

MAY 2016: END SEMESTER ASSESSMENT (ESA) B.TECH. IV SEMESTER

UE14MA251- LINEAR ALGEBRA

Time: 3 Hrs

Answer All Questions

Max Marks: 100

1.	a)	Use the method of Gaussian Elimination to decide if the planes $6x - 3y + 3z = -2$, $2x - y + z = 1$, $3x + 2y - 4z = 4$ have a common point of intersection in \mathbb{R}^3 . What happens if the right hand side of the second equation is changed to $-2/3$ instead of the present number 1? What are all the solutions of the system in that case?	7
	b)	Find the matrices L and U for $A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -8 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 6 & 7 & -3 & 1 \end{bmatrix}$. Write down the permutation matrix that is used in the process, if any.	7
	c)	Use the Gauss – Jordan method to invert $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$	6
2.	a)	Determine whether the matrices $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{bmatrix}$ have the same column space. If so, what is $C(A)$ or $C(B)$?	7
	b)	If $A = \begin{bmatrix} 2 & -6 & -8 \\ -4 & 12 & a \\ 1 & b & 2 \end{bmatrix}$ find the values of a and b so that the column space of A is (i) the whole of \mathbb{R}^3 (ii) a 2-dimensional subspace of \mathbb{R}^3 (iii) a 1-dimensional subspace of \mathbb{R}^3 . Find a basis for $N(A)$ in the second case choosing $a = 22$.	7
	c)	If a, b, c are linearly independent vectors determine whether the vectors $a - b$, $b - c$, $c - a$ are also linearly independent.	6
3.	a)	Find a basis for the subspace S spanned by all solutions of $x + y + z - t = 0$ and a basis for the orthogonal complement of S. Find a vector a in S and a vector b in S^\perp such that $a + b = (1, 1, 1, 1)$.	7
	b)	Find a least squares solution of the inconsistent system $Ax = b$ where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. Also find the error vector e.	7
	c)	Describe geometrically the range and kernel of the following linear transformations: (i) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (0, x, z)$ (ii) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x, 0)$ (iii) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (y, y, 0)$	6

4.	a)	Apply the Gram – Schmidt process to $(1, 0, 1)$, $(1, 0, 0)$ and $(2, 1, 0)$ to produce a set of three orthonormal vectors and write the result in the form $A = QR$.	7
	b)	Calculate seven iterations of the power method to approximate (correct to three decimal places) a dominant eigen value of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$. Use $(1, 1, 1)$ as the initial* approximation.	7
	c)	Find all the eigen values and eigen vectors of A and write two different eigen vector matrices , where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.	6
5.	a)	Solve (<u>only on a graph sheet</u>) the LPP : Maximize $Z = 3x + 2y$ subject to $2x + y \leq 18$, $2x + 3y \leq 42$, $3x + y \leq 24$ where $x \geq 0$, $y \geq 0$.	6
	b)	Find all the basic solutions of the system of equations $2x_1 + x_2 + 4x_3 = 11$, $3x_1 + x_2 + 5x_3 = 14$. Identify in each case the basic and non- basic variables.	6
	c)	Use the Simplex method to solve the LPP : Maximize $Z = x_1 + 3x_2$ subject to $x_1 + 2x_2 \leq 10$, $0 \leq x_1 \leq 5$, $0 \leq x_2 \leq 4$.	8