# UE20MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

.Unit 5: **Singular Value Decomposition**

Quadratic Forms, Definitions of Positive definite, negative definite, positive semi-definite negative semi-definite, indefinite forms of matrices, Tests for Positive Definiteness, Positive Definite Matrices and Least Squares, Semi Definite Matrices, Singular Value Decomposition, Applications.

| Class No. | Portions to be covered |
|---|---|
| 57-58 | Quadratic Forms Definitions of positive definite, negative definite, positive semi-definite, negative semi-definite, Indefinite forms of Matrices |
| 59 | Tests for Positive Definiteness |
| 60-61 | Problems on Positive Definite Matrices, Semi definite Matrices and Least Squares |
| 62 | **Matlab Class Number 10 - Eigen Values and Eigen Vectors** |
| 63-65 | The Singular Value Decomposition of a Matrix. Examples |
| 66 | **Applications** |
| 67-68 | **Matlab - In Semester Assessment** |

Classwork problems:

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| 1. | Write the symmetric matrix which corresponds to the following quadratic forms: $(i) Q(x) = 3x_1^2 - 5x_2^2 + 2x_3^2 - x_1x_3 + 4x_2x_3$<br>$(ii) Q(x) = 5x_1^2 + 7x_2^2 - 3x_3^2 - 4x_1x_2 + 3x_1x_3 - x_2x_3$<br>$(iii) Q(x) = 3x^2 - 4xy + 5y^2$ |
| 2. | Compute the quadratic form $x^TAx$ for $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ and<br>(a) $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ (b) $x = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ (c) $x = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ |
| 3. | Decide for or against the positive definiteness of these matrices and write the corresponding quadratic form f = $x^TAx$.<br>$\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ $\begin{pmatrix} -1 & 2 \\ 2 & 8 \end{pmatrix}$ $\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ $\begin{pmatrix} 5 & -4 & -4 \\ -4 & 6 & -4 \\ -4 & -4 & 7 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.<br>$\begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^3$ |

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| 4. | For which a and b are the matrices A and B have all $\lambda > 0$ and are therefore positive definite. $A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{pmatrix}$<br>Answer : For a > 2, A is positive definite, no value of b makes B positive definite . |
| 5. | With positive pivots in D. the factorization $A=LDL^T$ becomes $A = L\sqrt{D}\sqrt{D}L^T$, If $R = L\sqrt{D}$ gives $A=RR^T$ ($=CC^T$ Cholesky factorization), then from $R = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$ find A and from $A = \begin{pmatrix} 4 & 8 \\ 8 & 25 \end{pmatrix}$ find R.<br>Answer : $A = \begin{pmatrix} 9 & 3 \\ 3 & 5 \end{pmatrix}$; $R = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$. |
| 6. | Express A matrix below<br>$A = \begin{matrix} 4 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 5 \end{matrix}$<br>As sum of squares. |
| 7. | Write A= $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ as $R^TR$ in three ways ($L\sqrt{D}$)($\sqrt{D}L^T$),($Q\sqrt{\Lambda}$)($\sqrt{\Lambda}Q^T$) and ($Q\sqrt{\Lambda}Q^T$)($Q\sqrt{\Lambda}Q^T$) using pivots eigen values and eigen vectors. |
| 8. | Compute $A^TA$ and $A^TA$ and their eigen values and unit eigen vectors, for A= $\begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$ |
| 9. | Find SVD of the following matrices:<br>$\begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$ |
| 10. | Convert the matrix of observations to mean deviation form and construct the sample covariance matrix for the following data:<br><table><tr><td>1</td><td>5</td><td>2</td><td>6</td><td>7</td><td>3</td></tr><tr><td>3</td><td>11</td><td>6</td><td>8</td><td>15</td><td>11</td></tr></table> |