibrium in Finite Element Methods**.**

Unit-2: Determination of aerodynamic forces and moments, plotting streamlines using velocity fields. Understanding kinematic mechanism for velocity and acceleration analysis in Mechanics of Machines and Mechanisms, Strain analysis of structure in Theory of Elasticity.

Unit-3: Orthogonality of streamlines and equipotential lines, Solving equations involving load and displacement. Finding natural frequencies of Multi Degrees-of-Freedom in Mechanical Vibrations, Determination of resultant stress vector on any arbitrary plane with known cosine directions .Vehicle stability analysis for transient response,

Unit-4: Heat conduction as an Eigen value problem, Determination of Eigen values and Eigenvector of multi-degrees-of-freedom system using Orthogonality properties in Mechanical Vibrations State-space representation of system in solving control engineering problems.

Unit-5: Development of heat transfer correlations, Problem solving on semi-definite system in two degrees-of-freedom systems in Mechanical Vibrations.

Applications to Computer Science Engineers:

Unit-1. Matrices in Computer Graphics.

Unit-2: Vector space models for information retrieval application, Gaming applications,

Biometric Applications, Image Processing.
Unit-3 Application of Transformation (Translation, Rotation, Scaling) in Computer Graphics. Projections. Latent Semantic Analysis.
Unit-4: Finding most vital /influential person in the network/social platforms ,here we use Eigenvector centrality for undirected graph to find the important node/person. Face Recognition, Cryptographic techniques, Markov chains, Data Augmentation, Google's PageRank is a variant of the Eigenvector centrality measure for directed network.
Unit-5: Principal Component Analysis. Compress an image using SVD, Separate two audio sources using SVD.

Applications to Electronics Engineers:

Unit-1: Solving linear systems in image processing, speech processing, control systems, pattern recognitions, machine learning, Deep learning, Network analysis etc. Nodal analysis and Mesh analysis in "Network Analysis and Synthesis". VLSI arithmetic systems.
Unit-2: Understanding electromagnetic fields. Automating Analog filter design, Transfer function modelling of sequential circuits; all quantum spaces study depicted as vector spaces.
Unit-3: Solving over-determined and underdetermined systems, least square solutions for inverse problems like deconvolution (eg. image restoration), support vector machines, loss functions, regularization functions problems. Linear transformations to convert signal to noise ratio (SINR) expressions into a linear expression in wireless link scheduling problems. Linear transformations in minimizing FPGA design.
Unit-4: Solving RLC Circuits; developing and analysing on-chip devices for Photonics, Quantum optics, memristors etc. interpolation, polynomial curve fitting
Unit-5: SVD for compressing images, decomposing 2-D filters intosimpleouterproductsof1-D filters (much more efficient to implement), dimensionality reduction.

Applications to Electrical and Electronics Engineers:

Unit-1: Determine current and voltage in different RLC Circuits applying equations and converting to matrix.
Unit-2: Determine current and voltage in RLC Circuits.
Unit-3: Linear Transformations applied to time scaling in Audio, compression/expansion in pictures and different 3D transformations in Robotics .
Unit-4: Applying equations to RLC circuit, getting characteristic equations, eigen values and vectors to find the general solution; System Stability in Control System Applications.
Unit-5: Lyapunov System Stability in non-linear Control System Applications;
SVD application in Digital Image Processing

Text Book:

1: Linear Algebra and its Applications, Gilbert Strang, 4th Edition, Thomson Brooks/ Cole, Second Indian Reprint 2007.

Reference Books:

1: Linear Algebra and its Applications, David .C lay, Publication by Pearson, 5th Edition, 2015
2: Linear Algebra, Schaum's outlines, Seymour Lipschutz and Marc Lipson, 4th Edition, McGraw-Hill publications, 2009.
3**:** Higher Engineering Mathematics, B S Grewal, 44th Edition, 2020, Khanna Publishers.
4: Practical Linear Algebra, Gerald Farin and Dianne Hansford, 3rd Edition, CRC Press, Taylor & Francis Group, 2013.

MATLAB:

1. Introduction and Gaussian Elimination
2. Inverse of a Matrix by the Gauss- Jordan Method
3. The LU Decomposition
4. The Span of Column Space of a Matrix
5. The Four Fundamental Subspaces
6. The Gram-Schmidt Orthogonalization
7. QR factorization
8. Projections by Least Squares
9. Eigen values and Eigen Vectors of a Matrix.

Reference Books:

1: Getting started with MATLAB, Rudra Pratap, Oxford University Press, 7th Edition, 2016.
2: MATLAB for Engineers, Holly Moore, Pearson Publications, New Jersey, 5th Edition, 2018.