

Proof of Learning - Definitions and Practice

IE506 Course Project

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Presentation Outline

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Background

Proof of learning?

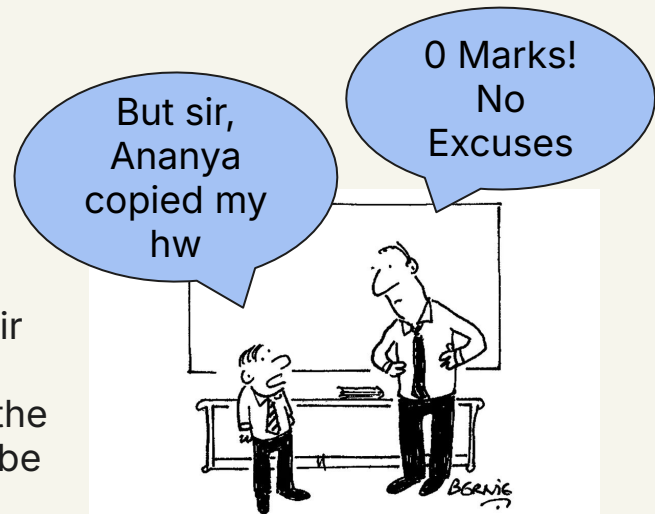
- Cryptographic framework that allows prover to convince a verifier that they have correctly trained an ML model from a given dataset
- Does so without revealing the model itself

Basic Cryptography

- Prover encrypts with verifier's public key and signs with their own private key
- Only the verifier can decrypt with their private key while at the same time ensuring that the signature of the prover cannot be tampered with

Byzantine workers

- malicious or faulty nodes in a distributed machine learning system that behave unpredictably
- They can send incorrect or manipulated updates to disrupt the



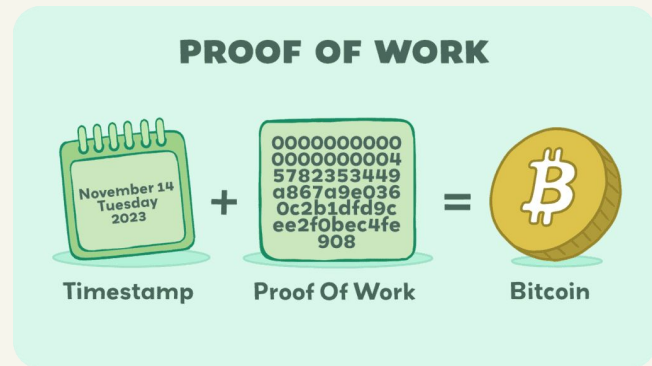
Background

Stochastic Gradient Descent

- The gradient is calculated for each training example (or a small subset of training examples) rather than the entire dataset
- Useful for larger datasets where computing gradient for all points will be computationally very expensive
- Noisy updates can help the model escape local minima or saddle points, potentially leading to better solutions in non-convex optimization problems (common in DL).

Proof of work in Bitcoin

- Ensures that miners must solve a computationally difficult problem before they can add a new block to the blockchain
- Must find a valid nonce, once found they broadcast it to all nodes, verified by other nodes



Problems Addressed

Model Ownership Verification: When multiple parties claim ownership of the same model, there's currently no way to verify who actually performed the training computation.

Byzantine-Resilient Distributed Training: In distributed training settings with untrusted workers, malicious participants could sabotage the process by returning incorrect model updates without detection.



Motivation

- **Shortcomings of ML models**
 - Intellectual Property Protection
 - High Training Costs
 - Distributed Training Security
 - Model Authenticity
 - Preventing Shortcuts
 - Establishing Provenance
- Enough work on watermarking and inference but **no work** on verifying who did the training



Past Work: Stealing Machine Learning Models via Prediction APIs

- The paper investigates how adversaries can "steal" machine learning models that are deployed as services (**ML-as-a-service**), even when they only have black-box access
- The attacks were successfully tested against real-world ML services including BigML and Amazon Machine Learning

Paper talks about:

- Simple equation-solving model extraction attacks
- A new path-finding algorithm for extracting decision trees
- Model extraction attacks against models that output only class labels

Past Work: Stealing Machine Learning Models via Prediction APIs

Simple equation-solving model extraction attacks

- Works for logistic regression and DNNs
- attacker can collect samples (input, class probabilities), treat them as equations, and solve for the model parameters, effectively extracting the model.

A new path-finding algorithm for extracting decision trees

- By modifying input values, the attacker finds decision predicates.
- APIs handling such inputs help reconstruct the tree structure using functions that return leaf or node identifiers.

Model extraction attacks against models that output only class labels

- Use line searches to find points near the decision boundary
- From these samples, it reconstructs the weight vector and bias.
- Uses retraining to achieve low training errors

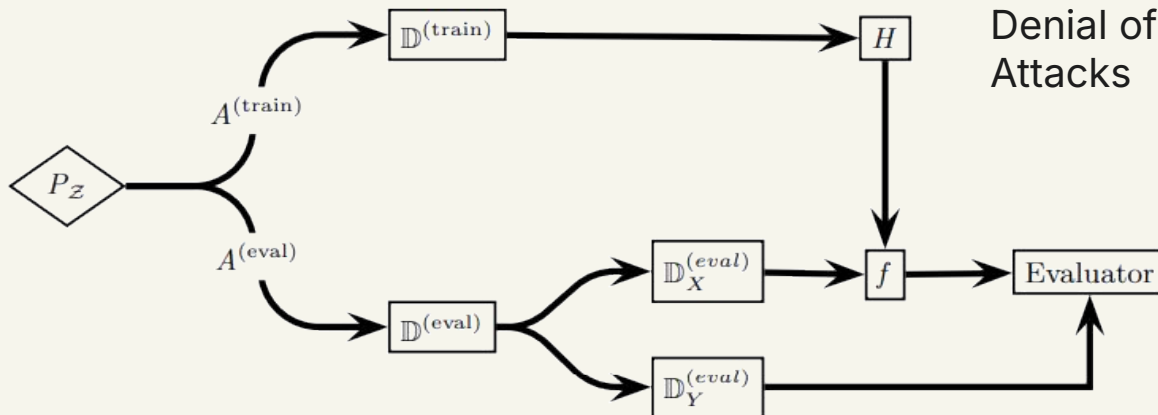
Drawing from this, our paper aims to curb the vulnerabilities discussed in this paper

Past Work : Adversarial Machine Learning

Players–

- **Defender**: Finds a learning model H that is immune to attacks
- **Attacker** : Adversary trying to disrupt the learning process by transforming Training $A^{(Train)}$ & Evaluation $A^{(Eval)}$ Datasets

**Model
as a
Game!**



Taxonomy of Attacks

- **Causative** :
Manipulate model by injecting malicious training data
- **Integrity** :
Giving false negatives during evaluation without being noticeable
- **Availability** :
Denial of Service (DoS) Attacks

Case-study: SpamBayes

Methodology of Filter

- Content-based statistical spam filter that classifies using token count
- Evaluates a spam score of the mail by considering the spam score of its tokens and running statistical tests for conclusive remarks
- The score is compared against 2 thresholds to classify the mail as spam, ham (not spam) or unsure

Attack Strategy

- Causative availability attack (**Denial of Service**)
- Adversary spams the receiver mails with a large set of tokens (**attack's dictionary**) that they believe are contained in the legitimate mails.
- After some training, the spam score of all those tokens will increase. This will cause the legitimate mails to get marked as spam (**false negative**) and potentially spam mails to get marked as ham (**false positive**).

Defence Strategy

- Uses **RONI** technique (**Reject on Negative Impact**)
- The model maintains of list of tokens that are considered harmful and **disregards them for spam score** useage.
- To find such tokens, it trains twice: once using a base training dataset and other including the token and calculates misclassification rate in both cases.
- If the addition of the token increases misclassification rate above a threshold, then that token is rejected for use.

Problem Setup

- Models are parameterized functions f_W that map from input space X to output space Y
- W represents the model parameters (weights).
- **Stochastic gradient descent** over T steps (E epochs); in each step:
 - Mini batch sampling
 - Gradient computation of loss function
 - Updating parameters according to:
$$W_i = W_{i-1} - \eta \nabla L_{i-1}$$
- SGD introduces **secret information** unique to training process due to **randomness**
- Proof-of-Learning (**Pol**) protocol between:
 - A **prover** T who trains the model and generates a certificate $P(T \rightarrow f_W T)$
 - A **verifier** V who analyzes this certificate
 - An **adversary** A who might try to claim ownership without performing training

Problem Setup

- Four types of spoofing scenarios are considered:
 - **Retraining-based spoofing**: Creating the exact same PoL
 - **Stochastic spoofing**: Creating a valid but different PoL
 - **Structurally correct spoofing**: Creating an invalid PoL that passes verification
 - **Distillation-based spoofing**: Creating a valid PoL for a modified model
- **W**: model-specific information from training; **I**: information about data points used; **H**: signatures of training data points; **A**: auxiliary info about hyperparams and architecture
- Verification cost should be less than training cost:
- Adversary's cost should be at least as high as honest training:

$$\begin{aligned} E[CV] &< E[CT] \\ E[CT] &\leq E[CA] \end{aligned}$$

PoL Generation

- PoL := (**W**, **I**, **H**, **A**)
- **During Training –**
 - Save **weights** (W_t) at every k steps
 - Save **data points** (I_t) corresponding to every step
 - Sign the data points and save the **signature** (H_t)
 - Save the corresponding **hyperparameter values** (A_t) at every step
- **During model release –**
 - Encrypt the PoL with verifier's (V) public key [$R := \text{enc}(P(f_{WT}), K_{V, \text{pub}})$]
 - **Sign** the encrypted proof with your **private key**
 - Publish/timestamp the signature to a **public ledger** (prevents replay attacks)

Algorithm for PoL Creation

- **init()** : sets initial weights –
 - Transfer Learning
 - Sampled from a distribution
- **getBatches()** : randomly distributes dataset indices into batches forming T sets of indices
- **update()** : updates $W_t \rightarrow W_{(t+1)}$ using gradient descent update rule

Algorithm 1 PoL Creation

Require: Dataset D , Training metadata M

Require: \mathcal{V} 's public key $K_{\mathcal{V}}^{pub}$

Require: E, S, k \triangleright Number of epochs, steps per epoch, checkpointing interval

Optional: W_0, ζ \triangleright Initialization weight and strategy

```
1:  $\mathbb{W} \leftarrow \{\}, \mathbb{I} \leftarrow \{\}, \mathbb{H} \leftarrow \{\}, \mathbb{M} \leftarrow \{\}$ 
2: if  $W_0 = \emptyset$  then
3:    $M_0 \leftarrow \zeta$ 
4:    $W_0 \leftarrow \text{init}(\zeta)$ 
5: for  $e \leftarrow 0, \dots, E - 1$  do  $\triangleright$  Training epochs
6:    $I \leftarrow \text{getBatches}(D, S)$ 
7:   for  $s \leftarrow 0, \dots, S - 1$  do  $\triangleright$  steps per epoch
8:      $t = e \cdot S + s$ 
9:      $W_{t+1} \leftarrow \text{update}(W_t, D[I_s], M_t)$ 
10:     $\mathbb{I}.\text{append}(I_t)$ 
11:     $\mathbb{H}.\text{append}(\text{h}(D[I_t]))$ 
12:     $\mathbb{M}.\text{append}(M_t)$ 
13:    if  $t \bmod k = 0$  then
14:       $\mathbb{W}.\text{append}(W_t)$ 
15:    else
16:       $\mathbb{W}.\text{append}(\text{nil})$ 
17:  $\mathbb{A} \leftarrow \{\mathbb{M}\}$ 
18:  $\mathcal{R} \leftarrow \text{enc}((\mathbb{W}, \mathbb{I}, \mathbb{H}, \mathbb{A}), K_{\mathcal{V}}^{pub})$ 
19: return  $\mathcal{R}, \text{h}(\mathcal{R}, K_{\mathcal{T}}^{priv})$ 
```

PoL Verification

- **PoL Extraction**

- Decrypt the PoL using V's private key to get PoL

- **Signature Verification (verifier require dataset)**

- Public Dataset: Immediate verification
- Private Dataset: Lazy Verification

Queries prover → gives dataset to verifier → verifies with published signature

- **Initial Weights Verification**

- Transfer Learning → Verifies PoL of prior model
- Distribution sampling → Using statistical tests (KS Test)

- **Verification of Weight update per checkpoint interval (k)**

- If $\text{distance}(\text{computed}(W_k), W_k) > \delta$: FAIL
- Focuses verification on Q largest model updates (sorted from mag list)
- Calibration of δ based on hardware, architecture, dataset, & learning parameters
- $\text{distance}(\text{computed}(W_k), W_k) < \delta \quad \forall t \text{ multiple of } k, \forall \text{ epochs} \Rightarrow$

#VERIFIED

Algorithm for PoL Creation

Please
Read Me :(

Verification Success Rate (VSR):

- Defines probability that verifier accepts a PoL
- Depends on probability of calculated updates landing within ε -ball of purported weights
- Models as product of independent probabilities due to Markovian nature of gradient descent

$$\Pr[\text{VERIFY}[\mathbb{W}, \phi] = 1] =$$

$$\prod_{e=1}^E \prod_{q=1}^Q \Pr[Tr_{e,q,k} \wedge \text{dist}_{e,q+k} \leq \delta \mid \phi]$$

$$= \prod_{e=1}^E \prod_{q=1}^Q \Pr[\text{dist}_{e,q+k} \leq \delta \mid \phi] \cdot \Pr[Tr_{e,q,k} \mid \phi]$$

Algorithm 2 Verifying a PoL

```

1: function VERIFY( $\mathcal{R}, \mathcal{R}^0, K_V^{priv}, f, D, Q, \delta$ )  $\triangleright$  encrypted
    $\mathcal{P}oLs, \mathcal{V}$ 's private key, model, dataset, query budget, slack parameter
2:    $\mathbb{W}, \mathbb{I}, \mathbb{H}, \mathbb{M} \leftarrow \text{dec}(\mathcal{R}, K_V^{priv})$ 
3:   if  $\mathcal{R}^0 = \emptyset$  then
4:     if VERIFYINITIALIZATION( $\mathbb{W}_0$ ) = FAIL then
5:       return FAIL
6:   else if VERIFYINITPROOF( $\mathcal{R}^0$ ) = FAIL then
7:     return FAIL
8:    $e \leftarrow 0$   $\triangleright$  Epoch counter
9:    $mag \leftarrow \{\}$   $\triangleright$  List of model update magnitudes
10:  for  $t \leftarrow 0, \dots, T-1$  do  $\triangleright$  training step
11:    if  $t \bmod k = 0 \wedge t \neq 0$  then
12:       $mag.append(d_1(\mathbb{W}_t - \mathbb{W}_{t-k}))$ 
13:     $e_t = \lfloor \frac{t}{S} \rfloor$   $\triangleright$  Recovering the epoch number
14:    if  $e_t = e + 1$  then
15:       $\triangleright$  New epoch started. Verify the last epoch
16:       $idx \leftarrow \text{sortedIndices}(mag, \downarrow)$ 
17:       $\triangleright$  get indices for decreasing order of magnitude
18:      if VERIFYEPOCH( $idx$ ) = FAIL then
19:        return FAIL
20:       $e \leftarrow e_t, mag \leftarrow \{\}$ 
21:  return Success
22:  function VERIFYEPOCH( $idx$ )
23:    for  $q \leftarrow 1, \dots, Q$  do
24:       $t = idx[q-1]$   $\triangleright$  index of  $q$ 'th largest update
25:       $H_t \leftarrow \mathbb{H}_t, I_t \leftarrow \mathbb{I}_t$ 
26:      VERIFYDATASIGNATURE( $H_t, D[I_t]$ )
27:       $W'_t \leftarrow \mathbb{W}_t$ 
28:      for  $i \leftarrow 0, \dots, k-1$  do
29:         $I_{t+i} \leftarrow \mathbb{I}_{t+i}, M_{t+i} \leftarrow \mathbb{M}_{t+i}$ 
30:         $W'_{t+i+1} \leftarrow \text{update}(W'_{t+i}, D[I_{t+i}], M_{t+i})$ 
31:         $W_{t+k} \leftarrow \mathbb{W}_{t+k}$ 
32:        if  $d_2(W'_{t+k}, W_{t+k}) > \delta$  then  $\triangleright$  Dist. func.  $d_2$ 
33:          return FAIL
34:  return Success

```


Practical Considerations

- **Private Dataset Handling:** When datasets are private, the trainer publishes data signatures rather than the data itself, enabling "lazy verification" where actual data is only revealed when verification is needed.
- **Data Transfer Requirements:** For lazy verification, the expected amount of data needed is calculated as $D[1-(1-Qk/S)^E]$, where Q is verifications per epoch, k is update steps, S is dataset size, and E is epochs.
- **Chain of Trust:** To prevent claims of "lucky initialization" with stolen models, proofs require reference to previous proofs (P_0) that verify initial weights, creating a verification chain with combined success rates.
- **Initialization Verification:** For models starting from random initialization, statistical methods (Kolmogorov-Smirnov test) verify if weights match claimed initialization distributions, with layer-by-layer testing.
- **Constraint on Initialization Strategies:** Both trainers and adversaries must use publicly known initialization strategies to prevent adversarial manipulation.

Kolmogorov–Smirnov test

- Compares empirical and theoretical cumulative distribution functions to determine if weights match claim.
- A **p-value** below the significance level indicates invalid initialization
- Assumes that **layer initialisations are independent**, Bonferroni's normalisation required if they are not independent

Stationary Markov Process

- **Markov process** i.e. future state depends only on its current state, not past states.
- It is also **stationary**, assuming fixed randomness
- This property enables in-place model updates in ML frameworks like PyTorch and TensorFlow.
- Model update is:

$$\tilde{W}_{t+1} = \tilde{W}_t - \eta \nabla_{\tilde{W}_t} \hat{\mathcal{L}}_t + z_t,$$

Entropy Growth

- uncertainty or variance in the gradient descent paths taken during training.
- entropy **increases linearly** with training steps.
- Because entropy is defined logarithmically in probability, this translates to an **exponential** increase in **possible training sequences**
- the presence of **small random variations** (e.g., hardware noise, cuDNN library behavior) makes exact model reproduction impossible

EXPERIMENTAL SETUP

- ResNet-20 and ResNet-50 were trained on CIFAR-10 and CIFAR-100 respectively
- CIFAR-10 only has 10 classes whereas CIFAR-100 has 100 classes
- Each of the two datasets is composed of 50,000 training images and 10,000 testing images, each of size 32 32 3
- Both models are trained for 200 epochs with batch size being 128

EVALUATION METRICS

- entropy growth in training makes exact reproduction of a model's training sequence impossible
- verification relies on checking small intervals where the reproduction error remains below a reference threshold.

DETERMINISTIC APPROACHES

- Using deterministic operations in PyTorch reduces training randomness and improves reproducibility but still results in significant errors, incurs high computational costs, and can lower model accuracy.

CHECKPOINTING INTERVAL

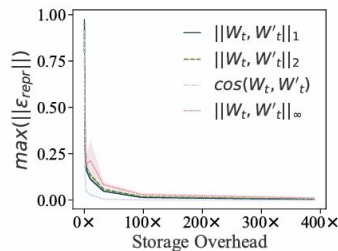
- The study finds that saving checkpoints at every step ($k=1$) is unnecessary, as using $k=S$ maintains accuracy while reducing storage needs
- To save storage costs save checkpoints in **float16** rather than float32

VARYING LEARNING RATE

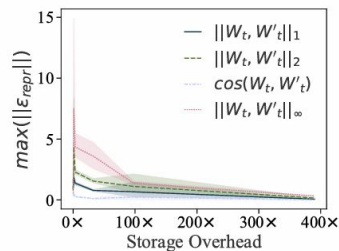
- Learning rate and reproduction error are correlated
- when **learning rate is too large, the training process is unstable** so a tiny difference may lead to distinct parameters after a few steps.

INITIALISATION VERIFICATION

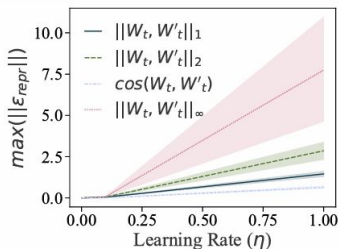
- For both models, the **minimum p-value** across all network layers **drops to 0 rapidly**.
- This means that the weight distribution for at least one of the layers is statistically different from the initialization distribution



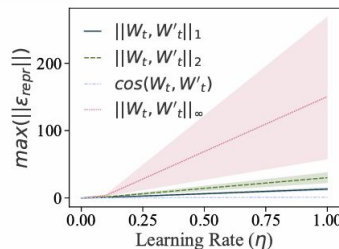
(a) CIFAR-10



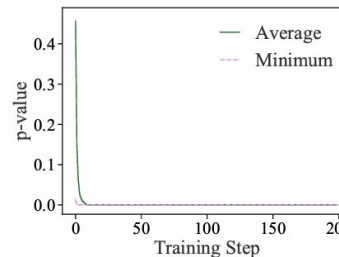
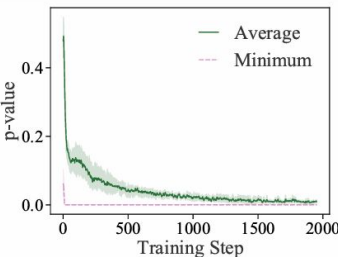
(b) CIFAR-100



(a) CIFAR-10



(b) CIFAR-100



INVERSE GRADIENT METHOD - ISSUES

- Attempts to reconstruct the initial weights W_0 from the final weights W_T by iteratively solving the inverse of the stochastic gradient descent (SGD) step

$$\beta(W_{t-1}) := W_{t-1} - W_t - \eta \nabla_{W_{t-1}} \mathcal{L} = 0$$

- Since different training paths can lead to the same final weights, an adversary trying to spoof training using the inverse gradient method will face high **uncertainty**
- Reversing a training sequence has at least as much entropy as forward training, making exact reconstruction nearly impossible due to exponentially many training paths as in DNNs (so **retraining based spoofing** is impractical)
- **Stochastic spoofing** won't work as the adversary still faces a computational cost at least as large as that for T and it is difficult to end in a suitable random initialization.

INVERSE GRADIENT METHOD - ISSUES

- Inverting a training step is at least as computationally expensive as training due to the non-linearity of DNNs and the lack of analytical solutions.
- Increasing learning rates to reduce computational costs leads to high reconstruction errors, making this approach ineffective

DIFFICULTY IN FINDING SUITABLE INITIALISATION

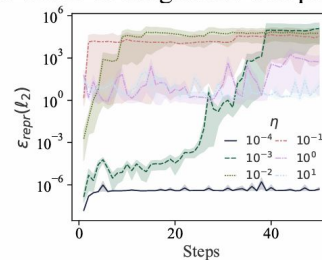
- Empirical tests on CIFAR-10 and CIFAR-100 show that inverse gradient methods fail to produce valid initializations, with p-values far below the threshold.
- Even directed approaches, fine-pruning, or sparsification fail to pass verification (KS test)

PoL CONCATENATION

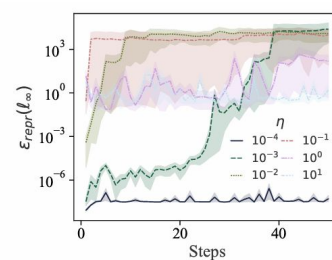
- The difference between WT and Ws is significantly larger than valid gradient updates, making it easy to detect when $Q=1$. Also, if discontinuity magnitude matches d_{ref} , indicating WT and Ws are unrelated.
- If the verifier randomly samples updates instead of selecting the largest, the probability of detecting the discontinuity is $1/S$, making verification unreliable
- To counter adversaries exploiting small Q values, potential solutions include increasing Q , random verification, or periodically checking model performance to catch large, unnatural updates.

DIRECTED WEIGHT MINIMIZATION

- An adversary may attempt to retrain toward WT by adding a regularization term that minimizes weight distance but they require knowledge of WT.
- The required information cannot be encoded in synthetic data, as no gradient exists for the regularization term. Tactics like adaptive learning rate tuning to reach W also fail verification.



(a) ℓ_2 distance



(b) ℓ_∞ distance

Dataset

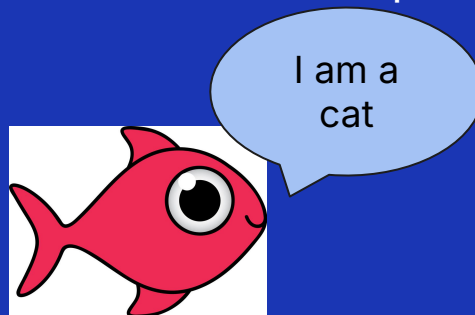
CIFAR-10

- ❖ Image Classification Dataset
- ❖ 32×32 RGB Images
- ❖ 60,000 Total Images
- ❖ 10 Classes (Plane, Bird, Cat, Frog,...)
- ❖ 6,000 Images per class



CIFAR-100

- ❖ Image Classification Dataset
- ❖ 32×32 RGB Images
- ❖ 60,000 Total Images
- ❖ 100 Classes
- ❖ 20 Superclasses
- ❖ 600 images per class
- ❖ Each image has 2 labels-
 - Fine : Its class
 - Coarse : Its superclass



Published Code

Language : **Python**
 Framework: **Pytorch**

Link: <https://github.com/cleverhans-lab/Proof-of-Learning>

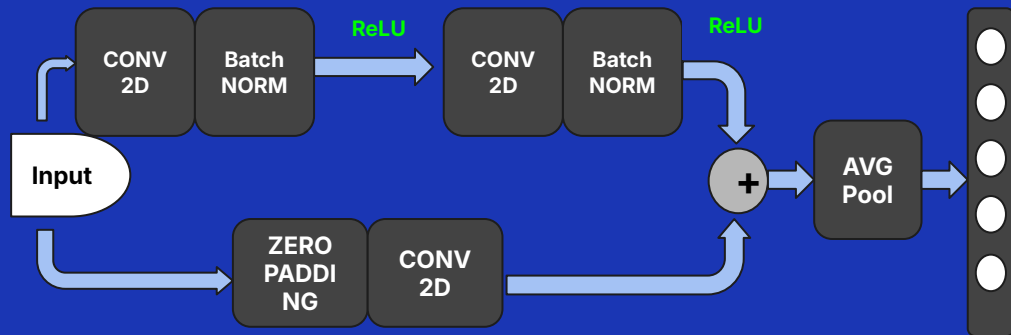
Weight Initialization: **Kaiming (He) Init.**

Model Architectures

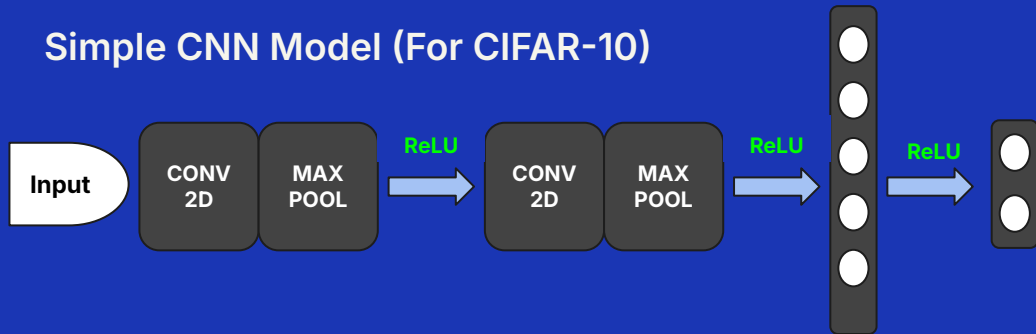
ResNet Model

For ResNet20:

- 1 Initial Layer: Conv2D → Batch Norm → ReLU
- 3 Layers, each with 3 ResNet Blocks (shown below)



Simple CNN Model (For CIFAR-10)



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Contribution

Ananya - understanding paper, theory slides, past work on model stealing attacks

Anuttar - understanding paper, code slides, finding new datasets, past work on Adversarial machine learning

AI Tools

- We used ChatGPT (GPT 4o) and Claude 3.7 in order to understand the paper better by comparing the summarisations it gave with our own understanding of the paper
- Asked it to generate examples that were helpful in understanding the meaning of the text
- Also used it to phrase our points more concisely in the presentation
- Used to understand the hard to decipher lines of code and class definitions formed in the code
- Used to understand the working and implementation of built-in functions of Pytorch and other unknown function