Large Deformation Modelling
in Thin, Flexible & Tunable Membranes

AE 493: B.Tech. Project - I

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Thin Membrane Characteristics

Bulk Body

- Compression and Tension dominate
- Transverse stresses balance transverse loads
- Needs complete 3D formulation
- Tougher to buckle

Thin Membrane

- Bending effects also dominate
- In-plane stresses balance transverse loads
- Dimension reduction to 2D using Plane Stress
- Buckles easily by wrinkling

Energy Relations

- The thickness is much smaller than the other dimensions.

- As a result,

  Energy with through-thickness deformations << Bending Energy << Energy with in-plane deformations

- In terms of thickness h,

  Energy associated with in-plane deformations $\sim O(h)$
  Energy associated with bending deformations $\sim O(h^3)$
  Energy associated with through-thickness def. $\sim O(h^5)$

Plane Stress Formulation

- Due to the negligible magnitudes of transverse normal and shear stresses, we neglect them, ending up with a **Plane Stress Formulation.**

- The matter of interest to us are the in-plane components, related as:

$$\boldsymbol{\sigma} = \frac{\mathrm{E}}{1-\nu^2}\left[\nu(tr\mathbf{E})\mathbf{I} + (1-\nu)\mathbf{E}\right]$$

  where E is Young's Modulus and $\boldsymbol{\nu}$ is the Poisson's Ratio

- From linear Isotropic relations,

  $\sigma_{xz} = 0 \rightarrow E_{xz} = 0$, $\sigma_{yz} = 0 \rightarrow E_{yz} = 0$, $\sigma_{zz} = 0 \rightarrow E_{zz} \neq 0$

  E_{zz} is the only non zero term, but we usually don't care about it. (small magnitude)

Assumptions

List of Assumptions

1. Membrane is flat in material frame

2. Kirchhoff - Love Assumptions

3. Total Strain Energy can be decoupled into Membrane and Bending parts

4. Material obeys linear isotropy

5. Linearised Curvature Tensor

6. Small Deformations *

*(is relaxed later)

Kirchhoff – Love Assumptions

1. Normals to the flat surface remain straight after deformation.

2. Normals remain of the same length and stay normal to the deformed surface after deformation.

Works in the same vein as the assumption we take in beam theory regarding cross-sectional area of the beam

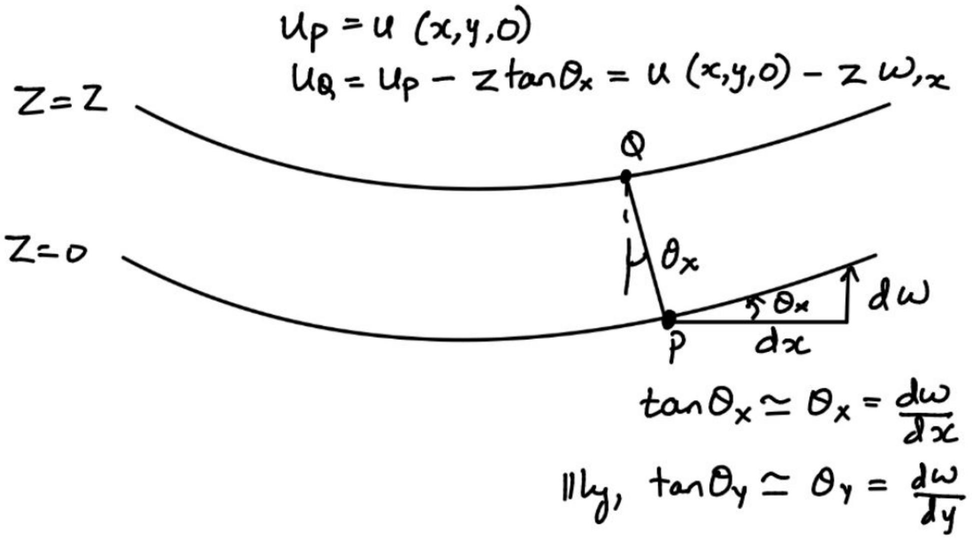
Kinematic Relations

- Based on Kirchhoff - Love Assumptions, we arrive at the following relations

$$u_1(x,y,z) = u_1(x,y,0) - z\frac{\partial w}{\partial x}\bigg|_{z=0}$$

$$u_2(x,y,z) = u_2(x,y,0) - z\frac{\partial w}{\partial y}\bigg|_{z=0}$$

$$w(x,y,z) = w(x,y,0)$$



Thin Membrane Models

List of Models Studied

- Kirchhoff - Love Model   (Classical Plate Theory)
- Foppl von-Karman Model
- Large Deformation Model

Classical Plate Theory

- Gives a completely **linear** model for thin membranes.
- One of the oldest models but is still widely accepted.
- Uses the following strain relation: $\mathbf{E} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$
- Plate Equations:

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}$$

$$D\nabla^4 w = q(x, y)$$

Foppl von – Karman Model

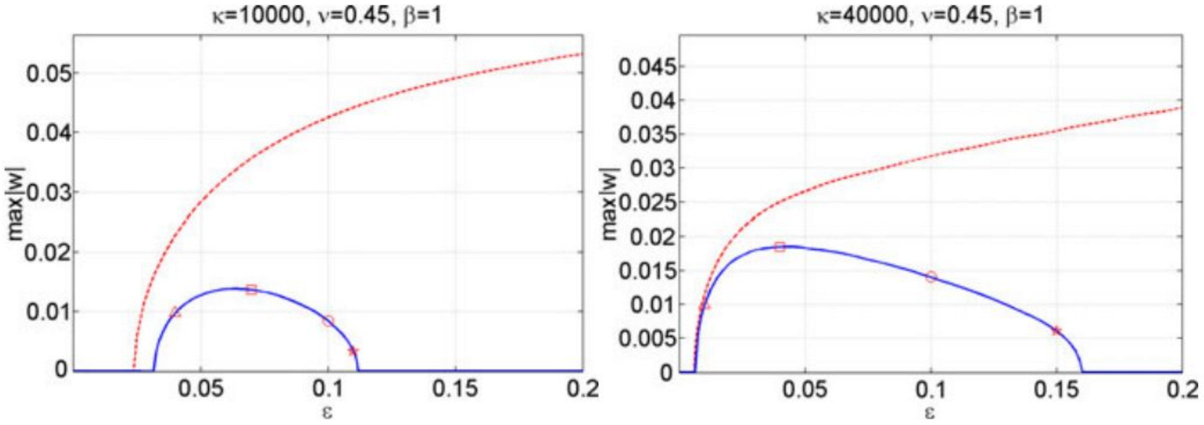
- An improvised version of Kirchhoff - Love Model involving non-linear terms.
- Works only in the small deformation regime.
- Strain Expression used: $\mathbf{E} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T + \nabla w \otimes \nabla w\right)$
- Plate Equations:

$$D\nabla^4 w \;-\; [\chi, w] = q(x,y)$$

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}$$

Drawbacks

- Fails under moderate to large deformations
- Does not bring any big advantage compared to Classical Plate Theory
- Gives incorrect bifurcation behaviour that does not match with experimental observations



(a) (b)

Large Deformation Model

- Strain Expression:

$$\mathbf{E}_m = \frac{1}{2}\left[\ \nabla\mathbf{u} + (\nabla\mathbf{u})^T + (\nabla\mathbf{u})^T(\nabla\mathbf{u}) + \nabla w \otimes \nabla w\right]$$

- Plate Equations:

$$\nabla \cdot [(\mathbf{I} + \nabla\mathbf{u})\mathbf{N}] + \mathbf{f}_m \quad = \mathbf{0}$$

$$\nabla \cdot [\nabla \cdot \mathbf{M} + \mathbf{N}\nabla w] + f_b = 0$$

- Natural Boundary Conditions:

$$\oint_{\Gamma_0} \left([(\mathbf{I} + \nabla\mathbf{u})\mathbf{N}]\,\mathbf{n} - \mathbf{t}_m \right) \cdot \eta \, d\Gamma = 0$$

$$\oint_{\Gamma_0} \Big(([\mathbf{N}\nabla w + \nabla \cdot \mathbf{M}] \cdot \mathbf{n} - t_b)\zeta - (\mathbf{M}\mathbf{n}) \cdot \nabla\zeta \Big) d\Gamma = 0$$

Large Deformation Model

PDE System

$$(1) \qquad \nabla \cdot [(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}] + \mathbf{f}_m \quad = \mathbf{0}$$

$$(2) \qquad \nabla \cdot [\nabla \cdot \mathbf{M} + \mathbf{N} \nabla w] + f_b = 0$$

$$(3) \qquad \mathbf{N} = \frac{\partial \Psi_m}{\partial \mathbf{E}}$$

$$(4) \qquad \mathbf{M} = \frac{\partial \Psi_b}{\partial \mathbf{K}}$$

$$(5) \qquad \mathbf{E} = \frac{1}{2}\left[\, \nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T (\nabla \mathbf{u}) + \nabla w \otimes \nabla w \right]$$

$$(6) \qquad \mathbf{K} = -\nabla^2 w$$

Large Deformation Model

Advantages

- Applicable under large deformations
- Able to predict post-buckling and bifurcation behaviours accurately
- Model is not confined for isotropic materials and can be generalised to any material, such as hyperelastic materials or magneto-elastic polymers
- Applicable for any flat geometry taken as initial configuration

Large Deformation Model

Disadvantages

- The plate equations are nonlinear PDEs and have no known analytical or semi-analytical solutions. The solution can only be found via numerical methods.

- Works only if the geometry is flat in material frame. Model will collapse under arbitrary surfaces.

- Assumes decoupling of membrane and bending energies in Cartesian coordinate system, which introduces inaccuracies. The decoupling assumption needs to adapt to curvilinear coordinate systems.

Derivation

Model Description

- Let there be a thin, flat membrane Ω_0, with thickness h, which is deformed to Ω_t under the deformation map φ: $\Omega_0 \to \Omega_t$
- Let **X** and **x** be the position vectors in undeformed and deformed configurations respectively.

$$\mathbf{X} = X_1\mathbf{e}_1 + X_2\mathbf{e}_2 \in \Omega_0 \qquad \mathbf{x} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3 \in \Omega_t$$

- Let **u**(**X**) and w(**X**) be the in-plane and out-of-plane displacements, resp.

$$\mathbf{u}(\mathbf{X}) = u_1(\mathbf{X})\mathbf{e}_1 + u_2(\mathbf{X})\mathbf{e}_2$$

- Under the deformation map φ,

$$\mathbf{x} = \varphi(\mathbf{X}) = \mathbf{X} + \mathbf{u}(\mathbf{X}) + w(\mathbf{X})\,\mathbf{e}_3$$

Deformation Gradient

$$\mathbf{F} = \nabla \varphi = \mathbf{I} + \nabla \mathbf{u} + \mathbf{e}_3 \otimes \nabla w.$$

Membrane Strain Tensor

$$\mathbf{E} = \frac{1}{2}(\ \mathbf{F}^T \mathbf{F} - \mathbf{I}\ )$$

$$\begin{aligned}\mathbf{E} = \frac{1}{2}[\ \nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T (\nabla \mathbf{u}) + (\nabla w \otimes \mathbf{e}_3)(\mathbf{e}_3 \otimes \nabla w)\ ] \\ + \frac{1}{2}[\ \nabla w \otimes \mathbf{e}_3 + (\nabla w \otimes \mathbf{e}_3) \nabla \mathbf{u} + \mathbf{e}_3 \otimes \nabla w + (\nabla \mathbf{u})^T (\mathbf{e}_3 \otimes \nabla w)\ ]\end{aligned}$$

$$\mathbf{E}_m = \frac{1}{2}\left[\ \nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T (\nabla \mathbf{u}) + \nabla w \otimes \nabla w\right]$$

Decoupling Total Strain Energy

- Total Strain Energy Density can be decomposed into Membrane and Bending components under small thickness assumption

$$\Psi(\mathbf{E}, \mathbf{K}) = \Psi_m(\mathbf{E}) + \Psi_b(\mathbf{K})$$

Derivation

- Substituting the kinematic relations obtained from Kirchhoff – Love assumptions into the membrane strain expression,

$$E_{\alpha\beta} \approx E_{\alpha\beta,0} + E_{\alpha\beta,z}\, z$$

$$E_{\alpha\beta,0} = \frac{1}{2}(u_{\alpha 0,\beta} + u_{\beta 0,\alpha} + w_{0,\alpha} w_{0,\beta})$$

$$E_{\alpha\beta,z} = -w_{0,\alpha\beta}.$$

- Substituting the Strain expression into the linear strain energy density expression,

$$\phi = \frac{1}{2}\mathbf{E} : \mathbb{C} : \mathbf{E}$$

$$\phi \approx \frac{1}{2}(\mathbf{E}_0 : \mathbb{C} : \mathbf{E}_0) + \frac{1}{2}(\mathbf{E}_0 : \mathbb{C} : \mathbf{E}_{,z} + \mathbf{E}_{,z} : \mathbb{C} : \mathbf{E}_0)z$$

- Integrating along thickness to get thickness-averages strain energy,

$$\Psi = \int_{-h/2}^{h/2} \phi \, dz.$$

$$\Psi = \frac{h}{2}(\mathbf{E}_0 : \mathbb{C} : \mathbf{E}_0) + \frac{h^3}{24}(\mathbf{E}_{,z} : \mathbb{C} : \mathbf{E}_{,z})$$

- General expression for the strain energies are:

$$\Psi_m = \frac{h}{2}(\mathbf{E}_0 : \mathbb{C} : \mathbf{E}_0), \qquad \Psi_b = \frac{h^3}{24}(\mathbf{E}_{,z} : \mathbb{C} : \mathbf{E}_{,z})$$

- These expressions, although derived under linear strain assumption, can (and will) be used for any Constitutive relation.

- Decoupling holds really good for Linear Isotropic Materials

$$\Psi_m = \frac{Eh}{2(1-\nu^2)}\left[\nu(\mathrm{tr}\mathbf{E})^2 + (1-\nu)\mathbf{E} : \mathbf{E}\right],$$
$$\Psi_b = \frac{Eh^3}{24(1-\nu^2)}\left[\nu(\mathrm{tr}\mathbf{K})^2 + (1-\nu)\mathbf{K} : \mathbf{K}\right].$$

Stress Tensors

$$\mathbf{N} = \frac{\partial \Psi_m(\mathbf{E})}{\partial \mathbf{E}}, \qquad \mathbf{M} = \frac{\partial \Psi_b(\mathbf{K})}{\partial \mathbf{K}}$$

$$\begin{aligned} D_{\mathbf{U}}[(\operatorname{tr}\mathbf{U})^2][\mathbf{V}] &= (\operatorname{tr}(\mathbf{U}+\mathbf{V}))^2 - (\operatorname{tr}\mathbf{U})^2 + O(\|\mathbf{V}\|^2) \\ &= 2(\operatorname{tr}\mathbf{U})(\operatorname{tr}\mathbf{V}) + O(\|\mathbf{V}\|^2) \\ &= 2(\operatorname{tr}\mathbf{U})\,\mathbf{I} : \mathbf{V}. \end{aligned}$$

$$\begin{aligned} D_{\mathbf{U}}(\mathbf{U}:\mathbf{U})[\mathbf{V}] &= (\mathbf{U}+\mathbf{V}) : (\mathbf{U}+\mathbf{V}) - \mathbf{U}:\mathbf{U} + O(\|\mathbf{V}\|^2) \\ &= 2\mathbf{U} : \mathbf{V} + O(\|\mathbf{V}\|^2). \end{aligned}$$

$$\mathbf{N} = \frac{\mathrm{E}h}{(1-\nu^2)}[\nu(tr\mathbf{E})\mathbf{I} + (1-\nu)\mathbf{E}]$$

$$\mathbf{M} = \frac{\mathrm{E}h^3}{12(1-\nu^2)}[\nu(tr\mathbf{K})\mathbf{I} + (1-\nu)\mathbf{K}]$$

Non-dimensionalisation

- Let “L” be the characteristic length of the model.

$$\mathbf{X} = L\mathbf{X}^*, \qquad \mathbf{u} = L\mathbf{u}^*, \qquad w = Lw^*, \qquad h = Lh^*$$

$$\nabla = \mathbf{e}_1 \frac{\partial}{\partial X_1} + \mathbf{e}_2 \frac{\partial}{\partial X_2} = \frac{1}{L}\left(\mathbf{e}_1 \frac{\partial}{\partial X_1^*} + \mathbf{e}_2 \frac{\partial}{\partial X_2^*}\right) = \frac{1}{L}\nabla^*$$

$$\nabla \mathbf{u} = \left(\frac{1}{L}\nabla^*\right)(L\mathbf{u}^*) = \nabla^* \mathbf{u}^*, \qquad \nabla w = \left(\frac{1}{L}\nabla^*\right)(Lw^*) = \nabla^* w^*$$

$$\mathbf{E} = \frac{1}{2}\left(\nabla^*\mathbf{u}^* + (\nabla^*\mathbf{u}^*)^T + (\nabla^*\mathbf{u}^*)^T(\nabla^*\mathbf{u}^*) + (\nabla^* w^* \otimes \mathbf{e}_3)(\mathbf{e}_3 \otimes \nabla^* w^*)\right) = \mathbf{E}^*$$

$$\mathbf{K} = -\nabla^2 w = -\left(\frac{1}{L}\nabla^*\right)^2 (Lw^*) = -\frac{1}{L}\nabla^{*,2} w^* = \frac{1}{L}\mathbf{K}^*$$

$$\lambda = \frac{Eh}{12(1-\nu^2)}$$

$$\Psi_m = 6\lambda \left[\nu(\mathrm{tr}\,\mathbf{E}^*)^2 + (1-\nu)\,\mathbf{E}^* : \mathbf{E}^*\right] = \lambda\Psi_m^*$$

$$\Psi_b = \frac{\lambda(Lh^*)^2}{2}\left[\nu\left(\mathrm{tr}\left(\frac{1}{L}\mathbf{K}^*\right)\right)^2 + (1-\nu)\left(\frac{1}{L}\mathbf{K}^*\right) : \left(\frac{1}{L}\mathbf{K}^*\right)\right] = \lambda\Psi_b^*$$

$$\Psi = \Psi_m + \Psi_b = \lambda\left(\Psi_m^* + \Psi_b^*\right) = \lambda\Psi^*(\mathbf{E}^*, \mathbf{K}^*)$$

$$\mathbf{N} = \frac{Eh}{1-\nu^2}\left[\nu(\mathrm{tr}\,\mathbf{E})\mathbf{I} + (1-\nu)\mathbf{E}\right] = 12\lambda\left[\nu(\mathrm{tr}\,\mathbf{E}^*)\mathbf{I} + (1-\nu)\mathbf{E}^*\right] = \lambda\mathbf{N}^*$$

$$\mathbf{M} = \frac{Eh^3}{12(1-\nu^2)}\left[\nu(\mathrm{tr}\,\mathbf{K})\mathbf{I} + (1-\nu)\mathbf{K}\right] = \lambda h^{*2}\left[\nu(\mathrm{tr}\,\mathbf{K}^*)\mathbf{I} + (1-\nu)\mathbf{K}^*\right] = \lambda\mathbf{M}^*$$

Variational Analysis

- Let $\mathbf{f}(\mathbf{X}) = \mathbf{f}_m(\mathbf{X}) + f_b(\mathbf{X})\mathbf{e}_3$ be the body force and $\mathbf{t}$ be the traction acting over the boundary Γ_0.
- Then for quasi-static loading, the Lagrangian can be written as

$$\Pi[\mathbf{u}, w] = V - W_{\text{ext}} = \int_{\Omega_0} \left[\Psi_{\text{Total}}(\mathbf{E}, \mathbf{K}) - \mathbf{f}_m \cdot \mathbf{u} - f_b\, w\right] \mathrm{d}\Omega - \oint_{\Gamma_0} \mathbf{t} \cdot (\mathbf{u} + w\mathbf{e}_3)\, \mathrm{d}\Gamma.$$

- Assume $\boldsymbol{\eta}$ and ζ be some arbitrary admissible functions. Then for some small parameter ε, the perturbed displacement vectors about the equilibrium position can be written as

$$\tilde{\mathbf{u}} = \mathbf{u} + \varepsilon\boldsymbol{\eta}, \qquad \tilde{w} = w + \varepsilon\zeta;$$

- Evaluating First Variation of Lagrangian and putting it to zero at equilibrium condition yields –

$$\int_{\Omega_0} \Big[- \big[\nabla \cdot [(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}] + \mathbf{f}_m\big] \cdot \eta - \big[\nabla \cdot (\mathbf{N}\nabla w) + \nabla \cdot (\nabla \cdot \mathbf{M}) + f_b\big]\zeta \Big] d\Omega$$

$$+ \oint_{\Gamma_0} \Big[\big([(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}]\, \mathbf{n} - \mathbf{t}_m \big) \cdot \eta + \big((\mathbf{N}\nabla w + \nabla \cdot \mathbf{M}) \cdot \mathbf{n} - t_b \big)\zeta - (\mathbf{M}\mathbf{n}) \cdot \nabla \zeta \Big] d\Gamma = 0$$

- Euler–Lagrange Equations:

$$\nabla \cdot [(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}] + \mathbf{f}_m \quad = \mathbf{0}$$

$$\nabla \cdot [\nabla \cdot \mathbf{M} + \mathbf{N}\nabla w] + f_b = 0$$

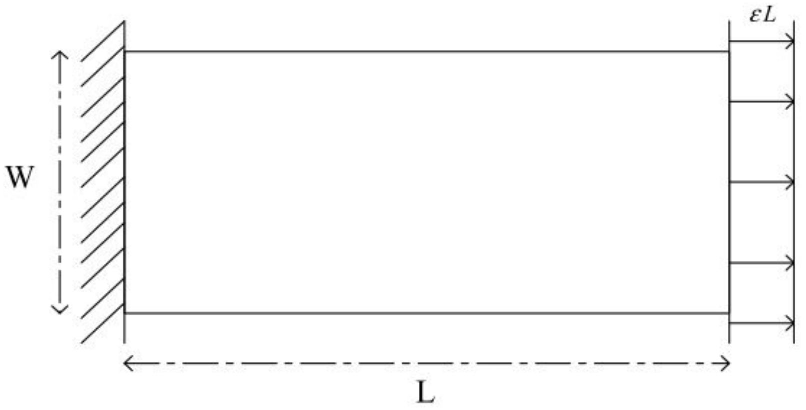
- Natural Boundary Conditions

$$\oint_{\Gamma_0} \big([(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}]\, \mathbf{n} - \mathbf{t}_m \big) \cdot \eta \; d\Gamma = 0$$

$$\oint_{\Gamma_0} \Big(([\mathbf{N}\nabla w + \nabla \cdot \mathbf{M}] \cdot \mathbf{n} - t_b)\zeta - (\mathbf{M}\mathbf{n}) \cdot \nabla \zeta \Big) d\Gamma = 0$$

Plate under Uniaxial Stretching

Problem Description



PDE System

$$(1)\quad \nabla \cdot [(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}] = \mathbf{0}$$

$$(2)\quad \nabla \cdot [\nabla \cdot \mathbf{M} + \mathbf{N}\nabla w] = 0$$

$$(3)\quad \mathbf{N} = 12\left[\nu(tr\mathbf{E})\mathbf{I} + (1-\nu)\mathbf{E}\right]$$

$$(4)\quad \mathbf{M} = h^2\left[\nu(tr\mathbf{K})\mathbf{I} + (1-\nu)\mathbf{K}\right]$$

$$(5)\quad \mathbf{E} = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T(\nabla \mathbf{u}) + \nabla w \otimes \nabla w\right]$$

$$(6)\quad \mathbf{K} = -\nabla^2 w$$

Geometric B.C.s

$$u_1(0,y) = 0 \qquad u_1(L,y) = \varepsilon L$$

$$u_2(0,y) = 0 \qquad u_2(L,y) = 0$$

$$w(0,y) = 0 \qquad w(L,y) = 0$$

Natural B.C.s

$$\mathbf{n} \cdot \mathbf{M}\mathbf{n} = 0 \qquad \text{along all 4 edges}$$

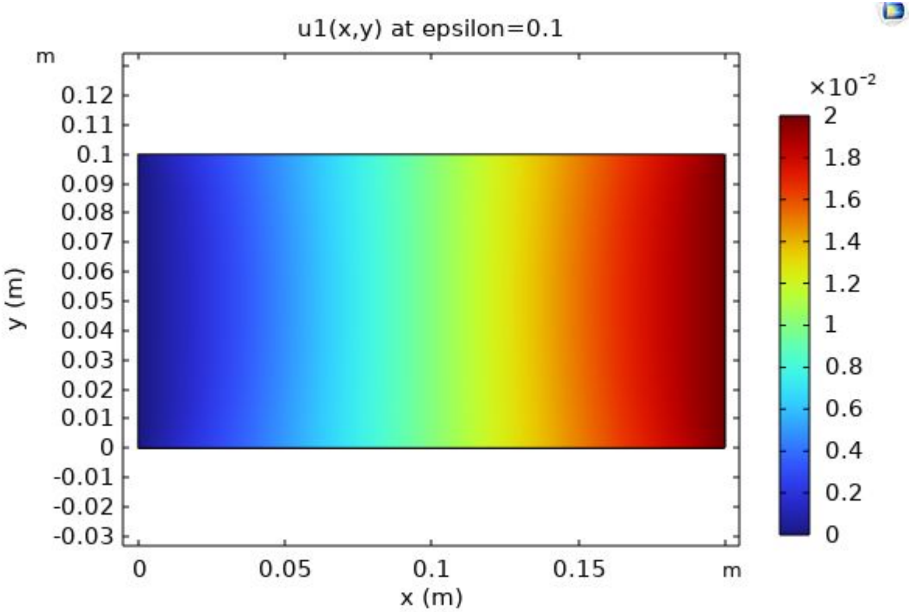
$$[(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}]\,\mathbf{e}_2\,|_{y=0} = 0 \qquad 0 < x < L$$

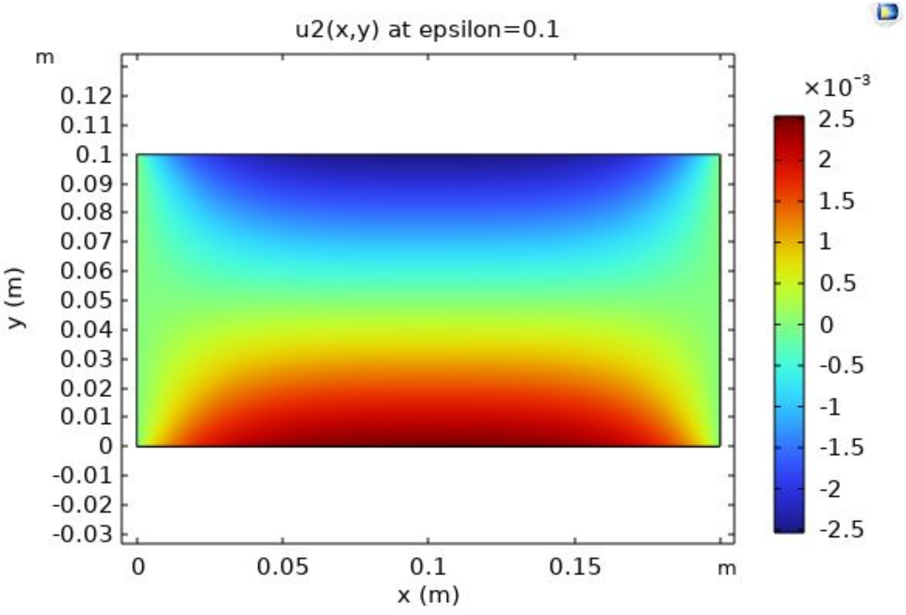
$$[(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}]\,\mathbf{e}_2\,|_{y=W} = 0 \qquad 0 < x < L$$

$$\left(\mathbf{N}\nabla w + \nabla \cdot \mathbf{M} + \frac{\partial \mathbf{M}\mathbf{e}_1}{\partial x}\right) \cdot \mathbf{e}_2 \Bigg|_{y-0} = 0 \qquad 0 < x < L$$

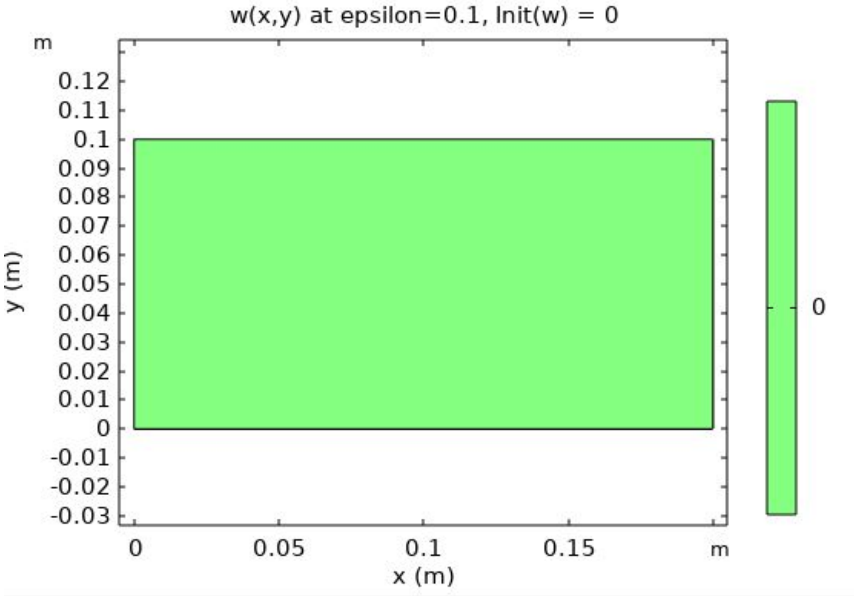
$$\left(\mathbf{N}\nabla w + \nabla \cdot \mathbf{M} + \frac{\partial \mathbf{M}\mathbf{e}_1}{\partial x}\right) \cdot \mathbf{e}_2 \Bigg|_{y=W} = 0 \qquad 0 < x < L$$

Plots of $u_1(x,y)$ and $u_2(x,y)$ at $\varepsilon = 0.1$





Plots of w(x,y) at ε = 0.1 under different initial conditions



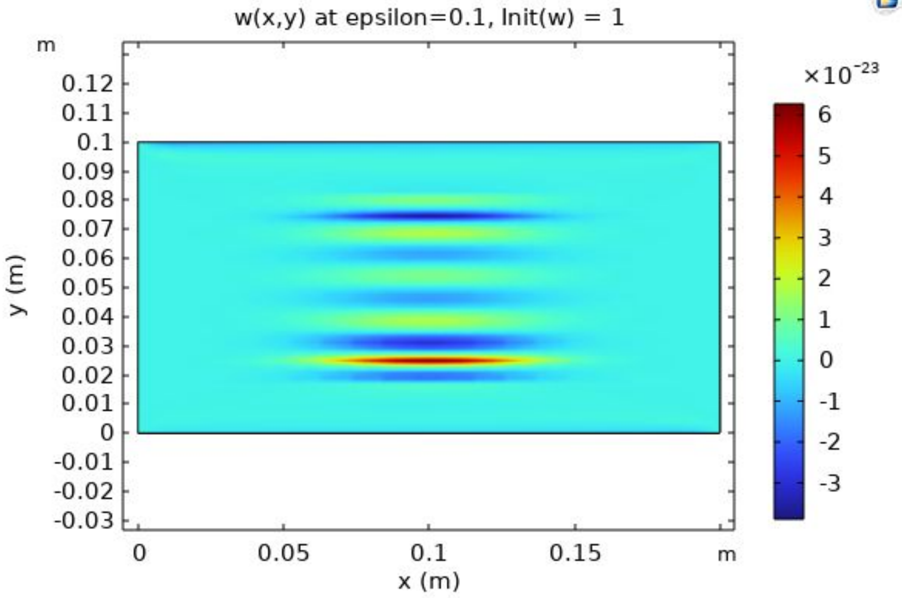
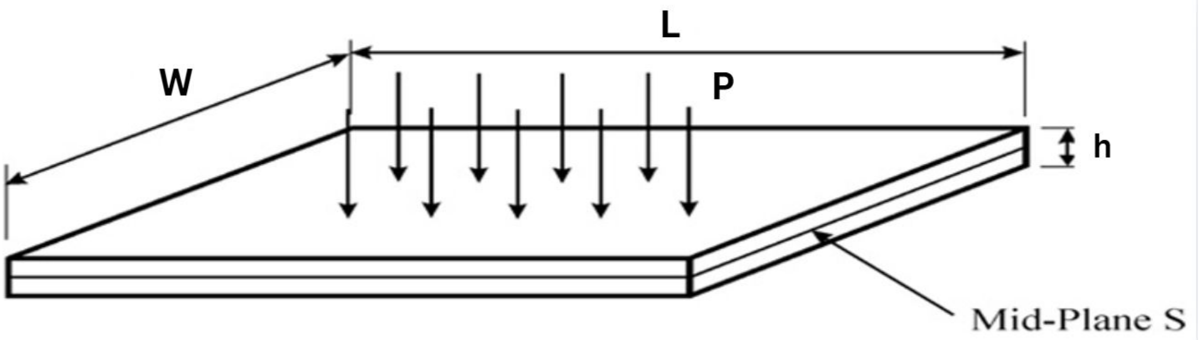


Plate under Transverse Loading

Problem Description

PDE System



(1) $\nabla \cdot [(\mathbf{I} + \nabla \mathbf{u})\mathbf{N}] \;\; = \mathbf{0}$

(2) $\nabla \cdot [\nabla \cdot \mathbf{M} + \mathbf{N}\nabla w] = \dfrac{PL}{\lambda}$

(3) $\mathbf{N} = 12\,[\nu(tr\mathbf{E})\mathbf{I} + (1-\nu)\mathbf{E}]$

(4) $\mathbf{M} = h^2\,[\nu(tr\mathbf{K})\mathbf{I} + (1-\nu)\mathbf{K}]$

(5) $\mathbf{E} = \dfrac{1}{2}\,[\nabla \mathbf{u} + (\nabla \mathbf{u})^T + (\nabla \mathbf{u})^T(\nabla \mathbf{u}) + \nabla w \otimes \nabla w]$

(6) $\mathbf{K} = -\nabla^2 w$

Dirichlet Boundary Conditions

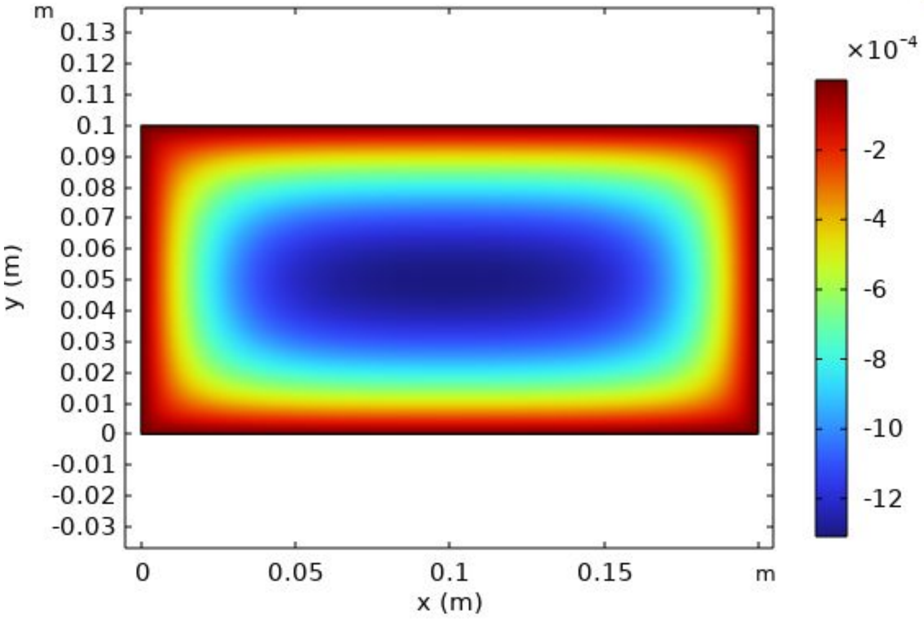
$\mathbf{u}(0,y) = \mathbf{u}(x,0) = \mathbf{u}(L,y) = \mathbf{u}(x,W) = \mathbf{0},$

$w(0,y) = w(x,0) = w(L,y) = w(x,W) = 0.$

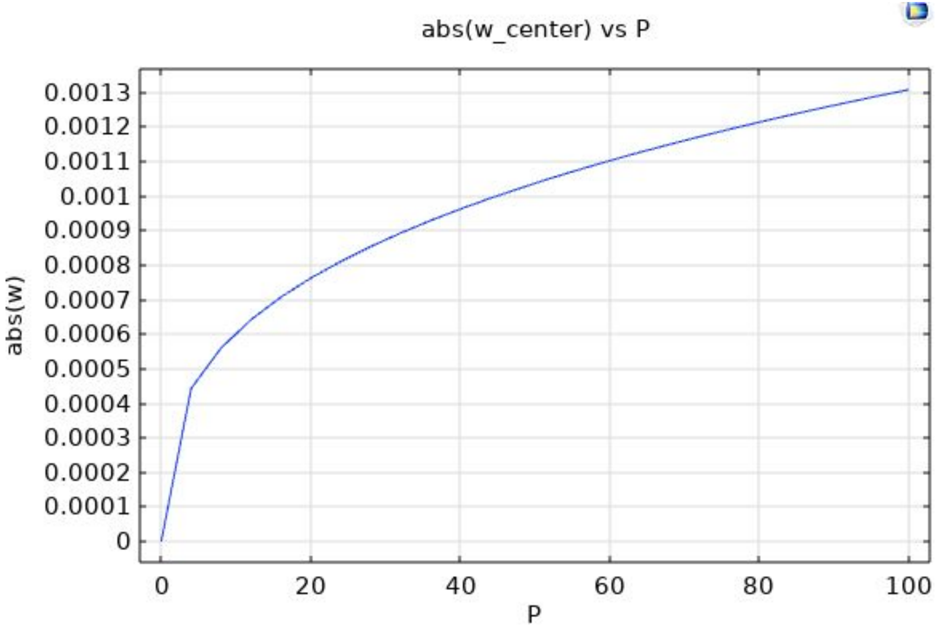
Natural Boundary Conditions

$\mathbf{n} \cdot \mathbf{M}\mathbf{n} = 0 \qquad$ along all 4 edges

Plots of w(x,y) and |w| vs P

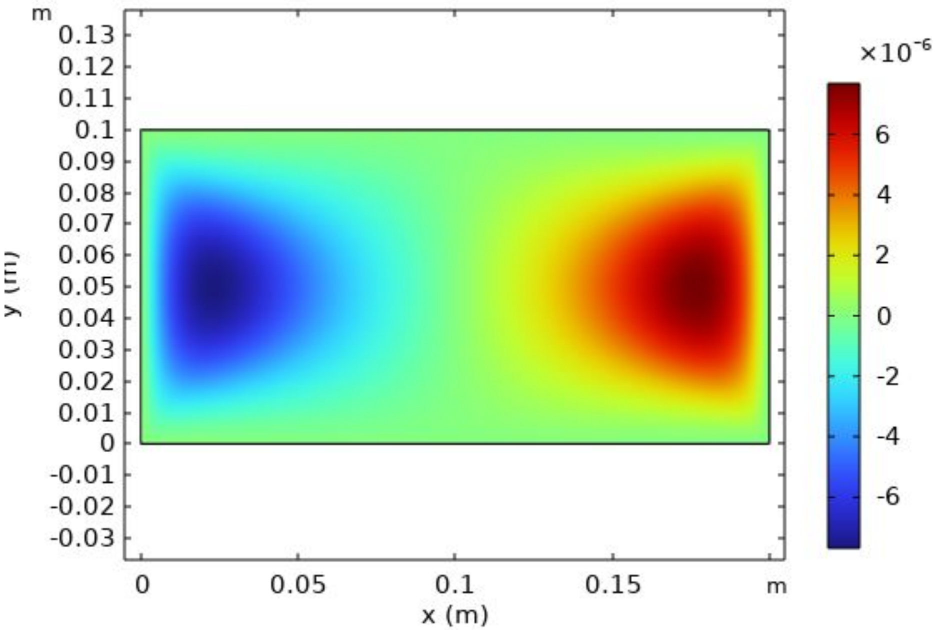


Plot of w(x,y)

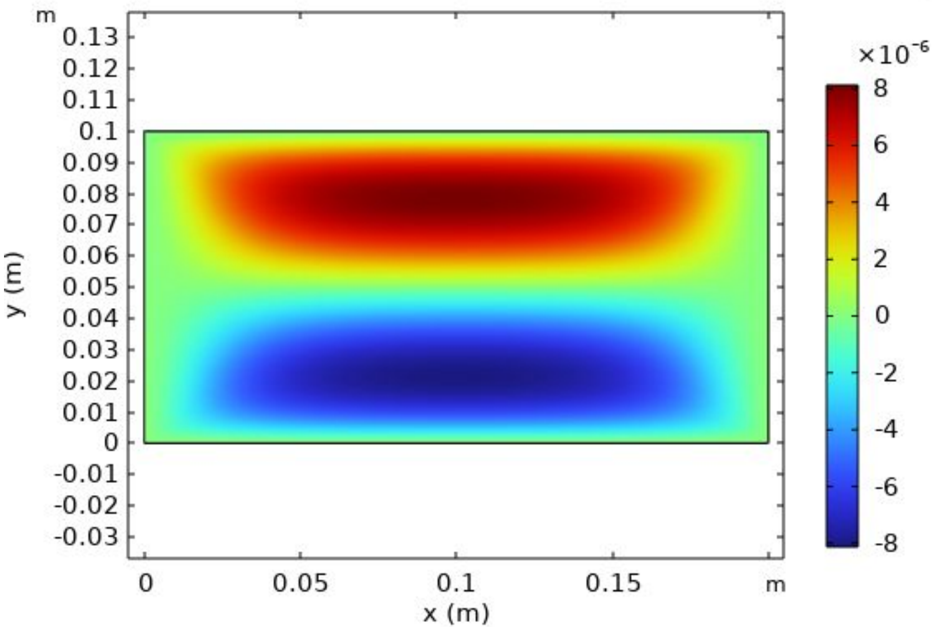


$|w_{center}|$ vs P

Plots of $u_1(x,y)$ and $u_2(x,y)$ at $\varepsilon = 0.1$

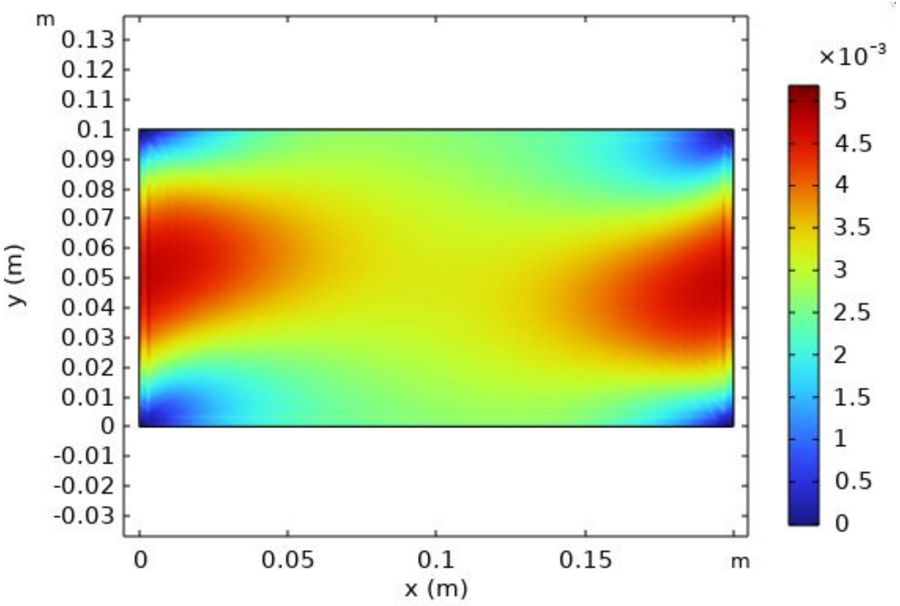


Plot of $u_1(x,y)$

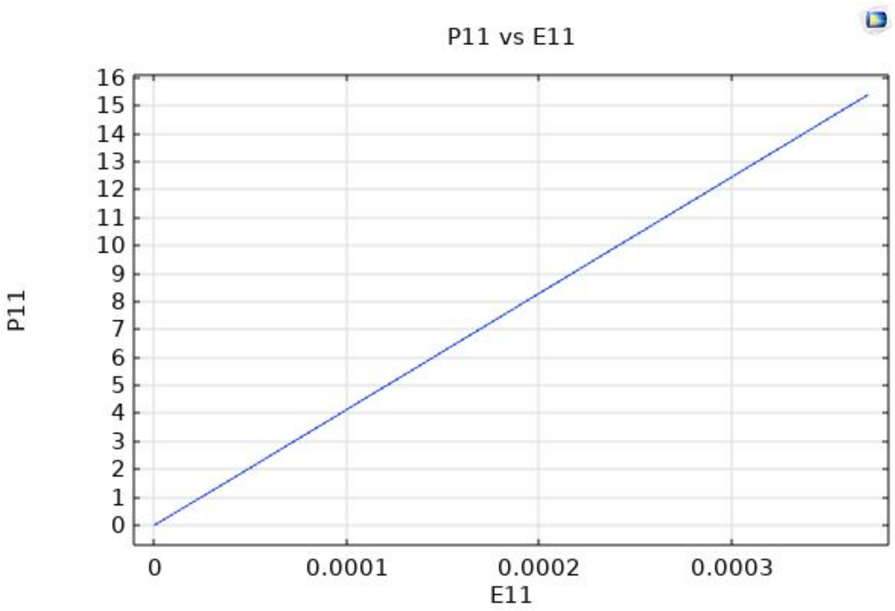


Plot of $u_2(x,y)$

Stress Plot and Stress - Strain Curve

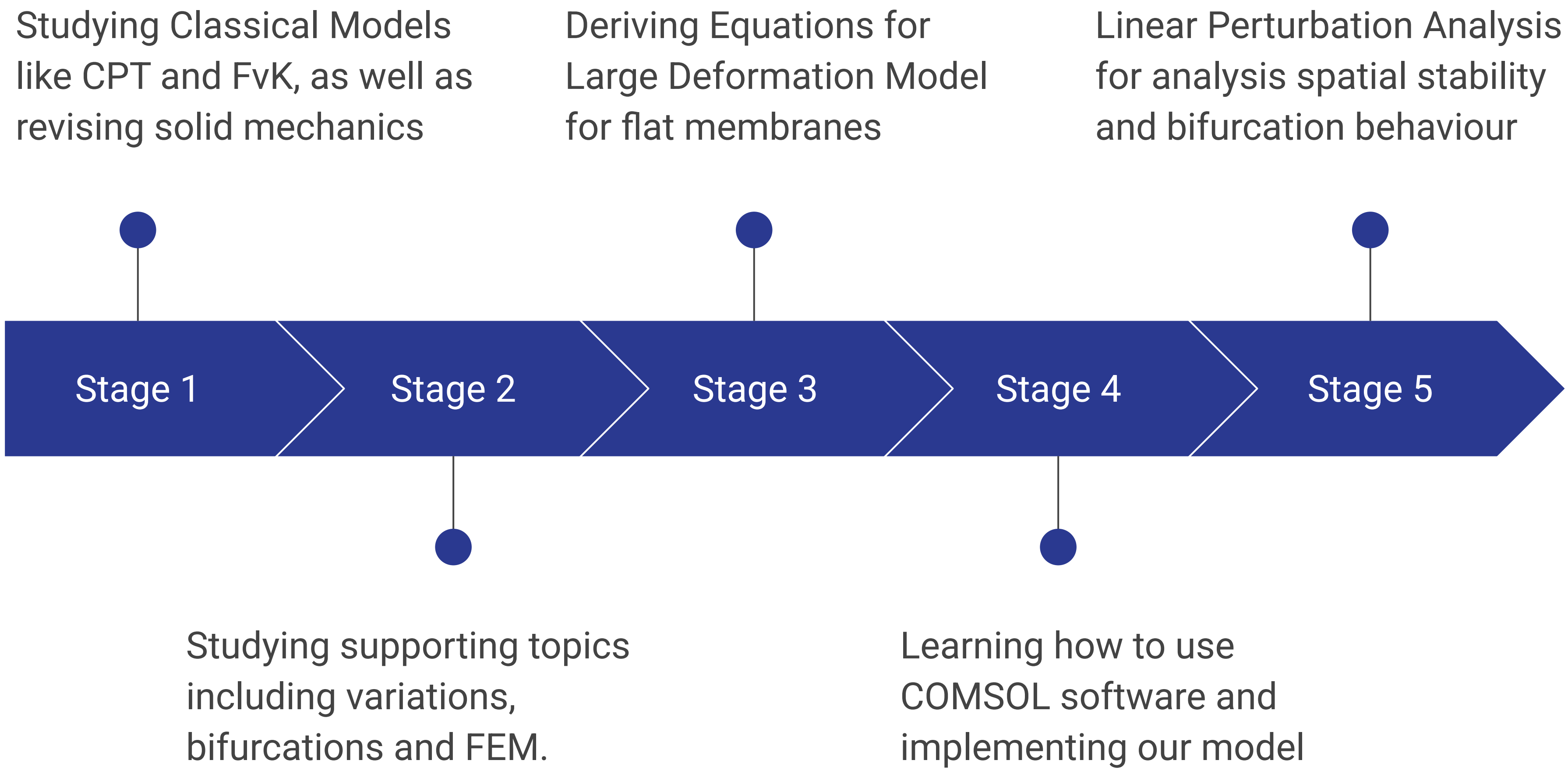


Plot of P_{11}



P_{11} vs E_{11}

Timeline


Studying Classical Models like CPT and FvK, as well as revising solid mechanics
Deriving Equations for Large Deformation Model for flat membranes
Linear Perturbation Analysis for analysis spatial stability and bifurcation behaviour
Stage 1
Stage 2
Stage 3
Stage 4
Stage 5
Studying supporting topics including variations, bifurcations and FEM.
Learning how to use COMSOL software and implementing our model

Future Direction

Future Directions

1. Adapting the model for thin magneto-elastic polymer membranes

2. Study the bifurcation for the thin membrane problem for hyperelastic materials and understand spatial stability

3. Formulate a model for thin membranes having some general curvilinear geometry in material frame

4. Build and implement a Finite element model for the problem