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PES University, Bengaluru
(Established under Karnataka Act No. 16 of 2013)

UE17/18/19MA251

Dec 2021: END SEMESTER ASSESSMENT (ESA) B TECH IV SEMESTER

UE17/18/19MA251 - Linear Algebra and Its Applications

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1	a)	Do the three planes $x + 2y + z = 4$, $y - z = 1$ and $x + 3y = 0$ have at least one common point of intersection ? Explain. Is the system consistent if the last equation is changed to $x + 3y = 5$? If so, solve the system completely.	7
	b)	Apply elimination to produce the factors L and U for A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & -2 & 4 \end{pmatrix}$. Is A=LU possible? Explain. Write down the permutation matrices if any.	6
	c)	Compute inverse of the following matrices by Gauss Jordan method. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$	7
2	a)	Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors: $\left\{t^2+t+2,\ 2t^2+t,\ 3t^2+2t+2\right\}$	7
	b)	Find a basis for the set of vectors in R^3 in the plane x+2y+z=0. Also find its dimension.	6
	c)	If the column space of A is spanned by the vectors (1,2,0), (-2,3,-7), (5,2,8) find all those vectors that span the left null space of A, Determine whether or not the vector b=(-4,2,2) is in that subspace What are the bases and dimensions of $C(A^T)$ and $N(A^T)$.	7
3	a)	Find the matrix P that projects every point in R^3 onto the line of intersection of the planes x+y+t=0 and x-t=0. What are the column space and row space of this matrix.	7
	b)	For each of the following linear transformations T, find a basis and the dimension of the range and kernel of $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by T(x, y, z)=(x+y, y-z)	6
	c)	Find the projection of b=(3,3,3) onto the column space of A spanned by (1,0,2) and (1,1,4).Split b into p+q with p in $C(A^T)$and q in N(A).	7
4	a)	Given the orthonomal basis $S=\left\{\left(\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}}\right),(0,1,0),\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\right\}$ for R^3. Express the vector (1,2,3) as a linear combination of the vectors in S.	7

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	b)	Factor $A=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ into $S\Delta S^{-1}$ and hence computeA^{85}.	6
	c)	Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}-6 & -1\\ 2 & -3\end{pmatrix}$. Verify that the trace equals the sum of eigenvalues and the determinant equals their product. If we shift A to A – 7 I what are the eigenvalues and eigenvectors and how are they related to those of A ?	7
5	a)	If A= $Q\Delta Q^T$ is symmetric positive definite, then R= $Q\sqrt{\Delta}Q^T$ is its symmetric positive definite square root. Why does R have positive eigen values? Compute R and verify R^2=A for $A=\begin{pmatrix}10 & 6\\ 6 & 10\end{pmatrix}$	7
	b)	Write the symmetric matrix which corresponds to the following quadratic forms: $(i) Q(x)=5x_1^2+3x_2^2+2x_3^2-x_1x_2+8x_2x_3$	4
	c)	Find SVD of the matrix $A=\begin{pmatrix}4 & 11 & 14\\ 8 & 7 & -2\end{pmatrix}$	9

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