AE339 Project
Rocket Nozzle Design using Method of Characteristics
Report

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Contents

1 Introduction

This report delves into the method of characteristics (MoC) and its application in the design of supersonic nozzles. We will explore the basic elements of the MoC, characteristics meshes, finite-difference methods, and the specific process of designing supersonic nozzles to ensure shock-free, isentropic flow.

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2 Elements of the Method of Characteristics

The method of characteristics is pivotal for solving hyperbolic partial differential equations that govern supersonic flows. It transforms these PDEs into a set of ordinary differential equations (ODEs) along characteristic lines where the flow variables are continuous.

2.1 Characteristic Lines

In a two-dimensional, steady, inviscid, supersonic flow, certain lines in the flow field make the derivatives of the flow variables indeterminate. These are known as characteristic lines. They play a critical role in determining the properties of the flow field.

$$\left(1-\frac{u^2}{a^2}\right)\frac{\partial^2\phi}{\partial x^2}+\left(1-\frac{v^2}{a^2}\right)\frac{\partial^2\phi}{\partial y^2}-\frac{2uv}{a^2}\frac{\partial^2\phi}{\partial x\partial y}=0 \tag{1}$$

where $u=\frac{\partial\phi}{\partial x}$ and $v=\frac{\partial\phi}{\partial y}$.

2.2 Characteristic Directions

The slopes of the characteristic lines can be determined by solving the quadratic equation:

$$\left(1-\frac{u^2}{a^2}\right)\left(\frac{dy}{dx}\right)^2+\frac{2uv}{a^2}\frac{dy}{dx}+\left(1-\frac{v^2}{a^2}\right)=0 \tag{2}$$

which simplifies to:

$$\left(\frac{dy}{dx}\right)_{char}=\tan(\theta\pm\mu) \tag{3}$$

where $\mu=\sin^{-1}\left(\frac{1}{M}\right)$ is the Mach angle.

2.3 Compatibility Equations

Along these characteristic lines, the governing PDEs reduce to ODEs known as compatibility equations. For a two-dimensional, steady, inviscid, supersonic flow, these equations are:

$$d\theta-\frac{\sqrt{M^2-1}}{M}\frac{dV}{V}=0 \quad \text{(along } C^- \text{ characteristics)} \tag{4}$$

$$d\theta+\frac{\sqrt{M^2-1}}{M}\frac{dV}{V}=0 \quad \text{(along } C^+ \text{ characteristics)} \tag{5}$$

3 Application to Supersonic Nozzle Design

Designing a supersonic nozzle involves ensuring that the flow within the nozzle is isentropic and shock-free. This can be achieved using the method of characteristics to determine the nozzle contour that will produce the desired exit Mach number.

3.1 Nozzle Flow Properties

The flow properties within the nozzle are defined along a series of characteristics lines that extend from the throat to the exit of the nozzle. These lines are computed using the compatibility equations.

3.2 Internal Points

For internal grid points within the nozzle, the properties are computed as follows:

1. Determine the intersection of the C^- characteristic from point 1 and the C^+ characteristic from point 2.
2. Use the compatibility equations to solve for the flow properties at the new point.

3.3 Wall Points

For points on the nozzle wall, the procedure is slightly different due to the known wall boundary conditions.

1. The slope of the wall (θ) is known at the wall points.
2. Use the C^- characteristic and the wall boundary condition to find the flow properties at the wall.

4 Supersonic Nozzle Design

To design a supersonic nozzle for a given exit Mach number (M_e), the method of characteristics is used to ensure that the nozzle contour produces isentropic, shock-free flow.

4.1 Initial Conditions

The design starts with the known flow properties at the throat, where the Mach number is 1. The limiting characteristic is used as the initial data line for the method of characteristics.

4.2 Expansion Section

The section of the nozzle where the flow expands from Mach 1 to the desired exit Mach number. The wall angle increases in this section.

4.3 Straightening Section

Beyond the expansion section, the wall contour is designed to straighten the flow, ensuring that no expansion waves reflect back into the nozzle.

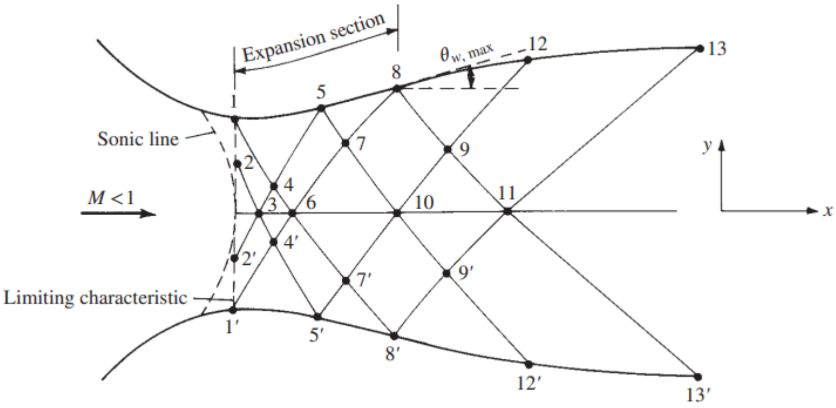

Figure 1: Schematic of supersonic nozzle design using the method of characteristics.

5 CFD Simulation

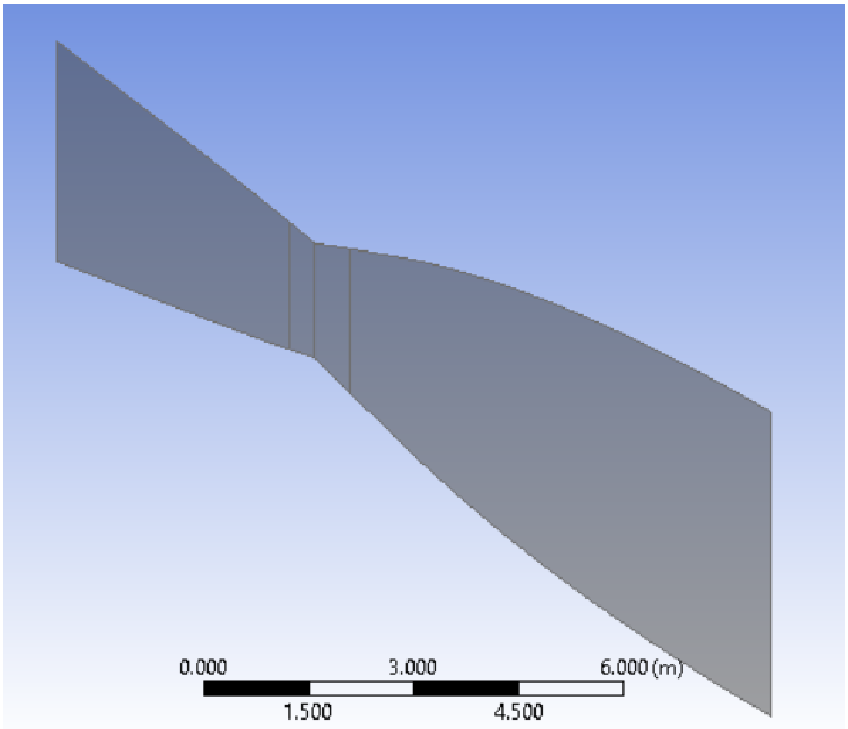

Figure 2: CAD Model of our Nozzle

Output Mach number is 2.47, close to our required design Mach Number of 2.5

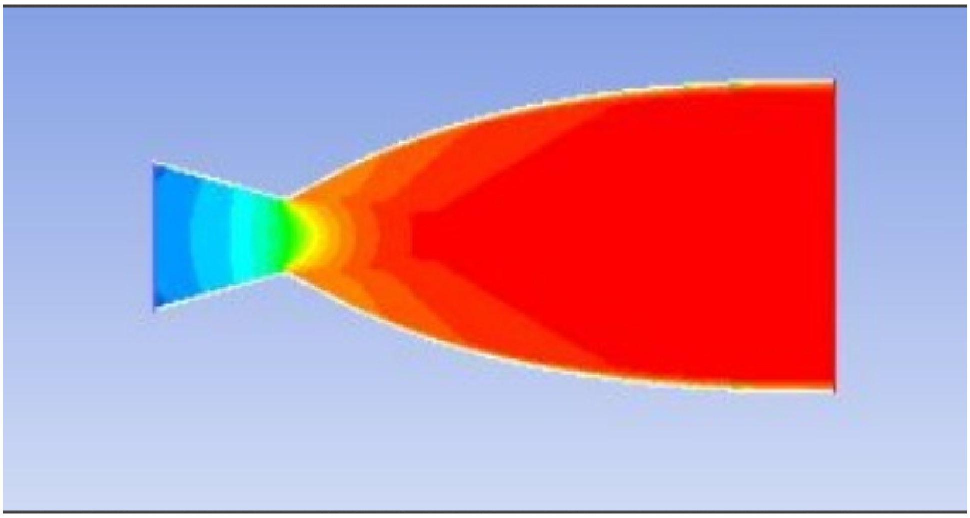

Figure 3: CFD Plot of Velocity Magnitude

6 Conclusion

The method of characteristics is a powerful tool for designing supersonic nozzles, ensuring shock-free, isentropic flow. By transforming the governing PDEs into ODEs along characteristic lines, the MoC provides a systematic approach to determine the nozzle contour that will achieve the desired flow properties.

References

1. Fundamentals of Aerodynamics, John D. Anderson Jr.

2. Computational Fluid Dynamics, J. Blazek