Unit-III

Knowledge Representation & Reasoning: Propositional logic, Theory of first order logic, Inference in First order logic, Forward & Backward chaining, Resolution, Probabilistic reasoning, Utility theory, Hidden Markov Models (HMM), Bayesian Networks.

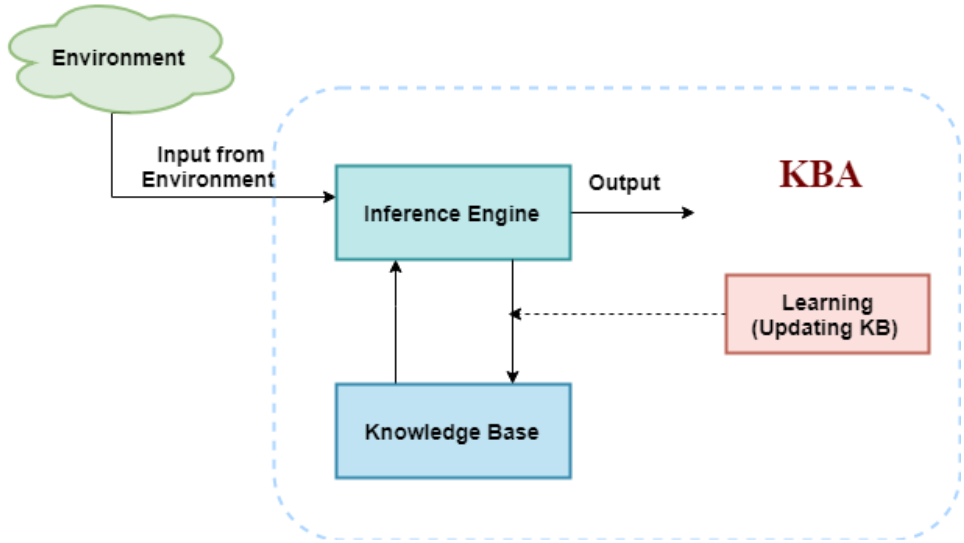
Knowledge-Based Agent in Artificial intelligence

- An intelligent agent needs **knowledge** about the real world for taking decisions and **reasoning** to act efficiently.
- Knowledge-based agents are those agents who have the capability of **maintaining an internal state of knowledge, reason over that knowledge, update their knowledge after observations and take actions. These agents can represent the world with some formal representation and act intelligently**.
- Knowledge-based agents are composed of two main parts:
  - **Knowledge-base and**
  - **Inference system**.

A knowledge-based agent must able to do the following:

- An agent should be able to represent states, actions, etc.
- An agent Should be able to incorporate new percepts
- An agent can update the internal representation of the world
- An agent can deduce the internal representation of the world
- An agent can deduce appropriate actions.

The architecture of knowledge-based agent:



The above diagram is representing a generalized architecture for a knowledge-based agent. The knowledge-based agent (KBA) takes input from the environment by perceiving the environment. The input is taken by the inference engine of the agent and which also communicate with KB to decide as per the knowledge store in KB. The learning element of KBA regularly updates the KB by learning new knowledge.

Knowledge base: Knowledge-base is a central component of a knowledge-based agent, it is also known as KB. It is a collection of sentences (here 'sentence' is a technical term and it is not identical to sentence in English). These sentences are expressed in a language which is called a knowledge representation language. The Knowledge-base of KBA stores fact about the world.

What is knowledge representation?

Humans are best at understanding, reasoning, and interpreting knowledge. Human knows things, which is knowledge and as per their knowledge they perform various actions in the real world. **But how machines do all these things comes under knowledge representation and reasoning**. Hence we can describe Knowledge representation as following:

- Knowledge representation and reasoning (KR, KRR) is the part of Artificial intelligence which concerned with AI agents thinking and how thinking contributes to intelligent behavior of agents.
- It is responsible for representing information about the real world so that a computer can understand and can utilize this knowledge to solve the complex real world problems such as diagnosis a medical condition or communicating with humans in natural language.
- It is also a way which describes how we can represent knowledge in artificial intelligence. Knowledge representation is not just storing data into some database, but it also enables an intelligent machine to learn from that knowledge and experiences so that it can behave intelligently like a human.

What to Represent:

Following are the kind of knowledge which needs to be represented in AI systems:

- **Object:** All the facts about objects in our world domain. E.g., Guitars contains strings, trumpets are brass instruments.
- **Events:** Events are the actions which occur in our world.
- **Performance:** It describe behavior which involves knowledge about how to do things.
- **Meta-knowledge:** It is knowledge about what we know.
- **Facts:** Facts are the truths about the real world and what we represent.
- **Knowledge-Base:** The central component of the knowledge-based agents is the knowledge base. It is represented as KB. The Knowledgebase is a group of the Sentences (Here, sentences are used as a technical term and not identical with the English language).

Knowledge: Knowledge is awareness or familiarity gained by experiences of facts, data, and situations. Following are the types of knowledge in artificial intelligence:

Types of knowledge

Following are the various types of knowledge:

1. Declarative Knowledge:

- Declarative knowledge is to know about something.
- It includes concepts, facts, and objects.
- It is also called descriptive knowledge and expressed in declarativesentences.
- It is simpler than procedural language.

2. Procedural Knowledge

- It is also known as imperative knowledge.
- Procedural knowledge is a type of knowledge which is responsible for knowing how to do something.
- It can be directly applied to any task.
- It includes rules, strategies, procedures, agendas, etc.
- Procedural knowledge depends on the task on which it can be applied.

3. Meta-knowledge:

- Knowledge about the other types of knowledge is called Meta-knowledge.

4. Heuristic knowledge:

- Heuristic knowledge is representing knowledge of some experts in a filed or subject.
- Heuristic knowledge is rules of thumb based on previous experiences, awareness of approaches, and which are good to work but not guaranteed.

5. Structural knowledge:

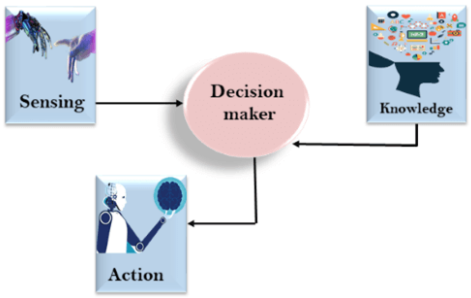
- Structural knowledge is basic knowledge to problem-solving.
- It describes relationships between various concepts such as kind of, part of, and grouping of something.
- It describes the relationship that exists between concepts or objects.

The relation between knowledge and intelligence:

Knowledge of real-worlds plays a vital role in intelligence and same for creating artificial intelligence. Knowledge plays an important role in demonstrating intelligent behavior in AI agents. An agent is only able to accurately act on some input when he has some knowledge or experience about that input.

Let's suppose if you met some person who is speaking in a language which you don't know, then how you will able to act on that. The same thing applies to the intelligent behavior of the agents.

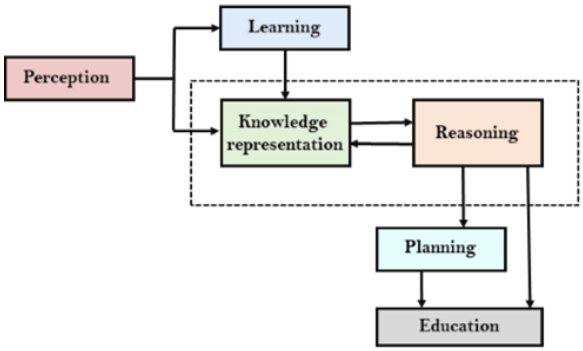
As we can see in below diagram, there is one decision maker which act by sensing the environment and using knowledge. But if the knowledge part will not present then, it cannot display intelligent behavior.



AI knowledge cycle:

An Artificial intelligence system has the following components for displaying intelligent behavior:

- Perception
- Learning
- Knowledge Representation and Reasoning
- Planning
- Execution



The above diagram is showing how an AI system can interact with the real world and what components help it to show intelligence. AI system has Perception component by which it retrieves information from its environment. It can be visual, audio or another form of sensory input. The learning component is responsible for learning from data captured by Perception comportment. In the complete cycle, the main components are knowledge representation and Reasoning. These two components are involved in showing the intelligence in machine-like humans. These two components are independent with each other but also coupled together. The planning and execution depend on analysis of Knowledge representation and reasoning.

Approaches to knowledge representation:

There are mainly four approaches to knowledge representation, which are givenbelow:

1. Simple relational knowledge:

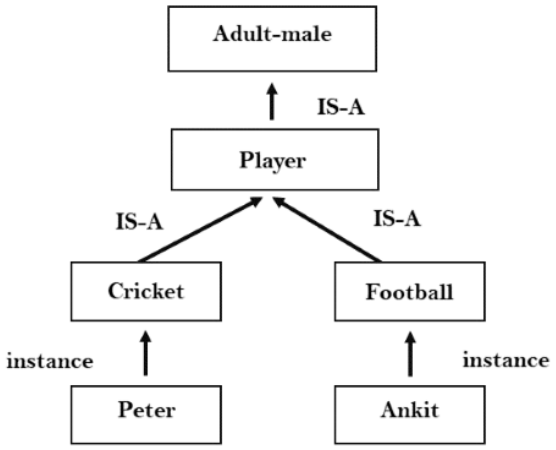
- It is the simplest way of storing facts which uses the relational method, and each fact about a set of the object is set out systematically in columns.
- This approach of knowledge representation is famous in database systems where the relationship between different entities is represented.
- This approach has little opportunity for inference.

Example: The following is the simple relational knowledge representation.

Player	Weight	Age
Player1	65	23
Player2	58	18
Player3	75	24

2. Inheritable knowledge:

- In the inheritable knowledge approach, all data must be stored into a hierarchy of classes.
- All classes should be arranged in a generalized form or a hierarchal manner.
- In this approach, we apply inheritance property.
- Elements inherit values from other members of a class.
- This approach contains inheritable knowledge which shows a relation between instance and class, and it is called instance relation.
- Every individual frame can represent the collection of attributes and its value.
- In this approach, objects and values are represented in Boxed nodes.
- We use Arrows which point from objects to their values.
- **Example:**



3. Inferential knowledge:

- Inferential knowledge approach represents knowledge in the form of formal logics.
- This approach can be used to derive more facts.
- It guaranteed correctness.
- **Example:** Let's suppose there are two statements:
    a. Marcus is a man
    b. All men are mortal
    Then it can represent as;

    man(Marcus)
    ∀x = man (x) ----------> mortal (x)s

4. Procedural knowledge:

- Procedural knowledge approach uses small programs and codes which describes how to do specific things, and how to proceed.
- In this approach, one important rule is used which is **If-Then rule**.

- In this knowledge, we can use various coding languages such as **LISP language** and **Prolog language**.
- We can easily represent heuristic or domain-specific knowledge using this approach.
- But it is not necessary that we can represent all cases in this approach.

Requirements for knowledge Representation system:

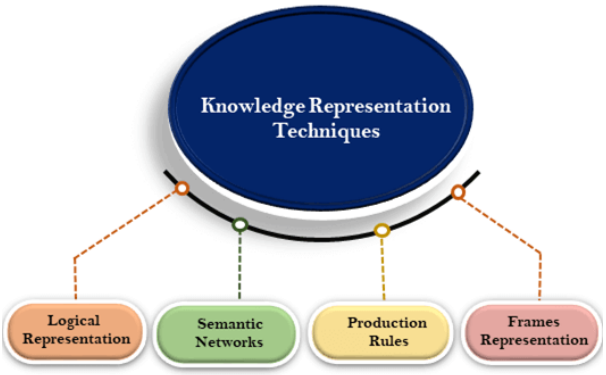
A good knowledge representation system must possess the following properties.

1. **1. Representational Accuracy:**
   KR system should have the ability to represent all kind of required knowledge.
2. **2. Inferential Adequacy:**
   KR system should have ability to manipulate the representational structures to produce new knowledge corresponding to existing structure.
3. **3. Inferential Efficiency:**
   The ability to direct the inferential knowledge mechanism into the most productive directions by storing appropriate guides.
4. **4. Acquisitional efficiency-** The ability to acquire the new knowledge easily using automatic methods.

Techniques of knowledge representation

There are mainly four ways of knowledge representation which are given as follows:

1. Logical Representation
2. Semantic Network Representation
3. Frame Representation
4. Production Rules



1. Logical Representation

Logical representation is a language with some concrete rules which deals with propositions and has no ambiguity in representation. Logical representation means drawing a conclusion based on various conditions. This representation lays down some important communication

rules. It consists of precisely defined syntax and semantics which supports the sound inference. Each sentence can be translated into logics using syntax and semantics.

Syntax:

- Syntaxes are the rules which decide how we can construct legal sentences in the logic.
- It determines which symbol we can use in knowledge representation.
- How to write those symbols.
-

Semantics:

- Semantics are the rules by which we can interpret the sentence in the logic.
- Semantic also involves assigning a meaning to each sentence.

Logical representation can be categorised into mainly two logics:

a. Propositional Logics
b. Predicate logics

Advantages of logical representation:

1. Logical representation enables us to do logical reasoning.
2. Logical representation is the basis for the programming languages.

Disadvantages of logical Representation:

1. Logical representations have some restrictions and are challenging to work with.
2. Logical representation technique may not be very natural, and inference may not be so efficient.

2. Semantic Network Representation

Semantic networks are alternative of predicate logic for knowledge representation. In Semantic networks, we can represent our knowledge in the form of graphical networks. This network consists of nodes representing objects and arcs which describe the relationship between those objects. Semantic networks can categorize the object in different forms and can also link those objects. Semantic networks are easy to understand and can be easily extended.

This representation consist of mainly two types of relations:

a. IS-A relation (Inheritance)
b. Kind-of-relation

Example: Following are some statements which we need to represent in the form of nodes and arcs.

Statements:

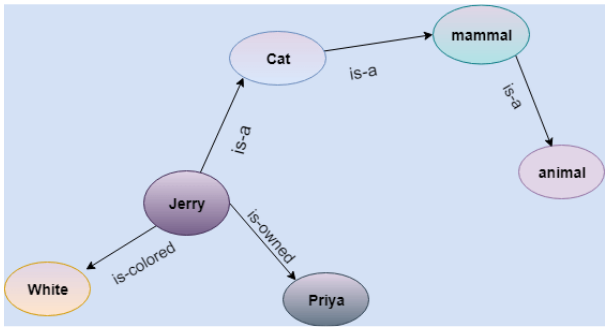
a. Jerry is a cat.

b. Jerry is a mammal

c. Jerry is owned by Priya.

d. Jerry is brown colored.

e. All Mammals are animal.



In the above diagram, we have represented the different type of knowledge in the form of nodes and arcs. Each object is connected with another object by some relation.

Drawbacks in Semantic representation:

1. Semantic networks take more computational time at runtime as we need to traverse the complete network tree to answer some questions. It might be possible in the worst case scenario that after traversing the entire tree, we find that the solution does not exist in this network.
2. Semantic networks try to model human-like memory (Which has 1015 neurons and links) to store the information, but in practice, it is not possible to build such a vast semantic network.
3. These types of representations are inadequate as they do not have any equivalent quantifier, e.g., for all, for some, none, etc.
4. Semantic networks do not have any standard definition for the link names.
5. These networks are not intelligent and depend on the creator of the system.

Advantages of Semantic network:

1. Semantic networks are a natural representation of knowledge.
2. Semantic networks convey meaning in a transparent manner.
3. These networks are simple and easily understandable.

3. Frame Representation

A frame is a record like structure which consists of a collection of attributes and its values to describe an entity in the world. Frames are the AI data structure which divides knowledge into substructures by representing stereotypes situations. It consists of a collection of slots and slot values. These slots may be of any type and sizes. Slots have names and values which are called facets.

Facets: The various aspects of a slot is known as **Facets**. Facets are features of frames which enable us to put constraints on the frames. Example: IF-NEEDED facts are called when data of any particular slot is needed. A frame may consist of any number of slots, and a slot may include any number of facets and facets may have any number of values. A frame is also known as **slot-filter knowledge representation** in artificial intelligence.

Frames are derived from semantic networks and later evolved into our modern-day classes and objects. A single frame is not much useful. Frames system consist of a collection of frames which are connected. In the frame, knowledge about an object or event can be stored together in the knowledge base. The frame is a type of technology which is widely used in various applications including Natural language processing and machine visions.

Example: 1

Let's take an example of a frame for a book

Slots	Filters
Title	Artificial Intelligence
Genre	Computer Science
Author	Peter Norvig
Edition	Third Edition
Year	1996
Page	1152

Example 2:

Let's suppose we are taking an entity, Peter. Peter is an engineer as a profession, and his age is 25, he lives in city London, and the country is England. So following is the frame representation for this:

Slots	Filter
Name	Peter
Profession	Doctor
Age	25
Marital status	Single
Weight	78

Advantages of frame representation:

1. The frame knowledge representation makes the programming easier by grouping the related data.
2. The frame representation is comparably flexible and used by many applications in AI.
3. It is very easy to add slots for new attribute and relations.
4. It is easy to include default data and to search for missing values.
5. Frame representation is easy to understand and visualize.

Disadvantages of frame representation:

1. In frame system inference mechanism is not be easily processed.
2. Inference mechanism cannot be smoothly proceeded by frame representation.
3. Frame representation has a much generalized approach.

4. Production Rules

Production rules system consist of (**condition, action**) pairs which mean, "If condition then action". It has mainly three parts:

- The set of production rules
- Working Memory
- The recognize-act-cycle

In production rules agent checks for the condition and if the condition exists then production rule fires and corresponding action is carried out. The condition part of the rule determines which rule may be applied to a problem. And the action part carries out the associated problem-solving steps. This complete process is called a recognize-act cycle.

The working memory contains the description of the current state of problems-solving and rule can write knowledge to the working memory. This knowledge match and may fire other rules.

If there is a new situation (state) generates, then multiple production rules will be fired together, this is called conflict set. In this situation, the agent needs to select a rule from these sets, and it is called a conflict resolution.

Example:

- **IF (at bus stop AND bus arrives) THEN action (get into the bus)**
- **IF (on the bus AND paid AND empty seat) THEN action (sit down).**
- **IF (on bus AND unpaid) THEN action (pay charges).**
- **IF (bus arrives at destination) THEN action (get down from the bus).**

Advantages of Production rule:

1. The production rules are expressed in natural language.
2. The production rules are highly modular, so we can easily remove, add or modify an individual rule.

Disadvantages of Production rule:

1. Production rule system does not exhibit any learning capabilities, as it does not store the result of the problem for the future uses.
2. During the execution of the program, many rules may be active hence rule-based production systems are inefficient.

Propositional logic in Artificial intelligence

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example:

1. a) It is Sunday.
2. b) The Sun rises from West (False proposition)
3. c) 3+3= 7(False proposition)
4. d) 5 is a prime number.

Following are some basic facts about propositional logic:

- Propositional logic is also called Boolean logic as it works on 0 and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function, and **logical connectives**.

- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.
- Connectives can be said as a logical operator which connects two sentences.
- A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**.
- A proposition formula which has both true and false values is called
- Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions.

Syntax of propositional logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

a. **Atomic Propositions**

b. **Compound propositions**

- **Atomic Proposition:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

1. a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.
2. b) "The Sun is cold" is also a proposition as it is a **false** fact.

- **Compound proposition:** Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

1. a) "It is raining today, and street is wet."
2. b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1. **Negation:** A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.
2. **Conjunction:** A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction.
   Example: Rohan is intelligent and hardworking. It can be written as,

P= Rohan is intelligent,
Q= Rohan is hardworking. → **P∧ Q**.

3. **Disjunction:** A sentence which has ∨ connective, such as **P ∨ Q**. is called disjunction, where P and Q are the propositions.
   Example: "Ritika is a doctor or Engineer",
   Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P ∨ Q**.
4. **Implication:** A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as
   If it is raining, then the street is wet.
   Let P= It is raining, and Q= Street is wet, so it is represented as P → Q
5. **Biconditional:** A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**
   P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Following is the summarized table for Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Truth Table:

In propositional logic, we need to know the truth values of propositions in all possible scenarios. We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called **Truth table**. Following are the truth table for all logical connectives:

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

For disjunction:

P	Q	P ∨ Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

For Biconditional:

P	Q	P⇔ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Truth table with three propositions:

We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of 8n Tuples as we have taken three proposition symbols.

P	Q	R	¬R	P∨ Q	P∨Q→¬R
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

Precedence of connectives:

Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators. This order should be followed while evaluating a propositional problem. Following is the list of the precedence order for operators:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Logical equivalence:

Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

Let's take two propositions A and B, so for logical equivalence, we can write it as A⇔B. In below truth table we can see that column for ¬A∨ B and A→B, are identical hence A is Equivalent to B

A	B	¬A	¬A∨ B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Properties of Operators:

- **Commutativity:**
  - P∧ Q= Q ∧ P, or
  - P ∨ Q = Q ∨ P.
- **Associativity:**
  - (P ∧ Q) ∧ R= P ∧ (Q ∧ R),
  - (P ∨ Q) ∨ R= P ∨ (Q ∨ R)
- **Identity element:**
  - P ∧ True = P,
  - P ∨ True= True.
- **Distributive:**
  - P∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R).
  - P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R).
- **DE Morgan's Law:**
  - ¬ (P ∧ Q) = (¬P) ∨ (¬Q)
  - ¬ (P ∨ Q) = (¬ P) ∧ (¬Q).
- **Double-negation elimination:**
  - ¬ (¬P) = P.

Limitations of Propositional logic:

- We cannot represent relations like ALL, some, or none with propositional logic. Example:
  a. **All the girls are intelligent.**
  b. **Some apples are sweet.**

  Propositional logic has limited expressive power.

  In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

Rules of Inference in Artificial intelligence

Inference:

In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, **so generating the conclusions from evidence and facts is termed as Inference**.

Inference rules:

Inference rules are the templates for generating valid arguments. Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.

In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:

- **Implication:** It is one of the logical connectives which can be represented as P → Q. It is a Boolean expression.
- **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as Q → P.
- **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as ¬ Q → ¬ P.
- **Inverse:** The negation of implication is called inverse. It can be represented as ¬ P → ¬ Q.

From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	P → Q	Q→ P	¬ Q → ¬ P	¬ P → ¬ Q.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Hence from the above truth table, we can prove that P → Q is equivalent to ¬ Q → ¬ P, and Q→ P is equivalent to ¬ P → ¬ Q.

Types of Inference rules:

1. Modus Ponens:

The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $\frac{P \rightarrow Q, \quad P}{\therefore Q}$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if P→ Q is true and P is true then Q will be true.

Proof by Truth table:

P	Q	$P \rightarrow Q$
0	**0**	**0**
0	**1**	**1**
1	**0**	**0**
1	**1**	**1**

2. Modus Tollens:

The Modus Tollens rule state that if P→ Q is true and **¬ Q is true, then ¬ P** will also true. It can be represented as:

Notation for Modus Tollens: $\frac{P \rightarrow Q, \ \sim Q}{\sim P}$

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I do not go to the bed."==> ~Q
Statement-3: Which infers that "**I am not sleepy**" => ~P

Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$
0	**0**	**1**	**1**	**1**
0	**1**	**1**	**0**	**1**
1	**0**	**0**	**1**	**0**
1	**1**	**0**	**0**	**1**

3. Hypothetical Syllogism:

The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:

Example:

Statement-1: If you have my home key then you can unlock my home. **P→Q**
Statement-2: If you can unlock my home then you can take my money. **Q→R**
Conclusion: If you have my home key then you can take my money. **P→R**

Proof by truth table:

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

4. Disjunctive Syllogism:

The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\frac{P \vee Q, \ \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>P∨Q
Statement-2: Today is not Sunday. ==> ¬P
Conclusion: Today is Monday. ==> Q

Proof by truth-table:

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

5. Addition:

The Addition rule is one the common inference rule, and it states that If P is true, then P∨Q will be true.

Notation of Addition: $\frac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P
Statement-2: I have Chocolate ice-cream.
Conclusion: I have vanilla or chocolate ice-cream. ==> (P∨Q)

Proof by Truth-Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

6. Simplification:

The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \wedge Q}{Q}$ Or $\frac{P \wedge Q}{P}$

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

7. Resolution:

The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. **It can be represented as**

Notation of Resolution $\frac{P \vee Q, \ \neg P \wedge R}{Q \vee R}$

Proof by Truth-Table:

P	¬ P	Q	R	$P \vee Q$	¬ P∧R	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1

First-Order Logic in Artificial intelligence

In the topic of Propositional logic, we have seen that how to represent statements using propositional logic. But unfortunately, in propositional logic, we can only represent the facts, which are either true or false. PL is not sufficient to represent the complex sentences or natural language statements. The propositional logic has very limited expressive power. Consider the following sentence, which we cannot represent using PL logic.

- **"Some humans are intelligent", or**
- **"Sachin likes cricket."**

To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

First-Order logic:

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as **Predicate logic or First-order predicate logic**. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.
- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
    - **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
    - **Relations: It can be unary relation such as:** red, round, is adjacent, **or n-any relation such as:** the sister of, brother of, has color, comes between
    - **Function:** Father of, best friend, third inning of, end of, ...
- As a natural language, first-order logic also has two main parts:
    a. **Syntax**
    b. **Semantics**

Syntax of First-Order logic:

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first-order logic are symbols. We write statements in short-hand notation in FOL.

Basic Elements of First-order logic:

Following are the basic elements of FOL syntax:

Constant	1, 2, A, John, Mumbai, cat,....

Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

Atomic sentences:

- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as **Predicate (term1, term2, ……, term n)**.

Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).
Chinky is a cat: => cat (Chinky).

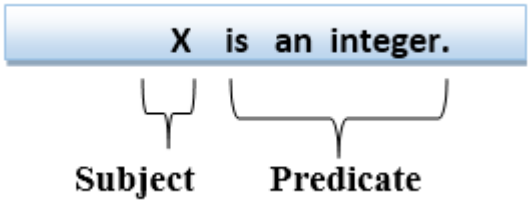
Complex Sentences:

- Complex sentences are made by combining atomic sentences using connectives.

First-order logic statements can be divided into two parts:

- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.

Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



Quantifiers in First-order logic:

- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:

a. **Universal Quantifier, (for all, everyone, everything)**

b. **Existential quantifier, (for some, at least one).**

Universal Quantifier:

Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.

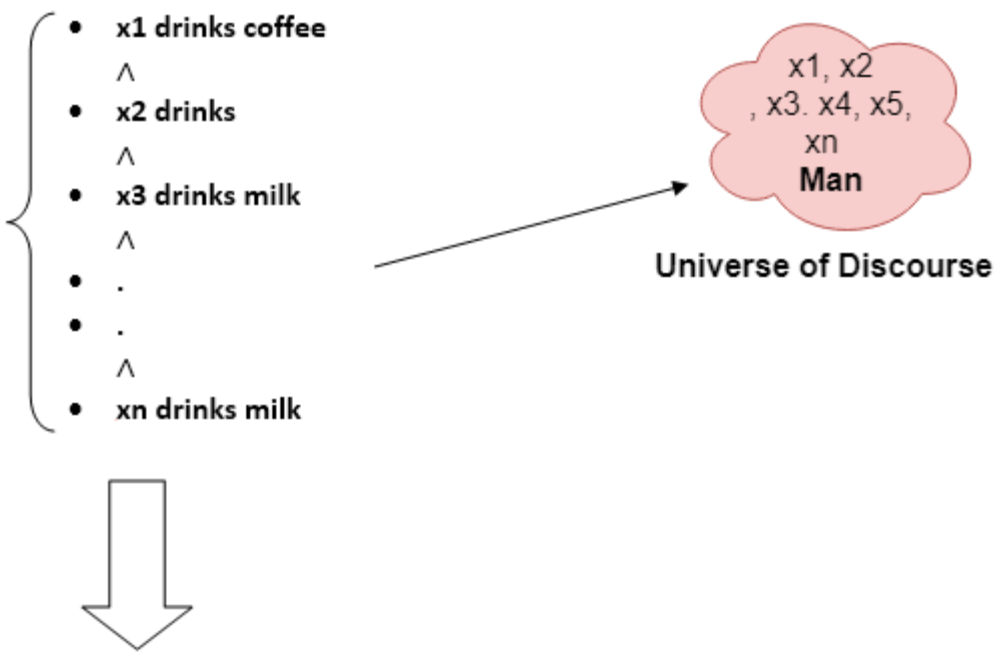
If x is a variable, then ∀x is read as:

- **For all x**
- **For each x**
- **For every x.**

Example:

All man drink coffee.

Let a variable x which refers to a cat so all x can be represented in UOD as below:



So in shorthand notation, we can write it as :

∀x man(x) → drink (x, coffee).

It will be read as: There are all x where x is a man who drink coffee.

Existential Quantifier:

Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.

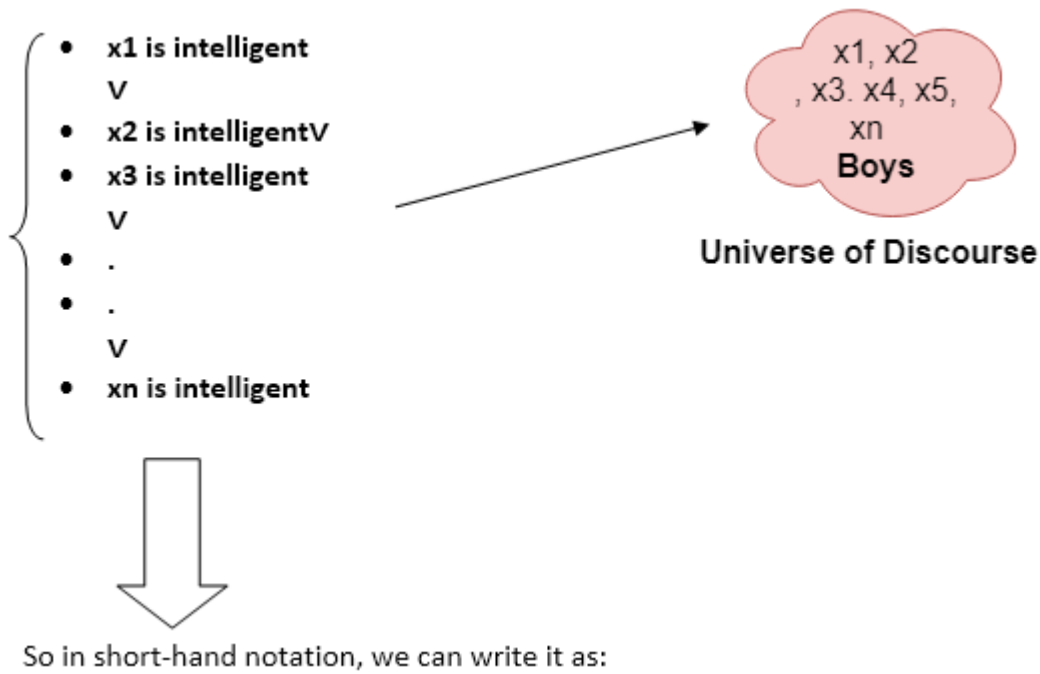
It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.

If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:

- **There exists a 'x.'**
- **For some 'x.'**
- **For at least one 'x.'**

Example:

Some boys are intelligent.



∃x: boys(x) ∧ intelligent(x)

It will be read as: There are some x where x is a boy who is intelligent.

Points to remember:

- The main connective for universal quantifier ∀ is implication →.
- The main connective for existential quantifier ∃ is and ∧.

Properties of Quantifiers:

- In universal quantifier, ∀x∀y is similar to ∀y∀x.
- In Existential quantifier, ∃x∃y is similar to ∃y∃x.
- ∃x∀y is not similar to ∀y∃x.

Some Examples of FOL using quantifier:

1. All birds fly.
In this question the predicate is "**fly(bird)**."
And since there are all birds who fly so it will be represented as follows.

∀x bird(x) →fly(x).

2. Every man respects his parent.
In this question, the predicate is "**respect(x, y)," where x=man, and y= parent**.
Since there is every man so will use ∀, and it will be represented as follows:

∀x man(x) → respects (x, parent).

3. Some boys play cricket.
In this question, the predicate is "**play(x, y)**," where x= boys, and y= game. Since there are some boys so we will use **∃, and it will be represented as**:

∃x boys(x) → play(x, cricket).

4. Not all students like both Mathematics and Science.
In this question, the predicate is "**like(x, y)," where x= student, and y= subject**.
Since there are not all students, so we will use **∀ with negation, so** following representation for this:

¬∀ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].

5. Only one student failed in Mathematics.
In this question, the predicate is "**failed(x, y)," where x= student, and y= subject**.
Since there is only one student who failed in Mathematics, so we will use following representation for this:

∃(x) [student(x) → failed (x, Mathematics) ∧∀ (y) [¬(x==y) ∧ student(y) → ¬failed (x, Mathematics)].

Free and Bound Variables:

The quantifiers interact with variables which appear in a suitable way. There are two types of variables in First-order logic which are given below:

Free Variable: A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

Example: ∀x ∃(y)[P (x, y, z)], where z is a free variable.

Bound Variable: A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

Example: ∀x [A (x) B(y)], here x and y are the bound variables.

Knowledge Engineering in First-order logic

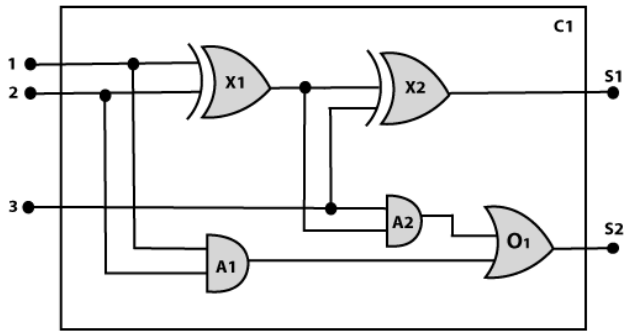
What is knowledge-engineering?

The process of constructing a knowledge-base in first-order logic is called as knowledge-engineering. In **knowledge-engineering**, someone who investigates a particular domain, learns important concept of that domain, and generates a formal representation of the objects, is known as **knowledge engineer**.

In this topic, we will understand the Knowledge engineering process in an electronic circuit domain, which is already familiar. This approach is mainly suitable for creating **special-purpose knowledge base**.

The knowledge-engineering process:

Following are some main steps of the knowledge-engineering process. Using these steps, we will develop a knowledge base which will allow us to reason about digital circuit (**One-bit full adder**) which is given below



1. Identify the task:

The first step of the process is to identify the task, and for the digital circuit, there are various reasoning tasks.

At the first level or highest level, we will examine the functionality of the circuit:

- **Does the circuit add properly?**
- **What will be the output of gate A2, if all the inputs are high?**

At the second level, we will examine the circuit structure details such as:

- **Which gate is connected to the first input terminal?**
- **Does the circuit have feedback loops?**

2. Assemble the relevant knowledge:

In the second step, we will assemble the relevant knowledge which is required for digital circuits. So for digital circuits, we have the following required knowledge:

- Logic circuits are made up of wires and gates.

- Signal flows through wires to the input terminal of the gate, and each gate produces the corresponding output which flows further.
- In this logic circuit, there are four types of gates used: **AND, OR, XOR, and NOT**.
- All these gates have one output terminal and two input terminals (except NOT gate, it has one input terminal).

3. Decide on vocabulary:

The next step of the process is to select functions, predicate, and constants to represent the circuits, terminals, signals, and gates. Firstly we will distinguish the gates from each other and from other objects. Each gate is represented as an object which is named by a constant, such as, **Gate(X1)**. The functionality of each gate is determined by its type, which is taken as constants such as **AND, OR, XOR, or NOT**. Circuits will be identified by a predicate: **Circuit (C1)**.

For the terminal, we will use predicate: **Terminal(x)**.

For gate input, we will use the function **In(1, X1)** for denoting the first input terminal of the gate, and for output terminal we will use **Out (1, X1)**.

The function **Arity(c, i, j)** is used to denote that circuit c has i input, j output.

The connectivity between gates can be represented by predicate **Connect(Out(1, X1), In(1, X1))**.

We use a unary predicate **On (t)**, which is true if the signal at a terminal is on.

4. Encode general knowledge about the domain:

To encode the general knowledge about the logic circuit, we need some following rules:

- If two terminals are connected then they have the same input signal, it can be represented as:

1. $\forall$ t1, t2 Terminal (t1) $\wedge$ Terminal (t2) $\wedge$ Connect (t1, t2) $\rightarrow$ Signal (t1) = Signal (2).

- Signal at every terminal will have either value 0 or 1, it will be represented as:

1. $\forall$ t Terminal (t) $\rightarrow$Signal (t) = 1 $\vee$Signal (t) = 0.

- Connect predicates are commutative:

1. $\forall$ t1, t2 Connect(t1, t2) $\rightarrow$ Connect (t2, t1).

- Representation of types of gates:

1. $\forall$ g Gate(g) $\wedge$ r = Type(g) $\rightarrow$ r = OR $\vee$r = AND $\vee$r = XOR $\vee$r = NOT.

- Output of AND gate will be zero if and only if any of its input is zero.

1. $\forall$ g Gate(g) $\wedge$ Type(g) = AND $\rightarrow$Signal (Out(1, g))= 0 $\Leftrightarrow$ $\exists$n Signal (In(n, g))= 0.

- Output of OR gate is 1 if and only if any of its input is 1:

1. ∀ g Gate(g) ∧ Type(g) = OR → Signal (Out(1, g))= 1 ⇔ ∃n Signal (In(n, g))= 1

- Output of XOR gate is 1 if and only if its inputs are different:

1. ∀ g Gate(g) ∧ Type(g) = XOR → Signal (Out(1, g)) = 1 ⇔ Signal (In(1, g)) ≠ Signal (In(2, g)).

- Output of NOT gate is invert of its input:

1. ∀ g Gate(g) ∧ Type(g) = NOT → Signal (In(1, g)) ≠ Signal (Out(1, g)).

- All the gates in the above circuit have two inputs and one output (except NOT gate).

1. ∀ g Gate(g) ∧ Type(g) = NOT → Arity(g, 1, 1)
2. ∀ g Gate(g) ∧ r =Type(g) ∧ (r= AND ∨r= OR ∨r= XOR) → Arity (g, 2, 1).

- All gates are logic circuits:

1. ∀ g Gate(g) → Circuit (g).

5. Encode a description of the problem instance:

Now we encode problem of circuit C1, firstly we categorize the circuit and its gate components. This step is easy if ontology about the problem is already thought. This step involves the writing simple atomics sentences of instances of concepts, which is known as ontology.

For the given circuit C1, we can encode the problem instance in atomic sentences as below:

Since in the circuit there are two XOR, two AND, and one OR gate so atomic sentences for these gates will be:

1. For XOR gate: Type(x1)= XOR, Type(X2) = XOR
2. For AND gate: Type(A1) = AND, Type(A2)= AND
3. For OR gate: Type (O1) = OR.

And then represent the connections between all the gates.

6. Pose queries to the inference procedure and get answers:

In this step, we will find all the possible set of values of all the terminal for the adder circuit. The first query will be:

What should be the combination of input which would generate the first output of circuit C1, as 0 and a second output to be 1?

1. ∃ i1, i2, i3 Signal (In(1, C1))=i1 ∧ Signal (In(2, C1))=i2 ∧ Signal (In(3, C1))= i3
2. ∧ Signal (Out(1, C1)) =0 ∧ Signal (Out(2, C1))=1

7. Debug the knowledge base:

Now we will debug the knowledge base, and this is the last step of the complete process. In this step, we will try to debug the issues of knowledge base.

In the knowledge base, we may have omitted assertions like $1 \neq 0$.

Inference in First-Order Logic

Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

Substitution:

Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write **F[a/x]**, so it refers to substitute a constant "**a**" in place of variable "**x**".

Equality:

First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality in FOL. For this, we can use **equality symbols** which specify that the two terms refer to the same object.

Example: Brother (John) = Smith.

As in the above example, the object referred by the **Brother (John)** is similar to the object referred by **Smith**. The equality symbol can also be used with negation to represent that two terms are not the same objects.

Example: $\neg(x=y)$ which is equivalent to $x \neq y$.

FOL inference rules for quantifier:

As propositional logic we also have inference rules in first-order logic, so following are some basic inference rules in FOL:

- **Universal Generalization**
- **Universal Instantiation**
- **Existential Instantiation**
- **Existential introduction**

1. Universal Generalization:

- Universal generalization is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as $\forall x\ P(x)$.

- It can be represented as: $\frac{P(c)}{\forall x\, P(x)}$.
- This rule can be used if we want to show that every element has a similar property.
- In this rule, x must not appear as a free variable.

Example: Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits**.", it will also be true.

2. Universal Instantiation:

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.
- The new KB is logically equivalent to the previous KB.
- As per UI, **we can infer any sentence obtained by substituting a ground term for the variable**.
- The UI rule state that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from **∀ x P(x) for any object in the universe of discourse**.
- It can be represented as: $\frac{\forall x\, P(x)}{P(c)}$.

Example:1.

IF "Every person like ice-cream"=> ∀x P(x) so we can infer that
"John likes ice-cream" => P(c)

Example: 2.

Let's take a famous example,

"All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

∀x king(x) ∧ greedy (x) → Evil (x),

So from this information, we can infer any of the following statements using Universal Instantiation:

- **King(John) ∧ Greedy (John) → Evil (John),**
- **King(Richard) ∧ Greedy (Richard) → Evil (Richard),**
- **King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John)),**

3. Existential Instantiation:

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.

- The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable.
- This rule states that one can infer P(c) from the formula given in the form of ∃x P(x) for a new constant symbol c.
- The restriction with this rule is that c used in the rule must be a new term for which P(c) is true.
- It can be represented as: $\frac{\exists x\, P(x)}{P(c)}$

Example:

From the given sentence: **∃x Crown(x) ∧ OnHead(x, John),**

So we can infer: **Crown(K) ∧ OnHead(K, John),** as long as K does not appear in the knowledge base.

- The above used K is a constant symbol, which is called **Skolem constant**.
- The Existential instantiation is a special case of **Skolemization process**.

4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P, then we can infer that there exists something in the universe which has the property P.
- It can be represented as: $\frac{P(c)}{\exists x P(x)}$
- **Example: Let's say that,**
  "Priyanka got good marks in English."
  "Therefore, someone got good marks in English."

Generalized Modus Ponens Rule:

For the inference process in FOL, we have a single inference rule which is called Generalized Modus Ponens. It is lifted version of Modus ponens.

Generalized Modus Ponens can be summarized as, " P implies Q and P is asserted to be true, therefore Q must be True."

According to Modus Ponens, for atomic sentences **pi, pi', q**. Where there is a substitution θ such that SUBST **(θ, pi',) = SUBST(θ, pi)**, it can be represented as:

$$\frac{p1', p2', \ldots, pn', (p1 \wedge p2 \wedge \ldots \wedge pn \Rightarrow q)}{SUBST(\theta, q)}$$

Example:

We will use this rule for Kings are evil, so we will find some x such that x is king, and x is greedy so we can infer that x is evil.

1. Here let say, p1' is king(John) p1 is king(x)
2. p2' is Greedy(y) p2 is Greedy(x)
3. θ is {x/John, y/John} q is evil(x)
4. SUBST(θ,q).

What is Unification?

- Unification is a process of making two different logical atomic expressions identical by finding a substitution. Unification depends on the substitution process.
- It takes two literals as input and makes them identical using substitution.
- Let Ψ_1 and Ψ_2 be two atomic sentences and σ be a unifier such that, $\boldsymbol{\Psi_1\sigma = \Psi_2\sigma}$, then it can be expressed as **UNIFY(Ψ_1, Ψ_2)**.
- **Example: Find the MGU for Unify{King(x), King(John)}**

Let Ψ_1 = King(x), Ψ_2 = King(John),

Substitution θ = {John/x} is a unifier for these atoms and applying this substitution, and both expressions will be identical.

- The UNIFY algorithm is used for unification, which takes two atomic sentences and returns a unifier for those sentences (If any exist).
- Unification is a key component of all first-order inference algorithms.
- It returns fail if the expressions do not match with each other.
- The substitution variables are called Most General Unifier or MGU.

E.g. Let's say there are two different expressions, **P(x, y), and P(a, f(z))**.

In this example, we need to make both above statements identical to each other. For this, we will perform the substitution.

P(x, y)..... (i)
P(a, f(z))..... (ii)

- Substitute x with a, and y with f(z) in the first expression, and it will be represented as **a/x** and f(z)/y.
- With both the substitutions, the first expression will be identical to the second expression and the substitution set will be: **[a/x, f(z)/y]**.

Conditions for Unification:

Following are some basic conditions for unification:

- Predicate symbol must be same, atoms or expression with different predicate symbol can never be unified.
- Number of Arguments in both expressions must be identical.
- Unification will fail if there are two similar variables present in the same expression.

Unification Algorithm:

Algorithm: Unify(Ψ_1, Ψ_2)

```
Step. 1: If Ψ1 or Ψ2 is a variable or constant, then:
      a) If Ψ1 or Ψ2 are identical, then return NIL.
      b) Else if Ψ1is a variable,
             a. then if Ψ1 occurs in Ψ2, then return FAILURE
             b. Else return { (Ψ2/ Ψ1)}.
      c) Else if Ψ2 is a variable,
             a. If Ψ2 occurs in Ψ1 then return FAILURE,
             b. Else return {( Ψ1/ Ψ2)}.
      d) Else return FAILURE.
Step.2: If the initial Predicate symbol in Ψ1 and Ψ2 are not same, then
return FAILURE.
Step. 3: IF Ψ1 and Ψ2 have a different number of arguments, then return
FAILURE.
Step. 4: Set Substitution set(SUBST) to NIL.
Step. 5: For i=1 to the number of elements in Ψ1.
      a) Call Unify function with the ith element of Ψ1 and ith element of
Ψ2, and put the result into S.
      b) If S = failure then returns Failure
      c) If S ≠ NIL then do,
             a. Apply S to the remainder of both L1 and L2.
             b. SUBST= APPEND(S, SUBST).
Step.6: Return SUBST.
```

Implementation of the Algorithm

Step.1: Initialize the substitution set to be empty.

Step.2: Recursively unify atomic sentences:

a. Check for Identical expression match.

b. If one expression is a variable v_i, and the other is a term t_i which does not contain variable v_i, then:

   a. Substitute t_i / v_i in the existing substitutions
   b. Add t_i /v_i to the substitution setlist.
   c. If both the expressions are functions, then function name must be similar, and the number of arguments must be the same in both the expression.

For each pair of the following atomic sentences find the most general unifier (If exist).

1. Find the MGU of {p(f(a), g(Y)) and p(X, X)}

Sol: S_0 => Here, Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(X, X)
SUBST θ= {f(a) / X}
S1 => Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(f(a), f(a))
SUBST θ= {f(a) / g(y)}, **Unification failed**.

Unification is not possible for these expressions.

2. Find the MGU of {p(b, X, f(g(Z))) and p(Z, f(Y), f(Y))}

Here, Ψ_1 = p(b, X, f(g(Z))) , and Ψ_2 = p(Z, f(Y), f(Y))
S_0 => { p(b, X, f(g(Z))); p(Z, f(Y), f(Y))}
SUBST θ={b/Z}

S_1 => { p(b, X, f(g(b))); p(b, f(Y), f(Y))}
SUBST θ={f(Y) /X}

S_2 => { p(b, f(Y), f(g(b))); p(b, f(Y), f(Y))}
SUBST θ= {g(b) /Y}

S_2 => { p(b, f(g(b)), f(g(b)); p(b, f(g(b)), f(g(b))} **Unified Successfully.**
And Unifier = { b/Z, f(Y) /X , g(b) /Y}.

3. Find the MGU of {p (X, X), and p (Z, f(Z))}

Here, Ψ_1 = {p (X, X), and Ψ_2 = p (Z, f(Z))
S_0 => {p (X, X), p (Z, f(Z))}
SUBST θ= {X/Z}
S1 => {p (Z, Z), p (Z, f(Z))}
SUBST θ= {f(Z) / Z}, **Unification Failed**.

Hence, unification is not possible for these expressions.

4. Find the MGU of UNIFY(prime (11), prime(y))

Here, Ψ_1 = {prime(11) , and Ψ_2 = prime(y)}
S_0 => {prime(11) , prime(y)}
SUBST θ= {11/y}

S_1 => {prime(11) , prime(11)} , **Successfully unified.**
Unifier: {11/y}.

5. Find the MGU of Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)}

Here, Ψ_1 = Q(a, g(x, a), f(y)), and Ψ_2 = Q(a, g(f(b), a), x)
S_0 => {Q(a, g(x, a), f(y)); Q(a, g(f(b), a), x)}
SUBST θ= {f(b)/x}
S_1 => {Q(a, g(f(b), a), f(y)); Q(a, g(f(b), a), f(b))}

SUBST θ= {b/y}
S_1 => {Q(a, g(f(b), a), f(b)); Q(a, g(f(b), a), f(b))}, **Successfully Unified.**

Unifier: [a/a, f(b)/x, b/y].

6. UNIFY(knows(Richard, x), knows(Richard, John))

Here, Ψ_1 = knows(Richard, x), and Ψ_2 = knows(Richard, John)
S_0 => { knows(Richard, x); knows(Richard, John)}
SUBST θ= {John/x}
S_1 => { knows(Richard, John); knows(Richard, John)}, **Successfully Unified.**
Unifier: {John/x}.

Resolution

Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions. It was invented by a Mathematician John Alan Robinson in the year 1965.

Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements. Unification is a key concept in proofs by resolutions. Resolution is a single inference rule which can efficiently operate on the **conjunctive normal form or clausal form**.

Clause: Disjunction of literals (an atomic sentence) is called a **clause**. It is also known as a unit clause.

Conjunctive Normal Form: A sentence represented as a conjunction of clauses is said to be **conjunctive normal form** or **CNF**.

The resolution inference rule:

The resolution rule for first-order logic is simply a lifted version of the propositional rule. Resolution can resolve two clauses if they contain complementary literals, which are assumed to be standardized apart so that they share no variables.

$$\frac{l_1 \vee \ldots\ldots \vee l_k, \qquad m_1 \vee \ldots\ldots \vee m_n}{\mathrm{SUBST}(\theta, l_1 \vee \ldots\ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots\ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

Where $\mathbf{l_i}$ and $\mathbf{m_j}$ are complementary literals.

This rule is also called the **binary resolution rule** because it only resolves exactly two literals.

Example:

We can resolve two clauses which are given below:

[Animal (g(x) V Loves (f(x), x)] **and** **[¬ Loves(a, b) V ¬Kills(a, b)]**

Where two complimentary literals are: **Loves (f(x), x) and ¬ Loves (a, b)**

These literals can be unified with unifier **θ= [a/f(x), and b/x]** , and it will generate a resolvent clause:

[Animal (g(x) V ¬ Kills(f(x), x)].

Steps for Resolution:

1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

To better understand all the above steps, we will take an example in which we will apply resolution.

Example:

a. **John likes all kind of food.**
b. **Apple and vegetable are food**
c. **Anything anyone eats and not killed is food.**
d. **Anil eats peanuts and still alive**
e. **Harry eats everything that Anil eats.**
Prove by resolution that:
f. **John likes peanuts.**

Step-1: Conversion of Facts into FOL

In the first step we will convert all the given statements into its first order logic.

a. ∀x: food(x) → likes(John, x)
b. food(Apple) ∧ food(vegetables)
c. ∀x ∀y: eats(x, y) ∧ ¬ killed(x) → food(y)
d. eats (Anil, Peanuts) ∧ alive(Anil).
e. ∀x : eats(Anil, x) → eats(Harry, x)
f. ∀x: ¬ killed(x) → alive(x) } **added predicates.**
g. ∀x: alive(x) →¬ killed(x)
h. likes(John, Peanuts)

Step-2: Conversion of FOL into CNF

In First order logic resolution, it is required to convert the FOL into CNF as CNF form makes easier for resolution proofs.

- **Eliminate all implication (→) and rewrite**
    a. ∀x ¬ food(x) V likes(John, x)
    b. food(Apple) Λ food(vegetables)
    c. ∀x ∀y ¬ [eats(x, y) Λ ¬ killed(x)] V food(y)
    d. eats (Anil, Peanuts) Λ alive(Anil)
    e. ∀x ¬ eats(Anil, x) V eats(Harry, x)
    f. ∀x¬ [¬ killed(x)] V alive(x)
    g. ∀x ¬ alive(x) V ¬ killed(x)
    h. likes(John, Peanuts).

Move negation (¬)inwards and rewrite

. ∀x ¬ food(x) V likes(John, x)
    a. food(Apple) Λ food(vegetables)
    b. ∀x ∀y ¬ eats(x, y) V killed(x) V food(y)
    c. eats (Anil, Peanuts) Λ alive(Anil)
    d. ∀x ¬ eats(Anil, x) V eats(Harry, x)
    e. ∀x ¬killed(x)] V alive(x)
    f. ∀x ¬ alive(x) V ¬ killed(x)
    g. likes(John, Peanuts).

Rename variables or standardize variables

. ∀x ¬ food(x) V likes(John, x)
    a. food(Apple) Λ food(vegetables)
    b. ∀y ∀z ¬ eats(y, z) V killed(y) V food(z)
    c. eats (Anil, Peanuts) Λ alive(Anil)
    d. ∀w¬ eats(Anil, w) V eats(Harry, w)
    e. ∀g ¬killed(g)] V alive(g)
    f. ∀k ¬ alive(k) V ¬ killed(k)
    g. likes(John, Peanuts).

Eliminate existential instantiation quantifier by elimination.

In this step, we will eliminate existential quantifier ∃, and this process is known as **Skolemization**. But in this example problem since there is no existential quantifier so all the statements will remain same in this step.

Drop Universal quantifiers.

In this step we will drop all universal quantifier since all the statements are not implicitly quantified so we don't need it.

. ¬ food(x) V likes(John, x)
    a. food(Apple)
    b. food(vegetables)
    c. ¬ eats(y, z) V killed(y) V food(z)

d. eats (Anil, Peanuts)
e. alive(Anil)
f. ¬ eats(Anil, w) V eats(Harry, w)
g. killed(g) V alive(g)
h. ¬ alive(k) V ¬ killed(k)
i. likes(John, Peanuts).

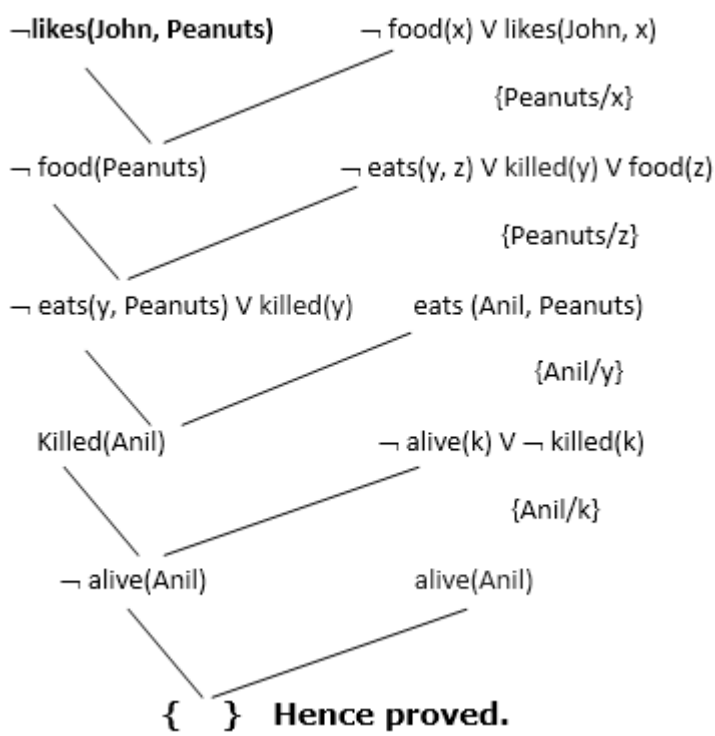
- **Distribute conjunction ∧ over disjunction ¬.**
  This step will not make any change in this problem.

Step-3: Negate the statement to be proved

In this statement, we will apply negation to the conclusion statements, which will be written as ¬likes(John, Peanuts)

Step-4: Draw Resolution graph:

Now in this step, we will solve the problem by resolution tree using substitution. For the above problem, it will be given as follows:



Hence the negation of the conclusion has been proved as a complete contradiction with the given set of statements.

<u>Explanation of Resolution graph:</u>

- In the first step of resolution graph, **¬likes(John, Peanuts)** , and **likes(John, x)** get resolved(canceled) by substitution of **{Peanuts/x}**, and we are left with **¬ food(Peanuts)**
- In the second step of the resolution graph, **¬ food(Peanuts)** , and **food(z)** get resolved (canceled) by substitution of **{ Peanuts/z}**, and we are left with **¬ eats(y, Peanuts) V killed(y)** .
- In the third step of the resolution graph, **¬ eats(y, Peanuts)** and **eats (Anil, Peanuts)** get resolved by substitution **{Anil/y}**, and we are left with **Killed(Anil)** .
- In the fourth step of the resolution graph, **Killed(Anil)** and **¬ killed(k)** get resolve by substitution **{Anil/k}**, and we are left with **¬ alive(Anil)** .
- In the last step of the resolution graph **¬ alive(Anil)** and **alive(Anil)** get resolved.

Forward Chaining and backward chaining in AI

In artificial intelligence, forward and backward chaining is one of the important topics, but before understanding forward and backward chaining lets first understand that from where these two terms came.

Inference engine:

The inference engine is the component of the intelligent system in artificial intelligence, which applies logical rules to the knowledge base to infer new information from known facts. The first inference engine was part of the expert system. Inference engine commonly proceeds in two modes, which are:

a. **Forward chaining**
b. **Backward chaining**

Horn Clause and Definite clause:

Horn clause and definite clause are the forms of sentences, which enables knowledge base to use a more restricted and efficient inference algorithm. Logical inference algorithms use forward and backward chaining approaches, which require KB in the form of the **first-order definite clause**.

Definite clause: A clause which is a disjunction of literals with **exactly one positive literal** is known as a definite clause or strict horn clause.

Horn clause: A clause which is a disjunction of literals with **at most one positive literal** is known as horn clause. Hence all the definite clauses are horn clauses.

Example: (¬ p V ¬ q V k). It has only one positive literal k.

It is equivalent to $p \wedge q \rightarrow k$.

A. Forward Chaining

Forward chaining is also known as a forward deduction or forward reasoning method when using an inference engine. Forward chaining is a form of reasoning which start with atomic

sentences in the knowledge base and applies inference rules (Modus Ponens) in the forward direction to extract more data until a goal is reached.

The Forward-chaining algorithm starts from known facts, triggers all rules whose premises are satisfied, and add their conclusion to the known facts. This process repeats until the problem is solved.

Properties of Forward-Chaining:

- It is a down-up approach, as it moves from bottom to top.
- It is a process of making a conclusion based on known facts or data, by starting from the initial state and reaches the goal state.
- Forward-chaining approach is also called as data-driven as we reach to the goal using available data.
- Forward -chaining approach is commonly used in the expert system, such as CLIPS, business, and production rule systems.

Consider the following famous example which we will use in both approaches:

Example:

"As per the law, it is a crime for an American to sell weapons to hostile nations. Country A, an enemy of America, has some missiles, and all the missiles were sold to it by Robert, who is an American citizen."

Prove that **"Robert is criminal."**

To solve the above problem, first, we will convert all the above facts into first-order definite clauses, and then we will use a forward-chaining algorithm to reach the goal.

Facts Conversion into FOL:

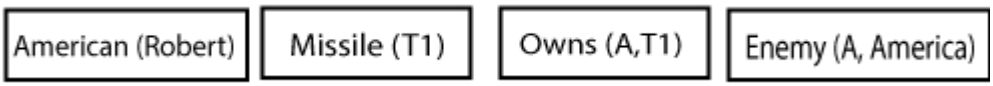
- It is a crime for an American to sell weapons to hostile nations. (Let's say p, q, and r are variables)
  American (p) ∧ weapon(q) ∧ sells (p, q, r) ∧ hostile(r) → Criminal(p) ...(1)
- Country A has some missiles. **?p Owns(A, p) ∧ Missile(p)**. It can be written in two definite clauses by using Existential Instantiation, introducing new Constant T1.
  Owns(A, T1) .....(2)
  Missile(T1) .....(3)
- All of the missiles were sold to country A by Robert.
  ?p Missiles(p) ∧ Owns (A, p) → Sells (Robert, p, A) .....(4)
- Missiles are weapons.
  Missile(p) → Weapons (p) .....(5)
- Enemy of America is known as hostile.
  Enemy(p, America) →Hostile(p) .....(6)

- Country A is an enemy of America.
  Enemy (A, America) **.....(7)**
- Robert is American
  American(Robert). **.....(8)**

Forward chaining proof:

Step-1:

In the first step we will start with the known facts and will choose the sentences which do not have implications, such as: **American(Robert), Enemy(A, America), Owns(A, T1), and Missile(T1)**. All these facts will be represented as below.



Step-2:

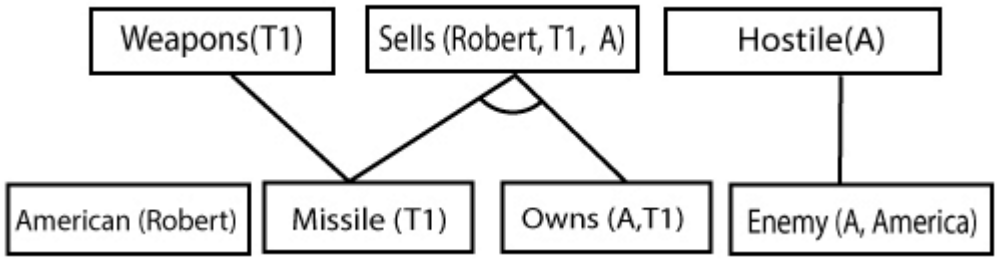
At the second step, we will see those facts which infer from available facts and with satisfied premises.

Rule-(1) does not satisfy premises, so it will not be added in the first iteration.

Rule-(2) and (3) are already added.

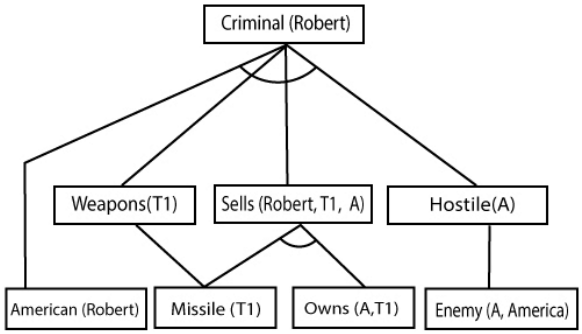
Rule-(4) satisfy with the substitution {p/T1}, **so Sells (Robert, T1, A)** is added, which infers from the conjunction of Rule (2) and (3).

Rule-(6) is satisfied with the substitution(p/A), so Hostile(A) is added and which infers from Rule-(7).



Step-3:

At step-3, as we can check Rule-(1) is satisfied with the substitution **{p/Robert, q/T1, r/A}, so we can add Criminal(Robert)** which infers all the available facts. And hence we reached our goal statement.



Hence it is proved that Robert is Criminal using forward chaining approach.

B. Backward Chaining:

Backward-chaining is also known as a backward deduction or backward reasoning method when using an inference engine. A backward chaining algorithm is a form of reasoning, which starts with the goal and works backward, chaining through rules to find known facts that support the goal.

Properties of backward chaining:

- It is known as a top-down approach.
- Backward-chaining is based on modus ponens inference rule.
- In backward chaining, the goal is broken into sub-goal or sub-goals to prove the facts true.
- It is called a goal-driven approach, as a list of goals decides which rules are selected and used.
- Backward -chaining algorithm is used in game theory, automated theorem proving tools, inference engines, proof assistants, and various AI applications.
- The backward-chaining method mostly used a **depth-first search** strategy for proof.

Example:

In backward-chaining, we will use the same above example, and will rewrite all the rules.

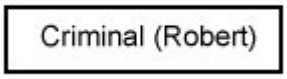
- **American (p) ∧ weapon(q) ∧ sells (p, q, r) ∧ hostile(r) → Criminal(p) ...(1)**
  Owns(A, T1) .....(2)
- **Missile(T1)**
- **?p Missiles(p) ∧ Owns (A, p) → Sells (Robert, p, A) .....(4)**
- **Missile(p) → Weapons (p) .....(5)**
- **Enemy(p, America) →Hostile(p) .....(6)**
- **Enemy (A, America) .....(7)**
- **American(Robert). .....(8)**

Backward-Chaining proof:

In Backward chaining, we will start with our goal predicate, which is **Criminal(Robert)**, and then infer further rules.

Step-1:

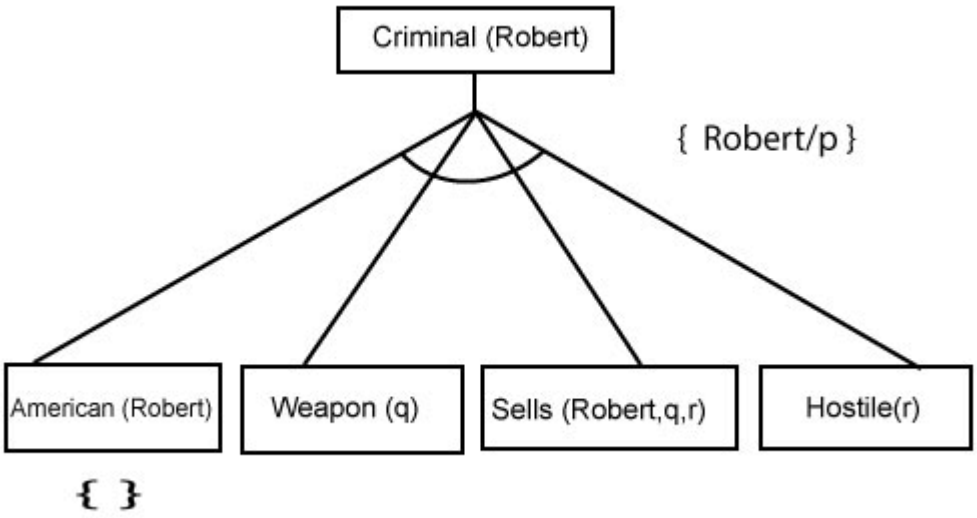
At the first step, we will take the goal fact. And from the goal fact, we will infer other facts, and at last, we will prove those facts true. So our goal fact is "Robert is Criminal," so following is the predicate of it.



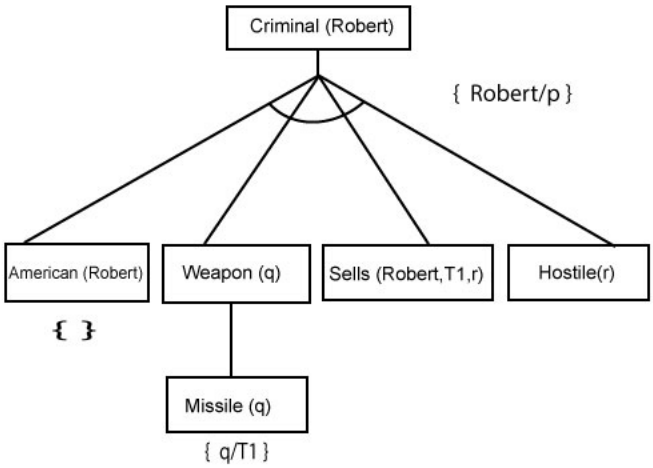
Step-2:

At the second step, we will infer other facts form goal fact which satisfies the rules. So as we can see in Rule-1, the goal predicate Criminal (Robert) is present with substitution {Robert/P}. So we will add all the conjunctive facts below the first level and will replace p with Robert.

Here we can see American (Robert) is a fact, so it is proved here.

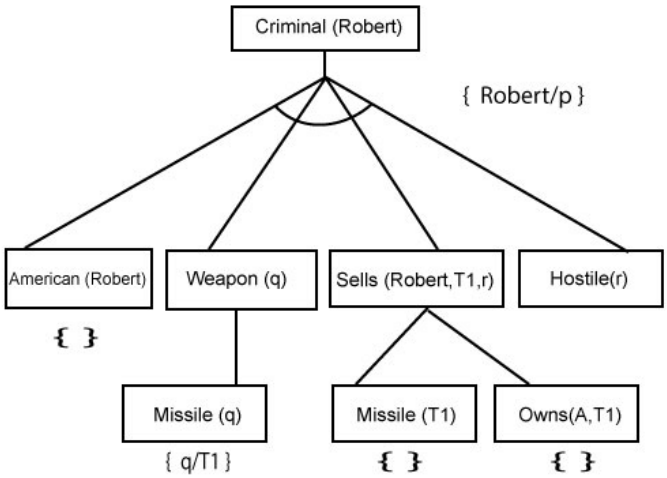


Step-3:t At step-3, we will extract further fact Missile(q) which infer from Weapon(q), as it satisfies Rule-(5). Weapon (q) is also true with the substitution of a constant T1 at q.



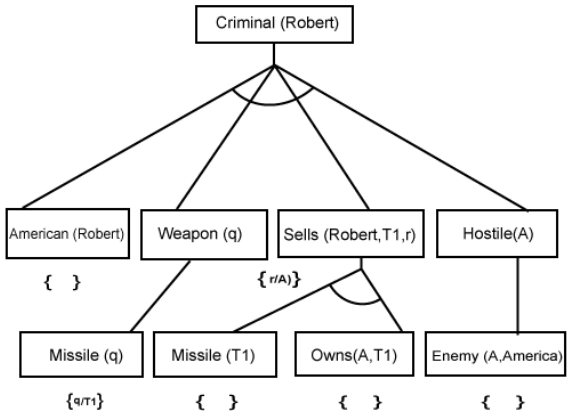
Step-4:

At step-4, we can infer facts Missile(T1) and Owns(A, T1) form Sells(Robert, T1, r) which satisfies the **Rule- 4**, with the substitution of A in place of r. So these two statements are proved here.



Step-5:

At step-5, we can infer the fact **Enemy(A, America)** from **Hostile(A)** which satisfies Rule-6. And hence all the statements are proved true using backward chaining.



Difference between backward chaining and forward chaining

Following is the difference between the forward chaining and backward chaining:

- Forward chaining as the name suggests, start from the known facts and move forward by applying inference rules to extract more data, and it continues until it reaches to the goal, whereas backward chaining starts from the goal, move backward by using inference rules to determine the facts that satisfy the goal.
- Forward chaining is called a **data-driven** inference technique, whereas backward chaining is called a **goal-driven** inference technique.
- Forward chaining is known as the **down-up** approach, whereas backward chaining is known as a **top-down** approach.
- Forward chaining uses **breadth-first search** strategy, whereas backward chaining uses **depth-first search** strategy.
- Forward and backward chaining both applies **Modus ponens** inference rule.
- Forward chaining can be used for tasks such as **planning, design process monitoring, diagnosis, and classification**, whereas backward chaining can be used for **classification and diagnosis tasks**.
- Forward chaining can be like an exhaustive search, whereas backward chaining tries to avoid the unnecessary path of reasoning.
- In forward-chaining there can be various ASK questions from the knowledge base, whereas in backward chaining there can be fewer ASK questions.
- Forward chaining is slow as it checks for all the rules, whereas backward chaining is fast as it checks few required rules only.

S. No.	Forward Chaining	Backward Chaining
1.	Forward chaining starts from known	Backward chaining starts from the goal

	facts and applies inference rule to extract more data unit it reaches to the goal.	and works backward through inference rules to find the required facts that support the goal.
2.	It is a bottom-up approach	It is a top-down approach
3.	Forward chaining is known as data-driven inference technique as we reach to the goal using the available data.	Backward chaining is known as goal-driven technique as we start from the goal and divide into sub-goal to extract the facts.
4.	Forward chaining reasoning applies a breadth-first search strategy.	Backward chaining reasoning applies a depth-first search strategy.
5.	Forward chaining tests for all the available rules	Backward chaining only tests for few required rules.
6.	Forward chaining is suitable for the planning, monitoring, control, and interpretation application.	Backward chaining is suitable for diagnostic, prescription, and debugging application.
7.	Forward chaining can generate an infinite number of possible conclusions.	Backward chaining generates a finite number of possible conclusions.
8.	It operates in the forward direction.	It operates in the backward direction.
9.	Forward chaining is aimed for any conclusion.	Backward chaining is only aimed for the required data.

Reasoning:

The reasoning is the mental process of deriving logical conclusion and making predictions from available knowledge, facts, and beliefs. Or we can say, "**Reasoning is a way to infer facts from existing data**." It is a general process of thinking rationally, to find valid conclusions.

In artificial intelligence, the reasoning is essential so that the machine can also think rationally as a human brain, and can perform like a human.

Types of Reasoning

In artificial intelligence, reasoning can be divided into the following categories:

- Deductive reasoning

- Inductive reasoning
- Abductive reasoning
- Common Sense Reasoning
- Monotonic Reasoning
- Non-monotonic Reasoning

1. Deductive reasoning:

Deductive reasoning is deducing new information from logically related known information. It is the form of valid reasoning, which means the argument's conclusion must be true when the premises are true.

Deductive reasoning is a type of propositional logic in AI, and it requires various rules and facts. It is sometimes referred to as top-down reasoning, and contradictory to inductive reasoning.

In deductive reasoning, the truth of the premises guarantees the truth of the conclusion.

Deductive reasoning mostly starts from the general premises to the specific conclusion, which can be explained as below example.

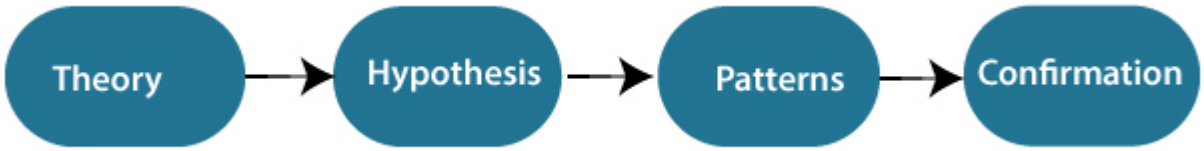
Example:

Premise-1: All the human eats veggies

Premise-2: Suresh is human.

Conclusion: Suresh eats veggies.

The general process of deductive reasoning is given below:



2. Inductive Reasoning:

Inductive reasoning is a form of reasoning to arrive at a conclusion using limited sets of facts by the process of generalization. It starts with the series of specific facts or data and reaches to a general statement or conclusion.

Inductive reasoning is a type of propositional logic, which is also known as cause-effect reasoning or bottom-up reasoning.

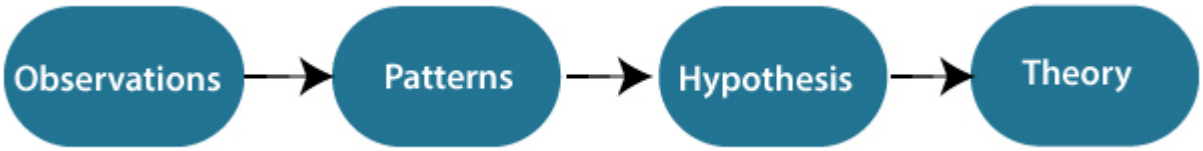
In inductive reasoning, we use historical data or various premises to generate a generic rule, for which premises support the conclusion.

In inductive reasoning, premises provide probable supports to the conclusion, so the truth of premises does not guarantee the truth of the conclusion.

Example:

Premise: All of the pigeons we have seen in the zoo are white.

Conclusion: Therefore, we can expect all the pigeons to be white.



3. Abductive reasoning:

Abductive reasoning is a form of logical reasoning which starts with single or multiple observations then seeks to find the most likely explanation or conclusion for the observation.

Abductive reasoning is an extension of deductive reasoning, but in abductive reasoning, the premises do not guarantee the conclusion.

Example:

Implication: Cricket ground is wet if it is raining

Axiom: Cricket ground is wet.

Conclusion It is raining.

4. Common Sense Reasoning

Common sense reasoning is an informal form of reasoning, which can be gained through experiences.

Common Sense reasoning simulates the human ability to make presumptions about events which occurs on every day.

It relies on good judgment rather than exact logic and operates on **heuristic knowledge** and **heuristic rules**.

Example:

1. **One person can be at one place at a time.**
2. **If I put my hand in a fire, then it will burn.**

The above two statements are the examples of common sense reasoning which a human mind can easily understand and assume.

5. Monotonic Reasoning:

In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base. In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.

To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.

Monotonic reasoning is not useful for the real-time systems, as in real time, facts get changed, so we cannot use monotonic reasoning.

Monotonic reasoning is used in conventional reasoning systems, and a logic-based system is monotonic.

Any theorem proving is an example of monotonic reasoning.

Example:

- **Earth revolves around the Sun.**

It is a true fact, and it cannot be changed even if we add another sentence in knowledge base like, "The moon revolves around the earth" Or "Earth is not round," etc.

Advantages of Monotonic Reasoning:

- In monotonic reasoning, each old proof will always remain valid.
- If we deduce some facts from available facts, then it will remain valid for always.

Disadvantages of Monotonic Reasoning:

- We cannot represent the real world scenarios using Monotonic reasoning.
- Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.
- Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

6. Non-monotonic Reasoning

In Non-monotonic reasoning, some conclusions may be invalidated if we add some more information to our knowledge base.

Logic will be said as non-monotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.

Non-monotonic reasoning deals with incomplete and uncertain models.

"Human perceptions for various things in daily life, "is a general example of non-monotonic reasoning.

Example: Let suppose the knowledge base contains the following knowledge:

- **Birds can fly**
- **Penguins cannot fly**
- **Pitty is a bird**

So from the above sentences, we can conclude that **Pitty can fly**.

However, if we add one another sentence into knowledge base "**Pitty is a penguin**", which concludes "**Pitty cannot fly**", so it invalidates the above conclusion.

Advantages of Non-monotonic reasoning:

- For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
- In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.

Disadvantages of Non-monotonic Reasoning:

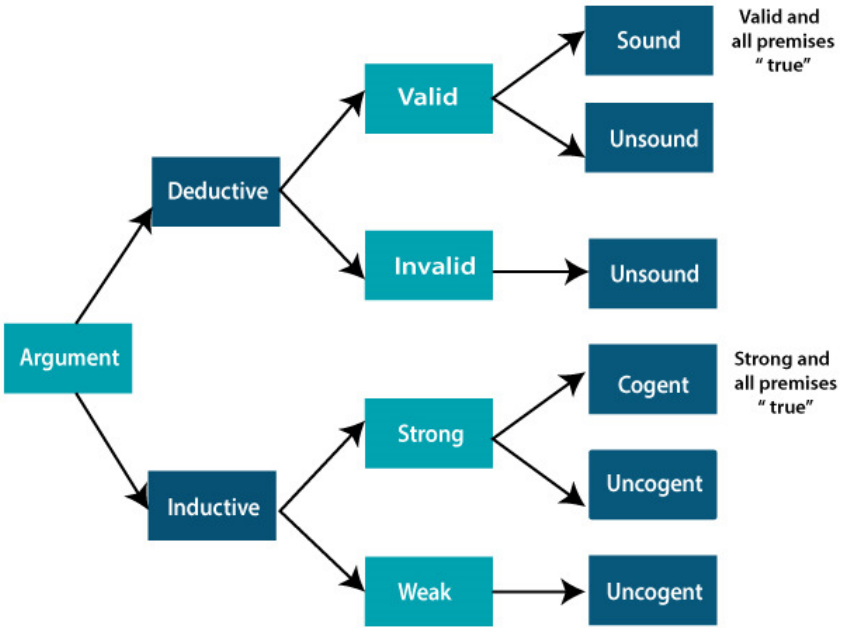
- In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
- It cannot be used for theorem **proving**.

Difference between Inductive and Deductive reasoning

Reasoning in artificial intelligence has two important forms, Inductive reasoning, and Deductive reasoning. Both reasoning forms have premises and conclusions, but both reasoning are contradictory to each other. Following is a list for comparison between inductive and deductive reasoning:

- Deductive reasoning uses available facts, information, or knowledge to deduce a valid conclusion, whereas inductive reasoning involves making a generalization from specific facts, and observations.
- Deductive reasoning uses a top-down approach, whereas inductive reasoning uses a bottom-up approach.
- Deductive reasoning moves from generalized statement to a valid conclusion, whereas Inductive reasoning moves from specific observation to a generalization.
- In deductive reasoning, the conclusions are certain, whereas, in Inductive reasoning, the conclusions are probabilistic.
- Deductive arguments can be valid or invalid, which means if premises are true, the conclusion must be true, whereas inductive argument can be strong or weak, which means conclusion may be false even if premises are true.

The differences between inductive and deductive can be explained using the below diagram on the basis of arguments:



Comparison Chart:

Basis for comparison	Deductive Reasoning	Inductive Reasoning
Definition	Deductive reasoning is the form of valid reasoning, to deduce new information or conclusion from known related facts and information.	Inductive reasoning arrives at a conclusion by the process of generalization using specific facts or data.
Approach	Deductive reasoning follows a top-down approach.	Inductive reasoning follows a bottom-up approach.
Starts from	Deductive reasoning starts from Premises.	Inductive reasoning starts from the Conclusion.
Validity	In deductive reasoning conclusion must be true if the premises are true.	In inductive reasoning, the truth of premises does not guarantee the truth of conclusions.
Usage	Use of deductive reasoning is difficult, as we need facts which must be true.	Use of inductive reasoning is fast and easy, as we need evidence instead of true facts. We often use it in our daily life.
Process	Theory→ hypothesis→	Observations-

	patterns→confirmation.	→patterns→hypothesis→Theory.
Argument	In deductive reasoning, arguments may be valid or invalid.	In inductive reasoning, arguments may be weak or strong.
Structure	Deductive reasoning reaches from general facts to specific facts.	Inductive reasoning reaches from specific facts to general facts.

Probabilistic reasoning in Artificial intelligence

Uncertainty:

Till now, we have learned knowledge representation using first-order logic and propositional logic with certainty, which means we were sure about the predicates. With this knowledge representation, we might write A→B, which means if A is true then B is true, but consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called uncertainty.

So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning or probabilistic reasoning.

Causes of uncertainty:

Following are some leading causes of uncertainty to occur in the real world.

1. Information occurred from unreliable sources.
2. Experimental Errors
3. Equipment fault
4. Temperature variation
5. Climate change.

Probabilistic reasoning:

Probabilistic reasoning is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge. In probabilistic reasoning, we combine probability theory with logic to handle the uncertainty.

We use probability in probabilistic reasoning because it provides a way to handle the uncertainty that is the result of someone's laziness and ignorance.

In the real world, there are lots of scenarios, where the certainty of something is not confirmed, such as "It will rain today," "behavior of someone for some situations," "A match between two teams or two players." These are probable sentences for which we can assume that it will happen but not sure about it, so here we use probabilistic reasoning.

Need of probabilistic reasoning in AI:

- When there are unpredictable outcomes.
- When specifications or possibilities of predicates becomes too large to handle.
- When an unknown error occurs during an experiment.

In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:

- **Bayes' rule**
- **Bayesian Statistics**

Note: We will learn the above two rules in later chapters.

As probabilistic reasoning uses probability and related terms, so before understanding probabilistic reasoning, let's understand some common terms:

Probability: Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 that represent ideal uncertainties.

1. $0 \leq P(A) \leq 1$, where P(A) is the probability of an event A.
1. $P(A) = 0$, indicates total uncertainty in an event A.
1. $P(A) = 1$, indicates total certainty in an event A.

We can find the probability of an uncertain event by using the below formula.

$$\textbf{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

- $P(\neg A)$ = probability of a not happening event.
- $P(\neg A) + P(A) = 1$.

Event: Each possible outcome of a variable is called an event.

Sample space: The collection of all possible events is called sample space.

Random variables: Random variables are used to represent the events and objects in the real world.

Prior probability: The prior probability of an event is probability computed before observing new information.

Posterior Probability: The probability that is calculated after all evidence or information has taken into account. It is a combination of prior probability and new information.

Conditional probability:

Conditional probability is a probability of occurring an event when another event has already happened.

Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

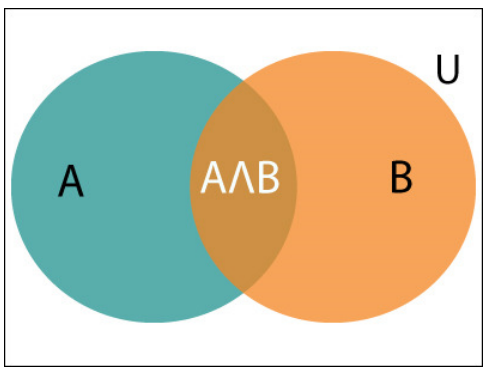
Where P(*A*∧*B*)= Joint probability of a and B

P(B)= Marginal probability of B.

If the probability of A is given and we need to find the probability of B, then it will be given as:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}$$

It can be explained by using the below Venn diagram, where B is occurred event, so sample space will be reduced to set B, and now we can only calculate event A when event B is already occurred by dividing the probability of **P(A∧*B*) by P(B)**.



Example:

In a class, there are 70% of the students who like English and 40% of the students who likes English and mathematics, and then what is the percent of students those who like English also like mathematics?

Solution:

Let, A is an event that a student likes Mathematics

B is an event that a student likes English.

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence, 57% are the students who like English also like Mathematics.

Bayes' theorem:

Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.

In probability theory, it relates the conditional probability and marginal probabilities of two random events.

Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

As from product rule we can write:

1. $P(A \wedge B) = P(A|B)\, P(B)$ or

Similarly, the probability of event B with known event A:

1. $P(A \wedge B) = P(B|A)\, P(A)$

Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)} \quad \text{....(a)}$$

The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

It shows the simple relationship between joint and conditional probabilities. Here,

P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.

P(A) is called the **prior probability**, probability of hypothesis before considering the evidence

P(B) is called **marginal probability**, pure probability of an evidence.

In the equation (a), in general, we can write P (B) = P(A)*P(B|Ai), hence the Bayes' rule can be written as:

$$P(A_i|B) = \frac{P(A_i)*P(B|A_i)}{\sum_{i=1}^{k} P(A_i)*P(B|A_i)}$$

Where A_1, A_2, A_3,....., A_n is a set of mutually exclusive and exhaustive events.

Applying Bayes' rule:

Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(*B*), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$\mathbf{P(cause|effect)} = \frac{P(effect|cause)\,P(cause)}{P(effect)}$$

Example-1:

Question: what is the probability that a patient has diseases meningitis with a stiff neck?

Given Data:

A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:

- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.

Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:

P(a|b) = 0.8

P(b) = 1/30000

P(a)= .02

$$\mathbf{P(b|a)} = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8*(\frac{1}{30000})}{0.02} = 0.001333333.$$

Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Example-2:

Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.

Solution:

$$\mathbf{P(king|face)} = \frac{P(\text{Face}|\text{king}) * P(\text{King})}{P(\text{Face})} \quad .....(i)$$

P(king): probability that the card is King= 4/52= 1/13

P(face): probability that a card is a face card= 3/13

P(Face|King): probability of face card when we assume it is a king = 1

Putting all values in equation (i) we will get:

$$P(\text{king}|\text{face}) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})} = \mathbf{1/3,}$$ **it is a probability that a face card is a king card.**

Application of Bayes' theorem in Artificial intelligence:

Following are some applications of Bayes' theorem:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

Bayesian Belief Network in artificial intelligence

Bayesian belief network is key computer technology for dealing with probabilistic events and to solve a problem which has uncertainty. We can define a Bayesian network as:

"A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a directed acyclic graph."

It is also called a **Bayes network, belief network, decision network**, or **Bayesian model**.

Bayesian networks are probabilistic, because these networks are built from a **probability distribution**, and also use probability theory for prediction and anomaly detection.

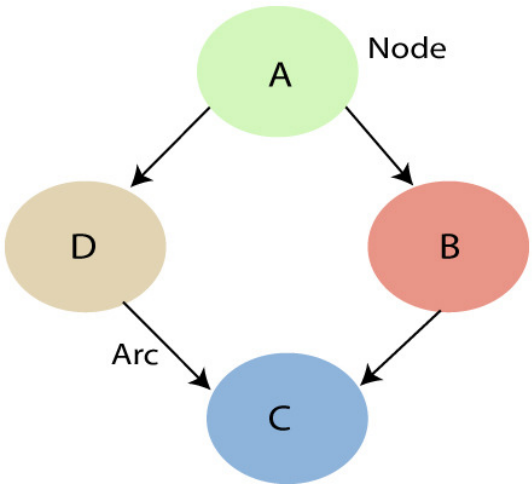
Real world applications are probabilistic in nature, and to represent the relationship between multiple events, we need a Bayesian network. It can also be used in various tasks including **prediction, anomaly detection, diagnostics, automated insight, reasoning, time series prediction**, and **decision making under uncertainty**.

Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:

- **Directed Acyclic Graph**
- **Table of conditional probabilities.**

The generalized form of Bayesian network that represents and solve decision problems under uncertain knowledge is known as an **Influence diagram**.

A Bayesian network graph is made up of nodes and Arcs (directed links), where:



- Each **node** corresponds to the random variables, and a variable can be **continuous** or **discrete**.
- **Arc or directed arrows** represent the causal relationship or conditional probabilities between random variables. These directed links or arrows connect the pair of nodes in the graph.
  These links represent that one node directly influence the other node, and if there is no directed link that means that nodes are independent with each other
  - **In the above diagram, A, B, C, and D are random variables represented by the nodes of the network graph.**
  - **If we are considering node B, which is connected with node A by a directed arrow, then node A is called the parent of Node B.**
  - **Node C is independent of node A.**

The Bayesian network has mainly two components:

- **Causal Component**
- **Actual numbers**

Each node in the Bayesian network has condition probability distribution **$P(X_i \mid Parent(X_i))$**, which determines the effect of the parent on that node.

Bayesian network is based on Joint probability distribution and conditional probability. So let's first understand the joint probability distribution:

Joint probability distribution:

If we have variables x1, x2, x3,....., xn, then the probabilities of a different combination of x1, x2, x3.. xn, are known as Joint probability distribution.

$P[x_1, x_2, x_3, \ldots, x_n]$, it can be written as the following way in terms of the joint probability distribution.

$= P[x_1 | x_2, x_3, \ldots, x_n]P[x_2, x_3, \ldots, x_n]$

$= P[x_1 | x_2, x_3, \ldots, x_n]P[x_2 | x_3, \ldots, x_n] \ldots P[x_{n-1} | x_n]P[x_n]$.

In general for each variable Xi, we can write the equation as:

$$P(X_i | X_{i-1}, \ldots\ldots\ldots, X_1) = P(X_i \mid Parents(X_i))$$

Explanation of Bayesian network:

Let's understand the Bayesian network through an example by creating a directed acyclic graph:

Example: Harry installed a new burglar alarm at his home to detect burglary. The alarm reliably responds at detecting a burglary but also responds for minor earthquakes. Harry has two neighbors David and Sophia, who have taken a responsibility to inform Harry at work when they hear the alarm. David always calls Harry when he hears the alarm, but sometimes he got confused with the phone ringing and calls at that time too. On the other hand, Sophia likes to listen to high music, so sometimes she misses to hear the alarm. Here we would like to compute the probability of Burglary Alarm.

Problem:

Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and David and Sophia both called the Harry.

Solution:

- The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but David and Sophia's calls depend on alarm probability.
- The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.
- The conditional distributions for each node are given as conditional probabilities table or CPT.
- Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.
- In CPT, a boolean variable with k boolean parents contains 2^K probabilities. Hence, if there are two parents, then CPT will contain 4 probability values

List of all events occurring in this network:

- **Burglary (B)**
- **Earthquake(E)**
- **Alarm(A)**
- **David Calls(D)**
- **Sophia calls(S)**

We can write the events of problem statement in the form of probability: **P[D, S, A, B, E]**, can rewrite the above probability statement using joint probability distribution:

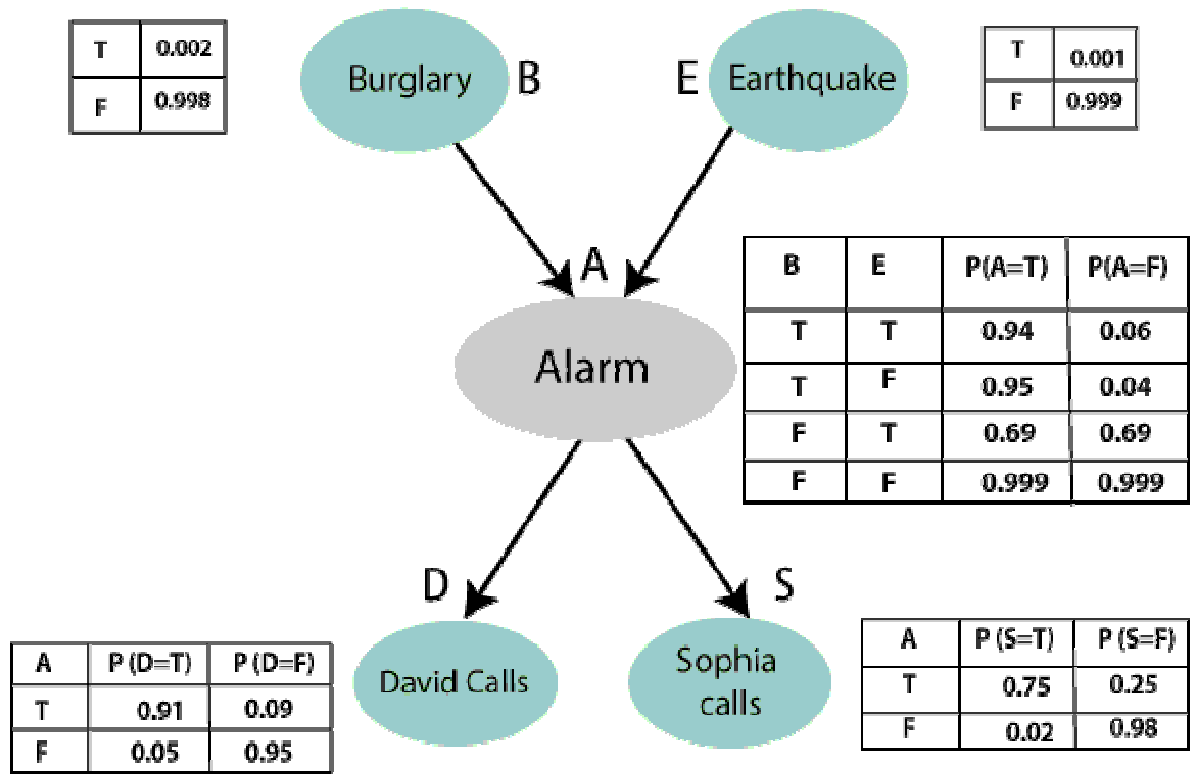
P[D, S, A, B, E]= P[D | S, A, B, E]. P[S, A, B, E]

=P[D | S, A, B, E]. P[S | A, B, E]. P[A, B, E]

= P [D| A]. P [S| A, B, E]. P[A, B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B |E]. P[E]



T	0.002
F	0.998

T	0.001
F	0.999

B	E	P(A=T)	P(A=F)
T	T	0.94	0.06
T	F	0.95	0.04
F	T	0.69	0.69
F	F	0.999	0.999

A	P (D=T)	P (D=F)
T	0.91	0.09
F	0.05	0.95

A	P (S=T)	P (S=F)
T	0.75	0.25
F	0.02	0.98

Let's take the observed probability for the Burglary and earthquake component:

P(B= True) = 0.002, which is the probability of burglary.

P(B= False)= 0.998, which is the probability of no burglary.

P(E= True)= 0.001, which is the probability of a minor earthquake

P(E= False)= 0.999, Which is the probability that an earthquake not occurred.

We can provide the conditional probabilities as per the below tables:

Conditional probability table for Alarm A:

The Conditional probability of Alarm A depends on Burglar and earthquake:

B	E	P(A= True)	P(A= False)
True	True	0.94	0.06
True	False	0.95	0.04
False	True	0.31	0.69
False	False	0.001	0.999

Conditional probability table for David Calls:

The Conditional probability of David that he will call depends on the probability of Alarm.

A	P(D= True)	P(D= False)
True	0.91	0.09
False	0.05	0.95

Conditional probability table for Sophia Calls:

The Conditional probability of Sophia that she calls is depending on its Parent Node "Alarm."

A	P(S= True)	P(S= False)
True	0.75	0.25

False	0.02	0.98

From the formula of joint distribution, we can write the problem statement in the form of probability distribution:

P(S, D, A, ¬B, ¬E) = P (S|A) *P (D|A)*P (A|¬B ^ ¬E) *P (¬B) *P (¬E).

= 0.75* 0.91* 0.001* 0.998*0.999

= 0.00068045.

Hence, a Bayesian network can answer any query about the domain by using Joint distribution.

The semantics of Bayesian Network:

There are two ways to understand the semantics of the Bayesian network, which is given below:

1. To understand the network as the representation of the Joint probability distribution.

It is helpful to understand how to construct the network.

2. To understand the network as an encoding of a collection of conditional independence statements.

It is helpful in designing inference procedure.

Hidden Markov Models

A **hidden Markov model (HMM)** is an augmentation of the Markov chain to include observations. Just like the state transition of the Markov chain, an HMM also includes observations of the state. These observations can be **partial** in that different states can map to the same observation and **noisy** in that the same state can stochastically map to different observations at different times.
The assumptions behind an HMM are that the state at time $t+1$ only depends on the state at time t, as in the Markov chain. The observation at time t only depends on the state at time t. The observations are modeled using the variable O_t for each time t whose domain is the set of possible observations. The belief network representation of an HMM is depicted in Figure 6.14. Although the belief network is shown for four stages, it can proceed indefinitely.

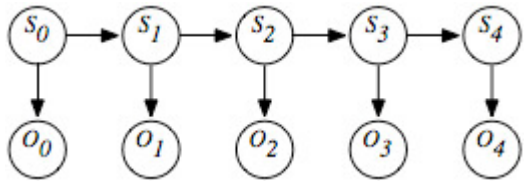


Figure 6.14: A hidden Markov model as a belief network

A stationary HMM includes the following probability distributions:

- $P(S_0)$ specifies initial conditions.
- $P(S_{t+1}/S_t)$ specifies the dynamics.
- $P(O_t/S_t)$ specifies the sensor model.

There are a number of tasks that are common for HMMs.

The problem of **filtering** or belief-state **monitoring** is to determine the current state based on the current and previous observations, namely to determine $P(S_i/O_0,\ldots,O_i)$.

Note that all state and observation variables after S_i are irrelevant because they are not observed and can be ignored when this conditional distribution is computed.

The problem of **smoothing** is to determine a state based on past and future observations. Suppose an agent has observed up to time k and wants to determine the state at time i for $i<k$; the smoothing problem is to determine $P(S_i/O_0,\ldots,O_k)$.

All of the variables S_i and V_i for $i>k$ can be ignored.