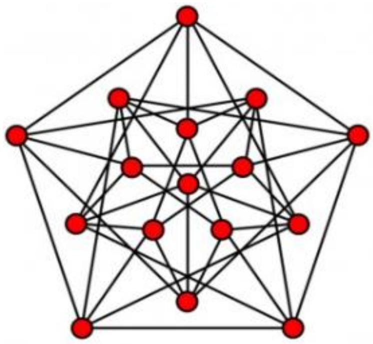
International Virtual Seminar on Exploring Graph Theory - The way we envision it

20th April 2025



Organised

by

Department of Mathematics,

School of Advanced Sciences,

VIT, Vellore

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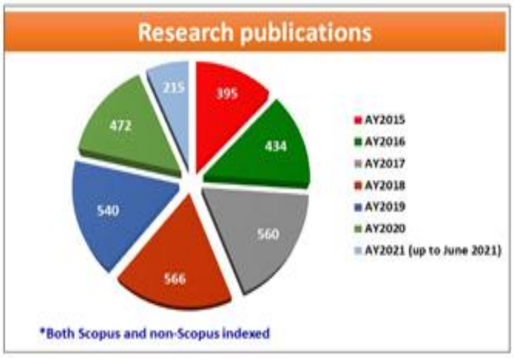
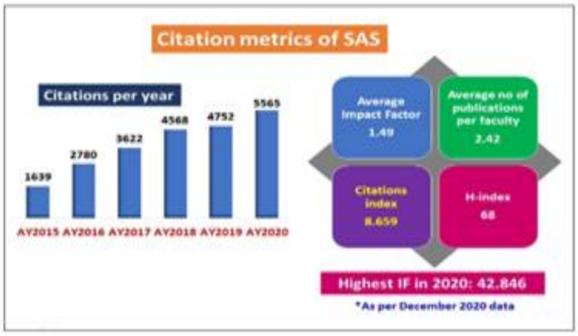
ABOUT VIT

VIT was established under Section 3 of the University Grants Commission (UGC) Act, 1956 and was founded in 1984 as a self-financing institution called the Vellore Engineering College. The Union Ministry of Human Resources Development conferred University status on Vellore Engineering College in 2001, in recognition of its excellence in academics, research and extracurricular initiatives. The University is headed by its founder and Chancellor, Dr. G. Viswanathan, a former Parliamentarian and Minister in the Tamilnadu Government. In recognition of his service to India in offering world class education, he was conferred an honorary doctorate by the West Virginia University, USA. Currently, VIT has 5 campuses in Vellore, Chennai, Amravati, Bhopal and Bangalore. NIRF of the MHRD, Government of India, has ranked VIT 10th best in university, 13th best in Research, 11th best in Engineering, 19th best in Overall category in 2024. VIT has gone for accreditation by NAAC [India], IET [UK], and ABET [USA] and follows world class academic processes. VIT is one of the top 2 institutions of India and within the top 501-600 Universities of the world as per Shanghai ARWU Ranking 2024. VIT is also ranked by QS World University Ranking, Times Higher Education (THE) World University Ranking, US News Ranking, Round University Ranking, Russia and others. VIT is the first institute of India to receive QS 4-Star rating in overall category and QS 5-Star rating in teaching, employability, facilities, innovation and inclusiveness.

SCHOOL OF ADVANCED SCIENCES

School of Advanced Sciences (SAS), which was established in the year 1984, comprises Mathematics, Physics, and Chemistry departments. The school consists of a total of 232 experienced and energetic faculties. The school has been receiving research grants from various funding agencies such as DST, CSIR, DRDO, NBHM, BRNS, IGCAR, AERBA, NRB and ARDB.





The school has also been recognized by DST for support under the FIST programme. The school conducts various programmes such as InoVIT, SciGATHER, and National Science Day.

The school offers University core and elective courses in Mathematics, Physics and Chemistry for all UG Programmes. The school offers M.Sc. programs in Chemistry, Physics and Mathematics. It also offers core and elective courses for all PG Programmes. The school offers Ph.D. programs in frontier research areas.

DEPARTMENT OF MATHEMATICS

The Department of Mathematics comprises 133 faculty members dedicated to fulfilling the needs of all the engineering departments. The faculty members of the department are specialized in frontier areas of mathematics such as Operations Research, Queuing Theory, Fuzzy Mathematics, Algebra, Analysis, Complex Analysis, Differential Equations, Fluid Dynamics, Graph Theory, Inventory Systems, Coding Theory, Mathematical Modeling, Numerical Methods, Mathematical Biology and others. Several research projects of the department are financially supported by some of the leading funding agencies such as CSIR, DRDO, DST and NBHM. The department offers Ph.D., Statistics and Data Science in addition to M.Sc. (Data Science), M.Sc. (Business Statistics), Integrated M.Sc. (Computational Statistics and Data Analytics) and Integrated

M.Sc. (Mathematics) programmes. The department is also serving the requirements of the leading industries through consultancy projects. The department of mathematics has a good record of frequently organizing seminars, workshops, symposiums and conferences in the recent emerging technical areas. VIT Mathematical Meet is conducted every year by the department, which is a south Indian level math and science contest for school children, to create and encourage mathematical and scientific innovations among the students.

SEMINAR FOCUS

Starting humbly with Euler in 1735, the knowledge of graph theory is advantageous as it offers a robust mathematical framework for analysing and identifying patterns and decision-making based on interconnected data. The core concepts of graph theory have disseminated their understanding to examine issues in molecular chemistry, communication networks, mathematical biology, genomics, drug development, machine learning and more. This seminar aims to unite individuals from various areas of graph theory, with the hope that the knowledge exchanged will inspire fruitful research outcomes.

TARGET AUDIENCE

Students of any UG / PG Degree/Research Scholar, Faculties, Academician and Researchers

IMPORTANT DATES

Last Date of abstract submission: 15 – 04 – 2025

Last Date of manuscript submission: 30 – 04 – 2025

SEMINAR SPEAKERS

	**Dr. H. S. Ramane,** Senior Professor, Department of Mathematics, Karnatak University, Dharwad, Karnataka, India
	**Dr. E. Suresh** Assistant Professor Department of Mathematics Faculty of Engineering and Technology SRM Institute of Science and Technology, Chennai.
	**Dr. Sourav Mondal** Research Fellow, RISE, Research Group MASEP, University of Sharjah, Sharjah, 27272, UAE. (Assistant Professor, Department of Mathematics, SRM IST, KTR, Tamilnadu, India).

SEMINAR COORDINATORS

Dr. M. Yamuna

Professor

Department of Mathematics

School of Advanced Sciences

Vellore Institute of Technology

Vellore

Tamilnadu

India

Dr. K. Karthika

Assistant Professor

Department of Mathematics

School of Advanced Sciences

Vellore Institute of Technology

Vellore

Tamilnadu

India

International Virtual Seminar on

Exploring Graph Theory

- The way we envision it

20th April 2025

Seminar Program

Join MS Tems Meeting: https://tinyurl.com/2kzhhu4z

Meeting Link: https://tinyurl.com/3mk6xemm

Total Number of Participants: 42

Total Number of Papers: 17

Total Number of Colleges: 34

Seminar Schedule

	**Dr. E. Suresh** **Title: On the Atom Bond Sum - Connectivity Index of Graphs** **Time: 10.30 am – 12.00 pm**
	**Dr. Sourav Mondal** **Title: Extremal Graphs for Degree – Based Topological Indices with Given Parameters** **Time: 12.00 pm – 1.00 pm**
	**Dr. H. S. Ramane** **Title: Hamming Index of a Graph** **Time: 2.00 pm – 3.30 pm**
Paper Presentations	**3.30 pm – 5.00 pm**

International Virtual Seminar on Exploring Graph Theory

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Date: 20 – 04 – 2025 **Time: 3.45 pm – 5.00 pm (India Time)**

Session 1 Link: https://tinyurl.com/5yswe392

Head of Session: Dr. M. Yamuna

S.NO.	TOPIC TITLE	PRESENTERS	AFFILIATION
1.	AUTOMORPHISM GROUP OF CNS CARTESIAN PRODUCT GRAPH	SUBHA A B	UNIVERSITY COLLEGE
2.	A MATHEMATICAL MODEL FOR THE PATTERNS OF ALCOHOL CONSUMPTION	A. AMALARATHINAM, SABARMATHI A.	AUXILIUM COLLEGE (AUTONOMOUS), VELLORE-632 006
3.	SUSTAINABLE STRATEGIES FOR MANAGING MANGO DISEASES	SARANYA P	AUXILIUM COLLEGE
4.	EVALUATING METHODS FOR INVENTORY DEMAND PREDICTION TO DETERMINE THE OPTIMAL APPROACH	T. KARPAGAVIGNESWARI	VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES
5.	ANALYSING SHORTEST PATHS IN A SELF-SIMILAR FRACTAL GRAPHS USING FLOYD-WARSHALL ALGORITHM WITH PYTHON IMPLEMENTATION	E. ROHIT AUXILIA	VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES.
6.	A STUDY OF ENCLAVE DOMINATION NUMBER AND ITS INTERPLAY WITH OTHER GRAPH PARAMETERS	M. SANTHOSH PRIYA, DR. A. MYDEEN BIBI	THE STANDARD FIREWORKS RAJARATNAM COLLEGE FOR WOMEN, SIVAKASI.
7.	CAYLEY DIGRAPHS OF THE GROUP GENERATED BY DIFFERENTIAL EQUATIONS	M. AISWARYA AND A. ANAT JASLIN JINI	HOLY CROSS COLLEGE(AUTONOMOUS), NAGERCOIL.
8.	CAYLEY DIGRAPHS OF THE GROUP GENERATED BY HIGHER ORDER DERIVATIVES	J. ABISHA AND A. ANAT JASLIN JINI	HOLY CROSS COLLEGE (AUTONOMOUS), NAGERCOIL.

International Virtual Seminar on Exploring Graph Theory

- The way we envision it

Date: 20 – 04 – 2025 **Time: 3.45 pm – 5.00 pm (India Time)**

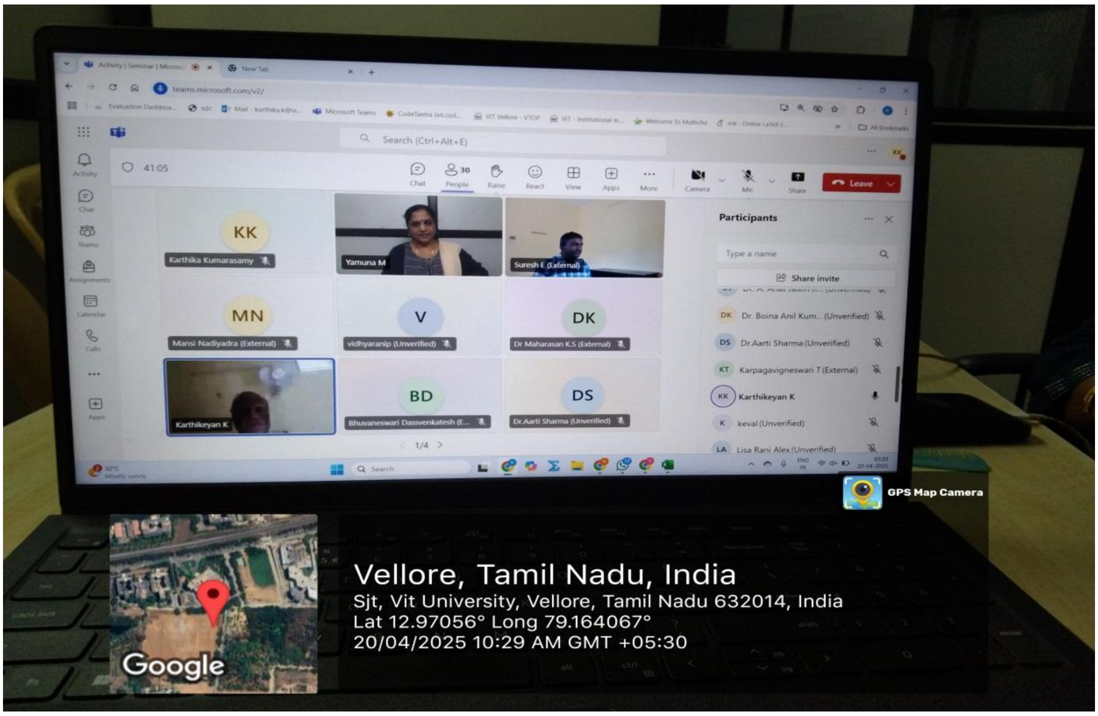
Session 2 Link: https://tinyurl.com/zha2wt98

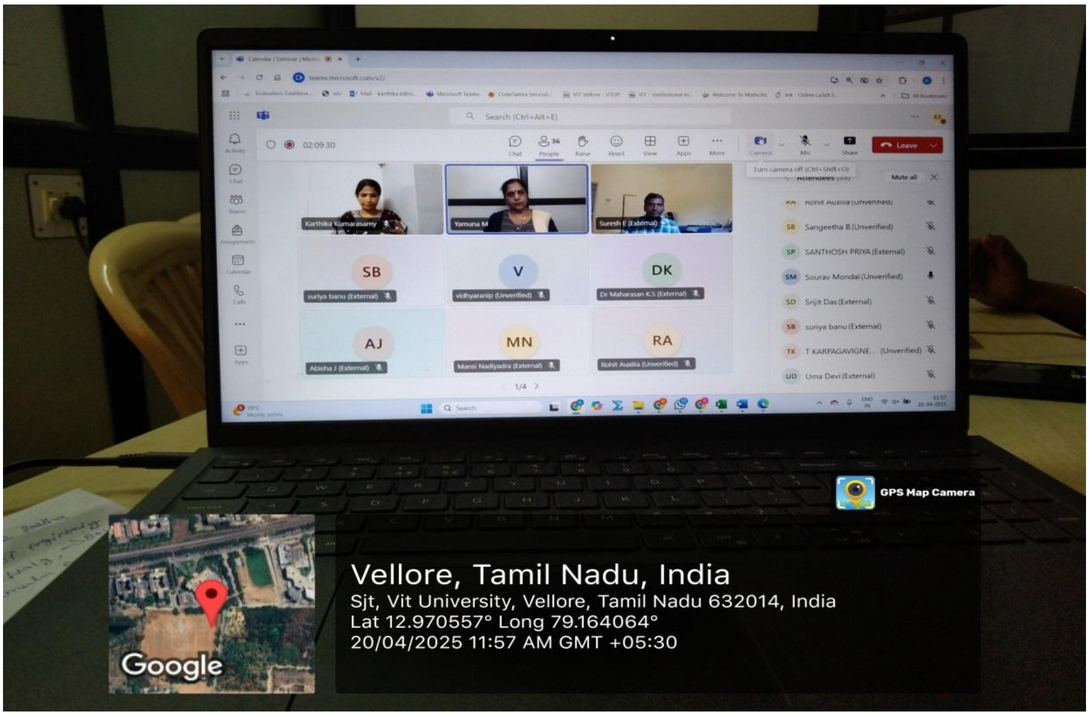
Head of Session: Dr. K. Karthika

S.NO.	TOPIC TITLE	PRESENTERS	AFFILIATION
1.	STUDIES ON RELATION BETWEEN GRAPH THEORY AND IMAGE PROCESSING	N.J.BHUVANESWARI,D P.KARTHICK	KALASALINGAM ACADEMY FOR RESEARCH AND EDUCATION
2.	BINOMIAL GRAPH	MD. SHAHIDUL ISLAM KHAN	PANDIT DEENDAYAL UPADHYAYA ADARSHA MAHAVIDYALAYA, AMJONGA
3.	A STUDY ON ANNIHILATOR IP DOMINATION NUMBER	AL. YAKAVI, DR. A. MYDEEN BIBI	THE STANDARD FIREWORKS RAJARATNAM COLLEGE FOR WOMEN, SIVAKASI.
4.	INVERSE COTOTAL DOMINATION ON GRAPHS	DR. A. PUNITHA THARANI, M.MARIYA EVILA.	ST. MARY'S COLLEGE(AUTONOMOUS)
5.	FUZZY LOGIC ENHANCED GRAPH CUT MODEL FOR LEAF IMAGE SEGMENTATION	MAGLIN HELINA X	KALASALINGAM ACADEMY OF RESEARCH AND EDUCATION
6.	AN ICT-BASED GRAPH THEORY FRAMEWORK FOR SMART TRIBAL FARMING	MAHARASAN K S	KG COLLEGE OF ARTS AND SCIENCE
7.	Z-FUZZY SUPERPIXEL HYPERGRAPHS AND RANDOM WALK DYNAMICS FOR ROBUST TUMOR SEGMENTATION	BRINTHAGURU T	KALASALINGAM ACADEMY OF RESEARCH AND EDUCATION
8.	LEVERAGING & HARNESSING BY 4.0 REVOLUTION FOR FOSTERING SUSTAINABILITY TECHNIQUES BEYOND THE AGRICULTURAL VALUE CHAIN	ANJALI PISE	TGPCET
9.	A THEORETICAL STUDY OF THE FRACTAL VON - KOCH CURVE GRAPH USING ECCENTRICITIES, TREE PROPERTIES AND BIPARTITE STRUCTURES	SURIYA BANU, KAMALI	VELS INSTITUTE OF SCIENCE, TECHNOLOGY AND ADVANCED STUDIES (VISTAS), CHENNAI-600117

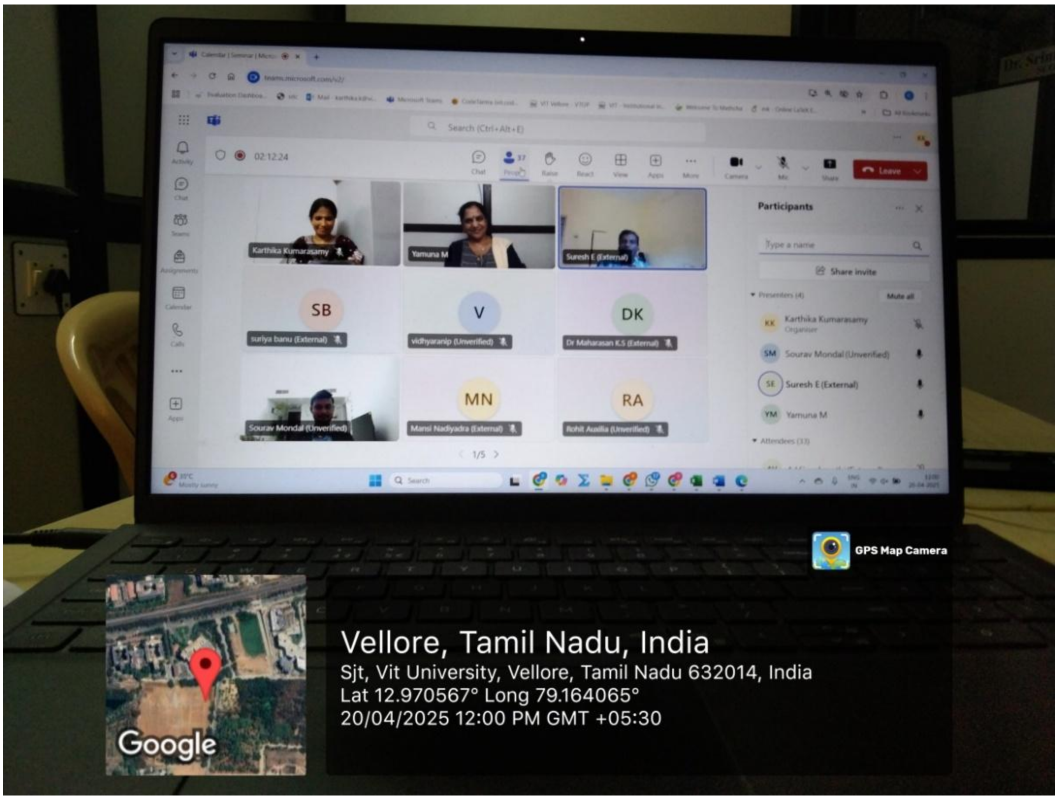
PHOTO GALLERY

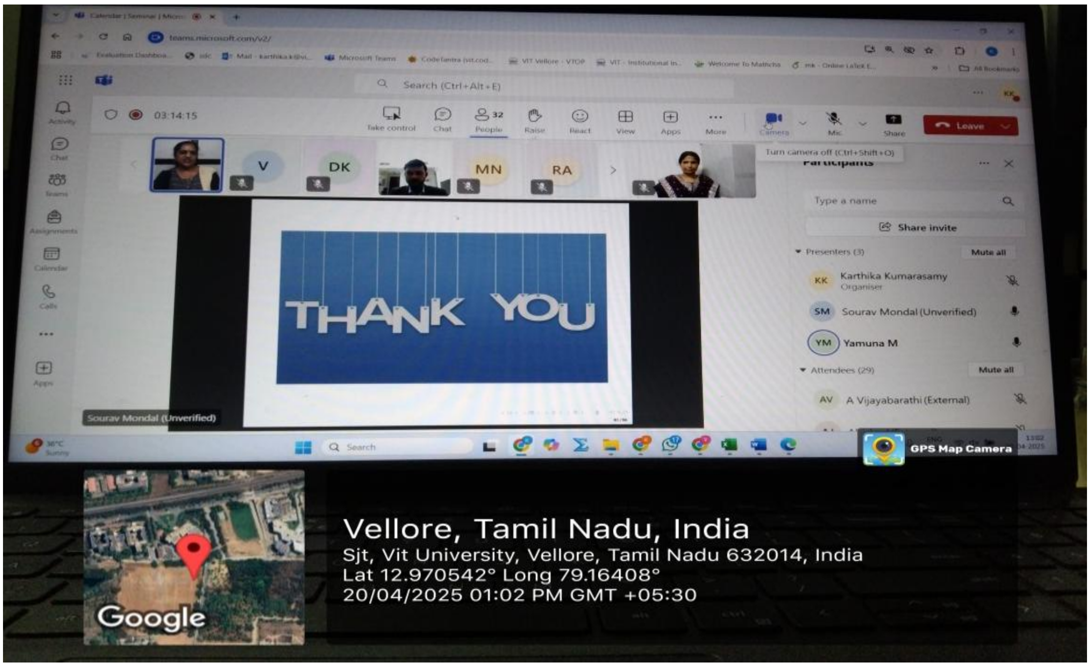
Session 1: 10.30 am - 12 noon



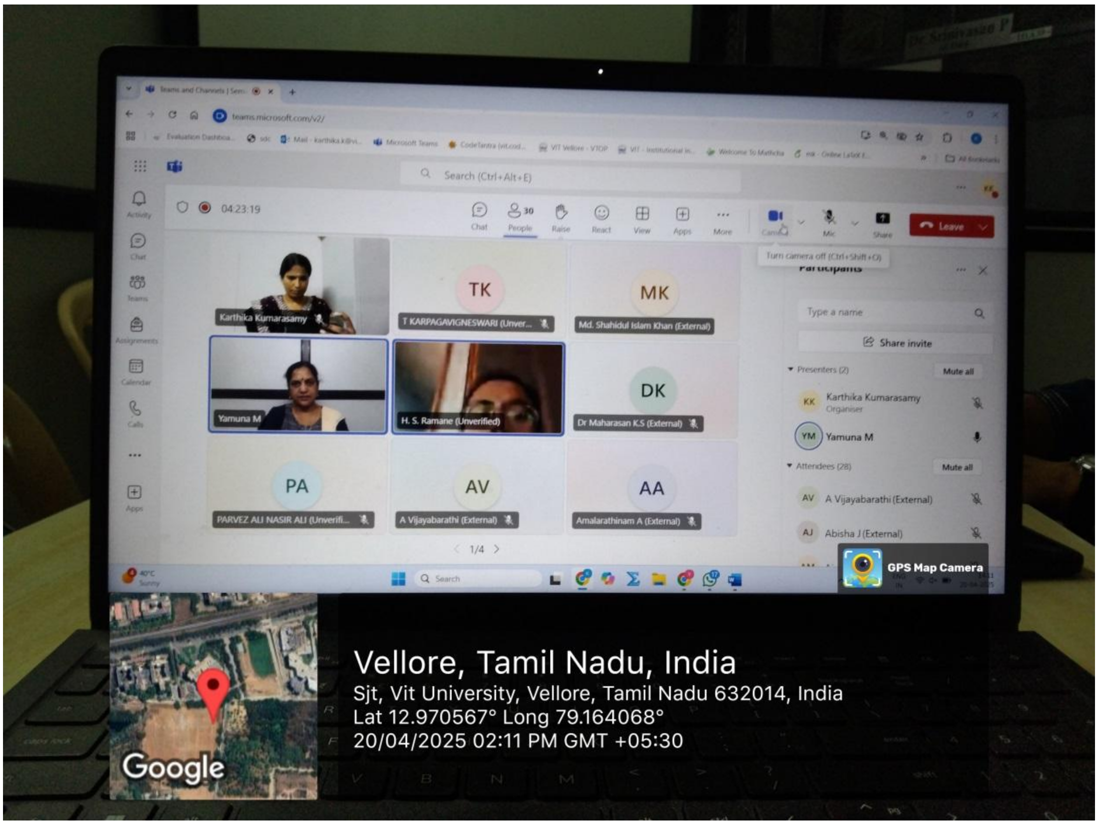


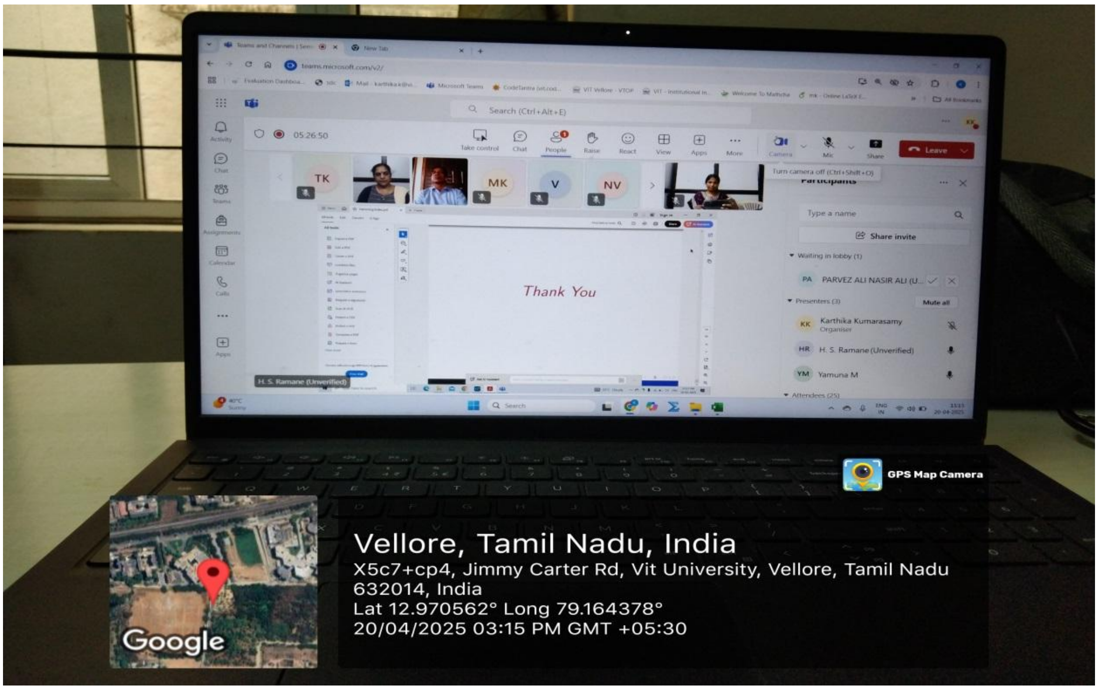
Session 2: 12.00 noon to 1.00 pm





Session 3: 2.00 pm to 3.30 pm





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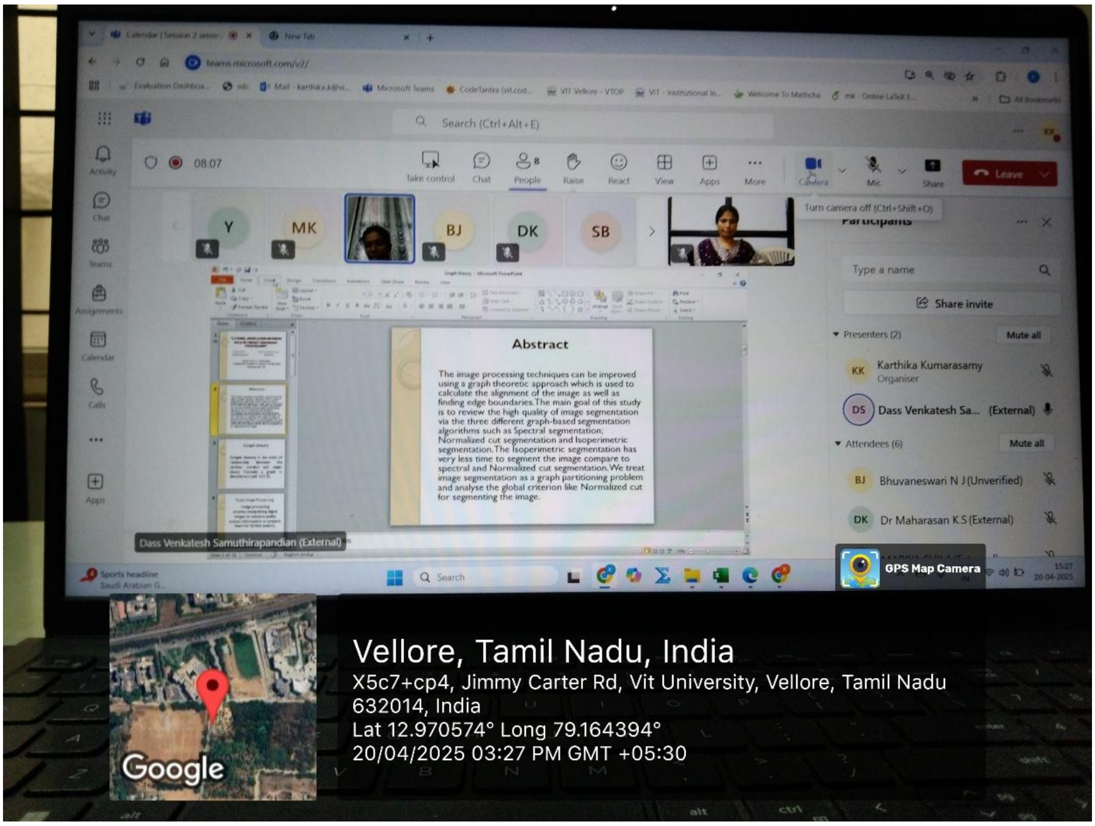
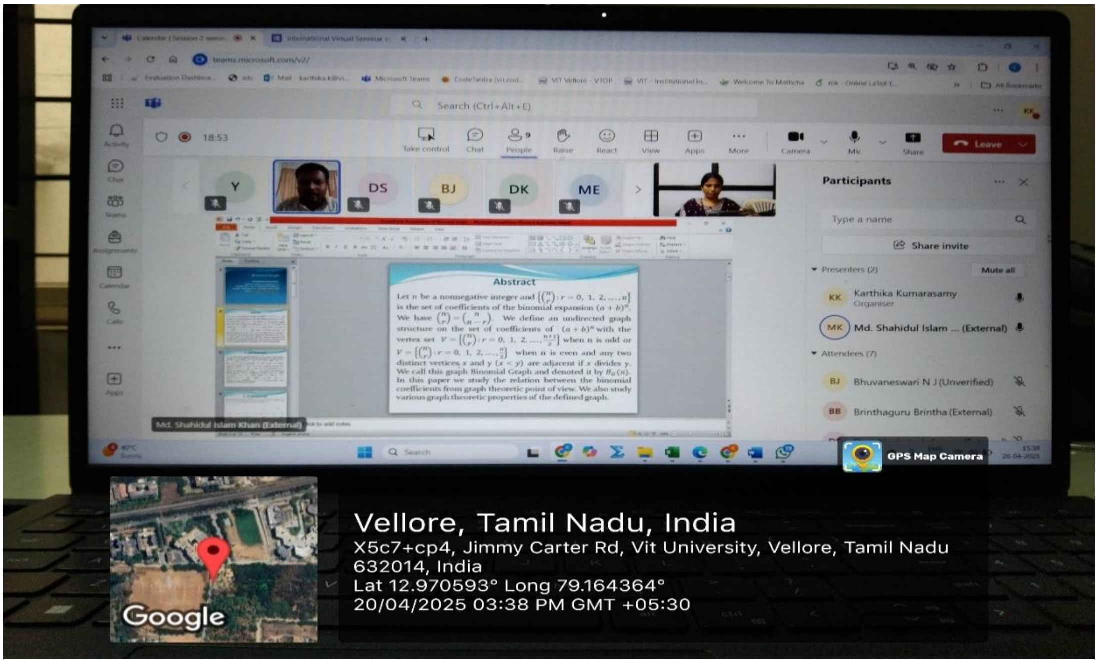
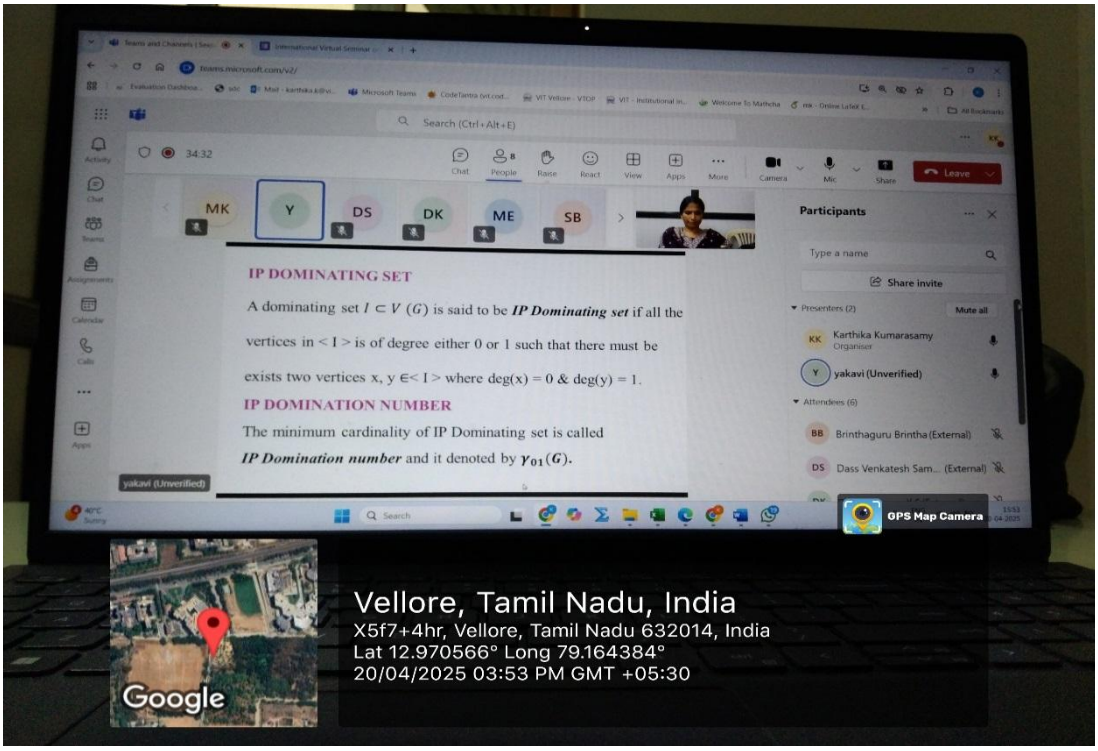
Presentation Session: 3.30 pm to 5 pm






IP DOMINATING SET
A dominating set I ⊂ V (G) is said to be IP Dominating set if all the vertices in < I > is of degree either 0 or 1 such that there must be exists two vertices x, y ∈< I > where deg(x) = 0 & deg(y) = 1.
IP DOMINATION NUMBER
The minimum cardinality of IP Dominating set is called IP Domination number and it denoted by γ01(G).
Participants
Karthika Kumarasamy
yakavi (Unverified)
Vellore, Tamil Nadu, India
X5f7+4hr, Vellore, Tamil Nadu 632014, India
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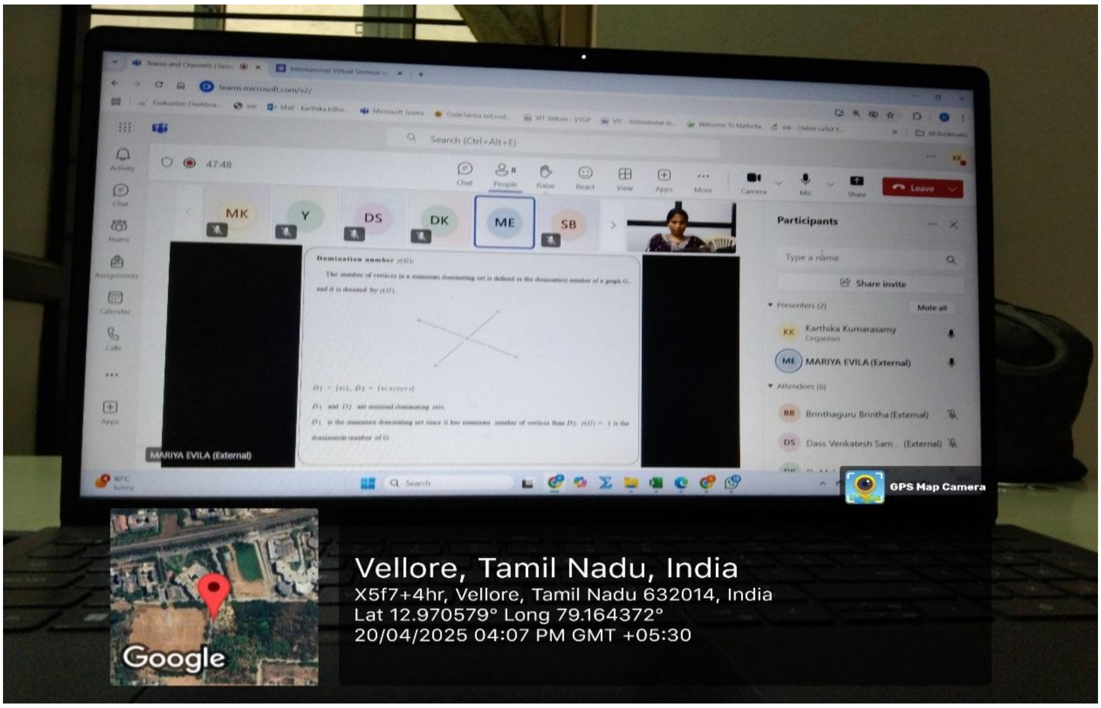
Participants
Karthika Kumarasamy
MARIYA EVILA (External)
Vellore, Tamil Nadu, India
X5f7+4hr, Vellore, Tamil Nadu 632014, India
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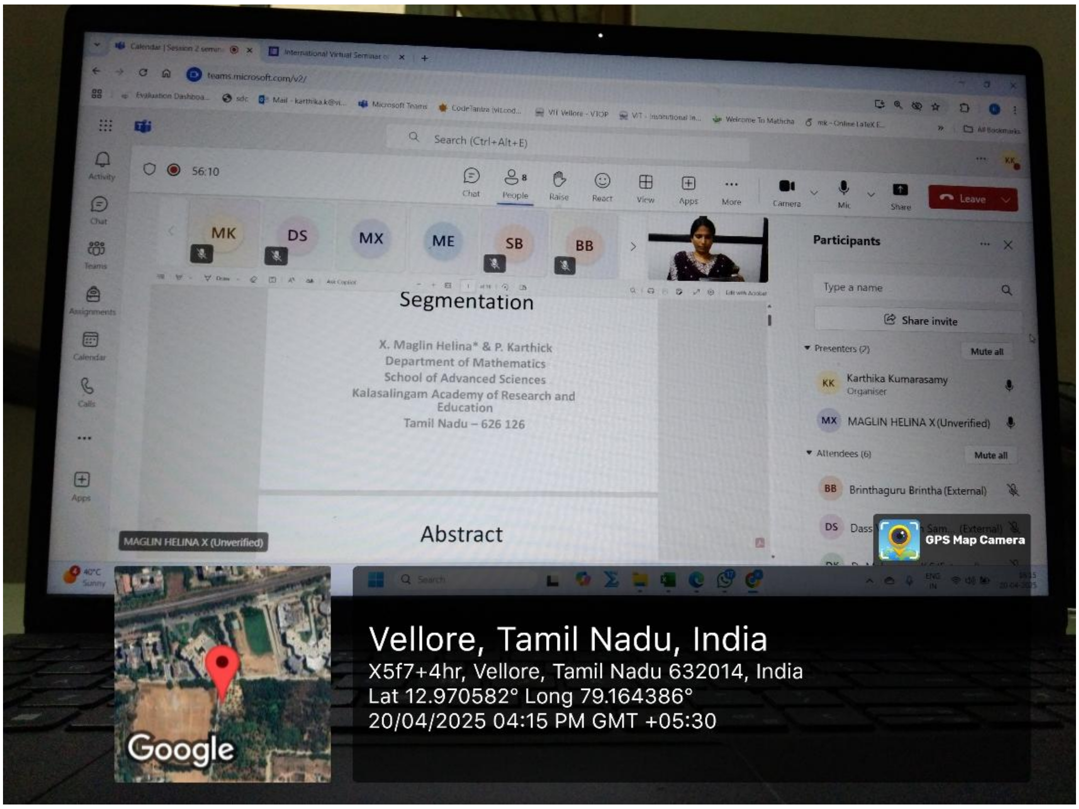
Segmentation
X. Maglin Helina* & P. Karthick
Department of Mathematics
School of Advanced Sciences
Kalasalingam Academy of Research and Education
Tamil Nadu – 626 126
Abstract
Participants
Karthika Kumarasamy
MAGLIN HELINA X (Unverified)
Brinthaguru Brintha (External)
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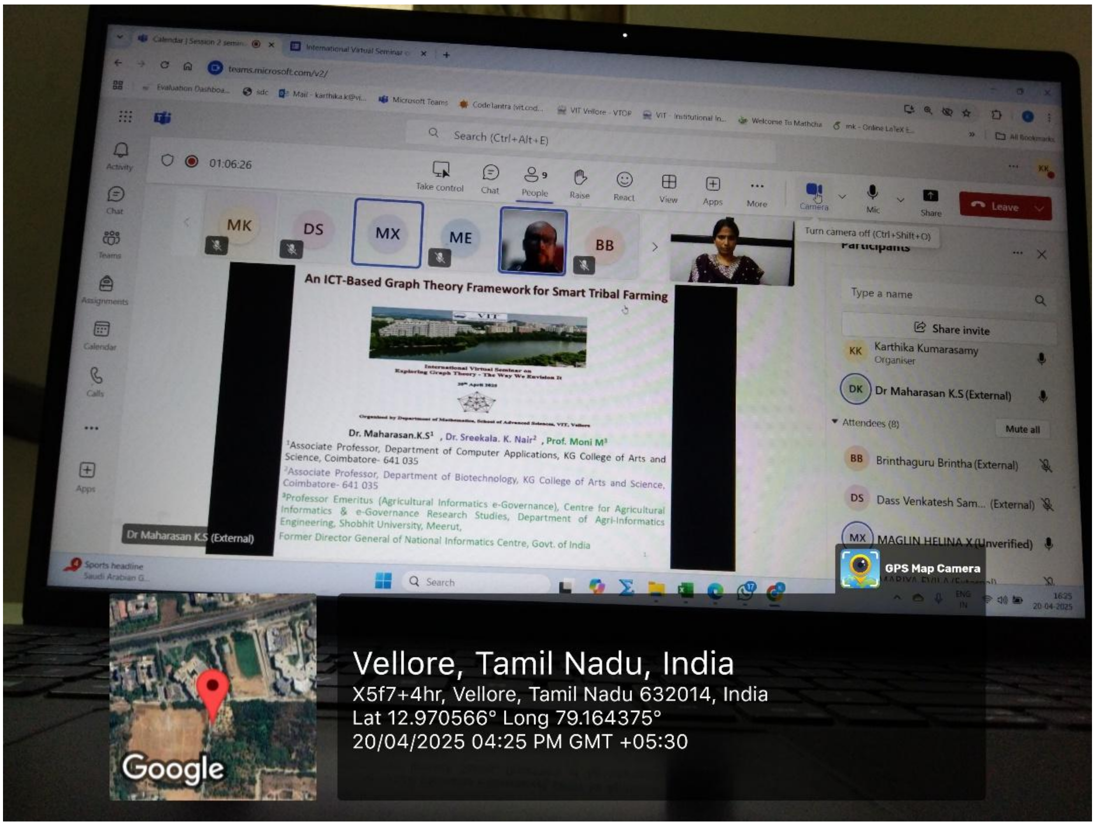
An ICT-Based Graph Theory Framework for Smart Tribal Farming
Participants
Karthika Kumarasamy
Dr Maharasan K.S (External)
Brinthaguru Brintha (External)
MAGLIN HELINA X (Unverified)
Dr Maharasan K.S (External)
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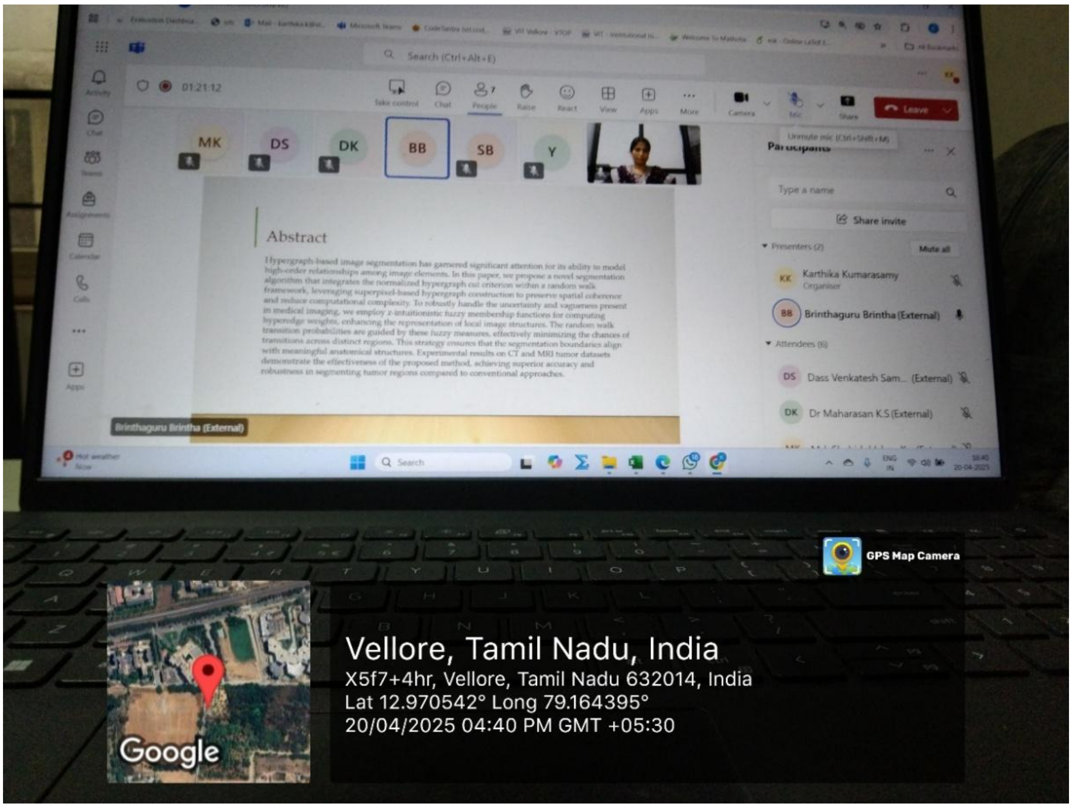
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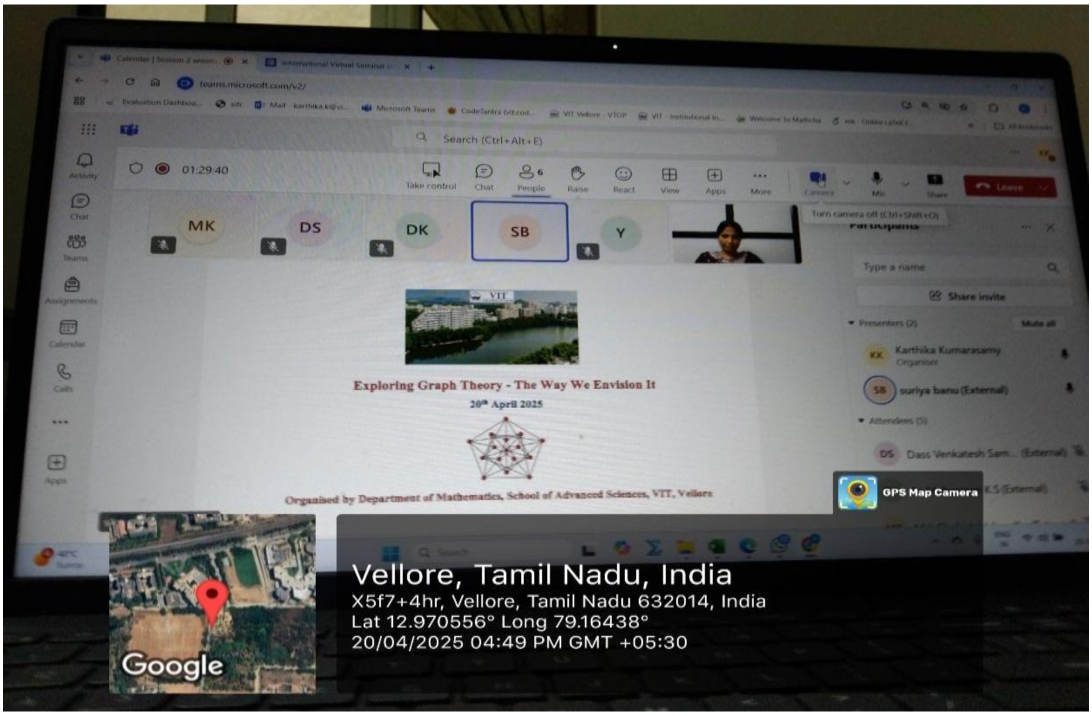
Exploring Graph Theory - The Way We Envision It
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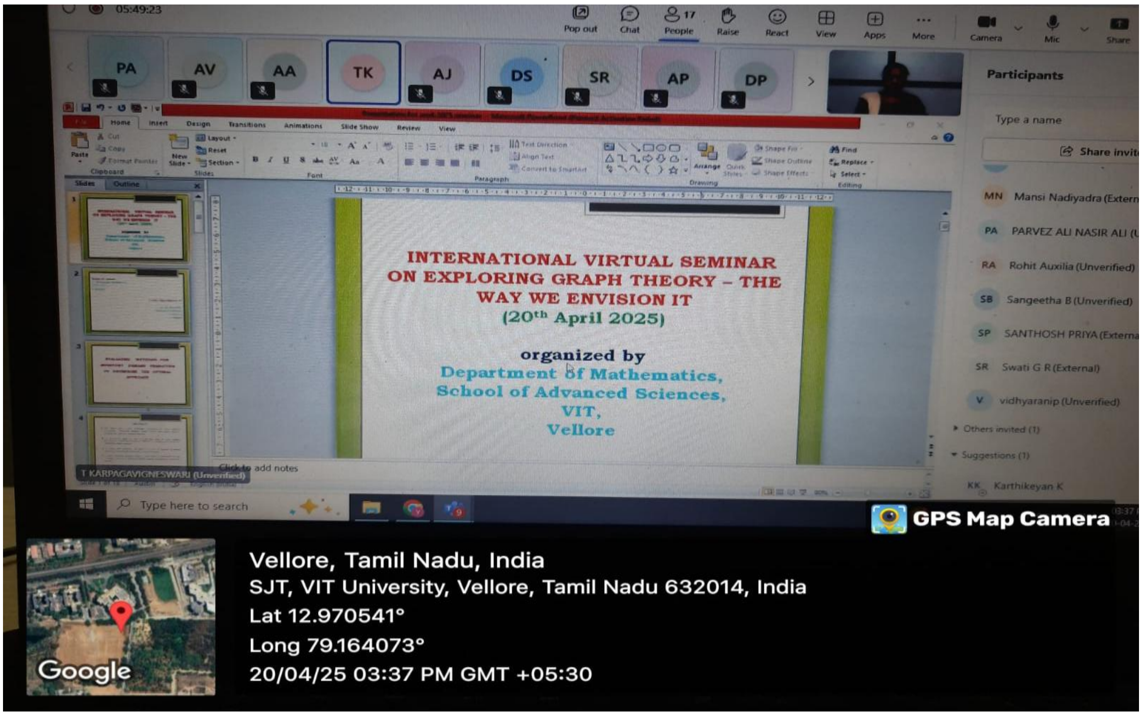
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ON EXPLORING GRAPH THEORY – THE
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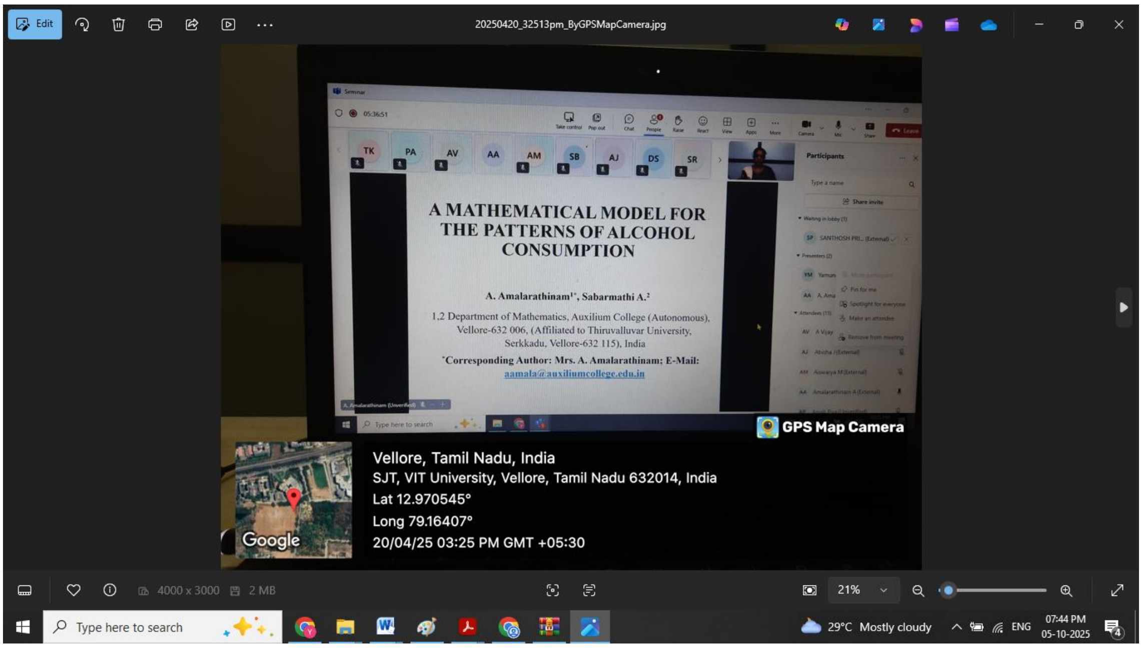
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A MATHEMATICAL MODEL FOR
THE PATTERNS OF ALCOHOL
CONSUMPTION
A. Amalarathinam1*, Sabarmathi A.2
1,2 Department of Mathematics, Auxilium College (Autonomous),
Vellore-632 006, (Affiliated to Thiruvalluvar University,
Serkkadu, Vellore-632 115), India
*Corresponding Author: Mrs. A. Amalarathinam; E-Mail:
aamala@auxiliumcollege.edu.in
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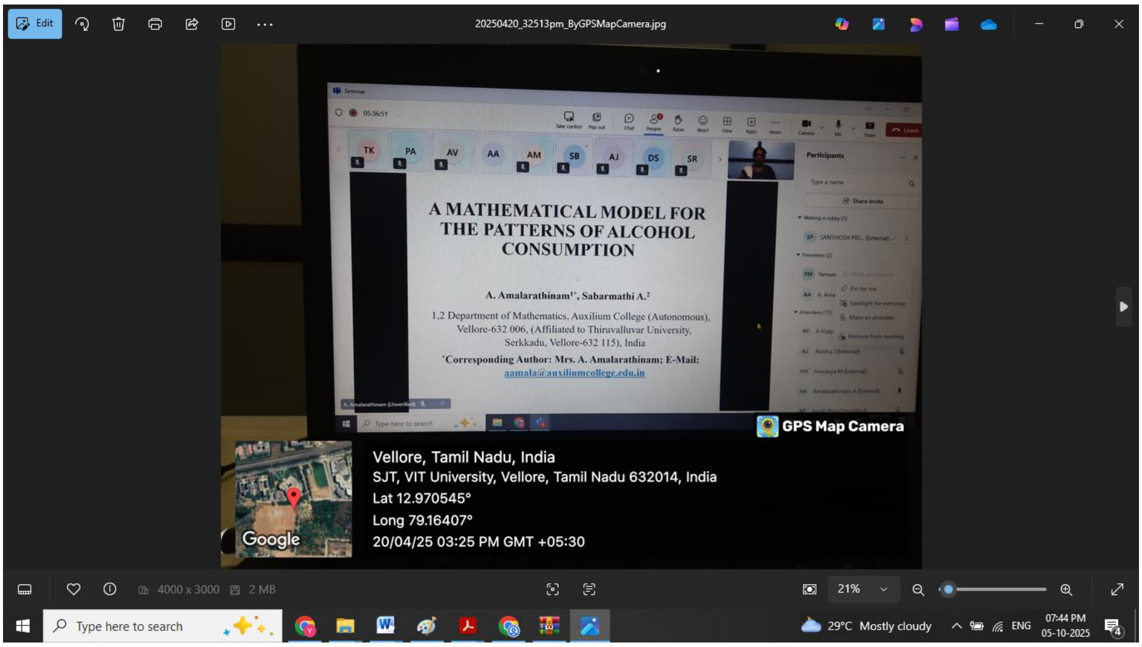

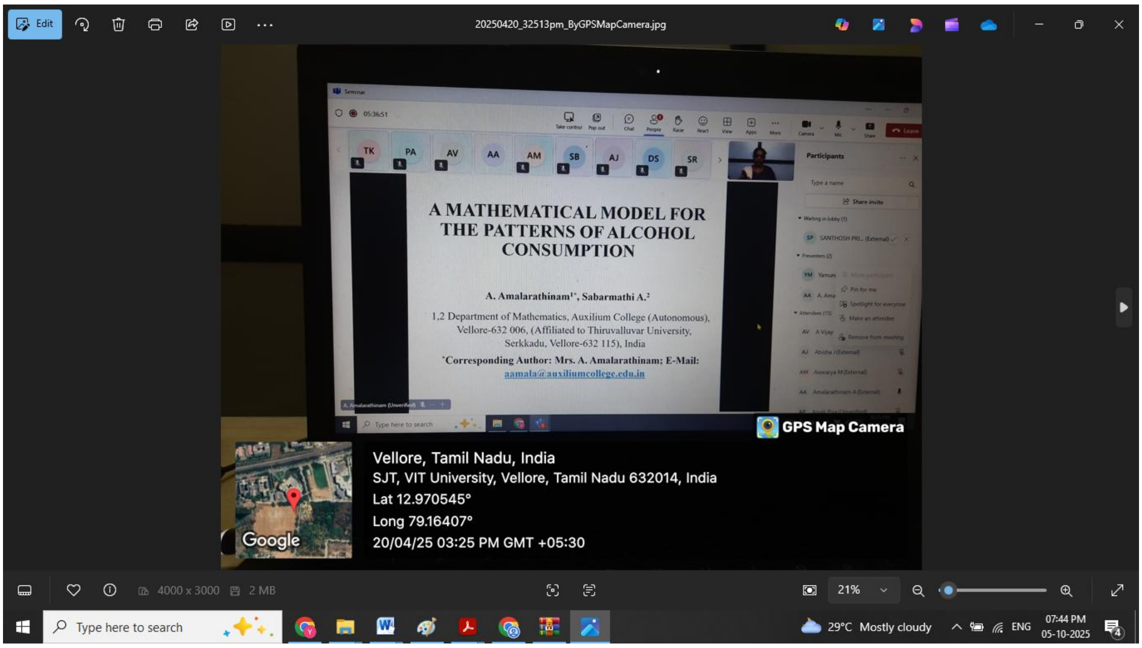

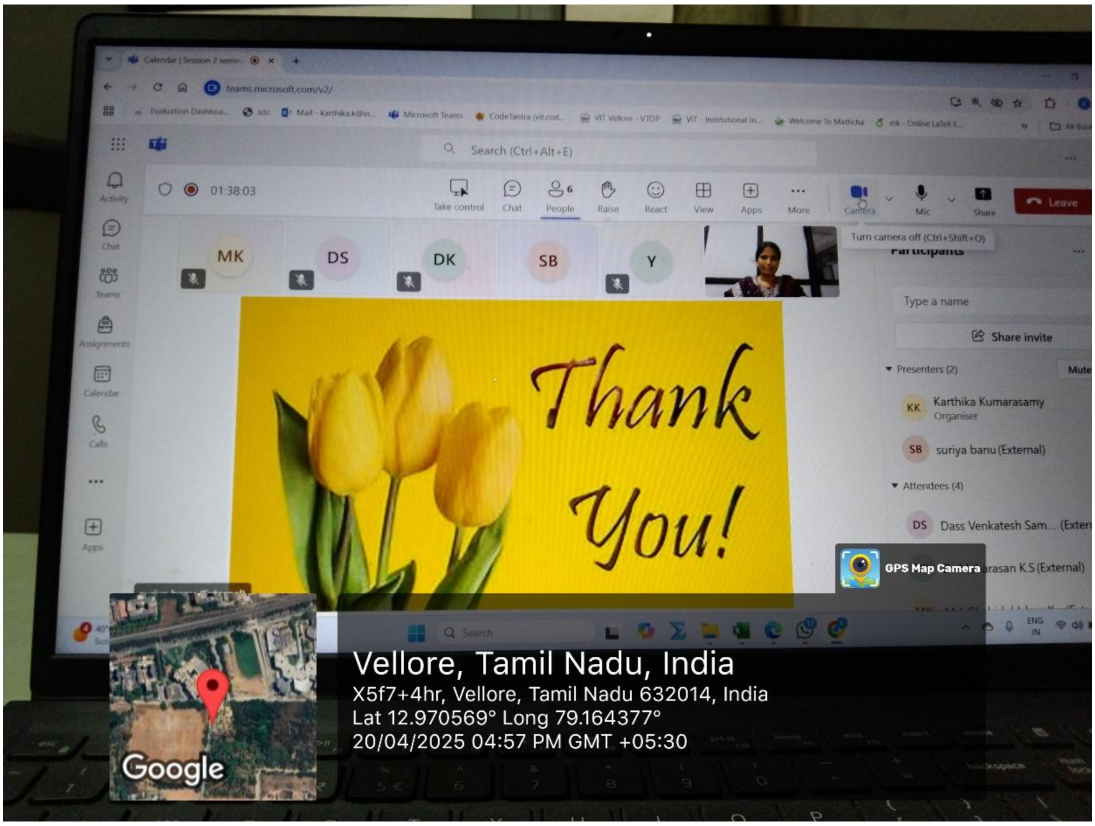

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CONTENTS

GUEST SPEAKER NOTES

On the Difference between Atom-Bond Sum-Connectivity Index and Randic Index

Suresh Elumalai[1], Sourav Mondal[2]

Department of Mathematics, College of Engineering and Technology, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chengalpet 603 203, India.

RISE, Department of Mathematics, University of Sharjah, Sharjah 27272, UAE.

Email: sureshkako@gmail.com; souravmath94@gmail.com

Abstract.

Topological indices are numerical invariants derived from the molecular graphs of chemical compounds to describe their structural characteristics. Among the most commonly studied are the atom–bond sum-connectivity (ABS) index and the Randic (R) index. This study focuses on analyzing the difference between these two indices, defined for a graph G as

$$ABS(G) - R(G) = \sum_{fg \in E(G)} \frac{1}{\sqrt{d_f d_g (d_f + d_g)}} \left(\sqrt{d_f d_g (d_f + d_g - 2)} - \sqrt{d_f + d_g} \right)$$

where d_f denotes the degree of the vertex f in G. The primary goal of this investigation is to explore the relevance of the ABS − R measure in modeling molecular structure property relationships.

Keywords: Randic(R) index, atom-bond sum-connectivity(ABS) index, QSPR analysis.

1. INTRODUCTION

Throughout this work, all graphs are assumed to be simple, connected, and undirected. For a graph G with vertex set V(G) and edge set E(G), the degree of a vertex x ∈ V(G) is denoted by d_x.

In chemical graph theory, hydrogen-suppressed molecular structures are represented as chemical graphs, where vertices correspond to atoms and edges denote chemical bonds.

One of the central challenges in theoretical chemistry is predicting the physicochemical

Properties of compounds from their molecular structures. Among several approaches, the use of topological indices has proven particularly effective.

The Randic index[22], introduced in1975, is defined as

$$R(G) = \sum_{fg \in E(G)} \frac{1}{\sqrt{d_f d_g}}$$

Later, the atom–bond connectivity(ABC) index [13], sum-connectivity index [26], and geometric–arithmetic(GA) index[24] were proposed as

$$ABC(G) = \sum_{fg \in E(G)} \sqrt{\frac{d_f + d_g - 2}{d_f d_g}},\ \chi(G) = \sum_{fg \in E(G)} \frac{1}{\sqrt{d_f + d_g}},\ GA(G) = \sum_{fg \in E(G)} \frac{2\sqrt{d_f d_g}}{d_f + d_g}.$$

The GA index has shown strong correlation with thermodynamic and molecular properties such as entropy, enthalpy, and acentric factors.

Recently, Akbar Alietal. [5] introduced the atom–bond sum-connectivity (ABS) index, combining the ideas of the ABC and sum-connectivity indices:

$$ABS(G) = \sum_{fg \in E(G)} \sqrt{\frac{d_f + d_g - 2}{d_f + d_g}}.$$

The ABS index has since been widely studied for its chemical significance ([1]–[9], [14], [15], [17], [18]–[21], [25], [27]).

Several comparative studies have explored relationships among these indices, such as and

$$GA(G) - ABC(G) = \sum_{fg \in E(G)} \left(\frac{2\sqrt{d_f d_g}}{d_f + d_g} - \sqrt{\frac{d_f + d_g - 2}{d_f d_g}} \right).$$

$$ABC(G) - R(G) = \sum_{fg \in E(G)} \frac{1}{\sqrt{d_f d_g}} \left(\sqrt{d_f + d_g - 2} - 1 \right).$$

and

$$AABS(G) - ABC(G) = \sum_{fg \in E(G)} \left(\sqrt{\frac{d_f + d_g - 2}{d_f + d_g}} - \sqrt{\frac{d_f + d_g - 2}{d_f d_g}} \right).$$

Motivated by these studies, this work investigates the role of the difference between the atom–bond sum-connectivity index and the Randic index in structure-property modeling.

2. MAIN RESULTS

Quantitative Structure–Property Relationship (QSPR) analysis is a computational technique used to relate molecular structure to physicochemical and biological properties. It establishes mathematical correlations between molecular descriptors—often topological indices—and experimentally measured properties. For recent advances in this area, readers may refer to [11,12,19].

In this section, we examine the potential of the ABS −R index in molecular property modeling. Following the methodology of Randic and Trinajstic [23], the significance of a graph invariant is evaluated by correlating its theoretical values with experimental data. We employ a benchmark dataset of octane isomers and correlate ABS − R with several physicochemical properties, including boiling point (BP), entropy (S), standard enthalpy of vaporization (DHVAP), and acentric factor (AF).

A linear regression model of the form

$$P = \alpha I + \beta, \tag{1}$$

is used, where P, α, β, and I denote the molecular property, slope, intercept, and index, respectively. The coefficient of determination (r^2), standard error (SE), F-test, and significance level (SF) are computed to assess correlation strength. The regression parameters are summarized in Table 1.

Table 1. Regression parameters for Eq. (1).

Property	A	β	r²	SE	F	SF
BP	-13.5595	132.4701	0.688	3.521	35.314	2.06×10^{-5}
S	-10.799	120.361	0.8	2.081	64.138	5.47×10^{-7}
DHVAP	-0.979	10.483	0.915	0.115	172.909	5.41×10^{-10}
AF	-0.084	0.453	0.794	0.016	61.848	6.92×10^{-7}

The results indicate that ABS − R shows strong correlations with BP ($r^2 = 0.69$), S ($r^2 = 0.80$), DHV AP ($r^2 = 0.92$), and AF ($r^2 = 0.79$). Among these, DHV AP exhibits the highest correlation and smallest error, signifying the best predictive performance. The corresponding linear fittings are illustrated in Figures 1 and 2, where the data points for DHVAP align closely with the regression line. Hence, the ABS−R index demonstrates strong potential for QSPR modeling, particularly in predicting the enthalpy of vaporization.

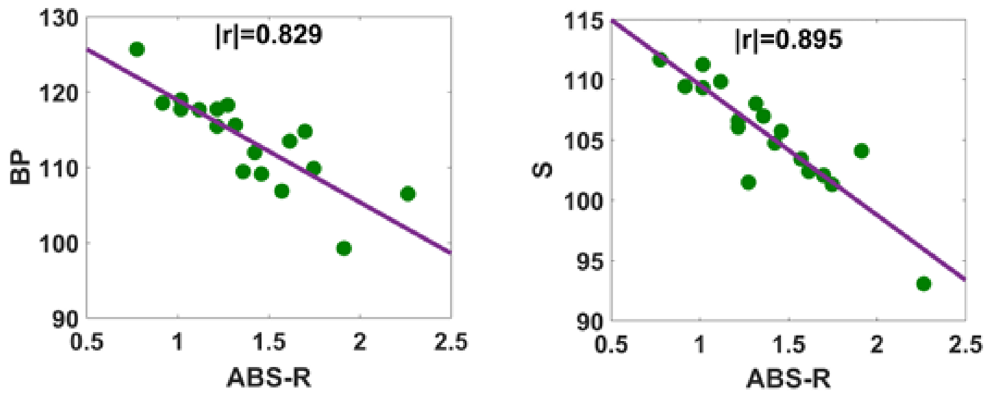

Fig. 1. Linear fitting of ABS −R with boiling point and entropy for octane isomers.

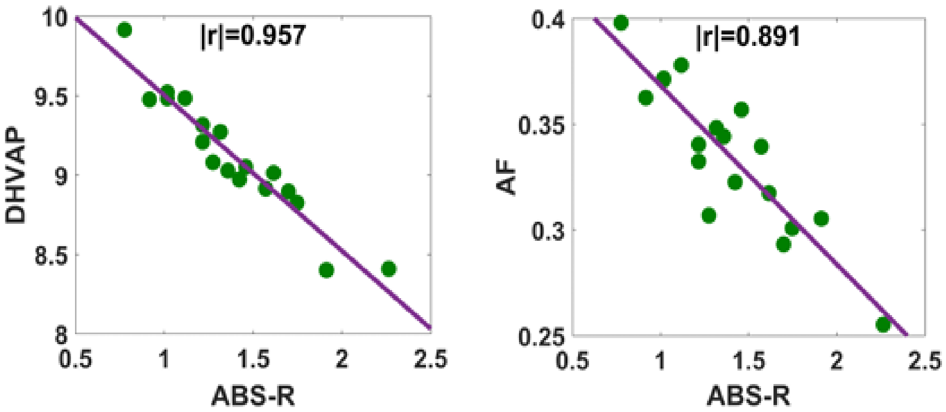

Fig. 2. Linear fitting of ABS −R with standard enthalpy of vaporization and acentric factor for octane isomers.

3. CONCLUDING REMARKS

In this work, we investigated the difference between the atom–bond sum-connectivity (ABS) index and the Randic (R) index, denoted by ABS − R, and explored its significance in structure–property modeling of molecules. The mathematical formulation of ABS −R was analyzed, and its chemical relevance was assessed through Quantitative Structure–Property Relationship (QSPR) analysis using a dataset of octane isomers.

The regression analysis revealed strong correlations of the ABS−R index with several physicochemical properties, particularly the standard enthalpy of vaporization (DHVAP), for which the coefficient of determination exceeded 0.91. This indicates that the ABS−R index effectively captures molecular features influencing thermodynamic behaviour. Overall, the findings demonstrate that ABS−R serves as a valuable structural descriptor with significant predictive capability in QSPR modeling. Future studies may extend this investigation to other classes of compounds or explore nonlinear and multi descriptor regression models to further enhance predictive accuracy.

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Hamming Index of a Graph

Harishchandra S. Ramane
Department of Mathematics, Karnatak University, Dharwad- 580003, India
Email: hsramane@yahoo.com

Abstract.
Let A(G) be the adjacency matrix of a graph G. Denote by s(v) the row of the adjacency matrix corresponding to the vertex v of G. It is a string in the set $\mathbb{Z}_2^n$ of all n-tuples over the field of order two. The Hamming distance between the strings s(u) and s(v) is the number of positions in which s(u) and s(v) differ. The Hamming index of a graph is the sum of Hamming distances between all pairs of strings generated by the adjacency matrix of a graph. In this article we survey the results on the Hamming distance between the strings generated by the adjacency matrix and Hamming index of graphs.
Keywords: Hamming distance, string, adjacency matrix, Hamming index.

1. INTRODUCTION

Hamming distance is employed in various applications such as cryptography and bioinformatics. In this paper we explore the results on the Hamming distances and Hamming index of a graph with respect to the strings generated by the adjacency matrix of a graph.

Let G be a simple undirected graph with n vertices and m edges. Let the vertex set of G be V(G) = $\{v_1, v_2, ..., v_n\}$ and edge set be E(G) = $\{e_1, e_2, ..., e_m\}$. The vertices adjacent to the vertex v are called the neighbours of v. If the vertices v_i and v_j are adjacent then we write $v_i \sim v_j$ and if they are not adjacent then we write $v_i \nsim v_j$. The degree of a vertex v_i, denoted by $\deg_G(v_i)$, is the number of edges incident to it. If all the vertices have same degree equal to r then the graph is called a regular graph of degree r. The set $\mathbb{Z}_2 = \{0,1\}$ is a group under binary operation ⊕ with addition modulo 2. Therefore for any positive integer n, $\mathbb{Z}_2^n = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ (n factors) is a group under the operation ⊕ defined by

$$(x_1, x_2, ..., x_n) \oplus (y_1, y_2, ..., y_n) = (x_1 + y_1, x_2 + y_2, ..., x_n + y_n),$$

with addition modulo2.

Element of $\mathbb{Z}_2^n$ is an n – tuple ($x_1, x_2, \ldots, x_n$) written as $x = x_1x_2 \ldots x_n$ where every x_i is either 0 or 1and is called a string. The number of 1 in $x = x_1x_2 \ldots x_n$ is called the weight of x and is denoted by wt(x).

Let $x = x_1x_2 \ldots x_n$ and $y = y_1y_2 \ldots y_n$ be the elements of $\mathbb{Z}_2^n$. Then the sum $x \oplus y$ is computed by adding the corresponding components of x and y under addition modulo 2. That is, $x_i + y_i = 0$ if $x_i = y_i$ and $x_i + y_i =1$ if $x_i \neq y_i$, i =1, 2, ..., n.

The Hamming distance $H_d(x, y)$ between the strings $x = x_1x_2 \ldots x_n$ and $y = y_1y_2 \ldots y_n$ is the number of is such that $x_i \neq y$, $1 \leq i \leq n$. Thus $H_d(x, y)$ = Number of positions in which x and y differ = $wt(x \oplus y)$ [5].

Example1.1. If x = 01001and y = 11010 are the strings, then $H_d(x, y) = 3$. The adjacency matrix of a graph G is a square matrix $A(G) = [a_{ij}]$ of order n, in which

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \sim v_j \\ 0 & \text{otherwise.} \end{cases}$$

Denote by s(v), the row of the adjacency matrix corresponding to the vertex v. It is a string in these $\mathbb{Z}_2^n$.

The Hamming index of a graph is the sum of Hamming distances between all pairs of strings generated by the adjacency matrix of a graph G and is denoted by $H_A(G)$.

That is,

$$H_A(G) = \sum_{1 \leq i < j \leq n} H_d(s(v_i), s(v_j)).$$

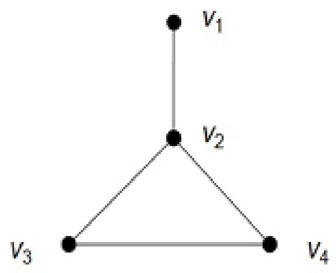

Fig. 1. Graph

Example1.2. For a graph G given in Fig. 1, the adjacency matrix is

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

and strings are s(v1) = 0100, $s(v_2) = 1011$, $s(v_3) = 0101$, $s(v_4) = 0110$. Therefore $H_d(s(v_1), s(v_2)) = 4$, $H_d(s(v_1), s(v_3)) = 1$, $H_d(s(v_1), s(v_4)) = 1$, $H_d(s(v_2), s(v_3)) = 3$, $H_d(s(v_2), s(v_4)) = 3$, and $H_d(s(v_3), s(v_4)) = 2$. Hence $H_A(G) = 4+1+1+3+3+2 = 14$.

2. HAMMING DISTANCE BETWEEN STRINGS

Ganagi and Ramane [2] obtained following result of Hamming distance between a given pair of strings generated by the adjacency matrix of a graph.

Theorem 2.1. [2] Let G be a graph with n vertices and u and v be the vertices of G. Let k be the number of common neighbours of u and v and l be the number of vertices which are neither neighbour of u nor the neighbour of v. Then

$$H_d\big(s(u), s(v)\big) = \begin{cases} n-k-l & \text{if } u \sim v \\ n-k-l-2 & \text{if } u \nsim v. \end{cases}$$

Proof. Let k be the number of common neighbours of u and v and l be the number of vertices which are neither neighbour of u nor the neighbour of v. Therefore, remaining $n - k - l - 2$ vertices other than u and v are adjacent to either u or v but not to both.

i. If u and v are adjacent vertices of G, then the strings of u and v from A(G) will be in the form

$s(u) = x_1x_2x_3 \ldots x_{k+1}x_{k+2}x_{k+3}\ldots x_{k+l+2}x_{k+l+3} \ldots x_n$ and

$s(v) = y_1y_2y_3 \ldots y_{k+1}y_{k+2}y_{k+3} \ldots y_{k+l+2}y_{k+l+3} \ldots y_n$

where $x_1 = 0$, $x_2 = 1$, $y_1 = 1$, $y_2 = 0$,

$x_i = y_i = 1$ for $i = 3, 4, \ldots, k + 2$,

$x_i = y_i = 0$ for $i = k + 3, k + 4, \ldots, k + l + 2$ and

$x_i \neq y_i$ for $i = k + l + 3, k + l + 4, \ldots, n$.

Therefore s(u) and s(v) differ at $n - k - l - 2 + 2 = n - k - l$ places.

Hence $H_d(s(u), s(v)) = n - k - l$.

ii. If u and v are nonadjacent vertices of G, then

$s(u) = x_1x_2x_3 \ldots x_{k+1}x_{k+2}x_{k+3}\ldots x_{k+l+2}x_{k+l+3} \ldots x_n$ and

$s(v) = y_1y_2y_3 \ldots y_{k+1}y_{k+2}y_{k+3} \ldots y_{k+l+2}y_{k+l+3} \ldots y_n$

where $x_1 = 0$, $x_2 = 0$, $y_1 = 0$, $y_2 = 0$,

$x_i = y_i = 1$ for $i = 3, 4, \ldots, k + 2$,

$x_i = y_i = 0$ for $i = k + 3, k + 4, \ldots, k + l + 2$ and

$x_i \neq y_i$ for $i = k + l + 3, k + l + 4, \ldots, n$.

Therefore s(u) and s(v) differ at $n - k - l - 2$ places.

Hence $H_d(s(u), s(v)) = n - k - l - 2$.

Lemma2.2. [2] Let G be a graph with n vertices. Let k be the number of common neighbours of u and v and l be the number of vertices which are neither neighbour of u nor the neighbour of v. Then

$$deg_G(u) + deg_G(v) = \begin{cases} n + k - l & \text{if} u \sim v \\ n + k - l - 2 & \text{if} u \nsim v. \end{cases}$$

Using above results, Pasaribu et al. [6] gave Hamming distance between two vertices in terms of the degrees and common neighbours.

Theorem 2.3. [6] Let k be the number of common neighbours of u and v. Then for any two vertices u and v,

$$H_d(s(u), s(v)) = deg_G(u) + deg_G(v) - 2k.$$

Corollary 2.4. [2]

Let G be an r-regular. If u and v have k common neighbours, then

$H_d(s(u),s(v)) = 2r - 2k$.

Following result gives the Hamming distance between the vertices in trees. Let $d_G(u, v)$ denotes the ordinary distance between the vertices u and v in a graph G.

Theorem 2.5. [2] Let G be a tree with n vertices. Let u and v be the distinct vertices of G. Then

$$H_d\big(s(u), s(v)\big) = \begin{cases} deg_G(u) + deg_G(v) & if\ d_G(u,v) \neq 2 \\ deg_G(u) + deg_G(v) - 2 & if\ d_G(u,v) = 2. \end{cases}$$

Theorem 2.6. [8] Let u and v be the nonadjacent vertices of G and G′ be the graph obtained from G by adding an edge between u and v. Then

$$H_{dG'}(s(u), s(v)) = H_{dG}(s(u),s(v)) + 2.$$

3. HAMMING INDEX

In this section we outline the results on the Hamming index of graphs.

Theorem 3.1. [1] Let G be a graph with n vertices and m edges. Let $V(G) = \{v_1, v_2, ..., v_n\}$ be the vertex set of G. Then

$$H_A(G) = 2mn - \sum_{i=1}^{n} d\,eg_G(v_i)^2.$$

Proof. Let $k = \left|n(v_i,\ v_j)\right|$ be the number of common neighbours of v_i and v_j. Then by the Theorem 2.3,

$$\mathbf{H_d}\left(\mathbf{s}(\mathbf{v_i}), \mathbf{s}(\mathbf{v_j})\right) = \mathbf{deg_G}(\mathbf{v_i}) + \mathbf{deg_G}(\mathbf{v_j}) - \mathbf{2}\left|\mathbf{n}(\mathbf{v_i}, \mathbf{v_j})\right|.$$

Therefore

$$
\begin{aligned}
H_A(G) &= \sum_{1 \le i < j \le n} H_d\left(s(v_i), s(v_j)\right) \\
&= \sum_{1 \le i < j \le n} \left[deg_G(v_i) + deg_G(v_j)\right] - 2 \sum_{1 \le i < j \le n} \left|n(v_i, v_j)\right| \\
&= (n-1) \sum_{i=1}^{n} deg_G(v_i) - 2 \sum_{i=1}^{n} \binom{deg_G(v_i)}{2} \\
&= 2(n-1)m - \left(\sum_{i=1}^{n} deg_G(v_i)^2 - 2m \right) \\
&= 2mn - \sum_{i=1}^{n} deg_G(v_i)^2.
\end{aligned}
$$

By above Theorem 3.1, we have

1. For an r-regular graph G, $H_A(G) = nr(n-r)$.
2. For a complete graph K_n, $H_A(K_n) = n(n-1)$.
3. For a cycle C_n, $H_A(C_n) = 2n(n-2), n \geq 3$.
4. For a complete bipartite graph $K_{p,q}$, $H_A(K_{p,q}) = pq(p+q)$.
5. For a path P_n, $H_A(P_n) = 2n^2 - 6n + 6$.

Following theorem gives the relation between $H_A(G)$ and $H_A(\overline{G})$, where $\overline{G}$ is the complement of G.

Theorem 3.2. [2] Let G be a graph with n vertices and m edges and $\overline{G}$ be the complement of G. Then

$$\mathbf{H_A(\overline{G}) = H_A(G) + n(n-1) - 4m.}$$

Theorem 3.3. [2] Let G be a graph with n vertices and m edges. Let $\overline{G}$ be the complement of G. Then $H_A(G) = H_A(\overline{G})$ if and only if G is self complementary graph.

Theorem 3.4. [2] Let G be a tree on n vertices. Then

$$\mathbf{H_A(G)} = \sum_{\mathbf{d_G(u,v)\neq 2}} [\mathbf{deg_G(u) + deg_G(v)}] + \sum_{\mathbf{d_G(u,v)=2}} [\mathbf{deg_G(u) + deg_G(v) - 2}].$$

4. HAMMING INDEX OF SOME GRAPH OPERATIONS

Let G_1 be the graph with vertex set V_1 and edge set E_1. Let G_2 be the graph with vertex set V_2 and edge set E_2. The graph $G = G_1 \cup G_2$ is graph with vertex set $V = V_1 \cup V_2$ and edge set $\mathrm{E} = \mathrm{E}_1 \cup \mathrm{E}_2$.

Theorem 4.1. [1,7] Let $\boldsymbol{G_1}$ be the graph with $\mathbf{n_1}$ vertices and $\mathbf{m_1}$ edges. Let $\mathbf{G_2}$ be the graph with $\mathbf{n_2}$ vertices and $\mathbf{m_2}$ edges. Then

$$\boldsymbol{H_A(G_1 \cup G_2) = H_A(G_1) + H_A(G_2) + 2(n_1 m_2 + n_2 m_1)}.$$

The join of two graph G_1 and G_2, denoted by $G_1 + G_2$ is a graph obtained from G_1 and G_2 by joining each vertex of G_1 to all vertices of G_2.

Theorem 4.2. [1,7] Let $\mathbf{G_1}$ be the graph with $\mathbf{n_1}$ vertices and $\mathbf{m_1}$ edges. Let $\mathbf{G_2}$ be the graph with $\mathbf{n_2}$ vertices and $\mathbf{m_2}$ edges. Then

$$\mathbf{H_A(G_1 + G_2) = H_A(G_1) + H_A(G_2) + n_1 n_2 (n_1 + n_2) - 2(n_1 m_2 + n_2 m_1)}.$$

Theorems 4.1 and 4.2 appeared in the papers and [1] and [7]. Approach of proofs in both papers are different.

The Cartesian product of two graphs $\boldsymbol{G}$ and $\boldsymbol{H}$ is the graph $\boldsymbol{G \times H}$ whose vertex set is $\boldsymbol{V(G) \times V(H)}$ and two vertices $(\boldsymbol{u}, \boldsymbol{v})$ and $(\boldsymbol{x}, \boldsymbol{y})$ are adjacent in $\boldsymbol{G \times H}$ if either $\boldsymbol{u = x}$ and $\boldsymbol{v}$ is adjacent to $\boldsymbol{y}$ in $\boldsymbol{H}$ or $\boldsymbol{u}$ is adjacent to $\boldsymbol{x}$ in $\boldsymbol{G}$ and $\boldsymbol{v = y}$.
Theorem 4.3. [1] Let $\mathbf{G_1}$ be the graph with $\mathbf{n_1}$ vertices and $\mathbf{m_1}$ edges. Let $\mathbf{G_2}$ be the graph with $\mathbf{n_2}$ vertices and $\mathbf{m_2}$ edges. Then

$$\begin{aligned}\mathbf{H_A(G_1 \times G_2)} &= \mathbf{n_2^2 H_A(G_1) + n_1^2 H_A(G_2) + \left(n_2^2 - n_2\right) \sum_{i=1}^{n_1} d\,eg_{G_1}(v_i)^2} \\ &\quad \mathbf{+ \left(n_1^2 - n_1\right) \sum_{j=1}^{n_2} d\,eg_{G_2}\left(u_j\right)^2 - 8 m_1 m_2}.\end{aligned}$$

The corona product $G \circ H$ of two graphs G and H is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining i-th vertex of G to each vertex in i-th copy of H, $i = 1, 2, \ldots, n$.

Theorem 4.4. [3] For two graphs $\mathbf{G_1(n_1, m_1)}$ and $\mathbf{G_2(n_2, m_2)}$,

$$\begin{aligned} \mathbf{H_A(G_1 \circ G_2)} &= \mathbf{H_A(G_1) + n_1 H_A(G_2) + n_1 n_2 (2n_1 n_2 + 2n_1 - n_2 - 1)} \\ &\quad \mathbf{+ 2n_1 m_2 (n_1 n_2 + n_1 - n_2 - 2) + 2m_1 n_2 (n_1 - 2).} \end{aligned}$$

Theorem 4.5. [4]. Let G be a graph with n vertices and m edges. The Hamming index of line graph L(G) of a graph G is

$$\mathrm{H_A(L(G)) = (m + 4)M_1(G) - 2M_2(G) - M_3(G) - 2m^2 - 4m.}$$

where

$$\mathrm{M_1} = \sum_{i=1}^{n} \deg_G(v_i)^2, \quad \mathrm{M_2} = \sum_{v_i \sim v_j} \deg_G(v_i)\deg_G(v_j), \quad \mathrm{M_3} = \sum_{i=1}^{n} \deg_G(v_i)^3.$$

The vertices and edges of G are referred as their elements. The total graph of G, denoted by $T(G)$, is a graph with vertex set $V(T(G)) = V(G) \cup E(G)$ and two vertices in $T(G)$ are adjacent if and only if they are adjacent elements or they are incident elements in G

Theorem 4.6. [4] Let G be a graph with n vertices and m edges. The Hamming index of total graph is

$$\mathbf{H_A(T(G)) = H_A(G) + 4m^2 + 2mn + (m + n - 3)M_1(G) - M_3(G) - 2M_2(G).}$$

The subdivision graph $S(G)$ is the graph obtained from G by inserting a new vertex into each edge of G.

Theorem 4.7. [4] Let **G** be a graph with n vertices and m edges. Then for a subdivision graph $\mathbf{S(G)}$ of **G**, $\mathbf{H_A(S(G)) = H_A(G) + 4m^2 - 4m + 2mn}$.

The splitting graph $Sp(G)$ is a graph obtained from G by adding a new vertex v_i' corresponding to the vertex v_i of G and the new vertex v_i' is adjacent to the vertices which are adjacent to v_i.

Theorem 4.8. [4] Let G be a graph with n vertices and m edges. Then for a splitting graph $\mathbf{Sp(G)}$ of $\mathbf{G}$, $\mathbf{H_A(Sp(G)) = 5H_A(G) + 2nm}$.

The semi-total line graph of G, denoted by $T_1(G)$, is a graph with vertex set $\left(T_1(G)\right) = V(G) \cup E(G)$ and two vertces in $T_1(G)$ are adjacent if they are adjacent edges in G or one is vertex and other is an edge, incident to it.

Theorem 4.9. [4] Let **G** be a graph with **n** vertices and **m** edges. Then for a semi-total line graph $\mathbf{T_1(G)}$ of $\mathbf{G}$,

$$\mathbf{H_A(T_1(G)) = H_A(G) + (m+n)M_1(G) - M^3(G) - 2M_2(G) + 2m^2.}$$

The semi-total point graph of G, denoted by $\mathrm{T_2(G)}$, is a graph with vertex set $\mathrm{V\left(T_2(G)\right)} = \mathrm{V(G) \cup E(G)}$ and two vertces in $\mathrm{T_2(G)}$ are adjacent if they are adjacent vertices in G or one is vertex and other is an edge, incident to it.

Theorem 4.10. [4] Let **G** be a graph with **n** vertices and **m** edges. Then for a semi-total point graph $\mathbf{T_2(G)}$ of $\mathbf{G}$,

$$\mathbf{H_A(T_2(G)) = H_A(G) - 3M_1(G) + 6m^2 + 4mn - 4m.}$$

5. BOUNDS FOR HAMMING INDEX

Theorem 5.1. [4] Let G be a graph with n vertices. Then $\mathrm{H_A(G)} \leq \mathrm{n^3/4}$. Equality holds for $\mathrm{(n/2)}$-regular graph with n vertices.

Proof. Let $\mathbf{k} = \left|\mathbf{n}(\mathbf{v_i}, \mathbf{v_j})\right|$ be the number of common neighbours of $\mathbf{v_i}$ and $\mathbf{v_j}$. Then by Theorem 2.3,

$$\mathbf{H_A(G)} = \sum_{\mathbf{1 \leq i < j \leq n}} \left[\mathbf{deg_G(v_i) + deg_G(v_j) - 2\left|N(v_i, v_j)\right|}\right]$$

But, $\left|\mathrm{N(v_i, v_j)}\right| = \sum_{\mathrm{i=1}}^{\mathrm{n}} \binom{\mathrm{deg_G(v_i)}}{2}$. Therefore,

$$\mathbf{H_A(G)} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{d\,eg_G(v_i)\left(n - deg_G(v_i)\right)}$$

Since, $\deg_G(v_i)(n - \deg_G(v_i))$ is maximum if $\deg_G(v_i) = n/2$.

Therefore, $\mathbf{H_A(G) \leq \frac{n^3}{4}}$.

If G is a $(n/2)$-regular graph of order n, then $H_A(G) = n^3/4$. □

Theorem 5.2. [4] Let $\mathbf{e}$ be an edge of a graph $\mathbf{G}$ of size $\mathbf{m}$ and order $\mathbf{n}$. Then,

$$\mathbf{H_A(G - e) - 2n + 6 \leq H_A(G) \leq H_A(G + e) + 2n - 6.}$$

Equality on LHS holds if the edge removed is adjacent to the vertices of degree $\mathbf{n - 1}$. Equality on RHS holds if the edge is added to the vertices of degree $\mathbf{n - 2}$.

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ABSTRACTS

A Mathematical Model for the Patterns of Alcohol Consumption

A. Amalarathinam[1*] and Sabarmathi A.[2]
Department of Mathematics, Auxilium College (Autonomous), Vellore-632 006, Tamilnadu, India.
Email: aamala@auxiliumcollege.edu.in

Abstract.
A mathematical model for the patterns of alcohol consumption is presented in this article. Equilibrium points are found. The boundedness of the system is verified using Gronwall's inequality. The local stability of the model is analysed around the equilibrium points using Routh-Hurwitz Criteria.

Keywords: Equilibrium points, Boundedness, Gronwall's inequality, Local stability, Routh-Hurwitz Criteria

Sustainable Strategies for Managing Mango Diseases

Saranya P [1*] and Sabarmathi A[2]
PG and Research Department of Mathematics, Auxilium College(Autonomous), Vellore-632006, Tamilnadu, India.
Email: saranyaperumal13@gmail.com

Abstract.
This study investigates sustainable strategies for managing mango diseases. The research explores the use of antagonistic yeasts for controlling postharvest diseases and the effectiveness of chitosan combined with phenolic compounds to manage anthracnose. It also examines the role of organic and biofertilizers in enhancing mango yield. Furthermore, the study evaluates organic alternatives, such as plant extracts and panchagavya, and yeast formulations for their ability to control mango diseases. These approaches aim to promote environmentally friendly and sustainable mango cultivation practices.

Keywords: Antagonistic yeasts, Chitosan, Biofertilizers, Panchagavya, Postharvest disease and Organic alternatives.

Evaluating Methods for Inventory Demand Prediction to Determine the Optimal Approach

T Karpagavigneswari[1] R Kamali[2]

[1]Research scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamilnadu, India.

[2]Assistant Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamilnadu, India.

Email: karpagavigneswarib@gmail.com

Abstract.

This study uses a few techniques, including the Least Squares Polynomial – Sinusoidal Method, ARIMA Model, and Logistic Map to evaluate the inventory demand forecast. By finding the trend line that best fits past data, the Least Squares Polynomial – Sinusoidal Method (LSPSM) is a statistical technique for forecasting future demand. By using this technique, the trend line's sum of squared deviations between observed and forecasted values is reduced. For the sequential structure of data, Auto Regressive Integrated Moving Average (ARIMA) is a better machine learning technique in predicting trends. The logistic map is used to evaluate these approaches and determine which one offers an optimal approach. Also, a numerical example revealed that the logistic map excels beyond other methods.

Keywords: Logistic map, Least Square Method, Sale Prediction, Time Series.

Analysing Shortest Paths in a Self-Similar Fractal Graphs using Floyd-Warshall Algorithm with Python Implementation

E. Rohit Auxilia[1], R. Kamali[2]

[1]Research Scholar, Department of Mathematics, VISTAS, Chennai

[2]Assistant Professor, Department of Mathematics, VISTAS, Chennai.

Email: rosyfrancis10@gmail.com

Abstract.

The paper explores the growth and structural properties of a recursively constructed fractal graphs exhibiting self-similar characteristics, where the number of vertices increases linearly by three at each iteration. At each step of fractal growth, the shortest path of all pairs of vertices are calculated using the Floyd-Warshall Algorithm. The Python solution offers an organised method to monitor this evolution of shortest pathways as the complexity of the fractal increases.

Keywords: Fractal Graphs, Floyd Warshall Algorithm, Python Coding.

A Study of Enclave Domination Number and its Interplay with Other Graph Parameters

M. Santhosh Priya[1], Dr. A. Mydeen Bibi[2]

[1]Research Scholar, PG & Research Department of Mathematics,

The Standard Firework Rajaratnam College for Women, Sivakasi, Tamilnadu.

[2]Assistant Professor, PG & Research Department of Mathematics,

The Standard Firework Rajaratnam College for Women, Sivakasi, Tamilnadu.

Email: santhoshpriya97@gmail.com

Abstract.

Let $G = (V, E)$ be a simple graph. A dominating subset E_u of $V(G)$ is said to be an enclave dominating set, if the set E_u has exactly one enclave vertex u in it. The vertex u is called as enclave dominating vertex. The minimum cardinality on all the enclave dominating sets, known as enclave domination number of G. It is denoted by $\gamma_s(G)$. We characterize the classes of graphs for which $4 \leq \gamma_\epsilon(G) + \chi(G) \leq 2n - 2$. Also, we investigate the bounds of enclave domination number and explore their relationship with other graph parameters.

Keywords: Domination number, Enclave dominating vertex, Enclave dominating set, Enclave domination number, Chromatic number.

Cayley Digraphs of the Group Generated by Differential Equations

M. Aiswarya[1] and A. Anat Jaslin Jini[2]

[1]Student, Department of Mathematics, Holy Cross College (Autonomous), Nagercoil - 4

[2]Assistant Professor of Mathematics, Holy Cross College (Autonomous), Nagercoil – 4

Email: anatjaslin@holycrossngl.edu.in

Abstract.

Differential equations are mathematical equation that relate a function to its derivatives. Graphs are mathematical representation of a set of objects connected by edges. Cayley graphs give a geometrical representation of groups by means of a set of generators. If $x \in \mathbb{R}$ and $n \in \mathbb{N}$ then the generalized Exponential function is defined as $e(x^m, n) = x^m/m! + x^{m+n}/(m+n)! + \cdots$, where m is the initial differential term and n represent the length of each differential term. The article focuses on using Cayley digraphs to visualize the structure of the group formed by differential equations. The vertices of Cayley digraphs are the elements of the group. This study verifies the general theorem for the higher order of exponential function, using addition and multiplication. For addition, the theorem is enabled by using two groups $\mathbb{Z}_6$ and $\mathbb{Z}_7$, where the graph theory and group concepts are used to find the Cayley digraphs using D operator. The general theorem for multiplication enables by using the group $\mathbb{Z}_7 - \{0\}$. Moreover, the article illustrates a C# program for finding the Cayley table for addition and multiplication modulo.

Key words: Differential equations, Cayley digraphs, C# program.

Cayley Digraphs of the Group Generated by Higher Order Derivatives

J. Abisha[1] and A. Anat Jaslin Jini[2]

[1]Student, Department of Mathematics, Holy Cross College (Autonomous), Nagercoil - 4

[2]Assistant Professor of Mathematics, Holy Cross College (Autonomous), Nagercoil – 4

Email: anatjaslin@holycrossngl.edu.in

Abstract.

Cayley digraphs serve as a powerful tool for visualizing the algebraic structure of groups, translating abstract algebraic concepts into concrete graphical representations. Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be any real valued function, then the delta difference operator on $f(x)$ is defined as $\Delta f(x)= f(x+1)-f(x)$. Let $x \in \mathbb{R}$ and $k \in \mathbb{N}$, then the generalized delta polynomial factorial function is given by $\Delta^s x^{(k)}= k^{(s)}x^{(k-s)}$, where $k^{(s)} = k(k-1)(k-2)\ldots(k-(s-1))$. This article focuses on using Cayley digraphs to visualize the structure of mathematical groups formed by higher-order derivatives. This study proves a general theorem for the higher-order derivative of extorial function, using two different approaches - addition and multiplication. For addition, the theorem is supported by using two groups $\mathbb{Z}_6$ and $\mathbb{Z}_7$, where graph theory and group concepts are used to find the cayley digraphs using delta operator. For multiplication, the general theorem is supported by using the group $\mathbb{Z}_7$-$\{0\}$. Additionally, this article describes a python program for finding the Cayley table for addition and multiplication modulo.

Key words: Cayley digraphs, Cayley table, Python program

Studies On Graph Theory Based Moving Object Segmentation

N. J. Bhuvaneswari* and P. Karthick
Department of Mathematics, School of Advanced Sciences, Kalasalingam Academy of Research and Education (Deemed to be University), Krishnankoil, Tamilnadu-626 126.
Email: bhuvana.nj@gmail.com

Abstract.

Image segmentation plays a crucial role in identifying and isolating moving objects within underwater images, particularly for applications like marine monitoring, object detection, and underwater robotics. Underwater environments present unique challenges due to factors like light scattering and absorption, making traditional segmentation methods less effective. The present research explores the utility of semantic segmentation algorithms for analyzing underwater photograph. The graph-based approach to image segmentation is fairly independent from distortion, colour alteration and other peculiar effects arising with light propagation in water medium. Moving object segmentation (MOS) using passive underwater image processing is an important technology for monitoring marine habitats. Besides that , to use the state-of-the-art segmentation method to face this problem, which are based on deep learning, an underwater image segmentation dataset must be proposed. In this work, concepts of graph signal processing are analyzed for MOS. Also, we study a new algorithm that is composed of segmentation, background initialization, graph construction, unseen sampling, and a semi-supervised learning method inspired by the theory of recovery of graph signals.

Keywords: Image processing, Graph theory, Moving object segmentation, Underwater image segmentation dataset, Graph signal processing

Binomial Graph

Md Shahidul Islam Khan
Assistant Professor, Department of Mathematics, Pandit Deendayal Upadhyaya Adarsha Mahavidyalaya, Amjonga, Assam, India
Email: rakhakshik786@gmail.com

Abstract.

Let n be a nonnegative integer and $\left\{\binom{n}{r} : r = 0,\ 1,\ 2, \dots, n\right\}$ is the set of coefficients of the binomial expansion $(a+b)^n$. We have $\binom{n}{r} = \binom{n}{n-r}$. We define an undirected graph structure on the collection of coefficients of $(a+b)^n$ with the set of vertex $V = \left\{\binom{n}{r} : 0 \le r \le \frac{n+1}{2}\right\}$ when n is odd or $V = \left\{\binom{n}{r} : 0 \le r \le \frac{n}{2}\right\}$ when n is even and any two vertices x and y $(x < y)$ are connected by an edge if x divides y. We call this graph Binomial Graph and denoted it by $B_G(n)$. In this present paper we investigate the relation between the binomial coefficients from graph theoretic point of view. We also study various graph theoretic properties of the binomial coefficients.

Keywords: Binomial coefficients, Binomial graph, Completeness.

A Study on Annihilator IP Domination Number

Al. Yakavi[1], Dr. A. Mydeen Bibi[2]

[1]Research Scholar,[2]Assistant Professor PG & Research Department of Mathematics,

The Standard Fireworks Rajaratnam College for Women, Sivakasi

Email: yakavi.al95@gmail.com

Abstract.

An Annihilator Isolate Pendant (AIP) Dominating set is defined as an IP dominating set I of G where the induced subgraph $\langle V - I\rangle$ is totally disconnected. The Annihilator Isolate Pendant Domination number, denoted by $\gamma_{a01}(G)$, is the minimum cardinality of such a set. This paper introduces the Annihilator Isolate Pendant Domination number, a novel graph parameter, and investigates its behaviour in various graph families and tensor product graphs.

Keywords: Isolate vertex, Pendant vertex, Isolate Pendant Dominating set (IPD-set), IP domination number, Annihilator Isolate Pendant (AIP) Dominating set, Annihilator Isolate Pendant (AIP) Domination number.

Inverse Cototal Domination on Graphs

Dr. A. Punitha Tharani, M.Mariya Evila
Associate Professor and Dean of Science, Department of Mathematics, St.Mary’s College (Autonomous), Thoothukudi.
Research Scholar, Department of Mathematics, St. Mary's College (Autonomous), Thoothukudi.

Abstract.
Let D be a minimum dominating set of a graph G. A subset D' ⊆ V - D is called an inverse cototal dominating set if D' is a dominating set and the induced subgraph <V - D'> contains no isolated vertices. The inverse cototal domination number of a graph G , denoted by $\gamma_{cot}^{-1}(G)$, is the cardinality of a minimum inverse cototal dominating set in G . In this paper, we introduce a new concept inverse cototal domination and discuss the bounds of inverse cototal number. Our paper focuses on studying the nature of the inverse cototal domination for some special types of graphs.
Keywords: Domination, Inverse Domination, Cototal Domination.

Fuzzy Logic Enhanced Graph Cut Model for Leaf Image Segmentation

X. Maglin Helina* and P. Karthick
Department of Mathematics, School of Advanced Sciences
Kalasalingam Academy of Research and Education (Deemed to be University)
Krishnankoil Tamilnadu - 626 126.
Email: helinaxavier0302@gmail.com

Abstract.

The incorporation of fuzzy graph theory into image processing has demonstrated significant potential for improving image segmentation. This study addresses the challenges of variability and uncertainty in leaf structures by utilizing a fuzzy graph model. By capturing the inherent imprecision in shape, texture, and color through fuzzy graph representations, it enables more robust and accurate segmentation. Furthermore, this paper proposes a graph-based optimization technique integrated with fuzzy membership functions to effectively model the uncertain boundaries and overlapping features common in plant leaves. This fuzzy graph-based method enhances leaf image segmentation, especially in complex natural scenes. Evaluation results demonstrate that fuzzy graph-cut segmentation offers improved performance in terms of accuracy, robustness, and computational efficiency when compared to traditional approaches.

Keywords: Fuzzy Graph-Cut, Image Processing, Image Segmentation, Leaf Images.

An ICT-Based Graph Theory Framework for Smart Tribal Farming

K S Maharasan[1][0000-0001-8591-0940]

Sreekala. K. Nair[2][0009-0008-7544-3144], Moni Madaswamy[3][0009-0009-4694-3999]

[1]Department of Computer Applications, KG College of Arts and Science, Coimbatore- 641035, India.

[2]Department of Biotechnology, KG College of Arts and Science, Coimbatore- 641035, India.

[3]Professor Emeritus (Agricultural Informatics & e-Governance), Centre for Agricultural Informatics & e-Governance Research Studies, Department of Agri-Informatics Engineering of Shobhit University, Meerut & Former Director General of National Informatics Centre, New Delhi, India

Email: maharasan.k.s@kgcas.com

Abstract.

This paper investigates the application of ICT-based graph theory concepts in smart tribal farming to significantly boost agricultural productivity and enhance the livelihoods of tribal farmers. By employing graph-based modeling and analytics, the proposed framework optimizes critical agricultural activities such as crop planning, irrigation management, and supply chain logistics, while addressing the distinct challenges and opportunities inherent in tribal agriculture. The idea of integrating ICT and graph theory facilitates informed decision-making and efficient resource allocation. This approach ultimately supports sustainable agricultural practices and fosters socio-economic development in tribal regions by suggesting a conceptual smart tribal farming framework supported by ICT enabled Graph theory concepts.

Keywords: Graph Theory, Smart Tribal Farming, Sustainable Development.

Z-Fuzzy Superpixel Hypergraphs and Random Walk Dynamics for Robust Tumour Segmentation

T. Brinthaguru* and P. Karthick
Department of Mathematics School of Advanced Sciences
Kalasalingam Academy of Research and Education (Deemed to be University)
Krishnankoil Tamilnadu-626 126.
Email: tbrindha121@gmail.com

Abstract.
Hypergraph-based image segmentation has garnered significant attention for its ability to model high-order relationships among image elements. In this paper, we propose a novel segmentation algorithm that integrates the normalized hypergraph cut criterion within a random walk framework, leveraging super pixel-based hypergraph construction to preserve spatial coherence and reduce computational complexity. To robustly handle the uncertainty and vagueness present in medical imaging, we employ z-intuitionistic fuzzy membership functions for computing hyperedge weights, enhancing the representation of local image structures. The random walk transition probabilities are guided by these fuzzy measures, effectively minimizing the chances of transitions across distinct regions. This strategy ensures that the segmentation boundaries align with meaningful anatomical structures. Experimental results on CT and MRI tumour datasets demonstrate the effectiveness of the proposed method, achieving superior accuracy and robustness in segmenting tumour regions compared to conventional approaches.

Keywords: Fuzzy Logic, Medical Image Analysis, Hypergraph Cut, Super pixels, Random Walk

Leveraging & Harnessing by 4.0 Revolution for Fostering Sustainability Techniques beyond the Agricultural Value Chain

Prof. Anjali Pise
Department of CSE,TGPCET, Nagpur, Maharashtra, India.
Email: avm.pise@gmail.com

Abstract.
An agro-industry is pivotal in sustaining the global population. The global agricultural industry is currently at a critical juncture where sustainability has become imperative rather than optional. Sustainability practices signify a fundamental change in the way humanity engages with the planet's resources. Adopting Blockchain-based sustainability techniques has substantial, lucrative, communal, and ecosystem ramifications and provides quality of life. Future developments in this developing industry are also scrutinized. Blockchain technology, recognized for its administrative decentralization security and lucid ledger system, presents significant potential for addressing these issues. Blockchain empowers the stakeholders from farmers to consumers by providing immutable records that enhance trust, mitigate fraud, and support informed decision-making. This kind of integration enhances the efficiency of resource use while also encouraging ethical sourcing and agriculture that is resilient to climate change. Through the lens of successful case studies, the practical application and impact of blockchain (BC) are illustrated. This research paper inquires into the certainty of blockchain (BC) technology to transform sustainability practices within the agro-industry. The findings suggest that while blockchain technology holds considerable promise for fostering sustainability in the agro-industry, its successful integration requires addressing existing challenges and promoting interdisciplinary collaboration among stakeholders. This study examined the integration of blockchain into agro-industry practices to enhance sustainability, traceability, and accountability. This study highlights blockchain's role as a digital enabler in fostering a more sustainable and resilient agro-industry.

Keywords: Ag-tech,4IR, Green IT.

A Theoretical Study of the Fractal Von-Koch Curve Graph using Eccentricities, Tree Properties and Bipartite Structures

Suriya Banu. R [1], Kamali. R [2*]

Research Scholar [1], Assistant Professor [2*]

Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS),Chennai

Email: Suriyabanu2020@gmail.com; kamali.sbs@vistas.ac.in

Abstract.

This research presents an innovative mathematical analysis of Fractal Von-Koch Curve Graph. This paper introduces the classes of graph such as eccentricities, radius, diameter, and degrees are calculated for the i^{th} iterations of the Curve. Mathematical expressions were formulated based on the findings. The obtained results indicate a correlation between the radius and diameter of the i^{th} iterations of the Curve. In each iteration of the Curve, there exists two leaf vertices with a degree of one. Additionally, a new formula will be proposed for the remaining two-degree vertices in every iteration of the Curve. Properties of trees and bipartite graphs are utilized to substantiate the theorem. Furthermore, relevant examples are taken to prove the theorem. Ultimately, this study aims to bridge two significant research domains of Fractals and Graph Theory.

Keywords: Eccentricities, Radius, Diameter, Tree, Coloring, Bipartite.

ACCEPTED MANUSCIRPTS

Evaluating Methods for Inventory Demand Prediction to Determine the Optimal Approach

T Karpagavigneswari[1] R Kamali[2]
[1]Research scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamilnadu, India.
[2]Assistant Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamilnadu, India.
Email: karpagavigneswarib@gmail.com; kamali.sbs@vistas.ac.in

Abstract.

This study uses a few techniques, including the Least Squares Polynomial – Sinusoidal Method, ARIMA Model, and Logistic Map to evaluate the inventory demand forecast. By finding the trend line that best fits past data, the Least Squares Polynomial – Sinusoidal Method (LSPSM) is a statistical technique for forecasting future demand. By using this technique, the trend line's sum of squared deviations between observed and forecasted values is reduced. For the sequential structure of data, Auto Regressive Integrated Moving Average (ARIMA) is a better machine learning technique in predicting trends. The logistic map is used to evaluate these approaches and determine which one offers an optimal approach. Also, a numerical example revealed that the logistic map excels beyond other methods.

Keywords: Logistic map, Least Square Method, Sale Prediction, Time Series.

1. INTRODUCTION

A time series is a type of series, series of data, series set of observations on the particular value, which is taken a different time frame. Here, data is collected by consistent time interval. The interval between $t2 - t1$, $t3 - t2$, $t4 - t3$ should be exactly equal, that means either it should be monthly, weekly, quarterly or annually. Time series are used in statistics, econometrics, mathematical finance, weather forecasting, earthquake prediction and many other applications.

Accurate forecasting enables businesses to align resources, strategies, and operations with future opportunities and challenges, which cannot be accomplished without analyzing data that shows change over time. The assessment of time series analysis suits obvious as a key tool for predicting and adapting to future trends. Quantifying particular variations across time is the aim of time series analysis. Time series analysis relieves the burden of comprehending the best aspects of phenomena. By characterizing previous movements and fluctuations and offering insights into historical trends, time series analysis is a potent tool for comprehending and interpreting data. It enables organizations to assess their progress and pinpoint areas for development by comparing their current achievements with historical results. Better planning and decision-making are made possible by its assistance in predicting future behavior, including price trends, output levels, and demand patterns. Time series improve the assessment of correlations and discrepancies between datasets by making it easier to compare two or more series at once. In the end, it is essential for evaluating accomplishments and formulating plans for future success [2].

The secular trend, seasonal variation, cyclical variation, and irregular variation are the four main components of time series. This study focuses on the secular trend, which is sometimes referred to as the long-term movement or trend. Over time (T), this trend is established. The main goal of trend measurement is to calculate the average growth or decline rate over a given time period. The data must be accessible for a considerable amount of time in order to measure secular trends. If not, it will be impossible to distinguish and gauge the trend's growth or fall. Few techniques are used intended for measuring trend. In this work, the method of least square is utilized to identify the best-fitting trend within the data. In order to determine the optimal function matching, the least squares approach minimizes the sum of the square errors between the measured points and the fitting straight lines [2].

The least squares approach is a data optimization technique that finds the data function that best fits the data by comparing the square of the difference between the estimated value and the actual value. Two scientists made significant contributions to the development of the least squares method: The least squares approach was initially proposed by Legendre as a way to solve linear equations; Gauss elaborated on the theory of the least squares method in great depth using the normal distribution. The segmented fitting approach is more representative of the true data relationship than the conventional direct fitting method, and curve fitting based on least squares is frequently used in data processing [3]. In a sense, the Curve Fitting Least Squares approach is an ideal. For structure prediction models, the Curve Fitting Least Squares approach is utilized [4].

By minimizing the sum of the square errors between the measured locations and the fitting straight lines, the least squares technique determines the optimal function matching. In order to determine the optimal function matching of data, the classic least squares method's main idea is to estimate the parameter vectors by minimizing the sum of squares of errors. The conventional least squares approach frequently takes into account removing the larger data while fitting regression. This significantly affects regression estimation (if the original data is accurate), but it will lose some information if the sample data is small [1].

Numerous natural phenomena may be simulated using logistic maps. Time is treated as continuous in the differential equation used by the logistic function. Instead, the logistic map examines discrete time steps using a nonlinear difference equation. Because it plots the population value at any given time step against its value at the subsequent time step, it is known as the logistic map. Furthermore, when the value of μ varies, this type of straight forward equation has a complicated character. After μ above a certain threshold, it ultimately results in chaos [5]. In data science and analysis, predictive modeling is a crucial stage that aims to create models that can forecast future behaviors or occurrences using existing data. Stated differently, it symbolizes the method by which a mathematical model that synthesizes a reality is discovered. Decisions may then be made using this model when faced with fresh information that would not have been utilized to create it in the first place [6].

Organizations have depended more and more on cutting-edge technology in recent years to recognize, anticipate, and reduce risks in a changing business environment. The use of statistical techniques and machine learning (ML) has become crucial for improving risk management procedures. By examining enormous volumes of historical data, finding patterns, and generating predictions in real time, machine learning techniques like predictive modeling, clustering, and anomaly detection help companies foresee possible dangers. These techniques support conventional statistical tools that offer exacting quantitative insights into risk variables, such as regression analysis, hypothesis testing, and time-series forecasting. When combined, machine learning and statistical models help decision-makers identify new risks, allocate resources as efficiently as possible, and put proactive risk mitigation plans into action [7]. The ultimate goal of this research is to offer insightful information on how logistic maps, as opposed to the other two approaches, might enable companies to make better predictions.

2. TECHNIQUES FOR PREDICTING INVENTORY DEMAND

2.1. Least Squares Method

The most popular technique for achieving trends is the least squares method. By reducing the sum of the squared differences between the observed values and the values predicted by the line, the least squares method is a statistical methodology that determines the best-fit line for a set of data points. Using this mathematical technique, a trend line is fitted to the data in such a way that satisfies the following two essential requirements.

i. $\sum(y - ye) = 0$ (1)

ii. $\sum(y - ye)^2$ is least. (2)

According to equation (1), there is no positive correlation between the calculated and real values of y. The approach is called least squares because equation (2) indicates that the sum of squared deviations between the actual values and the estimated values is minimized for this trend line.

2.2. Least Squares Polynomial-Sinusoidal Model

A quadratic polynomial model is a general from for fitting trends. $Z(X) = aX^2 + bX + c$ where a stands for curvature, b for linear trends and c for the constant. Sinusoidal model represents periodic oscillations, $Z(X) = de^{i\omega}$. This model combines sinusoidal oscillations with polynomial trends and operates in the complex domain. To construct this model with two components, trend with oscillations.

$$Z(X) = aX^2 + bX + c + de^{i\omega x} \quad (3)$$

Combining sinusoidal oscillations with a polynomial trend has a solid basis in signal processing and applied mathematics, and it is also utilized in a variety of domains, such as control systems and climate research.

2.3. Auto Regressive Integrated Moving Average (Arima)

ARIMA is a mathematical and economic instrument that uses historical data to forecast future events. It is employed to forecast and comprehend time series data. In a wide range of sectors, Autoregressive Integrated Moving Average (ARIMA) models are useful. It is frequently used to predict future values in a time series, such as stock prices, demand, sales, or gold rates. The standard notation for ARIMA models is ARIMA (p,d,q), where p represents the autoregressive model's order, d the degree of differencing, and q the moving- average model's order. The mathematical formulation of ARIMA model can be expressed as,

$$yp\ (\mathrm{B})\Delta^{d}\ \mathrm{X}t = \lambda q(B)\omega t \qquad (4)$$

Where:

yp (B): AR polynomial with coefficients$y1, y2 \ldots\ldots\ldots\ldots\ldots \lambda q$,

λ_q (B): MA polynomial with coefficientsλ_1, λ_2, ..., λ_q,

Δ^d X_t: Differenced time series of order d,

ωt: White noise error term (mean = 0, constant variance σ^2)

It generates accurate short-term projections by identifying patterns and relationships in past data.

2.4. Logistic Map

Despite its formal simplicity, the logistic map, a one-dimensional discrete-time map, reveals an astonishing level of complexity. In the early days of deterministic chaos research, it was one of the most significant and influential systems. The quadratic difference equation defines the logistic map as discrete dynamical system. A polynomial mapping of degree two and a recurrence relation are equivalent. The sequence's behavior across a range of parameter μ values. The first noteworthy finding is that, for μ between 0 and 4, the sequence does not diverge and stays finite. The logistic map is defined by the following equation:

$$X_{n+1} = \mu X(1 - X_n) \qquad (5)$$

represents the well-known logistic equation. Here, μ denotes the control parameter, Xn is the past value, and $Xn+1$ is the present value.

The following qualitative phenomena can be seen in chronological order

- Converges exponentially to zero.

* Convergence to a predetermined amount.
* A first oscillation followed by convergence. Steady variations between two numbers.
* Increasing oscillations among a range of numbers that are multiples of two, such 2, 4, 8, 16, and so forth.

2.5. Application Of The Models In Real Life Scenario

In this section the techniques utilized in 2.2, 2.3 and 2.4 are applied to real-life situation. Based on the main data provided, forecasting is predicted using all approaches. Mathematical approaches and Python scripts are used to determine the outcome of the provided data.

Table 1. Oil Production and Sale data

Month	Production	Sale
January	350	310.1
February	660	284.3
March	0	352.57
April	600	270.6
May	390	186.75

This statistics mentioned in the above Table 1 displays the details of oil production and sales. The Least Squares Polynomial-Sinusoidal Model, ARIMA and Logistic Map are used to calculate the oil production and selling value for the following six months based on this data.

2.6. For Least Squares Polynomial-Sinusoidal Model

It makes sense and works well to use the least squares polynomial-sinusoidal model for analyzing data that shows both long-term trends and oscillatory behavior. A polynomial component underlies a trend in the data, such as steady rises or falls over time (e.g., dropping output or expanding sales). Seasonality, cyclical patterns, or market dynamics can all produce oscillatory behavior, which is captured by the sinusoidal component. These elements work together to provide the model flexibility and represent both consistent patterns and sporadic variations.

2.7. Result of Least Squares Polynomial-Sinusoidal Model

Using a least squares polynomial sinusoidal model (LSPSM), the expected production and selling value are displayed in Table 2.

Table 2. Predicted Production and Sale data using LSPSM

Month	Production	Sale
June	494.18	312.58
July	91.50	253.58
August	526.56	401.60
September	-66.67	532.83
October	330.90	662.58
November	-184.67	986.06

2.8. For Arima Model

Another practical method for analyzing and predicting sales and production data is to use an ARIMA model. ARIMA was created especially for delayed data, such monthly sales and production, where values are impacted by earlier observations. It manages seasonality and patterns. For instance, ARIMA's delayed components can capture productivity during abrupt surges from January to February, which is a real-world occurrence. Drops (production from March 0) are regarded as variances or abnormalities. Because ARIMA is stronger at time series forecasting and fits better when oscillations are irregular rather than periodic, it may be used to anticipate future production and sales.

2.9. Result of Arima Model

The ARIMA model's estimated production and selling value are displayed in Table 3.

Table 3. Predicted Production and Sale data using ARIMA MODEL

Month	Production	Sale
June	410.83	204.73
July	398.65	214.67
August	405.77	220.17
September	401.60	223.21
October	404.04	224.89
November	402.62	225.82

2.10. For Logistic Map

By applying a data normalization approach similar to the logistic map, manufacturing and sales processes can be optimized. Scaling data to a 0-1 range reveals patterns and trends, enabling informed decision-making and increased efficiency. As noted in Equation-5, the logistic map can model fluctuations in production size and demand, anticipating future behavior and informing strategic planning.

2.11. Result of Logistic Map

Table 4 shows the logistic map's expected production and selling values.

Table 4. Predicted Production and Sale data using LOGISTIC MAP MODEL

Month	Production	Sale
June	390.00	518.00
July	638.18	815.82
August	84.39	186.75
September	294.39	403.27
October	652.32	832.78
November	30.38	186.75

3. NOVELTY IN APPLICATION

Based on the following criteria, the mathematical combination of polynomial and sinusoidal components is a new application in this study that may be regarded as a novel technique.

- *Inventory and Demand Forecasting*: It is not common practice to use this model to forecast sales and production with oscillatory behaviour, especially in a complicated least squares framework. Without specifically including oscillatory components, the majority of inventory prediction models depend on more straightforward linear regressions, exponential smoothing, or ARIMA.
- *Complex Numbers*: In demand forecasting and inventory management, it is rare to represent two interdependent variables (such as production and sale) as a single complicated variable.
- *Comparison with Logistic map:* The logistic map and other non-linear, bounded models may be directly compared with this combined least squares assigned to the

combination of polynomial and sinusoidal components-based methodology. This benchmarking is an innovative utilization case.

4. COMPARATIVE ANALYSIS

Discussing which strategy is best is based on the three models aspect and the numerical results of these three models. The comparative studies of three techniques are presented in Table 5.

The greatest method for forecasting production and sales is the Logistic Map Model as it can accurately depict seasonal patterns and natural variability. In contrast to the Least Squares Polynomial Sinusoidal Model, which generates irrational negative values (such as - 66.67 in production) and extreme spikes (such as 986.06 in sales), the Logistic Map Model keeps outputs within reasonable ranges while capturing dynamic patterns, like the August seasonal drop (186.75) and the October sales peak (832.78). More sensitivity to real- world behaviors, such as demand changes and seasonal impacts, is shown by the Logistic Map Model than by the ARIMA Model, which smooth's out oscillations and overlooks important seasonal variations.

Table 5: Comparison of Methods

ASPECT	LOGISTIC MAP	LSPSM	ARIMA
Model Suitability	Non-linear dynamics, Robust to randomness Realistic predictions.	Captures trends but over fits, unrealistic for production and sales.	Some trends, suitable for auto correlated data but lacks variability.
Realism of predictions	Positive and bounded values, gradual changes, variability.	Unrealistic negative production values and Overly high sales.	Conservative Predictions with Minimal variability.
Alignments with Actual data	Captures spikes and drops, closer to actual sales dynamics.	Overestimates or underestimates significantly for several months.	Underestimates variability smooth but Overly cautions.

formance metrics	Likely moderate MAPE due to variability And realistic trends.	High MAPE due to Extreme deviations And Unrealistic values.	Low MAPE but fails To represent true System dynamics.
riability and Dynamics	Captures variability Naturally, reflecting Real-world stochastic Behavior.	Artificial oscillations; Over fitting leads to Extremes.	Over-smooth's, failing to capture spikes or drops.
Visual Comparison	Shows variability and Spikes that align With real world Behavior.	Unrealistic peaks and thoughts, visually Inconsistent.	Smooth but oversimplified, missing Critical variations.

5. CONCLUSION

The logistic map model is the most dependable and effective tool for production and sales forecasting, as demonstrated by its moderate MAPE, which indicates that the forecasting model is expected to have a moderate level of error because it incorporates variability and realistic trends. It outperforms both the least squats polynomial- sinusoidal and ARIMA models by achieving superior accuracy, realism, and variability in predictions.

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Binomial Graphs

Md Shahidul Islam Khan

PDUAM, Amjonga, Assam, India

Email: rakhakshik786@gmail.com

Abstract.

Let n be a nonnegative integer and $\left\{\binom{n}{r} : r \in N \text{ and } 0 \leq r \leq n\right\}$ is the set of coefficients of the binomial expansion $(a+b)^n$. We have $\binom{n}{r} = \binom{n}{n-r}$. We define an undirected graph structure on the collection of coefficients of $(a+b)^n$ with the set of vertex $V = \left\{\binom{n}{r} : 0 \leq r \leq \frac{n+1}{2}\right\}$, when n is odd or $V = \left\{\binom{n}{r} : 0 \leq r \leq \frac{n}{2}\right\}$, when n is even and any two vertices x and y $(x < y)$ are connected by an edge if x divides y. We are denoted the graph by $B_G(n)$ and called Binomial Graph. In this present paper we investigate the relation between the binomial coefficients from graph theoretic point of view. We also study various graph theoretic properties of binomial graphs.

Keywords: Binomial coefficients, Binomial graph, Completeness.

1. INTRODUCTION

The Binomial Theorem is a powerful algebraic expressions of $(a+b)^n$, where a and b are any numbers or algebraic terms and n is a non-negative integer. The theorem narrate that

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r.$$

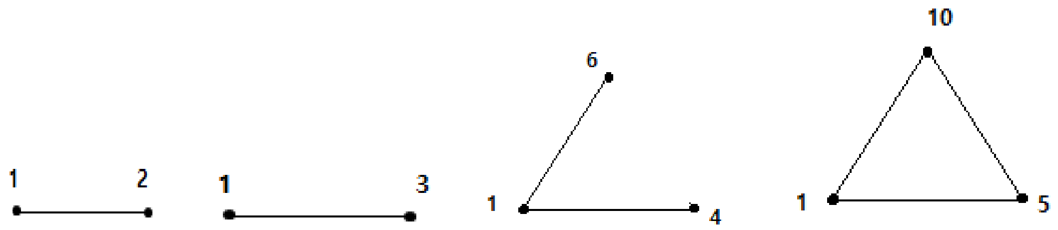
Here $\binom{n}{r}, 0 \leq r \leq n$ are called binomial coefficients. These coefficients are also obtained from Pascal's Triangle, which visually represents the coefficients for each power of the binomial expansion.

We apply binomial theorem in other branches of mathematics, namely, in algebra, probability, calculus and differential equations. It aids in simplifying complex exponential expressions, solving combinatorial problems, finding binomial distribution in probability. We also use binomial theorem for nonzero values of n to solve linear non-homogeneous differential equations with algebraic non-homogeneous terms.

In [1], Binomial graph *Bn* defined as "For each nonnegative integer *n*, *Bn* have the vertex set $V = \{v_i = 0, 1, 2, \ldots\ldots, 2^n - 1\}$ and edge set $E = \left\{(v_i, v_j) : \binom{\mathrm{i}+\mathrm{j}}{\mathrm{j}} \equiv 1(mod2)\right\}$". In this study we define binomial graph as: Let n be a nonnegative integer and $\left\{\binom{\mathrm{n}}{\mathrm{r}} : \mathrm{r} = 0,\ 1,\ 2, \ldots., \mathrm{n}\right\}$ is the collection of coefficients of the binomial expansion $(\mathrm{a}+\mathrm{b})^{\mathrm{n}}$. Then the binomial graph $B_G(n)$ is an undirected graph with the set of vertices $V = \left\{\binom{n}{r} : 0 \leq r \leq \frac{n+1}{2}\right\}$, when $\mathrm{n} = 1, 3, 5, \ldots.$ or $V = \left\{\binom{n}{r} : 0 \leq r \leq \frac{n}{2}\right\}$, when $\mathrm{n} = 2, 4, 6, \ldots.$ and set of edges $E = \{(x, y) : x < y,\ x \text{ divides y};\ x, y \in V\}$. We also study some important graph theoretic results of binomial graphs.

2. PRELIMINARIES

Definition 2.1. Let n be a nonnegative integer and $\left\{\binom{\mathrm{n}}{\mathrm{r}} : 0 \leq \mathrm{r} \leq \mathrm{n}\right\}$ is the set of coefficients of a binomial expansion $(\mathrm{a}+\mathrm{b})^{\mathrm{n}}$. We define the binomial graph $B_G(n)$ as an undirected graph with set of vertices $\mathrm{V} = \left\{\binom{\mathrm{n}}{\mathrm{r}} : 0 \leq \mathrm{r} \leq \frac{\mathrm{n}+1}{2}\right\}$, when $\mathrm{n} = 1, 3, 5, \ldots.$ or $\mathrm{V} = \left\{\binom{\mathrm{n}}{\mathrm{r}} : 0 \leq \mathrm{r} \leq \frac{\mathrm{n}}{2}\right\}$, when $\mathrm{n} = 2, 4, 6, \ldots.$ and set of edges $E = \{(x, y) : x < y,$ x divides y; $x, y \in V\}$.



(a) (b) (c) (d)

Fig. 2. (a) $B_G(2)$, (b) $B_G(3)$, (c) $B_G(4)$, (d) $B_G(5)$.

Definition 2.2. [3] A graph is called connected graph if every pair of vertices of the graph is connected by a path.

Definition 2.3. [3] Let G is a graph. We define diameter of G as

$$Diam(G) = \max_{u,v \in V}\{d(u, v)\}$$

where $d(u, v)$ denotes the shortest path between vertices v and u and V the set of all vertices of a graph. Thus the diameter of a graph G is the distance of the shortest, longest path connecting any two vertices of G. Diameter of a graph is a finite number if it is connected. Diameter of a disconnected graph is undefined.

Definition 2.4. [3] The largest distance between a vertex and any other vertex in a graph is known as its eccentricity. Formally, for a vertex *v*, its eccentricity *e(v)* is defined as:

$$e(v) = \max_{u \in V}\{d(u, v)\}$$

where $d(u, v)$ is denotes the shortest path between vertices v and u and V the set of all vertices of a graph. The eccentricity of a vertex is finite if the graph is connected. The eccentricity of a vertex is infinite (or undefined) if the graph is disconnected.

Definition 2.5. [3] The radius of a graph G is denoted by rad(G) and defined as rad(G)= $\min_{v \in V}\{e(v)\}$, where V is the set of vertices of graph G.

Definition 2.6 [3] A subgraph is said to be a spanning tree of a graph if it

1. contains every vertices of the original graph and
2. is a tree, which means it is connected and devoid of cycles.

To put it simply, a spanning tree avoids loops by joining all of the vertices with the fewest edges required to maintain the graph's connectivity.

A tree contains *n-1* edges will be a spanning tree for a graph with *n* vertices. A connected graph may has more than one spanning tree. Although each connected component may have its own spanning tree, the original graph cannot have a spanning tree if it is not connected. Spanning trees are important in network architecture, where you wish to connect all points with the least number of edges and no redundancy.

Definition 2.7 A series of natural numbers known as "catalan numbers" can be found in a variety of combinatorial problems, many of which involve recursively constructed objects. The Catalan number is defined as: $C_n = \frac{1}{n+1}\binom{2n}{n} \ \forall\, n \in N.$

They bear the name of Eugène Charles Catalan, a 19th-century Belgian mathematician who researched them.

3. MAIN RESULTS

Theorem 3.1. For all nonnegative integers *n*, $B_G(n)$ is connected graph.

Proof: For *n = 0, 1*; $B_G(n) \cong K_1$. For each positive integer $n \geq 2$, 1is adjacent to x in $B_G(n)$ for all $x \in V$. If x and y are any two nonadjacent vertices, then x - 1 - y is always a x - y path. Hence $B_G(n)$ is connected for all nonnegative integer n.

Theorem 3.2. $B_G(n)$ is tree for n = 0, 1, 2, 3, 4, 6.

Proof: For n = 0, 1; $B_G(n) \cong K_1$.

For n = 2, 3; $B_G(n) \cong K_2$. (Fig. 1)

For n = 4, 6; $B_G(n) \cong K_{1,2}$. (Fig. 1)

In each case $B_G(n)$ is tree.

Theorem 3.3. Diam($B_G(n)$) ≤ 2 for all nonnegative integers *n*.

Proof: For all n ≥ 4, 1 is adjacent to all vertices of $B_G(n)$. Thus for any two nonadjacent vertices x and y, x - 1 - y is always a path. But for n ≤ 3, the graphs are either K_1 or K_2 whose diameter is either 0 or 1. Hence the result.

Theorem 3.4. rad($B_G(n)$) ≤ 1 for all nonnegative integers n.

Proof: From Theorem **3.1** and Theorem **3.3,** it is clear that 1 ≤ e(v) ≤ 2, ∀ v ∈ V. For n ≥ 2, rad($B_G(n)$)=min{e(v) : v ∈ V } = e(1) = 1 and for n < 2, rad($B_G(n)$)=min{e(v) : v ∈ V } = e(1) = 0. Thus rad($B_G(n)$) ≤ 1 for all n.

Theorem 3.5. For all nonnegative integers n, $S_{[(n+1)/2]}$ is a spanning subgraph of $B_G(n)$.

Proof: From Theorem **3.3,** it is clear that 1 is adjacent to all v ∈ V and hence we have $K_{1,[(n-1)/2]}$ as a spanning subgraph of $B_G(n)$ for all n ≥ 2. Since $K_{1,[(n-1)/2]} \cong S_{[(n+1)/2]}$, therefore $S_{[(n+1)/2]}$ is a spanning subgraph of $B_G(n)$ for all nonnegative integers n.

Lemma 3.6. If p is an odd prime number, then $\binom{p}{r}$ is divisible by p for all $1 \le r \le (p+1)/2$.

Proof: Let p is an odd prime. Then for $1 \le r \le (p+1)/2$, $\binom{p}{r} = \frac{p!}{r!(p-r)!}$ is a multiple of p because p is not divisible by any integer less than p other than *1*.

Theorem 3.7. If p is prime, then deg(1)=deg(p) in $B_G(n)$.

Proof: If p=2, then it is obvious that deg(1)=deg(2)=1 in $B_G(n)$.

If p is an odd prime, then by Lemma, p divides $\binom{p}{r}$ for $1 \le r \le (p+1)/2$.

So, deg(p)= $(p+1)/2$= deg(1) since 1 divides $\binom{p}{r}$ for $1 \le r \le (p+1)/2$.

Theorem 3.8. $\mathrm{G_B}(\mathrm{n})$ is not completed for all n.

Proof: For $\mathrm{n} = 0, 1, 2, 3, 5$; $B_G(n)$ is complete graph.

For $\mathrm{n} = 4$, $\binom{4}{2} = \binom{4}{1} + 2 = \binom{4}{1} + C_2$

Similarly, for n $= 6, \binom{6}{3} = \binom{6}{2} + C_3$

for n $= 7, \binom{7}{3} = \binom{7}{2} + C_4$

for n $= 8, \binom{8}{4} = \binom{8}{3} + C_4$

for n $= 9, \binom{9}{4} = \binom{6}{3} + C_5$

for n $= 10, \binom{10}{5} = \binom{10}{4} + C_5$

and so on, where C_2, C_3, C_4, C_5, …. are the second, third, fourth, fifth, …. Catalan numbers respectively. So, from above we can write

$$\binom{n}{\frac{n-1}{2}} = \binom{n}{\frac{n-3}{2}} + C_{\frac{n+1}{2}}, \text{ if } n = 1, 3, 5, \ldots.$$

$$\binom{n}{\frac{n}{2}} = \binom{n}{\frac{n}{2}-1} + C_{\frac{n}{2}}, \text{ if } n = 2, 4, 6, \ldots.$$

Which implies $\binom{n}{\frac{n-1}{2}}$ is not divisible by $\binom{n}{\frac{n-3}{2}}$, if $n = 1, 3, 5, \ldots.$ and $\binom{n}{\frac{n}{2}}$ is not divisible by $\binom{n}{\frac{n}{2}-1}$, if $n = 2, 4, 6, \ldots.$. Hence we can conclude that $B_G(n)$ is not complete for all n.

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A Theoretical Study of the Fractal Von-Koch Curve Graph Using Eccentricities, Tree Properties and Bipartite Structures

Suriya Banu. R [1], Kamali. R[2]

Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Chennai.

Email: suriyabanu2020@gmail.com; kamali.sbs@vistas.ac.in

Abstract.

This research presents an innovative mathematical analysis of the Fractal Von-Koch Curve Graph. This article introduces the classes of graphs, such as eccentricities, radius, diameter, and degrees, which are calculated for the i^{th} iterations of the Curve. Mathematical expressions were formulated based on the findings. The obtained results indicate a correlation between the radius and the diameter of the i^{th} iterations of the Curve. In each iteration of the Curve, there exist two leaf vertices with a degree of one. Additionally, a new formula will be proposed for the remaining two-degree vertices in every iteration of the Curve. Properties of trees and bipartite graphs are utilized to substantiate the theorem. Furthermore, relevant examples are taken to prove the theorem. Ultimately, this study aims to bridge two significant research domains of Fractals and Graph Theory.

Keywords: Eccentricities, Radius, Diameter, Tree, Coloring, Bipartite.

1. INTRODUCTION

In the olden days, Euclidean Geometry played a vital role. Euclidean Geometry is used to analyze and find the dimension of a line, a circle, and a sphere. In 1975, the word Fractal was introduced by the famous Mathematician Benoit Mandelbrot [3,4,12]. Mandelbrot is known as the father of fractals. The properties of Fractals are characterized and traced back to the contributions of many mathematicians such as Cantor, Hausdorff, Koch, and Sierpinski [4,8,12].

Fractals occur in nature (e.g., bird feathers, tree branches, clouds, lightning, etc.). It is a branch of mathematics and belongs to the field of Fractal Geometry [8,9,12]. Fractal geometry is applied to find the F.D (fractal dimension). The fractal dimension of a river is approximately equal to 1.2, and the fractal dimension of a conical mountain lies between 2 and 3 [12]. The key properties of fractals are self-similarity, recursive iterations, and fractal dimension. These properties are mainly applied in the field of architectural designs, especially in temples and the wonders of the world. The applications of fractal geometry are extended to data analysis, 2D and 3D fractal interpolation, and aerospace engineering [3,9,12].

The major research area of Graph Theory, which started with the great Swiss mathematician Leonhard Euler, is considered the father of Graph Theory [1,2]. His contribution to the famous Konigsberg bridge problem in 1736 is recognized as one of the best applications of Graph Theory [10]. In recent years, the applications of Graph Theory have become increasingly significant across the various research domains [2,7]. Nowadays, many researchers who are exploring their ideas using Graph Theory parameters such as eccentricities, radius, clique, matching, energy, etc. This paper is a combination of two major research areas of Graph Theory and Fractals.

2. PRELIMINARIES

Definition 2.1: Graph

A Graph is a collection of non-empty sets of ordered pairs $G = (V, E)$, in which V *and* E represent a group of vertices and edges [1,2].

Definition 2.2: Degree of a Vertex in a Graph

The degree of a vertex in a Graph is defined as follows,

$$\deg(v) \quad = \quad \{e \in E \mid v \text{ is incident to } e\}$$

i.e., $\deg(v)$ Equals the number of edges incident to vertex v [2,5,10].

Definition 2.3: Proper Coloring of a Graph

The least number of colors required to color the graph so that no two adjacent vertices have the same color is called the proper coloring of a vertex in a graph [1,2,5,10].

Definition 2.4: Eccentricity of a Graph

The greatest possible shortest distance from a single vertex to all other vertices is known as the eccentricity of that vertex, and it is denoted as $\mathrm{E}(v)$ [2,6].

Definition 2.5: Radius and Diameter of a Graph

The radius is known as the minimum eccentricity in a graph, and the diameter is called the maximum eccentricity within a graph [1,2,5].

Definition 2.6: Bipartite Graph

A bipartite graph is known as one whose vertex set can be separated into two disjoint independent sets, and χ(G) = 2 [1,2,10].

Definition 2.7: Von-Koch Curve

This Curve is mathematically characterized as a fractal curve, and it is generated through a recursive iteration. The two key properties of the curve are as follows [3,8,11,12].

i. Self-similarity
ii. Infinite Perimeter

v_1 ($first\ iteraion$) Contains 5 vertices and 4 edges $\rightarrow$

v_2 ($second\ iteration$) has 17 vertices and 16 edges $\rightarrow$

3. MAIN RESULTS

Theorem 3.1

The Radius of the Fractal Von-Koch Curve Graph v_i is given by $Rad(v_i) = 2^{2i-1}$,
(where i = 1,2, 3,...,n,...)

Proof

This theorem is proved by the Induction Method.

Initially, let v_1 has 5 vertices and 4 edges.

Then find the minimum eccentricity of v_1, by using the definitions 2.4 and 2.5.

$$i.e., Rad(v_1) = 2$$

Similarly, the radius for v_2 is 8.

$$i.e., Rad\ (v_2) = 8$$

The calculated radii $Rad(v_i)$ for *i = 1,2, 3,* are tabulated in the following Table 1.

Table 1. The calculated radius values of a fractal von-koch curve graph

Von-Koch Curve Graph	Radius
v_1	$R(v_1) = 2$
v_2	$R(v_2) = 8$
v_3	$R(v_3) = 32$

By the induction hypothesis, assume that

$$Rad\ (v_n) = 2^{2n-1} \qquad (1)$$

The above equation (1) holds for iteration *i=n.*

At the iteration *n+1*, the Von-Koch Curve Graph expands by subdividing each edge into 4 new line segments. This increases the radius by a factor of 4.

$$Rad\ (\ v_{n+1}) = 4(R_n)$$

$$= 4(2^{2n-1}) \quad \text{by (1)}$$

$$Rad\ (\ v_{n+1}) = 2^{2(n+1)-1} \qquad (2)$$

$$\boxed{\boldsymbol{Rad(v_{n+1}) = 2^{2(n+1)-1}}}$$

Hence, the induction hypothesis is true for all iterations, and the theorem is proved.

Theorem 3.2

Prove that the Diameter of the Fractal Von-Koch Curve Graph v_i is $2(Rad\ (v_i\ ))$,

i.e., $Dia(v_i) = 2(Rad\ (v_i\ ))$, *(where i = 1,2, 3,…,n,…)*

Proof

Initially, the maximum eccentricity (the diameter) of v_1 is obtained by using definitions 2.4 and 2.5, and it is 4.

i.e., $Dia(v_1) = 4.$

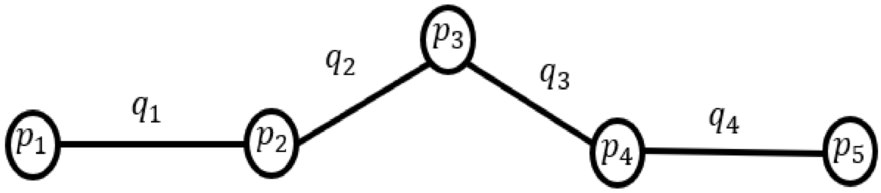

Fig 1. The diagrammatic representation of the von-koch curve graph (v_1) with 5 vertices and 4 edges.

Similarly, the diameter for v_2 is 16.

$$i.e., Dia(v_2) = 16.$$

The calculated diameters are given in Table 2.

Table 2. The calculated diameter values of a fractal von-koch curve graph

Von-Koch Curve Graph	Diameter
$\boldsymbol{v_1}$	$D(\boldsymbol{v_1}) = \mathbf{4}$
$\boldsymbol{v_2}$	$D(\boldsymbol{v_2}) = \mathbf{16}$
$\boldsymbol{v_3}$	$D(\boldsymbol{v_3}) = \mathbf{64}$

By the Induction hypothesis, assume that

$$Dia(v_n) = 2(Rad\ (v_n\ )) \tag{3}$$

The above (3) is true for iteration n.

$$Dia(v_n) = 2(Rad(V_{\ n}\ ))$$

$$= 2(2^{2n-1}) \text{ by } (1)$$

$$\boldsymbol{Dia(v_n\ ) = 2^{2n}} \tag{4}$$

At the iteration *n+1*, the Von-Koch Curve Graph expands by subdividing each edge into 4 new line segments. This increases the radius by a factor of 4.

$$Dia(v_{n+1}) = 4(Rad\ (v_n\ ))$$

$$Dia\ (v_{n+1}) = 2^{2(n+1)} \tag{5}$$

$$\boxed{\boldsymbol{Dia(\ v_{n+1}) = \ 2^{2(n+1)}}}$$

By the principle of mathematical induction hypothesis is true for all iterations n.

Hence, the theorem is proved.

Theorem 3.3

The cardinality of two-degree vertices of the Fractal Von-Koch Curve Graph occurred in v_i is given as follows,

$$|\deg\ (v_i\ )| = (2^{2i}) - 1,\ (where\ i = 1,2,\ 3,\dots,n,\dots)$$

Proof

Initially, for $i = 1,$ the degree of v_1 is obtained as

$$|\ \deg(v_1\ )| = (2^2) - 1$$
$$= 3$$

Similarly, for $i = 2,$ the degree of v_2 is obtained as

$$|\ \deg(v_2\ )| = 15.$$

The calculated two-degree vertices are tabulated in Table 3.

Table 3. The number of calculated two-degree vertices for a fractal von-koch curve graph

Von-Koch Curve Graph	Number of Two-Degree vertices
$\boldsymbol{v_1}$	3
$\boldsymbol{v_2}$	15
$\boldsymbol{v_3}$	63

By the Induction hypothesis, assume that

$$\deg(v_n\ ) = ((2^{2n}) - 1) \tag{6}$$

The above equation (6) holds good for iteration $i = n$.

Now, for the iteration $i = n+1$, the Von-Koch Curve Graph expands by subdividing each edge into 4 new line segments. This increases the radius by a factor of 4. Then

$$\left|\deg(v_{n+1})\right| = 4\left((2^{2n}) - 1\right)$$

$$= \left(2^{2(n+1)} - 1\right)$$

$$\left|\deg(v_{n+1})\right| = \left(2^{2(n+1)} - 1\right) \tag{7}$$

$$\boxed{\left|\mathbf{deg}(\boldsymbol{v}_{\boldsymbol{n+1}})\right| = (\mathbf{2}^{\mathbf{2(n+1)}} - 1)}$$

By the mathematical induction hypothesis, the theorem holds for all iterations *n.*

Corollary- 1:

The number of one-degree vertices in every iteration of the Fractal Von-Koch curve is always two.

Theorem 3.4

Prove that every fractal von-Koch curve is a Bipartite Graph.

Proof

Consider p_1, p_2, p_3, p_4, p_5 and q_1, q_2, q_3, q_4 be the five vertices and four edges of the Fractal Von-Koch Curve Graph v_1.

By using definition 2.6, the vertices are separated into two disjoint sets, A = { p_1, p_3, p_5 } and B = {p_2, p_4} respectively.

The diagrammatic representation is shown below.

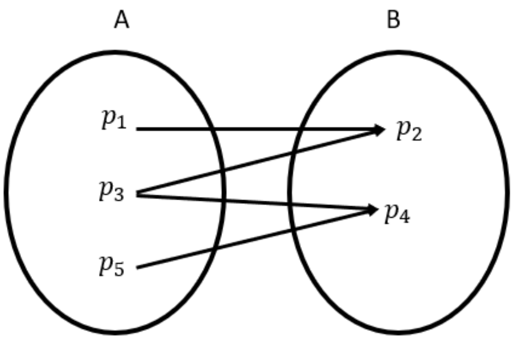

Fig 2. The diagrammatic representation of a bipartite graph for a fractal von-koch curve graph.

The following Fig. 3 shows that the separated vertices are properly colored.

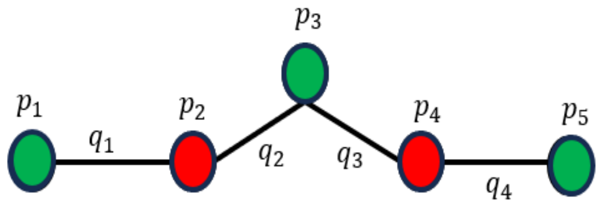

Fig 3. The two-colorable fractal von-koch curve graph

From Fig. 3, it is clear that every fractal von-Koch curve is two-colorable and hence it is a bipartite graph.

Hence, the theorem is proved.

4. CONCLUSION

In this paper, the fractal Von-Koch Curve iterations were taken with the essential properties of fractals and graphs. A Mathematical approach is done on the Von-Koch Curve to prove the theorem. The Empirical relationship between the radius and diameter is proved by the theorem. The investigation of Von-Koch Curve with other vital parameters of graphs will be included, and applications of this curve will be discussed in future research work.

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Leveraging & Harnessing by 4.0 Revolution for Fostering Sustainability Techniques beyond the Agricultural Value Chain

Prof. Anjali Pise
Department of CSE,TGPCET, Nagpur, Maharashtra, India.
Email: avm.pise@gmail.com

Abstract.

An agro-industry is pivotal in sustaining the global population. The global agricultural industry is currently at a critical juncture where sustainability has become imperative rather than optional. Sustainability practices signify a fundamental change in the way humanity engages with the planet's resources. Adopting Blockchain-based sustainability techniques has substantial, lucrative, communal, and ecosystem ramifications and provides quality of life. Future developments in this developing industry are also scrutinized. Blockchain technology, recognized for its administrative decentralization security and lucid ledger system, presents significant potential for addressing these issues. Blockchain empowers the stakeholders from farmers to consumers by providing immutable records that enhance trust, mitigate fraud, and support informed decision-making. This kind of integration enhances the efficiency of resource use while also encouraging ethical sourcing and agriculture that is resilient to climate change. Through the lens of successful case studies, the practical application and impact of blockchain (BC) are illustrated. This research paper inquires into the certainty of blockchain (BC) technology to transform sustainability practices within the agro-industry. The findings suggest that while blockchain technology holds considerable promise for fostering sustainability in the agro-industry, its successful integration requires addressing existing challenges and promoting interdisciplinary collaboration among stakeholders. This study examined the integration of blockchain into agro-industry practices to enhance sustainability, traceability, and accountability. This study highlights blockchain's role as a digital enabler in fostering a more sustainable and resilient agro-industry.

Keywords: Ag-tech,4IR, Green IT.

1. INTRODUCTION

Sustainability represents a crucial and increasingly emphasized objective in the context of agricultural development. In recent years, its importance has grown significantly due to the global emphasis on achieving the global goals (SDGs). Sustainable development is a multidimensional framework aimed at optimizing resource allocation to satisfy current socioeconomic demands without inducing irreversible degradation of ecological systems. It integrates three core pillars: economic viability, social, equity, and environmental stewardship, ensuring balanced progress. This Paradigm mandates the conservation of biodiversity and ecosystem services through the implementation of adaptive management practices and the mitigation of anthropological impacts. It advocates for the deployment of renewable energy technologies, the adoption of circular economy principles and enforcement of policies promoting equitable access to resources. The ultimate objective is to establishment resilient sociolect-ecological systems capable of sustaining human well- being across inter generational timelines.

On the quest of sustainable agriculture, attention must be directed towards interrelated concerns such as energy consumption, environmental preservation, and the responsible use of natural resources and the impacts of climate change. The intensification of agricultural production- particularly in irrigated zones and favourable rained areas- has at times been driven by misguided incentives and inappropriate policy frameworks. Such developments have led to considerable environmental degradation.

The expansion of cultivation into forested regions and steeper terrains has resulted in elevated levels of soil erosion. Furthermore, the intensification of livestock farming has contributed to the deterioration of both land and water quality. In the Indian context, soils are experience progressive degradation due to combination of factors, including erosion, depletion of organic carbon, nutrient imbalance and salinization. Additional problems such as water-logging and further erosion are also contributing to declining groundwater quality and land productivity.

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The global focus on sustainable agricultural practices is intensifying as awareness regarding the detrimental effects of conventional methods on the environment grows. Concerns about the deterioration of the environment containing sludge attrition, impregnate attrition, and diversity of deprivation coupled with the pressing need as in to assure to extended food invulnerability for a progressing inhabitants, necessitate a fundamental shift towards more sustainable approaches. Traditional industrial agriculture characterized by large-scale mono- cropping and heavy reliance on chemical pesticides and fertilizer has been identified as a system that depletes and degrades the very resources it depends on making it inherently unsustainable in response to these limitations various technological advancements are being explored to foster more sustainable food production. These include precision agriculture, which optimizes resources through data-driven insights, vertical farming that maximizes yield in a controlled environment with reduced land and water usage, and biotechnology, which develops crops with enhanced resilience and reduced need for chemical INPUT and smart irrigation systems. It conserves water using an optimized delivery method. The pursuit of sustainability in the agro- industry is not merely an environmental concern; it is also an economic and social imperative that shapes the future of food production and consumption.

1.2. Objectives and Scope

In border context, the agricultural sector directly influences a nation's trade dynamics due to the geographic and seasonal diversity in the production and consumption of commodities like crops, plants and livestock's. This research paper seeks to introduce a digital application that leverage's emerging technologies such as the Internet of Things(IoT), cloud computing and artificial intelligence to address everyday agricultural challenges. The proposed solution aims to bridge gaps in data accessibility, optimize input usage, and streamline farm management practices. Ultimately, the application is envisioned as a tool to empower farmers, increase resilience in the agriculture value chain, and contribute to the economic and sustainable development of the farming community [5].

Maharashtra stands out as one of the India's most prominent states in the agriculture sector. Agriculture plays a vital role in the state's economy, serving as a major source of income and employment. With its vast agro- climatic diversity, Maharashtra supports a wide range of crops, making it a significant player in both food and commercial crop production. The State economy is largely dependent on agriculture and its allied activities. A substantial portion of the population is directly or indirectly engaged in farming, animal husbandry and agri-based industries. This rural workforce forms the backbone of the state socio- economic structure, emphasizing the importance of policies and technologies that support sustainable agricultural practices. Maharashtra contributes significantly to the national agricultural output accounting for approximately 13.2% of India' s Agricultural gross domestic product(GDP).This highlights the state's crucial; role in ensuring food security and supporting the broader agricultural economy of the country. As a result, investments in agricultural innovation and infrastructure in Maharashtra have national level implications.

The objective of implementing blockchain in Indian smart farming is to revolutionize agriculture by increasing lucidity, outlining and trust throughout the agricultural value chain. By integrating BC with technologies like IoT, Deep learning and remote sensing, farmers can gain real time insights into soil health, weather patterns and crop performance. BC ensures that all collected data is securely recorded and cannot be tampered with which helps in better decision making, monitoring and accountability at every stage of production. Additionally BC aims to empower farmers economically by enabling direct transactions with buyers, eliminating the need for intermediaries, This is not only ensures fair pricing but also provides a tamper proof record of the origin, quality and movement of produce which is crucial for both export and domestic markets. The technology can also streamline access to crop insurance, subsidies and credit by providing verified records of farming activity thus promoting financial inclusion and reducing fraud in rural agriculture systems.

The main goal of this research paper is to thoroughly explore the application of blockchain technology as a means of enhancing sustainability practices within the agro-industry. This paper aims to analyze the multifaceted benefits that blockchain can offer, including improved traceability enhanced transparency increased efficiency, and the reliable verification of sustainable farming practices. Furthermore, it will delve into the significant challenges and limitations associated with the perpetration of 4IR technology in this context. Throughout an examination of real-world case studies, this research seeks to illustrate the practical application and impact of adopting sustainability agriculture. Finally, the paper will discuss the broader economic social, and environmental impacts of adopting blockchain-based sustainability practices and explore the future trends and potential advancement in this rapidly evolving field. The scope of this research encompasses the integration of blockchain technology across the entire agricultural value chain from primary production at the farm level to the final consumption of agricultural products with a specific focus on its role in fostering sustainability.

1.3. Challenges in Achieving Sustainability

Agriculture in Maharashtra faces several challenges while striving for sustainability. One major issue is water scarcity due to irregular and insufficient rainfall, especially in drought- prone regions like Marathwada and Vidarbha. Dependence on monsoons, over-extraction of groundwater, and poor irrigation infrastructure have led to declining water tables and crop failures, making it difficult for farmers to adopt sustainable practices. Another significant challenge is soil degradation caused by excessive use of chemical fertilizers and pesticides. Over the years, this has reduced soil fertility and biodiversity, making lands less productive and more dependent on external inputs, inadequate crop rotation and monoculture farming further exacerbate this issue, threatening long term sustainability and reducing farmers resilience to climate change.

Additionally, Socio-economic factors such as small landholdings, high input costs, and limited access to markets and technology hinder the adoption of sustainable practices, Many Farmers lack awareness or resources to invest in organic farming, precision agriculture, eco-friendly techniques. Without policy support, training and financial incentives, achieving sustainability in agriculture remains a major challenge in Maharashtra.

Maharashtra is one of India's most developed states, contributing significantly to the national economy. However, its rapid industrialization and urban growth have created serious environmental and social sustainability challenges. The green revolution supported sustainability by significantly increasing food production without expanding farmland, By introducing high-yielding crop varieties, chemical fertilizers, and improved irrigation methods, it allowed countries like India and Mexico to feed growing populations and avoid famine. This helped preserve natural ecosystems by reducing the pressure to convert forests and grasslands into agricultural land, thus supporting environmental sustainability to some extent. It also improved food security and economic development in many rural areas, contributing to social and economic sustainability.

However, while the Green Revolution brought short term gains, it also introduced environmental challenges such as soil degradation, water pollution, and loss of biodiversity due to the overuse of chemical inputs and monoculture farming. These negative impacts have prompted a shift towards more sustainable practices that build on the productivity gains of the green revolution while promoting long term ecological balance. Thus its support for sustainability lies in laying the groundwork for future innovations in sustainable agriculture. Maharashtra the green revolution supported sustainability by improving agricultural productivity especially in irrigated regions like western Maharashtra. An introduction of high yielding varieties of wheat, rice and later sugarcane along with chemical fertilizers and modern irrigation, helped farmers increase their crop output Without expanding agricultural land. This reduced pressure on forests and natural habitats, contributing to environmental sustainability. The improved yields also enhanced food security and rural incomes, supporting economic and social development in the region.

However, the benefits were not evenly distributed across Maharashtra. While western Maharashtra thrived due to better irrigation infrastructure, regions like vidarbha and Marathwada, which lacked sufficient water resource, were left behind. Over time the overuse of chemical inputs and excessive water extraction in more developed regions led to soil degradation, declining groundwater levels and other environmental issues. This has highlighted the need for more balanced and eco- friendly farming practices to ensure long term sustainability in Maharashtra's agriculture.

Despite the development of various sustainable agricultural practices significant challenges remain in their widespread adoption and effective implementation. A primary obstacle is the lack of transparency and traceability within complex agricultural supply chains. The opacity makes it difficult to track products from their origin to the consumer to leading issues such as food fraud and the inability to verify sustainability claims. Verifying whether farming practices are indeed sustainable and preventing fraudulent activities such as mislabeling of organic products poses another considerable challenge. The absence of a reliable tracking mechanism can undermine consumer trust in sustainably labeled products. Further inefficiencies within a supply chain often result in significant food waste and mismanagement of valuable resources. Traditional systems may lack in real data and coordination necessary to optimize logistics and minimize spoilage. Smallholder farmers who constitute a significant portion of global agricultural production often face exclusion from fair markets and limited access to financing and technology hindering their ability to adopt sustainable practices. The existing agricultural system frequently lacks the integrated mechanism required to effectively monitor, verify, and reward sustainable behavior across all stakeholders.

2. INTRODUCTION TO BLOCKCHAIN TECHNOLOGY

The blockchain technology has come up as a conceivable informative solution for addressing many of the challenges associated with achieving sustainability in the agro-industry. The primary blockchain is a segregated, distributed ledger that records deal across numerous computers, assure lucidity, safety, and immutability of data. This distributed nature means that no single entity controls the ledger making it resistant to tampering and a single point of failure. Each block in the chain essence is a set of verified deals, linked chronologically and crypt-graphically to the antecedent block creating an audible and permanent record. Beyond its function as a secure data repository, blockchain also enables the use of smart contracts. These are contract-based agreements written into code on the blockchain that automatically plan turn of phrase of the ledger simultaneously predetermined circumstances are met.

Digital contracts have the potential to automate various processes within the agricultural supply chain, such as payment verification of conditions and compliance with regulations. The fundamental characteristics of blockchain technology align remarkably well with the need for enhanced trust and transparency in tackling the complex challenges of sustainability in the agro-industry sector.

Beyond traceability, blockchain can significantly promote sustainable farming practices. By allowing farmers to record data on their adoption of environmentally friendly methods—such as soil health management, reduced chemical inputs, and water conservation techniques—blockchain creates a verifiable "sustainability passport" for their produce. This verifiable data can unlock access to niche markets that value sustainable production, potentially leading to better prices and even opportunities for carbon credits. Furthermore, the integration of blockchain with other emerging technologies like IoT (Internet of Things) sensors can enable precise monitoring of resources, leading to optimized water and fertilizer use and reduced environmental impact. Smart contracts, a core feature of blockchain, can also streamline financial transactions, ensuring transparent and timely payments to farmers upon delivery and quality verification, thereby improving their financial inclusion and reducing the reliance on intermediaries. The Maharashtra government's new 'Krishi Samruddhi' scheme, with a Rs 25,000 crore outlay over five years, aims to encourage sustainable farming practices through financial assistance for technology adoption, soil health improvement, and water-use efficiency, creating fertile ground for blockchain implementation. While the potential of blockchain in Maharashtra's sustainable agriculture is immense, widespread adoption necessitates addressing challenges such as digital literacy among farmers, initial infrastructure costs, and the need for robust interoperability between different platforms. However, Maharashtra's proactive policy measures and focus on agri-tech position it as a front runner in leveraging blockchain for a more sustainable and prosperous agricultural future.

Maharashtra, a leading agricultural state in India, is increasingly recognizing the critical role of technology in fostering a more sustainable and resilient farming sector. Facing challenges like climate variability, water scarcity, and market access issues for its large farming population, the state is actively exploring innovative solutions. Among these, blockchain technology stands out as a powerful tool to drive sustainability across the agricultural value chain. In Maharashtra's agriculture, this translates into unprecedented opportunities for enhanced traceability and transparency. Imagine a farmer in Nashik growing grapes for export: blockchain can record every step from seed procurement and pesticide application to harvesting, storage, and transport. This "farm-to-fork" visibility not only assures consumers of product authenticity and sustainable practices (e.g., precise water usage, organic inputs) but also empowers farmers by providing verifiable data for higher prices in premium markets. The Maharashtra government, through its ambitious Maha Agri-AI Policy 2025–29, is already prioritizing QR code-based blockchain traceability for key export crops like grapes, bananas, and pomegranates, aiming to bolster food safety and export compliance. This policy, with a substantial budget outlay, also focuses on establishing an Agricultural Data Exchange (ADeX) for secure, consent-based data sharing, further strengthening the data backbone for blockchain applications. When considering the state-wise use of sustainability in Indian agriculture and its readiness for blockchain integration, it's crucial to acknowledge that "sustainability use" is a broad concept encompassing various practices. A precise, real-time quantifiable pie chart for this specific metric is not readily available, as blockchain adoption in sustainable agriculture is still evolving and often project-specific rather than a uniformly implemented state-level phenomenon. However, based on the leading states in organic farming, climate-smart agriculture initiatives, and overall agricultural technology adoption, we can present a conceptual distribution illustrating which states are likely at the forefront of embracing sustainable practices that can then be transparently verified and managed by blockchain technology. For instance, states with significant organic cultivation areas, like Madhya Pradesh and Maharashtra, are inherently strong contenders. Madhya Pradesh has historically led in terms of certified organic farming area, while Maharashtra has emerged as the second-largest in organic farm production and is making rapid strides in climate-smart agriculture and digital initiatives. States like Rajasthan and Karnataka are also making notable progress in sustainable practices and technology integration. Therefore, a conceptual pie chart representing the leading states in embracing sustainable agriculture practices (and thus, more amenable to blockchain integration for transparency and verification) might look like: Madhya Pradesh (27%), Maharashtra (22%), Rajasthan (13%), Gujarat (15%), Karnataka (7%), Northeastern States (collectively) (10%), and Other States (6%). This conceptual breakdown highlights the varying degrees of focus and progress states are making towards a more sustainable and technologically advanced agricultural landscape, paving the way for wider blockchain adoption.

3. SUSTAINABILITY PRACTICES IN THE AGRICULTURAL INDUSTRY

A review of existing academic research and industry reports reveals a broad spectrum of sustainability practices currently employed or under investigation within the agriculture industry. These practices range from the adoption of advanced technologies to the implementation of holistic management strategies. Precision agriculture, for instance, leverages equipment such as navigator, detectors, drones and data analysis increase farming practices leading to reduced resource waste and increased yields. Verticals farming offers another innovative approach to growing crops in the vertically stacked layers in controlled environments thereby minimizing land and water use and reducing the need for pesticides. Biotechnology plays a role in developing crops that are more resistant to pests, diseases, and environmental stressors, potentially leading to higher yields and reduced chemical usage. Precision irrigates devices utilize sensors and climate information to improve watering catalog preserve water and degrade nature prices. Beyond technological solutions, sustainable practices also encompass management planning for livestock producers, focusing on the animal section, nutrition, reproduction, herd health, and grazing management. The overreaching goals of these diverse approaches are to integrate biological and ecological processes into food production minimize harmful non- renewable inputs and improve natural capital. Industry stakeholders, including major agricultural companies, are also increasingly reporting on their sustainability efforts, outlining targets for reducing their environmental footprint and committing to sustainable business practices. The USDA emphasizes the foundational role of sustainability agricultural productivity growth in building more sustainable farming practices in the peanut industry focusing on resource optimization and biodiversity preservation while these existing practices offer significant contributions to specific aspects of sustainability their effectiveness in achieving comprehensive sustainability goals can be limited by a lack of integrated data and transparency across the entire supply chain.

3.1. Application of Digital ledger Technology in Agriculture

Digital Ledger is being explored and implemented across various facets of the agriculture industry, offering novel solutions to enhance operational security, transparency, and efficiency. Its application spans supply chain management where it provides end-to-end traceability and transparency, allowing stakeholders to track products from farm to consumer. Academic papers and industry reports highlight blockchain's potential in addressing critical challenges such as ensuring food safety through rapid recall capabilities, combating food frauds by verifying product authenticity, and improving market access for smallholder farmers by enabling direct transactions and fairer pricing. The technology's decentralized and immutable nature fosters trust among supply chain participants reducing the inadequacy for go-between and streamlining processes. Crypto contract, a primary feature of BC are also being explored for their ability to automate payments enforce agreements and ensure compliance, With various standards. The growing body of literature underscores the significant potential of blockchain to metamorphose the agriculture sector by intensifying limpidness and trust across the value chain.

4. IMPROVED TRACEABILITY AND TRANSPARENCY IN SUPPLY CHAIN

Blockchain technology offers a powerful mechanism for achieving unprecedented levels of as in tracking and translucent throughout the agro-industry supply chain. Through noting each and every sales operation of agricultural products. An immutable and distributed ledger, blockchain creates a transparent and trustworthy record accessible to all authorized stakeholders. This end-to-end tracking capability allows consumers to trace the origin of their food ensuring, they know exactly where it comes from and how it was produced. The integration of technologies like QR codes and RFID labels associated with blockchain further enhanced product tracking, allowing consumers to effortlessly way in descriptive data with regards to the product's expedition by simply scanning the packaging. This level of transparency significantly enhances consumer trust, as they can verify the origin quality and even the sustainability practices associated with the product they purchase. Moreover, improved traceability through blockchain can significantly aid in rapid recalls of contaminated or defective products by enabling quick identification of their exact source. This targeted approach reduced food waste and minimized the economic burden on retailers and producers by limiting the scope of recall to only the affected products. The enhanced transparency offered by Blockchain not only benefits consumers but also incentivizes producers to adopt better more sustainable practices due to the increased accountability and potential to command premium prices for transparently sourced goods.

4 .1. Verification of sustainable farming practices

Blockchain technology provides a robust platform for recording and verifying data related to various sustainable farming practices while offering immutable proof of compliance with environmental and ethical standards. Farmers can record data on their farming methods including details about pesticides and fertilizer usage, water conservation techniques, crop rotation practices, and other environmentally friendly actions directly onto the blockchain. The tamper-proof nature of this distributed ledger assure the capability, reliability of this information assembling it a very valuable gadgets for verifying sustainability claims. Furthermore, smart contracts, a key attributes of blockchain, can play a very pivotal role in automating incentives or subsidies for farmers who adhere to sustainable protocols. For example, smart contracts can be programmed to automatically release payment of issued certification to farmers upon the verification of specific sustainable practices, such as achieving a certain level of water reduction or adopting organic farming methods. This transparent and automated verification process can significantly build trust among consumers who are increasingly demanding ethically and sustainably sourced products. By providing consumers with verifiable proof of sustainable practices, blockchain can effectively reduce green washing and promote genuine environmental and ethical stewardship within the agro-industry.

4.2. Enhanced food safety and quality assurance

The immutable record-keeping capabilities of blockchain technology offer significant advantages for enhancing foodstuff invulnerability and excellence assurance from beginning to end of the agricultural supply chain. Blockchain enables the tracking of critical circumstances such as climate and stuffiness during the storage and transportation of agricultural products. Sensors integrated with the blockchain can automatically record these data points, ensuring that products are maintained within safe parameters and that any deviations are immediately flagged. This continuous monitoring helps to prevent spoilage and ensure that the food products meet required safety standards throughout their journey from farm to table. In the event of a food-borne illness outbreak, blockchain's propensity is to provide a detailed unalterable chronicle of products allowing for quick and efficient tracing of contaminated products back to their origin. This rapid tracing capability minimizes health risks to consumers and reduces the economic burden on retailers and producers by enabling targeted recalls of only the affected batches. Furthermore, blockchain can be used to record a wide range of quality assurance data at each stage of the supply chain, including information on pesticide usage, handling procedures, processing methods, and product specifications. This comprehensive record-keeping enhances accountability and allows for easy verification of product quality claims, ultimately building greater trust in the food system.

4.3. Efficient Resources Management

Integrating blockchain technology with the Internet of Things devices and sensors presents significant opportunities for optimizing the management of critical agricultural resources such as water, energy, and fertilizers. IoT sensors deployed in the farmhand can accumulate actual interval details regarding diverse criteria involving soil dampness levels, weather patterns, and plant health. This information can be securely marked with regard to blockchain, handling over lucid and perpetual annals of environmental conditions and resources of consumption. Farmers can then leverage these insights to make more informed decisions regarding irrigation schedules, fertilization needs, and pest management strategies, thereby minimizing waste and maximizing the efficiency of resource utilization. The transparent and audible complex nature of blockchain ensures that information on resource consumption is accurate and reliable, encouraging more sustainable practices and reducing the overall environmental footprint of agricultural operations.

4.4. Empowering Smallholder Farmers and Promoting Fair Trade

Blockchain technology holds significant potential for empowering smallholder farmers and fostering fairer trade practices within the agricultural division. Through distributing see-through an invariable saga of sale, blockchain can enable small farmers to connect directly with buyers and bypass traditional intermediaries who often take significant cuts of the profits. This direct market access allows farmers to receive fairer prices for their produce, improving their livelihoods and promoting economic sustainability. Smart contracts further enhance this by automating payments and ensuring that farmers are compensated fairly and promptly upon meeting agreed-upon conditions. The increased transparency provided by blockchain also enables consumers to have greater confidence in the ethical sourcing of their food, allowing them to support farmers who adhere to sustainable and fair labor practices. By fostering more direct and transparent relationships between producers and consumers, blockchain can contribute to a more equitable and sustainable agricultural ecosystem.

4.5. Streamlining Financial Transactions and Insurance

Blockchain technology offers significant potential for streamlining financial transactions and improving access to insurance within the agro-industry. Its dispersed cosmos can speed up unassailable and efficient adequate transactions between various stakeholders, reducing the vital for traditional go-between mitigating affiliated charges and deferrals. By leveraging their digital identities and transaction history recorded on the blockchain, farmers can build creditworthiness and access to traditional banking services, blockchain-based platforms can provide opportunities for micro-loans and other essential financial services. Furthermore, blockchain-based smart contracts have the potential to revolutionize crop insurance by automating the claims process. These contracts can be programmed to automatically trigger a payout to farmers based on variable data such as weather conditions or crop yields recorded on the blockchain streamlining the process and ensuring timely compensation for losses due to unforeseen events. This improved access to financial services and insurance can significantly enhance the economic resilience of farmers and promote greater sustainability within the agricultural sector.

5. CHALLENGES AND LIMITATIONS OF IMPLEMENTING BLOCKCHAIN FOR SUSTAINABILITY

5.1. Technical Challenges

Despite its numerous potential benefits, the enactment of blockchain technology for long-lasting in the agro-industry faces several technical challenges. One significant hurdle is the lack of data standardization and interoperability across different blockchain platforms and existing agricultural systems. Ensuring that diverse data from various sources can be seamlessly exchanged and understood by different blockchain networks and legacy systems remains a complex task. Scalability presents another considerable challenge, as approximately blockchain in a netting may stumble to operate the enormous volume of data and sales inherent in the agriculture sector without experiencing a slowdown and increased costs. Security and privacy concerns related to the data stored on the blockchain also need careful consideration. Overcoming these technical complexities and ensuring seamless integration along with the current process are crucial to the flourishing and widespread adoption of blockchain in the agro-industry.

5.2. Economic Challenges

The enforcement of Blockchain technology in the agro-industry for sustainability purposes also faces several economic challenges. The high initial investment and implementation costs associated with setting up blockchain infrastructure including hardware, software network development, and the training of personnel, can be significant barriers, particularly for smallholders, farmers, and smaller agricultural businesses. There is often uncertainty about the return on investment and long-term economic benefits that blockchain adoption will yield for farmers making them hesitant to invest in new technologies yield for farmers without clear evidence of profitability. Additionally, the potential costs associated with ongoing data collection, storage, and management on the blockchain can add to the overall expenses for participants of the supply chain. Setting the seal on the quality and its standardization of data input onto the blockchain can also involve additional costs. The economic viability and affordability of blockchain solution for all stakeholders, especially those with limited resources need to be carefully considered to ensure equitable access and broad adoption across the economic viability and affordability of blockchain solutions for all stakeholders, especially those with limited resources, need to be carefully considered to ensure equitable access and broad adoption across the agro-industry.

5.3. Social and Organizational Challenges

Several social and organizational challenges can obstruct the extensive inoperative of blockchain technology for sustainability in the agro-industry a significant barrier is the lack of awareness, understanding, and digital literacy among many farmers and other stakeholders regarding blockchain and its potential benefits. Many individuals involved in agriculture particularly in rural areas and developing countries, may lack the necessary digital skills and access to information to effectively utilize blockchain-based solutions. Resistance to challenge change and the adoption of new technologies within the agricultural community which often relies on traditional farming practices and established supply chain relationships can also present considerable challenges, building trust and fostering collaboration among diverse stakeholders across the agricultural supply chain who may have varying level of technological understanding and potentially conflicting interests is crucial for the successful implementation of blockchain initiatives. Effective collaboration and communication are essential for establishing and maintaining a functional and beneficial blockchain network. Addressing these social and organizational challenges through education, training and the promotion of collaborative partnerships is vital for the successful merging of blockchain technology in the agricultural sector.

5.4. Standardized and legal challenges

The operations of blockchain technology for sustainability in the agro- industry also faces several regulatory and authorized obstacles. The significant hurdles are the precise scarcity of clear international standards and industry best practices specifically governing the adoption and use of blockchain in agriculture. This regulatory uncertainty can hinder widespread adoption is create complexities for businesses operating across different jurisdictions, furthermore, there is often an inadequacy of precision with respect to the legal interpretation and affordability of smart contracts which are a key factor of multiple types of blockchain based solutions. The legal framework surrounding the governance of blockchain transactions and the resolution of potential disputes is still evolving in many regions. The complacency with the diverse and often complex agricultural regulations that very vary significantly across significantly across different regions and countries can also pose a challenge for the implementation of blockchain-based solutions that may need to operate across borders. Establishing clear regulatory frameworks, standards and legal clarity for blockchain technology in agriculture is necessary to provide a stable and predictable environment that encourages innovation and wider adoption within the industry. The authors emphasize that sustainability is an interconnected concept economic, environmental, and social aspects are interdependent and must be balanced for true sustainability to be achieved. For instance, a farm that fails to maintain economic viability cannot be expected to uphold environmental standards in the long term. [12].

6. CASE STUDIES FOR SUCCESSFUL IMPLEMENTATION

The successful implementation blockchain technology for sustainable agriculture practices is evidences by a growing number of real world case studies across the globe.

1. IBM food Trust stands out as a prominent example providing a blockchain founded podium is enhanced transparency and food security by tracking the roots and journey of different types of foodstuff products for major players like Walmart and nestle.

Agri Digital in Australia has successfully implemented blockchain streamline gain transaction and supply chain management improving efficiency and trust among stakeholders. Provenance has focused on tracing fish and other food products, enabling client to ingress of details in reference to their inception and feasibility practices.

2. Yara International's "Thanks My Farmer" App utilizes blockchain to allows consumers to trace the origin and quality of their coffee while also enabling direct financial support to the farmers when grew it. A Walmart 's implementation of blockchain based port traceability system serves as a compelling demonstration of the technology ability to quickly trace the source of food products, significantly enhancing food safety.
3. Agi Ledger is a UK -based social enterprise project that supports farmers in developing countries by enabling them to document the quality.[11]
4. Ag Unity has developed a blockchain based smartphone app designed to empower small farmers with tools for planning, trading and tracking their transactions. Farm Fresh produce improved traceability for consumers by developing a user-friendly mobile app that connect them directly to farmers through QR codes. The Green Farm Network utilize s blockchain to support fair trade principals by allowing farmers to securely record their sales and the prices they received .
5. Eco Cycle developed a blockchain solution to monitor sustainable practices by collecting real-time data on resource use, enabling farmers to showcase their commitment to environmental stewardship.
6. Crop Tech Partnered with logistics companies tom use blockchain for instant tracking of shipment status, reducing spoilage and improving the quality of agricultural products. Other notable examples include Beef-ledger for cattle tracking.

7. TE-FOOD for livestock registration and supply chain monitoring,
   - Farm2Kitchen for farm -to-fork traceability in India.
   - Demeter for promoting sustainable practices through transparency.
   - Agri chain for peer-to-peer agricultural transactions.
   - Grain-chain for optimizing production processes.
   - Etheric for decentralized crop insurance.
8. Bayer Crop Science's Trace Harvest Network for tracking the life cycle of agricultural products from seed. These diverse case studies illustrate the practical applications of block chain technology in addressing specific sustainability challenges across various agricultural sectors, demonstrating its capability to improve inadequacy, symphony and trust within the agro-industry.

7. DIAGRAMMATIC REPRESENTATION OF BLOCKCHAIN IN SUSTAINABLE AGRICULTURE

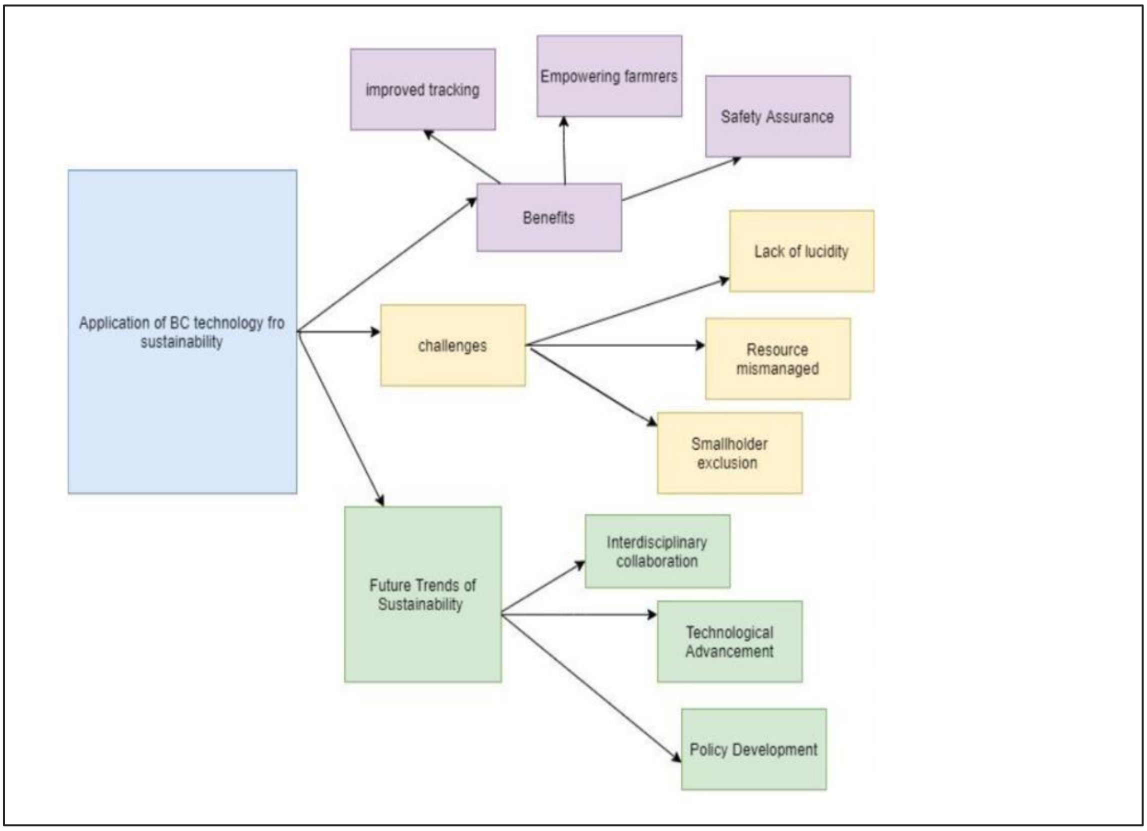

Fig. 1. Blockchain in Sustainable Agriculture

8. ECONOMIC, SOCIAL AND ENVIRONMENTAL IMPACTS

8.1. Economic Impacts

The embrace of blockchain-based devices used for agro-industry sustainability has the potential to generate significant positive economic impacts. Increased efficiency and automation of processes, such as supply-chain tracking, payments, and contract enforcement through smart contracts, can lead to reduced operational costs for all stakeholders involved. By providing direct access to markets and ensuring fairer pricing for their products, blockchain can significantly improve the income of farmers, particularly smallholder farmers who often face exploitative practices in traceability offered by block-chain can also create new market opportunities for sustainably produced goods, as consumers are increasingly willing to pay a premium for products with verified sustainable attributes.

8.2. Social Impacts

An acceptance of blockchain technology for sustainability in the agricultural industry can also lead to significant positive social impacts. Enhanced consumer trust is a key outcome, resulting from the increased lucidity and tracking that blockchain provides regarding the inception, quality, and sustainability attributes of food products. Blockchain can also play a crucial role in promoting fair trade practices and empowering smallholder farmers by providing them with direct market access, fairer pricing mechanisms, and reduced reliance on exploitative intermediaries. The enhanced traceability and quality assurance enabled by blockchain can lead to improved food security and contribute to better public health outcomes by facilitating rapid recalls and preventing the spread of food-borne illnesses. Moreover, blockchain plays a very vital role in building trust and accountability within the food system by contributing an invariable and transparent record of all operation and product information.

8.3. Environmental Impacts

The agro-industry holds significant promise for positive environmental impacts. Better supply chain management and enhanced traceability facilitated by blockchain can contribute to a reduction in food waste by enabling more efficient logistics and quicker responses to potential spoilage issues. Furthermore, the integration of blockchain with IoT devices and sensors allows for the optimization of resource use including water, energy, and fertilizers, leading to a lower overall environmental footprint of agricultural practices. Blockchain can also facilitate the tracking of carbon emissions associated with agricultural production and distribution, potentially enabling the development and trading of carbon credits, thereby incentive practices that reduce green grouse gas emissions by contribute a lucid and verifiable ledger of farming practices, blockchain can support and promote environmentally friendly methods, contributing to the preservation of biodiversity and the long term health of ecosystems.

9. FUTURE TRENDS AND POTENTIAL ADVANCEMENTS

Blockchain technology for sustainability in the agricultural sectors is a swiftly progressing field, with several emerging trends and potential advancements on the horizon. One significant trend is the increasing unification of blockchain with other pioneering techniques advancement for instance the Internet of Things(IoT), artificial intelligence (AI), and big data analytic. It convergence will enable more sophisticated data tracking, analysis, and automated decision-making for sustainable agriculture. The potential for decentralized autonomous organizations(DAOs) to manage aspects of agricultural supply chains and sustainability initiatives is also being explored. Greater standardization and interoperability among different blockchain solutions are expected to emerge, facilitating seamless data exchange and broader adoption. The role of governments and industry collaborations will be crucial in fostering innovation, setting standards, and creating supportive regulatory frameworks to encourage the adoption of blockchain for sustainable agriculture is likely to be characterized by increased integration with other advanced technologies, a move towards more standardized and interoperable solutions, and greater support from regulatory bodies and industry stakeholders.

10. CONCLUSION

This research has explored the significant latent of blockchain technology to enhance sustainable practices within the agro-industry. The analysis indicates that blockchain offers numerous benefits including improved traceability and transparency in supply chains, reliable verification of sustainable farming practices, enhanced food safety and quality assurance, more efficient resource management, empowerment of smallholder farmers, and streamlined financial transactions. These advantages collectively contribute to the financial, communal, and ecological dimensions of sustainability in agriculture. Despite these challenges, the growing number of successful case studies demonstrate the practical viability and positive impacts of blockchain technology in various agricultural contexts. These examples highlight the diverse applications and tangible benefits achieved in areas such as traceability, food safety, and supply chain efficiency.

The future of blockchain in sustainable agriculture appears promising with emerging trends pointing towards greater integration with other advanced technologies, increased standardization, and more supportive regulatory environments.

Interdisciplinary collaboration among researchers, technology developers, farmers, commerce, contributors, associates, and legislators will be pivotal in harnessing the full latent of blockchain to formulate a more prolong and resilient future for the agro-industry. Further research is needed to explore specific use cases across different agricultural sectors, address the challenges of scalability and interoperability, and investigate the long-term economic and environmental impacts of blockchain adoption.

In conclusion, blockchain technology offers a transformation moment to advance prolong in the agro-industry, contributing to a more secure, equitable, and environmentally responsible food system for the future.

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