Massive MIMO-Based Underlay Spectrum Access Under Incomplete and/or Imperfect Channel State Information

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***Abstract*—We investigate use of massive number of antennas at cognitive base station (BS) in reducing interference caused to primary users (PUs) under incomplete and/or imperfect channel state information (CSI) without deteriorating sum spectral efficiency of cognitive users (CUs). We derive analytical expressions for complement of interference outage probability with incomplete and/or imperfect CSI. These novel expressions provide ballpark number for the amount of back-off required to meet interference constraint. We deduce new expressions for sum spectral efficiency of CUs with MR beamforming under incomplete and/or imperfect CSI. We prove that under imperfect CSI, by deploying more antennas at cognitive BS, interference outage probability can be reduced while keeping sum spectral efficiency of CUs fixed. And larger number of PUs can be accommodated in the network while keeping interference outage probability at PUs and sum spectral efficiency of CUs fixed. Furthermore, outage probability reduces as number of channels *S* to which CSI is available increases, since the constraint is violated less often. And by deploying more antennas at cognitive BS, sum spectral efficiency of CUs can be kept fixed even if *S* decreases. Impact of spatial correlation on outage probability at PUs and sum spectral efficiency of CUs is also elucidated.**

***Index Terms*—Massive MIMO, concurrent spectrum access, imperfect CSI, incomplete CSI, achievable rate, interference outage probability.**

I. INTRODUCTION

THE NEXT generation wireless systems must be designed to provide extremely high data rates and efficient connectivity to massive number of Internet-of-Things (IoT) devices. However, the data rates and number of users that can be served depend heavily on the available wireless spectrum. And most of the spectrum below 6 GHz is already allocated to support diverse cellular technologies such as 2G-5G and WiFi services such as IEEE 802.11ac and IEEE 802.11ax. This is the reason why spectrum regulatory authorities around the world are now deliberating on allowing concurrent sharing of spectrum by unlicensed cognitive users (CUs) in order to boost spectrum utilization efficiency [2], [3]. Thus, these CUs must co-exist with the high-priority licensed primary users (PUs) who continue to use the spectrum [2].

To prevent adverse effects of concurrent spectrum sharing on PUs, the power of the interference at the PUs due to downlink (DL) transmissions by the cognitive base station (BS) must be below an acceptable threshold [4], [5]. To ensure this, the cognitive BS must adapt its transmit power depending on the available channel state information (CSI). However, in practice, the available CSI to all PUs may either be imperfect due to channel estimation errors or incomplete and may not be available a priori since the acquisition of CSI to all the PUs in a timely and scalable manner becomes practically demanding as the number of PUs increases. Therefore, the nature of interference constraint together with accuracy and availability of the CSI from the cognitive BS to all the PUs restrict the power with which a cognitive BS can transmit. And this constraint limits the data rate that can be offered to the CUs in underlay spectrum access networks.

Massive MIMO has already been identified as a key component of 5G cellular systems, in which hundreds of antennas at the BS can simultaneously serve tens of users through spatial multiplexing at the same time and over the same frequency [6], [7]. Since it offers huge array gain, multiplexing gain, favorable propagation, high spatial resolution and reduced inter-user interference, the integration of massive MIMO in underlay spectrum access networks can significantly enhance the data rates of users in unlicensed cognitive network, where interference caused to the PUs, and the accuracy and the availability of the CSI at the cognitive BS are of prime concern and limit the power with which a cognitive BS can transmit [8].

A. Related Literature

The authors in [9] considered an underlay cognitive massive MIMO network and optimized the number of CUs that can be served on the DL under power, rate and a peak interference constraint and with imperfect CSI. A multiuser massive MIMO primary network and a multiple-input single-output cognitive network was considered in [10] and achievable rate was derived for a CU under a peak interference constraint. The authors in [11] considered a scenario where CUs harvest energy from PU transmissions and analyzed signal-to-interference-plus-noise ratio (SINR), energy-rate trade-off and sum rate. In [12], the authors studied uplink (UL) of

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an underlay cognitive massive MIMO network and optimized energy efficiency while ensuring fairness among CUs. A cognitive massive MIMO relay network was studied in [13] and asymptotic SINR and the sum rate under different antenna configurations at the BS and the relay were analyzed.

In [14], the authors considered UL of a cognitive massive MIMO network under imperfect CSI and studied joint pilot and data power allocation while ensuring max-min fairness to users. In [15], the authors investigated effects of pilot contamination on a random CR network underlaid upon a random primary network. In [16], the authors analyzed low-complexity transmission in massive MIMO systems by exploiting their tolerance to incomplete CSI. The authors in [17] analyzed relay selection for a massive MIMO underlay CR two-way relay network to maximize sum rate under peak interference constraint at the PU. The authors in [18] studied an underlay cooperative relay system in which the source and the relays back-off their transmit power based on the available CSI to meet an interference outage constraint.

The authors in [1] investigated the performance of underlay cognitive massive MIMO network with imperfect and complete CSI. However, this analysis was done considering only one CU in the network and no analysis was done for incomplete CSI. In [19], the authors investigated the achievable rates for multiuser massive MIMO relay networks operating in the underlay mode under both perfect and imperfect CSI. In [20], the authors analyzed UL of a cognitive massive MIMO network under imperfect CSI where the CUs adapt their power based on statistical CSI in order to satisfy the interference constraint. In [21], the statistics of the aggregate interference at any CU due to both primary and cognitive transmissions was derived.

In [22], expressions for beamforming gain and distribution of signal and interference powers were derived in terms of gamma random variables (rvs) with zero-forcing (ZF) beamforming at the BS in a non-cognitive radio setting. In [23], average interference at the PUs and complementary cumulative distribution function (CCDF) of the achievable rate of the CUs in massive MIMO underlay cognitive radio networks were analyzed under imperfect CSI and with ZF beamforming. The authors in [24] proposed deep reinforcement learning algorithms for user selection and power allocation in cognitive massive MIMO networks. In [25], the authors determined power allocation policy to maximize sum-rate of CUs while meeting total transmit power constraint at the BS and SINR constraints at the PUs. In [26], data rates in a wireless powered underlay cognitive massive MIMO system over spatially correlated channels were analyzed.

Based on the discussion above and to the best of the authors' knowledge, no work in the existing literature has comprehensively analyzed the sum spectral efficiency of a massive MIMO based underlay spectrum access network in which the cognitive BS is subject to a *practically motivated interference outage probability constraint.* This stochastic constraint markedly changes the mathematical analysis as shown in detail in Theorems 1-6 in Section III and Section IV. Furthermore, most of the literature focuses on perfect/imperfect CSI and peak interference constraint only. Our new results, which deal with the practically relevant *incomplete CSI* scenario as well and analyze the system with *multiple CUs*, therefore, distinguish the paper all the more. In other words, this is the first work that comprehensively analyzes implications of power control under the novel and practically relevant interference outage probability constraint in massive MIMO based underlay spectrum access with incomplete and/or imperfect CSI.

B. Focus and Our Contributions

In this paper, we consider a scenario where an unlicensed N-antenna cognitive BS serves K_c CUs and accesses the spectrum concurrently with the incumbent M-antenna primary BS serving K_p PUs. The wireless channels to each of the PUs or the CUs are independent and non-identically distributed (i.n.i.d.). The main objective of this work is to ascertain if we can exploit the huge array gain and high spatial resolution offered by massive MIMO in underlay spectrum access networks to reduce interference outage probability at the PUs or accommodate more number of PUs without degrading sum spectral efficiency of CUs under incomplete and/or imperfect CSI. This is an essential first step in understanding the robustness of any wireless system in practical scenarios, in which the CSI is likely to be imperfect and/or incomplete. The channel knowledge may be imperfect due to errors incurred during channel estimation. And the channel knowledge may be incomplete since the process of acquiring CSI to all the users in a timely and scalable manner is a practically challenging task. We summarize our key contributions below.

- *Modeling of Underlay Cognitive Massive MIMO Network Under Interference Outage Probability Constraint:* We consider a novel model in which the cognitive BS with massive number of antennas operates in concurrent spectrum access mode and is subject to a well-motivated and a practically relevant interference outage probability constraint under imperfect and/or incomplete CSI. This constraint mandates that the power of the interference generated due to transmissions by the cognitive BS does not exceed a target threshold at any of the PUs more than a certain fraction of time [4], [18], [27]–[29]. We consider this stochastic constraint since it is well-suited for primary systems that can tolerate co-channel interference, deep fades and offer voice and data services robust to delays and disruptions [27], [28].
- *Interference Outage Probability Analysis Under Imperfect and/or Incomplete CSI:* We propose simple back-off factor based power adaptation policies and derive new analytical expressions for interference outage probability at the PUs when maximum ratio (MR) beamforming is used the BS under three different scenarios, namely, imperfect with complete CSI, perfect with incomplete CSI and imperfect with incomplete CSI. This involves finding joint probability of the events that interference caused to each of the PUs is below a certain threshold depending on whether imperfect and/or incomplete CSI is available. And we use Bonferroni's inequality to obtain closed-form analytical bounds that are very tight in the regime of interest. These results can be used to determine the power margin required to meet the constraint under each of the three scenarios.

- *Sum Spectral Efficiency Analysis Under Imperfect and/or Incomplete CSI:* We also derive novel closed-form analytical expressions for the DL sum spectral efficiency of the CUs with MR beamforming for each of the three scenarios. This involves finding statistical mean of the cognitive BS transmit power and also the expected value of its square root. It also entails finding order statistics of strength of the estimated channels to all the PUs.
- *Performance Analysis Under Spatial Correlation:* We elucidate the impact of spatial correlation on interference outage probability at PUs and sum spectral efficiency of CUs.
- *Extensive Set of Numerical Results:* We present numerical results to confirm our mathematical analysis and also to obtain insights into the impact of different system parameters such as the back-off factor, channel estimation error, the number of PUs, the number of cognitive BS antennas, the incompleteness of CSI and the spatial correlation on interference outage probability at PUs and the DL sum spectral efficiency of CUs.

Key Design Insights: We infer that interference outage probability (P_o) is independent of channel estimates from cognitive BS to CUs. It is independent of number of cognitive BS antennas and depends only on the strength of the channel estimates from cognitive BS to PUs. We observe that for larger estimation errors along the link from cognitive BS to the PUs, a larger power margin is required at the cognitive BS to maintain P_o at the same level. We show that under imperfect CSI, by deploying more antennas at the cognitive BS, (1) sum spectral efficiency of CUs can be maintained at a constant level while reducing P_o, and (2) larger number of PUs can be accommodated in the network while keeping P_o and sum spectral efficiency of CUs fixed. Furthermore, we show that P_o reduces as the number of channels to which CSI is known perfectly or imperfectly increases. And by deploying more antennas at the cognitive BS, sum spectral efficiency of CUs can be kept fixed even if *S* decreases. And as spatial correlation increases, a larger power margin is required at cognitive BS to keep P_o fixed.

Notations: A circular symmetric complex Gaussian random vector $\boldsymbol{X}$ with zero mean vector and covariance matrix $\boldsymbol{\Lambda}$ is denoted by $\boldsymbol{X} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Lambda})$. We denote conjugate operator by $(\cdot)^*$, transpose by $(\cdot)^{\mathrm{T}}$ and the Euclidean norm by $||\cdot||$. Furthermore, $\mathbb{E}(\cdot)$ denotes the expectation operator, $\boldsymbol{I}_N$ denotes an $N \times N$ identity matrix, $f_X(x)$ denotes the probability density function (pdf) of a rv *X*, $F_X(x)$ denotes its CDF, $f_{X,Y}(x,y)$ denotes the joint pdf of two rvs *X* and *Y* and the joint probability of events $A_1, A_2 \dots A_n$ is denoted by $\Pr(A_1, A_2 \dots A_n)$. The cardinality of a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$ and $k \in \mathcal{S}$ indicates that the element *k* belongs to the set $\mathcal{S}$.

II. System Model

We consider DL of a massive MIMO based underlay spectrum access system as illustrated in Figure 1. The model comprises of the incumbent high priority primary system who are licensed owners of the spectrum and the unlicensed cognitive system that accesses the spectrum concurrently subject to an interference outage probability constraint set by the PUs. The primary system comprises of a primary BS with *M* co-located antennas providing service to K_p single-antenna PUs. The cognitive system comprises of a cognitive BS with N co-located antennas providing service to K_c single-antenna CUs. For the primary system, the channel vector from the primary BS to the k^{th} PU is denoted by $\boldsymbol{h}_{p_k} = [h_{p_{k,1}}, \dots, h_{p_{k,M}}]^{\mathrm{T}} \in \mathcal{C}^{M\times 1}$, where $\boldsymbol{h}_{p_k} \sim \mathcal{CN}(\mathbf{0}, \beta_k \boldsymbol{I}_M)$ and $h_{p_{k,i}}$ represents the small-scale fading coefficient from the i^{th} primary BS antenna to the k^{th} PU and β_k denotes the path loss from the primary BS to the k^{th} PU.

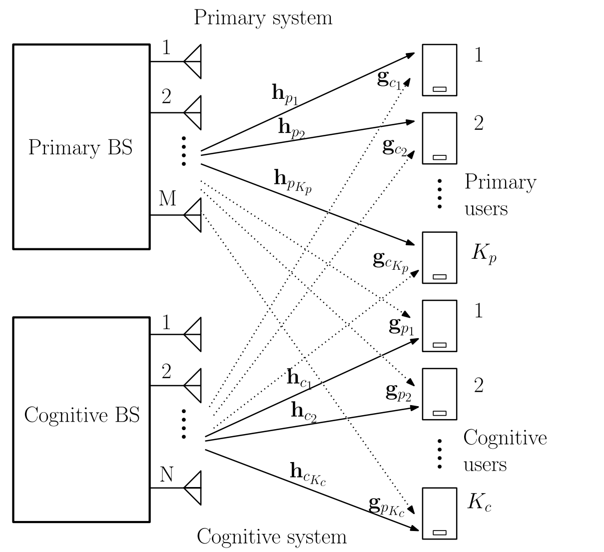


Fig. 1. System Model: Massive MIMO Based Underlay Spectrum Access System.

The channel vector of the interference link from the primary BS to the k^{th} CU is denoted by $\boldsymbol{g}_{p_k}$, where $\boldsymbol{g}_{p_k} = [g_{p_{k,1}}, \dots, g_{p_{k,M}}]^{\mathrm{T}} \in \mathcal{C}^{M\times 1}$, $\boldsymbol{g}_{p_k} \sim \mathcal{CN}(\mathbf{0}, \phi_k \boldsymbol{I}_M)$ and ϕ_k denotes the large-scale fading coefficient corresponding to this link. For the cognitive system, the channel vector from the cognitive BS to the k^{th} CU is $\boldsymbol{h}_{c_k} = [h_{c_{k,1}}, \dots, h_{c_{k,N}}]^{\mathrm{T}} \in \mathcal{C}^{N\times 1}$ and $\boldsymbol{h}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \gamma_k \boldsymbol{I}_N)$ where γ_k captures the path loss. The channel vector of the interference link from the cognitive BS to the k^{th} PU is denoted by $\boldsymbol{g}_{c_k} = [g_{c_{k,1}}, \dots, g_{c_{k,N}}]^{\mathrm{T}} \in \mathcal{C}^{N\times 1}$, where $\boldsymbol{g}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \alpha_k \boldsymbol{I}_N)$ and $g_{c_{k,j}}$ represents the small-scale fading coefficient from the j^{th} cognitive BS antenna to the k^{th} PU and α_k denotes the path loss between cognitive BS and the k^{th} PU.

A. Data Transmission

When both the cognitive BS and the primary BS employ MR transmission,[1] then the signal received at the k^{th} CU on the DL equals

$$y_{c_k} = \sqrt{\frac{P_c}{K_c}} \sum_{k'=1}^{K_c} \frac{\boldsymbol{h}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_{k'}}^* q_{k'}}{||\boldsymbol{h}_{c_{k'}}||} + \sqrt{\frac{P_p}{K_p}} \sum_{j=1}^{K_p} \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^* s_j}{||\boldsymbol{h}_{p_j}||} + n_k, \quad (1)$$

where P_c is the cognitive BS transmit power, $q_{k'} \sim \mathcal{CN}(0,1)$ is the data symbol intended for the k'^{th} CU, P_p is the primary

[1]We consider MR transmission since it is computationally less intensive and asymptotically optimal [10], [25], [30].

BS transmit power, $s_j \sim \mathcal{CN}(0,1)$ is the data symbol corresponding to the j^{th} PU and $n_k \sim \mathcal{CN}(0,\sigma^2)$ denotes additive white Gaussian noise at the k^{th} CU. Furthermore, the symbols transmitted to distinct PUs or CUs are mutually uncorrelated to each other, i.e., $\mathbb{E}\{q_i q_j^*\} = 0$ and $\mathbb{E}\{s_i s_j^*\} = 0$ for $i \neq j$. Due to transmissions from the cognitive BS, the interference seen by the k^{th} PU equals

$$y_{p_k} = \sqrt{\frac{P_c}{K_c}} \boldsymbol{g}_{c_k}^{\mathrm{T}} \sum_{i=1}^{K_c} \frac{\boldsymbol{h}_{c_i}^*}{||\boldsymbol{h}_{c_i}||} q_i. \tag{2}$$

Based on (2), the instantaneous power of the interference caused to the k^{th} PU equals

$$I_{p_k} = \frac{P_c}{K_c} \sum_{i=1}^{K_c} \left| \frac{\boldsymbol{g}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_i}^*}{||\boldsymbol{h}_{c_i}||} \right|^2, \text{ for all } 1 \leq k \leq K_p. \tag{3}$$

B. Interference Outage Probability Constraint

In this paper, we consider massive MIMO based concurrent spectrum access system in which the cognitive BS transmits subject to an *interference outage probability constraint.* This constraint ensures that the interference power due to transmissions from the cognitive BS does not exceed a threshold $\overline{I}_p$ at any of the K_p PUs more than P_o fraction of the time. Mathematically, this can be written as

$$\Pr\Big(I_{p_1} \leq \overline{I}_p, I_{p_2} \leq \overline{I}_p, \ldots, I_{p_{K_p}} \leq \overline{I}_p\Big) \geq (1 - P_o), \tag{4}$$

where P_o is referred to as the interference outage probability. We consider this stochastic constraint since (i) it is practically relevant and its implications on a massive MIMO based concurrent spectrum access system has not been thoroughly studied before; (ii) it is justified to use this constraint when the services, for example, user datagram protocol (UDP) services like voice and video offered by the primary system are robust to delays or disruptions [4], [18], [27], [28]; (iii) it need not harm primary systems that are robust to deep fades or co-channel interference [4], [18], [27]; (iv) this practically relevant constraint is widely used to design primary exclusive zones to protect the PUs in spectrum sharing networks [29] and (v) this constraint is generic and a more appropriate constraint to consider in practical scenarios where the cognitive BS has imperfect and/or incomplete CSI of its channels to the PUs. This is because under perfect and complete CSI, the peak interference caused to the PUs can be strictly kept below a threshold so that P_o is zero. However, in practical scenarios where the CSI is likely to be imperfect and/or incomplete, P_o cannot be made zero. It can however be constrained to meet a certain target level by controlling the fraction of time, the interference power exceeds the given interference threshold.

1) Perfect and Complete CSI: If the cognitive BS has perfect and complete knowledge of its channels to the K_p PUs and the K_c CUs and adapts its power P_c as follows:

$$P_c = \frac{\overline{I}_p}{\max_{1 \leq k \leq K_p} \left\{ \frac{1}{K_c} \sum_{i=1}^{K_c} \left| \frac{\boldsymbol{g}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_i}^*}{||\boldsymbol{h}_{c_i}||} \right|^2 \right\}}, \tag{5}$$

it can be shown that P_o equals zero and $\Pr(I_{p_1} \leq \overline{I}_p, I_{p_2} \leq \overline{I}_p, \ldots, I_{p_K} \leq \overline{I}_p) = 1$, i.e., the power of the interference caused by the cognitive BS is always below $\overline{I}_p$ at each of the K_p PUs.

2) Imperfect and Complete CSI: In practical wireless systems, CSI is estimated every coherence interval and is thus known imperfectly. With imperfect CSI, P_o will not be zero with the power adaptation policy in (5), where the true channel vectors are replaced by their estimates. It can, however, be constrained to satisfy a certain specified target P_o. In this work, we consider the generic imperfect CSI model based on Gauss-Markov uncertainty [31]. Based on this model, the true interference channel vector $\boldsymbol{g}_{c_k}$ between cognitive BS and the k^{th} PU can be written as

$$\boldsymbol{g}_{c_k} = s_p \ \widehat{\boldsymbol{g}}_{c_k} + \sqrt{1 - s_p^2} \ \widetilde{\boldsymbol{g}}_{c_k}, \ k = 1, \ldots, K_p, \tag{6}$$

where $\widehat{\boldsymbol{g}}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \alpha_k \mathbf{I}_N)$ is the estimate of the channel vector from the cognitive BS to the k^{th} PU, $\widetilde{\boldsymbol{g}}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \alpha_k \mathbf{I}_N)$ denotes the Gaussian noise which is uncorrelated to $\widehat{\boldsymbol{g}}_{c_k}$ and s_p captures the channel estimation error. Based on this widely accepted generic imperfect CSI model, $s_p = 1$ corresponds to the case when the available CSI is perfect/error-free, $0 < s_p < 1$ corresponds to imperfect CSI case and $s_p = 0$ corresponds to no CSI scenario, in which case the available CSI is only noise that is uncorrelated and independent of the channel estimate and has the same statistics as the true channel [10], [31]–[34]. Similarly, the true channel vector $\boldsymbol{h}_{c_k}$ between cognitive BS and the k^{th} CU can be written as

$$\boldsymbol{h}_{c_k} = s_s \ \widehat{\boldsymbol{h}}_{c_k} + \sqrt{1 - s_s^2} \ \widetilde{\boldsymbol{h}}_{c_k}, \ k = 1, \ldots, K_c, \tag{7}$$

where $\widehat{\boldsymbol{h}}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \gamma_k \mathbf{I}_N)$ is the estimate of the channel vector from the cognitive BS to the k^{th} CU, $\widetilde{\boldsymbol{h}}_{c_k} \sim \mathcal{CN}(\mathbf{0}, \gamma_k \mathbf{I}_N)$ denotes the Gaussian noise which is uncorrelated to $\widehat{\boldsymbol{h}}_{c_k}$ and as before, s_s captures the channel estimation error.

3) Incomplete CSI: In practice, it is possible that the cognitive BS has the CSI to only few of the PUs due to limitation of resources available for learning the wireless channel. In this work, we also consider the scenario where the cognitive BS knows the CSI perfectly or imperfectly only to $|\mathcal{S}|$ out of K_p PUs. Even in such scenarios, where the cognitive BS has perfect and incomplete CSI or imperfect and incomplete CSI of its links to the K_p PUs, P_o will not be exactly zero. It can, however, be constrained to meet a certain target level.

III. Analysis With Imperfect and Complete CSI

In this section, we first analyze the interference outage probability with imperfect and complete CSI. Thereafter, we analyze the DL sum spectral efficiency that can be achieved by the CUs using the proposed power policy while satisfying the interference outage probability constraint at the PUs. As mentioned above in Section II-B2, with imperfect and complete CSI, P_o will not be zero. However, P_o can be constrained to meet a certain target level. To this end, we propose a back-off factor based power adaptation policy in which the cognitive BS adapts its transmit power as a function of channel estimates from the cognitive BS to the K_p PUs and from the cognitive BS to the K_c CUs. Under this policy, the cognitive BS transmits with power[2]

$$P_{ic} = \frac{\overline{I}_p}{\eta \max_{1\leq k\leq K_p}\left\{\frac{1}{K_c}\sum_{i=1}^{K_c}\left|\frac{\widehat{\boldsymbol{g}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||}\right|^2\right\}}, \tag{8}$$

where the back-off factor η (> 1) is chosen such that the interference outage probability constraint under imperfect CSI is met with equality for a target P_o. In other words, given s_p, P_o, K_c, K_p and α_k for all PUs, the back-off factor η is chosen such that

$$\Pr\Big(\widehat{I}_{p_1} \leq \overline{I}_p, \widehat{I}_{p_2} \leq \overline{I}_p, \ldots, \widehat{I}_{p_{K_p}} \leq \overline{I}_p\Big) = 1 - P_o\ , \tag{9}$$

where $\widehat{I}_{p_k} = \frac{P_{ic}}{K_c}\sum_{i=1}^{K_c}|\frac{\boldsymbol{g}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||}|^2$, for all $1 \leq k \leq K_p$. We next state a lower bound on the complement of the interference outage probability.

Theorem 1: A lower bound on the complement of interference outage probability $P_{L_{(ic)}}$ with imperfect and complete CSI is given by

$$\begin{aligned}(1-P_o) \geq P_{L_{(ic)}} = 1 - \sum_{i=1}^{K_p}\sum_{l=1}^{L}\frac{w_l\, {x_l}^{K_c-1}}{\Gamma(K_c)}\prod_{j\neq i}^{K_p}\frac{\gamma\Big(K_c, \frac{x_l\alpha_i}{\eta\alpha_j}\Big)}{\Gamma(K_c)} \\ \times\left(1 - Q_{K_c}\left(\sqrt{\frac{2\rho x_l}{(1-\rho)}}, \sqrt{\frac{2x_l}{\eta(1-\rho)}}\right)\right),\end{aligned} \tag{10}$$

where $\rho = s_p^2$, $\{x_l\}$ are the integration points, $\{w_l\}$ are the corresponding weights obtained via Gauss-Laguerre integration, *L* is the total number of integration points, $\gamma(\cdot)$ denotes the lower incomplete Gamma function [35, eq. (8.350.1)] and $Q_{K_c}(\cdot,\cdot)$ denotes the generalized Marcum-Q function of order K_c [36, eq. (4.59)].

Proof: The derivation is given in Appendix A. ■

Remark: The integral-free expression for the lower bound $P_{L_{(ic)}}$ above brings out the dependence of P_o on the back-off factor (η), number of CUs (K_c), number of PUs (K_p), channel estimation error s_p and the path loss α_j for $1 \leq j \leq K_p$. An interesting insight that we obtain based on the analytical expression in (10) is that P_o is independent of s_s since it does not depend on the estimated channels to the CUs as highlighted in Appendix A. Furthermore, it is also independent of the number of cognitive BS antennas *N* because of the scaling by the norm of the channel vector in the MR precoder used at the cognitive BS. It, however, depends on the strength of the estimated channels from the cognitive BS to the PUs.

We next analyze the achievable sum spectral efficiency of CUs in this scenario. The signal received at the k^{th} CU with imperfect and complete CSI at the cognitive BS is given by[3]

$$y_{c_k} = \sqrt{\frac{P_{ic}}{K_c}}\sum_{k'=1}^{K_c}\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_{k'}}^{*}q_{k'}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||} + \sqrt{\frac{P_p}{K_p}}\sum_{j=1}^{K_p}\frac{\boldsymbol{g}_{p_k}^{\mathrm{T}}\boldsymbol{h}_{p_j}^{*}s_j}{||\boldsymbol{h}_{p_j}||} + n_k. \tag{11}$$

Using the fact that $\boldsymbol{h}_{c_k} = s_s\widehat{\boldsymbol{h}}_{c_k} + \sqrt{1-s_s^2}\widetilde{\boldsymbol{h}}_{c_k}$ and considering availability of only statistical CSI at the CUs, (11) can be re-written as[4]

$$\begin{aligned} y_{c_k} = {} & \underbrace{\mathbb{E}\left(\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||\right)q_k}_{1} + \underbrace{\sqrt{\frac{P_{ic}(1-s_s^2)}{K_c}}\frac{\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_k}^{*}q_k}{||\widehat{\boldsymbol{h}}_{c_k}||}}_{2} \\ & + \underbrace{\sqrt{\frac{P_{ic}}{K_c}}\sum_{k'\neq k}^{K_c}\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_{k'}}^{*}q_{k'}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||}}_{3} \\ & + \underbrace{\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||q_k - \mathbb{E}\left(\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||\right)q_k}_{4} \\ & + \underbrace{\sqrt{\frac{P_p}{K_p}}\sum_{j=1}^{K_p}\frac{\boldsymbol{g}_{p_k}^{\mathrm{T}}\boldsymbol{h}_{p_j}^{*}s_j}{||\boldsymbol{h}_{p_j}||}}_{5} + \underbrace{n_k}_{6}. \end{aligned} \tag{12}$$

Based on the use-and-forget bound [37] and corresponding to the proposed power adaptation policy in (8), the SINR at the k^{th} CU is written in (13) at the bottom of the page.

We can clearly observe that SINR_k^{ic} is a function of I_p, s_s and the back-off factor η, which in turn depends on s_p, K_p and the target P_o. The achievable sum spectral efficiency of the CUs is obtained by simplifying the SINR expression in (13) and is stated below as a Theorem.

Theorem 2: A lower bound SE^{ic} on the DL achievable sum spectral efficiency of the CUs with imperfect and complete CSI and corresponding to a back-off factor based power adaptation policy while satisfying an interference outage probability of P_o at PUs is given by

$$\mathrm{SE}^{ic} = \sum_{k=1}^{K_c}\log_2\Big(1 + \mathrm{SINR}_k^{ic}\Big), \tag{14}$$

[2]To compute P_{ic}, $K_c(6N+2)+1$ real multiplications, $5NK_c - 1$ real additions, $K_c + 2$ divisions and the max operation that scales as $\mathcal{O}(K_p)$ would be required.

[3]In the analysis that ensues, our focus is on evaluating the performance of interference outage probability constrained CUs under imperfect and/or incomplete CSI at the cognitive BS. Therefore, we assume that primary BS know its channels perfectly to the K_p PUs [10].

[4]In the massive MIMO regime, the channel hardens and the instantaneous values of the channel coefficient approaches its statistical mean. This is the reason why the sum spectral efficiency obtained based on statistical CSI will be very close to that based on instantaneous CSI [37].

$$\mathrm{SINR}_k^{ic} = \frac{\left|\mathbb{E}\Big(\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||\Big)\right|^2\mathbb{E}\Big(|q_k|^2\Big)}{\mathbb{E}\left|\sqrt{\frac{P_{ic}(1-s_s^2)}{K_c}}\frac{\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_k}^{*}q_k}{||\widehat{\boldsymbol{h}}_{c_k}||}\right|^2 + \mathbb{E}\left|\sum_{k'\neq k}^{K_c}\sqrt{\frac{P_{ic}}{K_c}}\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_{k'}}^{*}q_{k'}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||}\right|^2 + \mathbb{E}\left|\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||q_k - \mathbb{E}\Big(\sqrt{\frac{P_{ic}}{K_c}}s_s||\widehat{\boldsymbol{h}}_{c_k}||\Big)q_k\right|^2 + \mathbb{E}\left|\sum_{j=1}^{K_p}\sqrt{\frac{P_p}{K_p}}\frac{\boldsymbol{g}_{p_k}^{\mathrm{T}}\boldsymbol{h}_{p_j}^{*}s_j}{||\boldsymbol{h}_{p_j}||}\right|^2 + \mathbb{E}|n_k|^2} \tag{13}$$

where SINR_k^{ic} is given in (15) at the bottom of the page. And $\zeta_{ic} = \mathbb{E}(\sqrt{P_{ic}}) = \mathbb{E}(\sqrt{\frac{\overline{I}_p}{\eta X}}) \approx \sqrt{\frac{\overline{I}_p}{\eta}}(\frac{1}{\mathbb{E}(\sqrt{X})} + \frac{\mathbb{V}\mathrm{ar}(\sqrt{X})}{(\mathbb{E}(\sqrt{X}))^3})$ [38] , $X = \max_{1\leq k\leq K_p}(\widehat{Z}_k)$, $\mathbb{E}(\sqrt{X}) = \int_0^\infty \frac{1}{2\sqrt{x}}(1 - F_X(x))dx$, and $\mathbb{V}\mathrm{ar}(\sqrt{X}) = \int_0^\infty(1 - F_X(x))dx - (\mathbb{E}(\sqrt{X}))^2$ [39]. Note that $F_X(x) = \frac{1}{(\Gamma(K_c))^{K_p}}\prod_{i=1}^{K_p}\gamma(K_c, \frac{K_c x}{\alpha_i})$. Furthermore, $\psi_{ic} = \mathbb{E}(P_{ic}) = \mathbb{E}(\frac{\overline{I}_p}{\eta X}) \approx \frac{\overline{I}_p}{\eta}(\frac{1}{\mathbb{E}(X)} + \frac{\mathbb{V}\mathrm{ar}(X)}{(\mathbb{E}(X))^3})$ [38], $\mathbb{E}(X) = \int_0^\infty(1 - F_X(x))dx$ and $\mathbb{V}\mathrm{ar}(X) = \int_0^\infty 2x(1 - F_X(x))dx - (\mathbb{E}(X))^2$ [39].

Proof: The derivation is given in Appendix B. ■

Please note that the sum spectral efficiency above is a function of only the deterministic constants such as s_s, γ_k, K_c, N, ϕ_k, P_p and also depends on deterministic statistical parameters such as ζ_{ic} and ψ_{ic}. It is analytically intractable to obtain ζ_{ic} and ψ_{ic} in closed-form. Thus, we obtain the sum spectral efficiency analytically using (15) by evaluating ζ_{ic} and ψ_{ic} through numerical integration [40], [41]. We also cross-validate this against sum spectral efficiency obtained via Monte Carlo simulations in Section V.

Asymptotic Analysis: We next analyze the sum spectral efficiency of the CUs in the asymptotic regime, i.e., in the regime where the number of cognitive BS antennas and the number of primary BS antennas grow to infinity. As $N \to \infty$, by weak law of large numbers, $\frac{1}{N}\widehat{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*} \to \gamma_k$ for $k = i$ and $\frac{1}{N}\widehat{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*} \to 0$ for $k \neq i$. Furthermore, $\frac{1}{N}\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_k}^{*} \to 0$, since channel estimate is uncorrelated to estimation error and both are zero mean random vectors and $\frac{1}{N}\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*} \to 0$ for $k \neq i$. Furthermore, $\frac{1}{M}\boldsymbol{g}_{p_k}^{\mathrm{T}}\boldsymbol{h}_{p_k}^{*} \to 0$ as $M \to \infty$, since $\boldsymbol{g}_{p_k}$ corresponds to the channel from the primary BS to the k^{th} CU and this is independent to the channel $\boldsymbol{h}_{p_k}$ from the primary BS to the k^{th} PU. Therefore, in the asymptotic regime, the sum spectral efficiency of the CUs under imperfect and complete CSI simplifies to

$$\mathrm{SE}^{ic} = K_c \log_2\left(1 + \frac{\zeta_{ic}^2}{\psi_{ic} - \zeta_{ic}^2}\right). \tag{16}$$

We observe that in the asymptotic regime, the effects of channel estimation error, co-channel interference among CUs, noise and the interference at the CUs due to primary transmissions vanishes.

IV. Analysis With Incomplete CSI

In a massive MIMO based underlay spectrum access system, the cognitive BS operating in the unlicensed frequency band must acquire knowledge of its interference channels to the K_p PUs in order to control interference caused to the PUs. When there are multiple PUs operating in the network, the process of acquiring CSI to all the PUs in a timely and scalable manner is a practically challenging task. This is because as the number of PUs in the network increases, more and more resources need to be devoted to estimate the CSI [42]. In this section, in order to obtain insights, we first analyze the perfect and incomplete CSI scenario in which the cognitive BS knows the channels to only a subset of the PUs but knows them perfectly, i.e., without any channel estimation errors. We then generalize the analysis to the imperfect and incomplete CSI scenario in which the cognitive BS knows only a noisy version of the CSI to a subset of PUs.

A. Impact of Perfect and Incomplete CSI

Let $\mathcal{S}$ denote the set of PUs to which the cognitive BS knows the channels perfectly, i.e., in this scenario, the cognitive BS knows the channels to only $S = |\mathcal{S}|$ out of the K_p PUs, where $S \leq K_p$. We propose the use of the following back-off factor based power adaptation policy in which the cognitive BS transmits with power P_{pi} given by[5]

$$P_{pi} = \frac{\overline{I}_p}{\eta \max_{k\in S}\left\{\frac{1}{K_c}\sum_{i=1}^{K_c}\left|\frac{\boldsymbol{g}_{c_k}^{\mathrm{T}}\boldsymbol{h}_{c_i}^{*}}{||\boldsymbol{h}_{c_i}||}\right|^2\right\}}. \tag{17}$$

Under this policy, the back-off factor η is chosen to ensure that

$$\Pr\Big(I_{p_1} \leq \overline{I}_p, I_{p_2} \leq \overline{I}_p, \ldots, I_{p_{K_p}} \leq \overline{I}_p\Big) = 1 - P_o, \tag{18}$$

where the interference power I_{pk} at the k^{th} PU equals $I_{p_k} = \frac{P_{pi}}{K_c}\sum_{i=1}^{K_c}|\frac{\boldsymbol{g}_{c_k}^{\mathrm{T}}\boldsymbol{h}_{c_i}^{*}}{||\boldsymbol{h}_{c_i}||}|^2$. We next state a mathematical expression to evaluate P_o in this scenario.

Theorem 3: The complement of interference outage probability under perfect and incomplete CSI is given by

$$(1 - P_o) = \sum_{k\in\mathcal{S}}\sum_{l=1}^{L}\frac{w_l\ y_l^{K_c-1}}{(\Gamma(K_c))^{K_p}}\prod_{k'\notin\mathcal{S}}\gamma\left(K_c, \frac{y_l\alpha_k\eta}{\alpha_{k'}}\right) \times \left[\prod_{\substack{i\in\mathcal{S}\\ i\neq k}}\gamma\left(K_c, \frac{y_l\alpha_k}{\alpha_i}\right)\right]. \tag{19}$$

Proof: The derivation is given in Appendix C. ■

Remark: Based on this theorem, given a target P_o and the number $S = |\mathcal{S}|$ of PUs to which the channels are known perfectly, the wireless system designer can decide on the back-off factor/power margin η which ensures that the interference power does not exceed the threshold $\overline{I}_p$ more than P_o fraction of the time at any of the K_p PUs. Note that $(1 - P_o)$ equals 1 when $S = K_p$ and when perfect CSI is available.

[5]To compute P_{pi}, $K_c(6N + 2) + 1$ real multiplications, $5NK_c - 1$ real additions, $K_c + 2$ divisions and the max operation that scales as $\mathcal{O}(|\mathcal{S}|)$ would be required.

$$\mathrm{SINR}_k^{ic} = \frac{\frac{s_s^2\gamma_k}{K_c}\left(\zeta_{ic}\frac{\Gamma(N+\frac{1}{2})}{\Gamma(N)}\right)^2}{\frac{(1-s_s^2)\gamma_k\psi_{ic}}{K_c} + \frac{\psi_{ic}\gamma_k(K_c-1)}{K_c} + \frac{s_s^2\gamma_k}{K_c}\left[N\psi_{ic} - \left(\zeta_{ic}\frac{\Gamma(N+\frac{1}{2})}{\Gamma(N)}\right)^2\right] + P_p\phi_k + 1} \tag{15}$$

The signal received at the k^{th} CU with perfect and incomplete CSI at the cognitive BS is

$$y_{c_k} = \sqrt{\frac{P_{pi}}{K_c}} \sum_{k'=1}^{K_p} \frac{\boldsymbol{h}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_{k'}}^{*} q_{k'}}{||\boldsymbol{h}_{c_{k'}}||} + \sqrt{\frac{P_p}{K_p}} \sum_{j=1}^{K_c} \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^{*} s_j}{||\boldsymbol{h}_{p_j}||} + n_k. \tag{20}$$

Considering availability of statistical CSI, signal received at the k^{th} CU can be re-arranged as

$$\begin{aligned} y_{c_k} &= \underbrace{\mathbb{E}\left(\sqrt{\frac{P_{pi}}{K_c}}||\boldsymbol{h}_{c_k}||\right) q_k}_{①} + \underbrace{\sqrt{\frac{P_{pi}}{K_c}}||\boldsymbol{h}_{c_k}|| q_k - \mathbb{E}\left(\sqrt{\frac{P_{pi}}{K_c}}||\boldsymbol{h}_{c_k}||\right) q_k}_{②} \\ &+ \underbrace{\sqrt{\frac{P_{pi}}{K_c}} \sum_{k' \neq k}^{K_p} \frac{\boldsymbol{h}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_{k'}}^{*}}{||\boldsymbol{h}_{c_{k'}}||} q_{k'}}_{③} + \underbrace{\sqrt{\frac{P_p}{K_p}} \sum_{j=1}^{K_c} \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^{*} s_j}{||\boldsymbol{h}_{p_j}||}}_{④} + \underbrace{n_k}_{⑤}. \end{aligned} \tag{21}$$

All the terms in the equation above are mutually uncorrelated. Thus, SINR at the k^{th} CU equals (22), shown at the bottom of the page. Note that SINR_k^{pi} is a function of η which in turn depends on the target P_o and the number of channels S that are known perfectly to the cognitive BS. We use this expression to compute the DL sum spectral efficiency of the CUs in this scenario and state next as Theorem.

Theorem 4: A lower bound SE^{pi} on the DL achievable sum spectral efficiency of CUs with perfect and incomplete CSI and corresponding to a back-off factor based power adaptation policy while satisfying an interference outage probability of P_o at the PUs equals

$$\text{SE}^{pi} = \sum_{k=1}^{K_c} \log_2\Big(1 + \text{SINR}_k^{pi}\Big), \tag{23}$$

where SINR_k^{pi} is given in (24), shown at the bottom of the page. And ζ_{pi} and ψ_{pi} are computed the same way as in Theorem 2 with 'ic' replaced with 'pi', K_p replaced with $S = |\mathcal{S}|$ and $X = \max_{k \in \mathcal{S}}(Z_k)$.

Proof: The derivation is given in Appendix D. ■

Remark: The lower the value of S relative to K_p, the greater is the back-off needed in order to ensure that P_o is maintained at the same level. The reduction in the power due to back-off results in a loss in sum spectral efficiency of the CUs, however this loss can be recovered by deploying more antennas at the cognitive BS. In the asymptotic regime, based on results outlined in Section III, sum spectral efficiency of CUs under perfect and incomplete CSI simplifies to

$$\text{SE}^{pi} = K_c \log_2\left(1 + \frac{\zeta_{pi}^2}{\psi_{pi} - \zeta_{pi}^2}\right). \tag{25}$$

B. Impact of Imperfect and Incomplete CSI

We now consider a scenario where the cognitive BS has an estimate of the interference channels to only a subset of the PUs. As stated before, let $\mathcal{S}$ be the set of PUs to which the cognitive BS knows the channels with estimation errors where $|\mathcal{S}| \leq K_p$. Under this scenario, the power P_{ii} with which the cognitive BS essentially transmits is given by[6]

$$P_{ii} = \frac{\overline{I}_p}{\eta \max_{k \in \mathcal{S}} \left\{ \frac{1}{K_c} \sum_{i=1}^{K_c} \left| \frac{\widehat{\boldsymbol{g}}_{c_k}^{\mathrm{T}} \widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||} \right|^2 \right\}}, \tag{26}$$

where η is chosen such that the interference outage constraint under incomplete and imperfect CSI is met with equality for a target P_o. In other words, given $P_o,\ K_p\ \text{and} s_p$, the back-off factor η is chosen such that

$$\Pr\Big(\widehat{I}_{p_1} \leq \overline{I}_p, \widehat{I}_{p_2} \leq \overline{I}_p, \ldots, \widehat{I}_{p_{K_p}} \leq \overline{I}_p\Big) = 1 - P_o, \tag{27}$$

where the interference power $\widehat{I}_{p_k} = \frac{P_{ii}}{K_c} \sum_{i=1}^{K_c} |\frac{\boldsymbol{g}_{c_k}^{\mathrm{T}} \widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||}|^2$ in this scenario. The power with which cognitive BS transmits now depends on the channel estimates of the S available links. We state a lower bound on the complement of P_o next as a Theorem.

Theorem 5: A lower bound on the complement of interference outage probability $P_{L_{(ii)}}$ under imperfect and incomplete CSI is given by

$$\begin{aligned} (1 - P_o) &\geq P_{L_{(ii)}} \\ &= \Bigg[1 - \sum_{k \in \mathcal{S}} \sum_{l=1}^{L} \frac{w_l\, {x_l}^{K_c - 1}}{\Gamma(K_c)} \prod_{\substack{j \in \mathcal{S} \\ j \neq k}} \frac{\gamma\Big(K_c, \frac{x_l \alpha_k}{\eta \alpha_j}\Big)}{\Gamma(K_c)} \\ &\quad \times \left(1 - Q_{K_c}\left(\sqrt{\frac{2\rho x_l}{(1-\rho)}}, \sqrt{\frac{2 x_l}{\eta(1-\rho)}}\right)\right)\Bigg] \end{aligned}$$

[6]As before, to compute P_{ii}, $K_c(6N+2)+1$ real multiplications, $5NK_c-1$ real additions, K_c+2 divisions and the max operation that scales as $\mathcal{O}(|\mathcal{S}|)$ would be required.

$$\text{SINR}_k^{pi} = \frac{\left|\mathbb{E}\left(\sqrt{\frac{P_{pi}}{K_c}}||\widehat{\boldsymbol{h}}_{c_k}||\right)\right|^2 \mathbb{E}\Big(|q_k|^2\Big)}{\mathbb{E}\left|\sum_{k' \neq k}^{K_c} \sqrt{\frac{P_{pi}}{K_c}} \frac{\boldsymbol{h}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_{k'}}^{*} q_{k'}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||}\right|^2 + \mathbb{E}\left|\sqrt{\frac{P_{pi}}{K_c}} s_s ||\boldsymbol{h}_{c_k}|| q_k - \mathbb{E}\left(\sqrt{\frac{P_{pi}}{K_c}} s_s ||\boldsymbol{h}_{c_k}||\right) q_k\right|^2 + \mathbb{E}\left|\sum_{j=1}^{K_p} \sqrt{\frac{P_p}{K_p}} \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^{*} s_j}{||\boldsymbol{h}_{p_j}||}\right|^2 + \mathbb{E}|n_k|^2} \tag{22}$$

$$\text{SINR}_k^{pi} = \frac{\frac{1}{K_c}\left(\zeta_{pi} \frac{\sqrt{\gamma_k}\ \Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2}{\frac{1}{K_c}\left[\psi_{pi} N \gamma_k - \left(\zeta_{pi} \frac{\sqrt{\gamma_k}\ \Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2\right] + \frac{\psi_{pi}}{K_c}(K_c - 1)\gamma_k + P_p \phi_k + 1} \tag{24}$$

$$\times \left[\sum_{k\in\mathcal{S}} \sum_{l=1}^{L} \frac{w_l \, y_l^{K_c-1}}{(\Gamma(K_c))^{K_p}} \prod_{k'\notin\mathcal{S}} \gamma\left(K_c, \frac{y_l \alpha_k \eta}{\alpha_{k'}}\right) \times \left[\prod_{\substack{i\in\mathcal{S}\\ i\neq k}} \gamma\left(K_c, \frac{y_l \alpha_k}{\alpha_i}\right) \right]\right]. \quad (28)$$

Proof: The derivation is given in Appendix E. ■

With imperfect and incomplete CSI and with the proposed power adaptation policy, the SINR at the k^{th} CU is identical to (13) with P_{ic} replaced by P_{ii}. And the achievable sum spectral efficiency of the CUs under this scenario is stated next as a Theorem.

Theorem 6: A lower bound SE^{ii} on the DL achievable sum spectral efficiency of the CUs with imperfect and incomplete CSI and corresponding to a back-off factor based power adaptation policy while satisfying an interference outage probability of P_o at the PUs is given by

$$\text{SE}^{ii} = \sum_{k=1}^{K_c} \log_2\left(1 + \text{SINR}_k^{ii}\right), \quad (29)$$

where SINR_k^{ii} is given in (30), shown at the bottom of the page. And ζ_{ii} and ψ_{ii} are computed the same way as in Theorem 2 with 'ic' replaced with 'ii' and K_p replaced with $S = |\mathcal{S}|$.

Proof: This can be derived along similar lines as is done in Appendix B. We do not show the proof to conserve space. ■

In the asymptotic regime, based on results outlined in Section III, sum spectral efficiency of CUs under imperfect and incomplete CSI simplifies to

$$\text{SE}^{ii} = K_c \log_2\left(1 + \frac{\zeta_{ii}^2}{\psi_{ii} - \zeta_{ii}^2}\right). \quad (31)$$

C. Spatially Correlated Channel Scenario

In order to analyze the effect of spatial correlation on interference outage probability at the PUs and the sum spectral efficiency at the CUs, we generate the channel covariance matrices for the channels from cognitive BS to CUs, cognitive BS to PUs, primary BS to PUs and primary BS to CUs using the Gaussian local scattering model [43]. For this model, the spatial correlation matrix for the channel from the cognitive BS to the k^{th} user is given by

$$[\boldsymbol{R}_k^{cu}]_{l,m} = \varpi_k^c e^{j2\pi d_H (l-m)\sin\left(\theta_k^{cu}\right)} e^{-\frac{\sigma_\theta^2}{2}\left(2\pi d_H (l-m)\cos\left(\theta_k^{cu}\right)\right)^2}, \quad (32)$$

where $u \in \{c, p\}$, the large-scale fading coefficient $\varpi_k^c \in \{\gamma_k, \alpha_k\}$, and the nominal angle of arrival (AoA) in the azimuthal plane $\theta_k^{cu} \in \{\theta_k^{cc}, \theta_k^{cp}\}$, depending on whether we consider the channel to the CU or to the PU. Furthermore, $l, m \in \{1, 2 \ldots, N\}$, where *N* denotes the number of antennas at the cognitive BS, d_H denotes the spacing among the antenna elements in the array and the parameter σ_θ referred to as the angular standard deviation captures the spatial correlation. A lower value of σ_θ signifies larger spatial correlation [43]. Similarly, based on this model, the spatial correlation matrix for the channel from the primary BS to the k^{th} user is given by

$$\left[\boldsymbol{R}_k^{pu}\right]_{l,m} = \varpi_k^p e^{j2\pi d_H (l-m)\sin\left(\theta_k^{pu}\right)} e^{-\frac{\sigma_\theta^2}{2}\left(2\pi d_H (l-m)\cos\left(\theta_k^{pu}\right)\right)^2}, \quad (33)$$

where $u \in \{c, p\}$, the large-scale fading coefficient $\varpi_k^p \in \{\phi_k, \beta_k\}$, and the nominal AoA in the azimuthal plane $\theta_k^{pu} \in \{\theta_k^{pc}, \theta_k^{pp}\}$, depending on whether we consider the channel to the CU or to the PU. Furthermore, $l, m \in \{1, 2 \ldots, M\}$, where *M* denotes the number of antennas at the primary BS.

V. Numerical Results and Discussion

In this section, we present numerical results to illustrate the impact of different system parameters such as η, s_p, s_s, *S*, *N* and K_p on P_o at the PUs and on the sum spectral efficiency of the CUs under different scenarios. Unless mentioned otherwise, for illustration,[7] we generate the path loss values γ_k, α_j, ϕ_k and β_j independently and uniformly over the interval -30 dB to -60 dB for $1 \leq k \leq K_c$ and for $1 \leq j \leq K_p$, $\sigma^2 = 1$, $M = 100$, $K_c = 10$ and $K_p = 10$ [10], [37]. Furthermore, we take $\overline{I}_p = 10$ dB, $P_p = 10$ dB, $s_s \in \{0.4, 0.6, 0.8, 1\}$, $s_p \in \{0.4, 0.6, 0.8, 1\}$ [10], $S \in \{5, 7, 10\}$ [18] and $\sigma_\theta \in \{10^\circ, 30^\circ\}$ [43].

Figure 2 plots $(1 - P_o)$ vs. η for $K_p = 10$, $K_c = 10$ and for two different values of s_p, namely 0.6 and 0.8. We obtain exact curves through Monte Carlo simulation of (9). We also plot the lower bound obtained through (10). Please note that the lower bound is quite tight for $P_o \leq 0.2$ and we expect the target P_o at PUs to be lower than 20% in underlay spectrum access networks.[8] As the back-off factor increases, P_o decreases because the cognitive BS transmits at a lower power. Furthermore, with the increase in s_p, the quality of channel estimates improves. Therefore, the cognitive BS violates the target P_o less often. Thus, for larger estimation errors, greater amount of back-off in power is required at cognitive BS to keep P_o at the same level. Please note that η is not affected by changes in s_s for a given P_o, since the transmit power of

[7] Please note that our mathematical analysis is in general valid for any practical simulation parameter values.

[8] The lower bound is very tight in the regime of interest where these underlay spectrum access networks are expected to operate, i.e., in the regime ($P_o \leq 0.2$) where the interference outage probability is small enough to ensure minimal interference to PUs.

$$\text{SINR}_k^{ii} = \frac{\frac{s_s^2 \gamma_k}{K_c}\left(\zeta_{ii}\frac{\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2}{\frac{(1-s_s^2)\gamma_k \psi_{ii}}{K_c} + \frac{\psi_{ii}\gamma_k (K_c-1)}{K_c} + \frac{s_s^2 \gamma_k}{K_c}\left[N\psi_{ii} - \left(\zeta_{ii}\frac{\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2\right] + P_p \phi_k + 1} \quad (30)$$

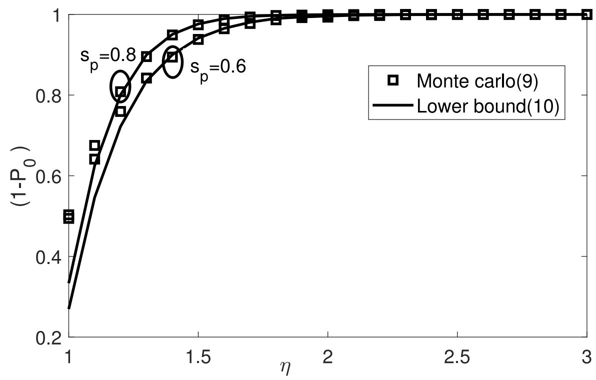


Fig. 2. Imperfect and Complete CSI: Impact of s_p and η on P_o ($K_p = 10$, $N = 100$ and $K_c = 10$).

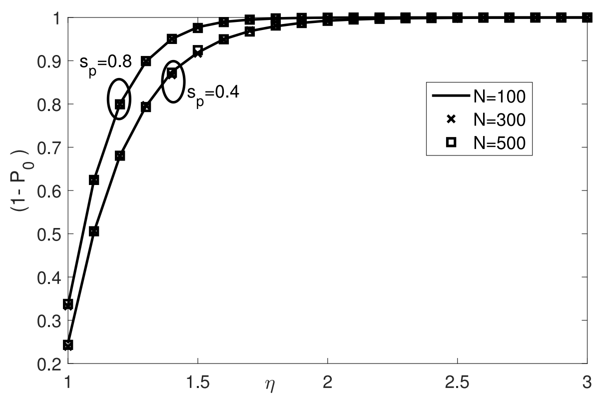


Fig. 3. Imperfect and Complete CSI: Impact of N on P_o ($K_p = 10$ and $K_c = 10$).

the cognitive BS is a function of only the channels between the cognitive BS and the K_p PUs and is independent of the channel to the CUs as also outlined in Theorem 1.[9]

Figure 3 plots $(1-P_o)$ vs. η for $s_p = 0.4$ and 0.8, $K_c = 10$, $K_p = 10$ and for three different values of the number of cognitive BS antennas *N*, namely 100, 300 and 500 based on the lower bound in Theorem 1. We observe that for a given s_p, P_o is independent of N and depends only on η because of scaling by the norm of the channel vector from the cognitive BS to CUs in the MR precoder used at the cognitive BS. This result gives us an interesting design insight that the interference outage probability P_o at the PUs remains unaffected to changes in the number of cognitive BS antennas *N*. As before, a larger back-off is required to maintain P_o at the same level as s_p decreases, i.e., as the channel estimation error increases.

Figure 4 plots the sum spectral efficiency SE^{ic} vs. N for different $(\eta, 1-P_o)$ sampled from Figure 2. We show SE^{ic} using the analytical expression for SINR in (15) and cross-validate through Monte Carlo simulations based on SINR expression given in (13). The design insight that we obtain based on this result is that, for a given value of s_p and s_s, we can reduce P_o while maintaining a fixed value of SE^{ic} by deploying more antennas at the cognitive BS. For example, by increasing N from 107 to 116, we can reduce P_o from 20% to 3% while keeping the sum spectral efficiency fixed at 25 (bits/s/Hz) for

[9] P_o is independent of P_p and $\overline{I}_p$.

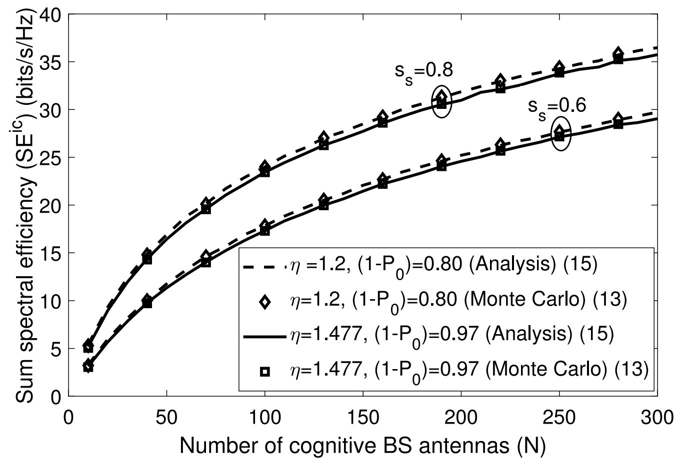
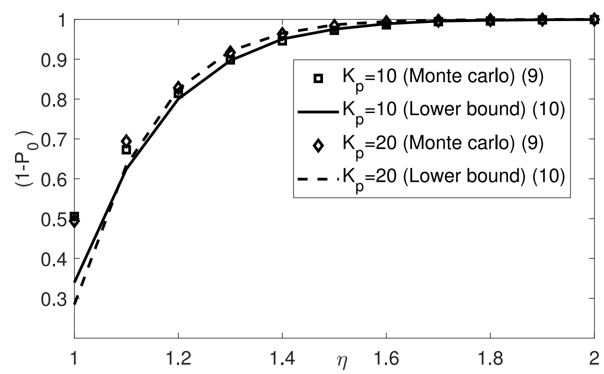
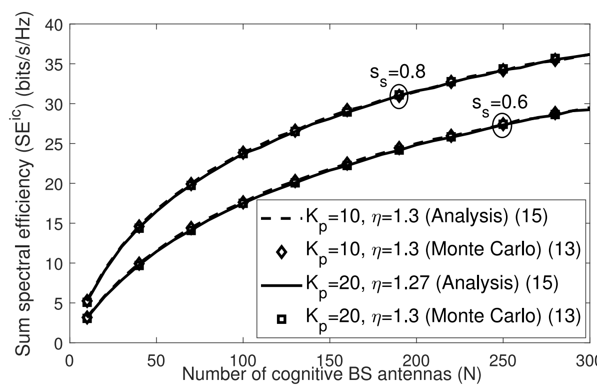


Fig. 4. Imperfect and Complete CSI: Impact of s_s, P_o and N on SE^{ic} ($s_p = 0.8$, $K_p = 10$, $K_c = 10$, $M = 100$, $\overline{I}_p = 10$ dB and $P_p = 10$ dB.)



(a)



(b)

Fig. 5. Imperfect and Complete CSI (a) Impact of K_p on P_o ($s_p = 0.8$, $N = 100$ and $K_c = 10$). (b) Impact of K_p, N and η on SE^{ic} ($(1 - P_o) = 0.9$, $s_p = 0.8$, $K_c = 10$, $M = 100$, $\overline{I}_p = 10$ dB and $P_p = 10$ dB).

$s_s = 0.8$. In order to reduce P_o, the cognitive BS lowers its transmit power based on the proposed back-off power policy. The subsequent reduction in SE^{ic} due to a larger back-off at the cognitive BS is compensated by deploying more antennas at the cognitive BS. Furthermore, as s_s reduces, estimation error increases and sum spectral efficiency of the CUs decreases.

Figure 5 (a) plots $(1 - P_o)$ vs. η for $s_p = 0.8$ and for different values of K_p under imperfect and complete CSI. We observe that for a fixed value of η, P_o is equal to or higher with

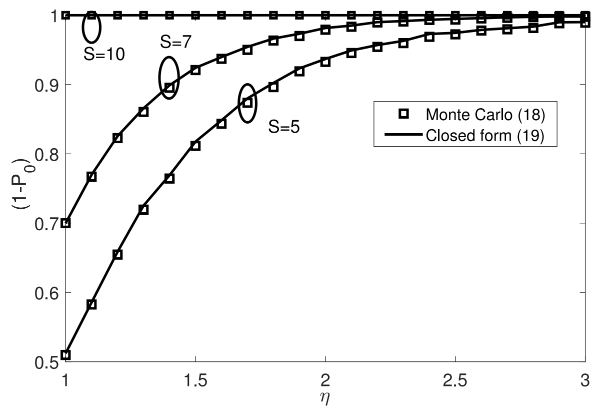
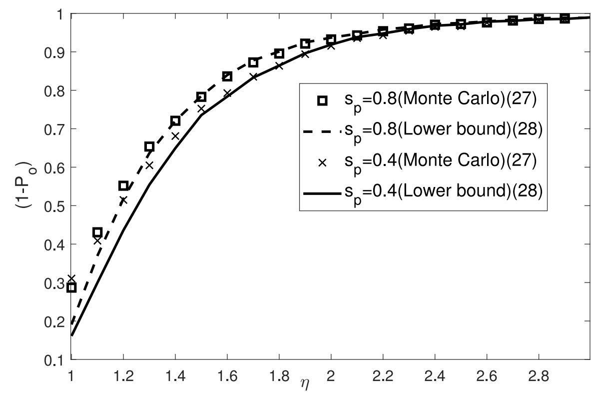
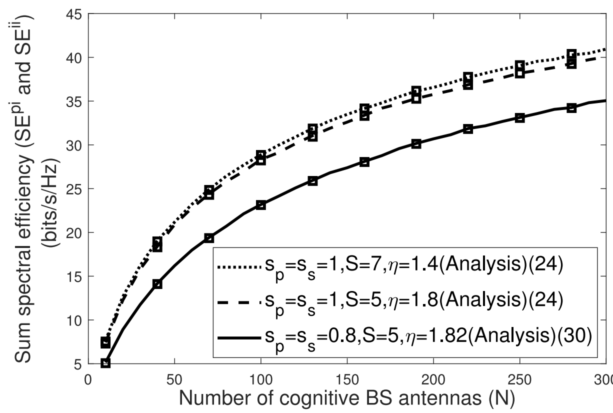


Fig. 6. Perfect and incomplete CSI: Impact of S on $(1-P_o)$ ($s_p = 0.8$, $K_p = 10$, $K_c = 10$ and $N = 100$).



(a)



(b)

Fig. 7. Perfect/Imperfect and incomplete CSI (a) Impact of s_p and η on $1 - P_o$ ($S = 5$, $K_p = 10$, $K_c = 10$ and $N = 100$). (b) Impact of S, s_s, s_p and N on SE^{pi} and SE^{ii} ($(1 - P_o) = 0.9$, $K_p = 10$, $K_c = 10$, $M = 100$, $\overline{I}_p = 10$ dB and $P_p = 10$ dB). Corresponding Monte Carlo simulations are shown using the marker □.

$K_p = 10$ when compared to $K_p = 20$. This is because with $K_p = 20$, the cognitive BS transmits at a reduced power as can be seen from (8), since the maximum value of a set consisting of 20 random channel coefficients is likely to be larger than the maximum value of a set with 10 random channel coefficients corresponding to $K_p = 10$.

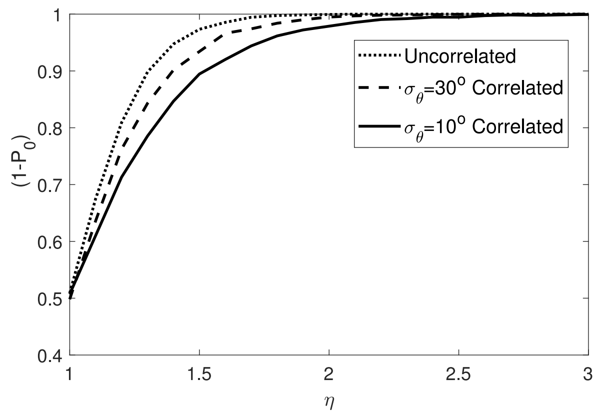
Figure 5 (b) plots SE^{ic} vs. N for different (K_p, η) sampled from Figure 5 (a). We obtain SE^{ic} using the analytical expression for SINR in (15) and cross-validate through Monte Carlo simulations based on SINR expression given in (13). We observe as K_p increases, SE^{ic} decreases marginally since the cognitive BS transmits at a reduced power. Also, SE^{ic} decreases as s_s decreases, due to degradation in the quality of the channel estimates. Another important insight that we obtain is that even if the number of PUs increases, $(1 - P_o)$ and SE^{ic} can be kept fixed by deploying slightly more number of antennas at the BS. For example, for $s_s = 0.8$ and $P_o = 0.1$, we can accommodate ten more PUs in the network while keeping the DL sum spectral efficiency of the CUs fixed at 20 (bits/s/Hz) by increasing N from 69 to 72.

Figure 6 plots $(1 - P_o)$ vs. η for $K_p = 10$ and for different S considering that the cognitive BS has perfect CSI to S out of the K_p PUs. We obtain the curves corresponding to the analytical expression in (19) and cross-validate it via Monte Carlo simulation based on (18). We observe that as the number of channels in set $\mathcal{S}$ (known perfectly to the cognitive BS) increases, P_o reduces. This is primarily because as S increases, the cognitive BS violates the interference outage probability constraint less often. In other words, the lesser the number of channels that are known perfectly to the cognitive BS, the larger is the back-off required to maintain P_o at the same level.

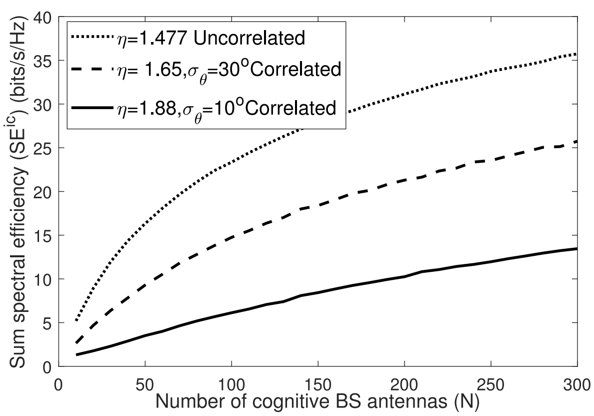
Figure 7(a) plots $(1 - P_o)$ vs. η for $S = 5$, $K_p = 10$, $N = 100$ and for two different values of s_p, namely, 0.4 and 0.8. We obtain exact curves through Monte Carlo simulation of (27) with imperfect incomplete CSI. We also plot the lower bound obtained through the analytical expression in (28). We observe that even under incomplete CSI, as s_p increases, the quality of channel estimates improves and interference outage probability constraint is violated less often.

Figure 7 (b) plots the sum spectral efficiency SE^{pi} under perfect and incomplete CSI (simulations based on SINR in (22) and analysis based on SINR in (24)) with respect to N for $s_s = 1$, for two different values of S, namely, 5 and 7 and $(\eta, 1 - P_o)$ sampled from Figure 6. We also plot the sum spectral efficiency SE^{ii} under imperfect and incomplete CSI (simulations based on SINR in (13) with P_{ic} replaced by P_{ii} and the analysis based on SINR in (30)), where, $(\eta, 1 - P_o)$ is sampled from Figure 7 (a) for $s_s = 0.8$ and $S = 5$. We observe that simulations and the analysis are in close agreement with each other. An interesting design insight that we obtain based on this result is that the sum spectral efficiency of CUs and P_o can be maintained at the same level even under incomplete CSI as s_s decreases by deploying a higher N at the cognitive BS. Furthermore, the sum spectral efficiency improves as S increases.

Figure 8 (a) plots $(1 - P_o)$ as a function of η or two different values of σ_θ, namely 10° and 30°. As a benchmark, we plot $(1 - P_o)$ for the uncorrelated case as well. As the angular standard deviation σ_θ decreases from 30° to 10°, the spatial correlation increases. This results into an increase in the power of the interference caused to the PUs due to transmission by the cognitive BS. This is the reason why a larger back-off is



(a)



(b)

Fig. 8. Impact of spatial correlation on (a) interference outage probability ($s_p = 0.8$, $S = K_p = 10$, $K_c = 10$ and $\theta_k^{cu} = \theta_k^{pu} = 30^\circ$ for all k). (b) sum spectral efficiency ($(1 - P_o) = 0.97$, $s_s = s_p = 0.8$, $K_p = 10$, $K_c = 10$, $M = 100$, $\overline{I}_p = 10$ dB, $P_p = 10$ dB and $\theta_k^{cu} = \theta_k^{pu} = 30^\circ$ for all k).

required as spatial correlation increases in order to maintain interference outage probability at the PUs at the same target level.

Figure 8 (b) plots the sum spectral efficiency of the CUs as a function of N for two different values of σ_θ, namely 10° and 30°. The corresponding back-off factors obtained from the result in Figure 8 (a) for $P_o = 0.03$ have also been marked in the legend. We observe that the sum spectral efficiency degrades as spatial correlation increases due to decrease in the angular standard deviation σ_θ from 30° to 10°. This is because as the spatial correlation increases, the cognitive BS transmits at a lower power in order to maintain P_o at the same level. In other words, by deploying more antennas at the cognitive BS, a higher spatial correlation may be tolerated for a given P_o at the PUs while maintaining the sum spectral efficiency of the CUs at a fixed level. For example, in order to maintain the sum spectral efficiency of CUs at $\mathrm{SE}^{ic} = 10$(bits/s/Hz), for $P_o = 0.03$ at the PUs, the uncorrelated scenario requires $N = 23$, the mildly correlated scenario with $\sigma_\theta = 30^\circ$ requires $N = 56$ and the severely correlated wireless channel scenario with $\sigma_\theta = 10^\circ$ requires $N = 193$ antennas at cognitive BS.

VI. Conclusion

We considered a massive MIMO based underlay spectrum access network in which the cognitive BS has an imperfect and/or incomplete estimate of its channels to the K_p PUs and employs MR beamfomer to transmit under an interference outage probability constraint. We developed novel back-off factor based power control policies for the cognitive BS such that the interference power exceeds an interference threshold no more than P_o fraction of the time at any of the K_p PUs under either incomplete and/or imperfect CSI. The new analytical expressions for $(1 - P_o)$ offer the wireless system designer a ballpark number for η that must be chosen to satisfy the constraint under imperfect and/or incomplete CSI. We then derived new expressions for sum spectral efficiency of CUs under each of the scenarios.

We observed and quantified that by exploiting the huge array gain and the high spatial resolution offered by a massive MIMO cognitive BS, P_o can be reduced without incurring a degradation in the sum spectral efficiency of the CUs under imperfect CSI. To be specific, P_o reduces since the cognitive BS transmits at a reduced power, and due to this the CUs may suffer loss in the sum spectral efficiency. However, this loss can be recovered by deploying more cognitive BS antennas. Also, more number of PUs can be served while keeping P_o at the PUs and the sum spectral efficiency of the CUs fixed at a constant level by deploying a larger number of antennas at the cognitive BS. Furthermore, we observed that P_o increases as S reduces and in order to keep the sum spectral efficiency of CUs fixed as S reduces, more antennas need to be deployed at the cognitive BS. The impact of spatial correlation on P_o at PUs and sum spectral efficiency of CUs was also elucidated. The analysis of underlay cognitive massive MIMO networks under sparse channels requires an independent study and is an interesting avenue for future work.

Appendix A
Proof of Theorem 1

Let $u_{k,i} = \frac{1}{\sqrt{K_c}} \frac{\boldsymbol{g}_{c_k}^{\mathrm{T}} \widehat{\boldsymbol{h}}_{c_i}^*}{||\widehat{\boldsymbol{h}}_{c_i}||}$ and $\widehat{u}_{k,i} = \frac{1}{\sqrt{K_c}} \frac{\widehat{\boldsymbol{g}}_{c_k}^{\mathrm{T}} \widehat{\boldsymbol{h}}_{c_i}^*}{||\widehat{\boldsymbol{h}}_{c_i}||}$ for $1 \leq k \leq K_p$. It can be shown that conditioned on $\widehat{\boldsymbol{h}}_{c_i}$, both $u_{k,i}$ and $\widehat{u}_{k,i}$ are $\mathcal{CN}(0, \frac{\alpha_k}{K_c})$ random variables and are independent of $\widehat{\boldsymbol{h}}_{c_i}$. We need to find the distribution of $Z_k = \sum_{i=1}^{K_c} |u_{k,i}|^2$ and $\widehat{Z}_k = \sum_{i=1}^{K_c} |\widehat{u}_{k,i}|^2$. To do so, we note that both $\chi_k = \frac{2K_c}{\alpha_k} Z_k$ and $\widehat{\chi}_k = \frac{2K_c}{\alpha_k} \widehat{Z}_k$ are chi-square distributed rvs with $2K_c$ degrees of freedom, since each of them is the sum of absolute squares of $2K_c$ i.i.d. $\mathcal{N}(0,1)$ rvs. Based on this and by transformation of rvs, the pdf of Z_k equals $f_{Z_k}(z_k) = \frac{K_c}{\alpha_k} \frac{1}{\Gamma(K_c)} (\frac{K_c}{\alpha_k} z_k)^{K_c - 1} \mathrm{e}^{-\frac{K_c}{\alpha_k} z_k}$ and the pdf of $\widehat{Z}_k$ is given by $f_{\widehat{Z}_k}(\widehat{z}_k) = \frac{K_c}{\alpha_k} \frac{1}{\Gamma(K_c)} (\frac{K_c}{\alpha_k} \widehat{z}_k)^{K_c - 1} \mathrm{e}^{-\frac{K_c}{\alpha_k} \widehat{z}_k}$. Note that Z_k and $\widehat{Z}_k$ are correlated chi-square distributed rvs and the joint pdf is given by [44]

$$f_{Z_k, \widehat{Z}_k}(z_k, \widehat{z}_k) = \frac{\left(\frac{2K_c}{\alpha_k}\right)^{K_c+1} (z_k \widehat{z}_k)^{\frac{K_c - 1}{2}} I_{K_c - 1}\left(\frac{2K_c \sqrt{\rho z_k \widehat{z}_k}}{\alpha_k (1-\rho)}\right)}{\Gamma(K_c) 2^{K_c+1} (1-\rho) \rho^{\frac{K_c - 1}{2}}}$$

$$\times \exp\left(\frac{-1}{1-\rho}\left(\frac{K_c z_k}{\alpha_k}+\frac{K_c\widehat{z}_k}{\alpha_k}\right)\right), \tag{34}$$

where $\rho = s_p^2$ is the correlation coefficient between Z_k and $\widehat{Z}_k$ and $I_{K_c-1}(\cdot)$ is the modified Bessel function of the first kind and order $K_c - 1$ [36]. Thus, (9) can be written as

$$(1-P_o) = \Pr\left(Z_1 \le \eta \max_{1\le k\le K_p}\{\widehat{Z}_k\} \ldots Z_{K_p} \le \eta \max_{1\le k\le K_p}\{\widehat{Z}_k\}\right). \tag{35}$$

Using Bonferroni's inequality [39], a lower bound on (35) is given by

$$\begin{aligned}(1-P_o) &\ge \sum_{i=1}^{K_p}\Pr\left(Z_i \le \eta \max_{1\le k\le K_p}\{\widehat{Z}_k\}\right) - K_p + 1,\\ &= 1-\sum_{i=1}^{K_p}\Pr\left(\widehat{Z}_1 \le \frac{Z_i}{\eta}, \widehat{Z}_2 \le \frac{Z_i}{\eta}\ldots \widehat{Z}_{K_p} \le \frac{Z_i}{\eta}\right).\end{aligned}$$

This can be rewritten as

$$(1-P_o) \ge 1-\sum_{i=1}^{K_p}\int_{z_i=0}^{\infty}\int_{\widehat{z}_i=0}^{z_i/\eta}\prod_{j\ne i}^{K_p}\Pr\left(\widehat{Z}_j \le \frac{z_i}{\eta}\right) f_{Z_i,\widehat{Z}_i}(z_i,\widehat{z}_i)\,d\widehat{z}_i\,dz_i. \tag{36}$$

Using the pdf $f_{\widehat{Z}_k}(\widehat{z}_k)$, it can be shown that

$$\Pr\left(\widehat{Z}_j \le \frac{z_i}{\eta}\right) = \frac{1}{\Gamma(K_c)}\gamma\left(K_c, \frac{K_c z_i}{\eta\alpha_j}\right), \tag{37}$$

where $\gamma(\cdot)$ is the lower incomplete Gamma function [35]. Substituting (37) in (36), we get

$$(1-P_o) \ge 1-\sum_{i=1}^{K_p}\int_{z_i=0}^{\infty}\int_{\widehat{z}_i=0}^{z_i/\eta}\prod_{j\ne i}^{K_p}\frac{\gamma\left(K_c, \frac{K_c z_i}{\eta\alpha_j}\right) f_{Z_i,\widehat{Z}_i}(z_i,\widehat{z}_i)\,d\widehat{z}_i\,dz_i}{\Gamma(K_c)}. \tag{38}$$

Upon re-arrangement, (38) can be written as

$$\begin{aligned}(1-P_o) \ge 1-\sum_{i=1}^{K_p}\int_{z_i=0}^{\infty}\prod_{j\ne i}^{K_p}\frac{\gamma\left(K_c, \frac{K_c z_i}{\eta\alpha_j}\right)}{\Gamma(K_c)}\\ \times\left(f_{Z_i}(z_i) - \int_{\widehat{z}_i=z_i/\eta}^{\infty} f_{Z_i,\widehat{Z}_i}(z_i,\widehat{z}_i)\,d\widehat{z}_i\right)dz_i.\end{aligned} \tag{39}$$

Substituting for the joint pdf from (34) and simplifying using the identity [36, eq. (4.59)], the lower bound in (39) simplifies to

$$\begin{aligned}(1-P_o) \ge P_{L_{(ic)}} = 1-\sum_{i=1}^{K_p}\int_{z_i=0}^{\infty}\prod_{j\ne i}^{K_p}\frac{\gamma\left(K_c, \frac{K_c z_i}{\eta\alpha_j}\right)}{\Gamma(K_c)} f_{Z_i}(z_i)\\ \times\left(1-Q_{K_c}\left(\sqrt{\frac{2K_c\rho z_i}{\alpha_i(1-\rho)}}, \sqrt{\frac{2K_c z_i}{\eta\alpha_i(1-\rho)}}\right)\right)dz_i.\end{aligned} \tag{40}$$

This is further simplified using Gauss Laguerre integration to obtain (10).

Appendix B
Proof of Theorem 2

We are interested in simplifying the SINR expression in (13) obtained from (12). To this end, we compute the variance of each of the terms in (12). The variance of term ① is given by

$$\begin{aligned}\mathbb{V}\mathrm{ar}(\text{term}①) &= \left|\mathbb{E}\left(\sqrt{\frac{P_{ic}}{K_c}}\left(s_s||\widehat{\boldsymbol{h}}_{c_k}||\right)\right)\right|^2\mathbb{E}\left(|q_k|^2\right),\\ &= \frac{s_s^2}{K_c}\left(\mathbb{E}\left(\sqrt{P_{ic}}\right)\right)^2\left(\mathbb{E}\left(||\widehat{\boldsymbol{h}}_{c_k}||\right)\right)^2,\\ &= \frac{s_s^2}{K_c}\left(\zeta_{ic}\sqrt{\gamma_k}\frac{\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2.\end{aligned} \tag{41}$$

The second equality above follows from the fact that $\mathbb{E}(|q_k|^2) = 1$ and the fact that P_{ic} depends only on $\widehat{\boldsymbol{g}}_{c_k}$ and is independent and uncorrelated to $\widehat{\boldsymbol{h}}_{c_k}$. Note that $\zeta_{ic} = \mathbb{E}(\sqrt{P_{ic}}) = \mathbb{E}(\sqrt{\frac{\overline{I}_p}{\eta X}}) \approx \sqrt{\frac{\overline{I}_p}{\eta}}(\frac{1}{\mathbb{E}(\sqrt{X})} + \frac{\mathbb{V}\mathrm{ar}(\sqrt{X})}{(\mathbb{E}(\sqrt{X}))^3})$ [38], where $X = \max_{1\le k\le K_p}(\widehat{Z}_k)$, and its CDF $F_X(x) = \frac{1}{(\Gamma(K_c))^{K_p}}\prod_{i=1}^{K_p}\gamma(K_c, \frac{K_c x}{\alpha_i})$. Furthermore, $\mathbb{E}(\sqrt{X}) = \int_0^\infty \frac{1}{2\sqrt{x}}(1-F_X(x))dx$ and $\mathbb{V}\mathrm{ar}(\sqrt{X}) = \int_0^\infty(1-F_X(x))dx - (\mathbb{E}(\sqrt{X}))^2$ [39]. Also, let $U = ||\widehat{\boldsymbol{h}}_{c_k}||^2$. Since, $\widehat{h}_{c_{k_i}} \sim \mathcal{CN}(0,\gamma_k)$, it can be shown that $f_U(u) = \frac{1}{(\gamma_k)^N}\frac{1}{\Gamma(N)}(u)^{N-1}\mathrm{e}^{-\frac{u}{\gamma_k}}$, $\mathbb{E}(U) = N\gamma_k$ and $\mathbb{E}(\sqrt{U}) = \sqrt{\gamma_k}\frac{\Gamma(N+\frac{1}{2})}{\Gamma(N)}$ [39].

The variance of the second term is given by

$$\begin{aligned}\mathbb{V}\mathrm{ar}(\text{term}②) &= \frac{(1-s_s^2)\mathbb{E}(P_{ic})}{K_c}\mathbb{E}\left|\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\frac{\widehat{\boldsymbol{h}}_{c_k}^*}{||\widehat{\boldsymbol{h}}_{c_k}^*||}\right|^2\mathbb{E}|q_k|^2,\\ &= \frac{(1-s_s^2)\mathbb{E}(P_{ic})}{K_c}\mathbb{E}\left[\frac{\widehat{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widetilde{\boldsymbol{h}}_{c_k}^*\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_k}^*}{||\widehat{\boldsymbol{h}}_{c_k}^*||^2}\right],\\ &= \frac{(1-s_s^2)}{K_c}\psi_{ic}\gamma_k.\end{aligned} \tag{42}$$

The first equality above is due to the fact that $\widehat{P}_{ic}$ is independent and uncorrelated to both $\widetilde{\boldsymbol{h}}_{c_k}$ and $\widehat{\boldsymbol{h}}_{c_k}$. And the last equality follows since $\widetilde{\boldsymbol{h}}_{c_k}$ is uncorrelated and independent of $\widehat{\boldsymbol{h}}_{c_k}$ and the fact that $\mathbb{E}(\widetilde{\boldsymbol{h}}_{c_k}^*\widetilde{\boldsymbol{h}}_{c_k}^{\mathrm{T}}) = \gamma_k\boldsymbol{I}_N$. Note that $\psi_{ic} = \mathbb{E}(P_{ic}) = \mathbb{E}(\frac{\overline{I}_p}{\eta X}) \approx \frac{\overline{I}_p}{\eta}(\frac{1}{\mathbb{E}(X)} + \frac{\mathbb{V}\mathrm{ar}(X)}{(\mathbb{E}(X))^3})$ [38], $\mathbb{E}(X) = \int_0^\infty(1-F_X(x))dx$ and $\mathbb{V}\mathrm{ar}(X) = \int_0^\infty 2x(1-F_X(x))dx - (\mathbb{E}(X))^2$ [39].

To compute the variance of the third term, we use the facts that P_{ic}, $\boldsymbol{h}_{c_k}$ and $\widehat{\boldsymbol{h}}_{c_{k'}}$ are mutually independent for any k or k' and that $\mathbb{E}\{q_i q_j^*\} = 0$ for $i \ne j$ to simplify as follows:

$$\begin{aligned}\mathbb{V}\mathrm{ar}(\text{term}③) &= \mathbb{E}\left|\sqrt{\frac{P_{ic}}{K_c}}\sum_{k'\ne k}^{K_c}\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_{k'}}^* q_{k'}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||}\right|^2,\\ &= \frac{\mathbb{E}(P_{ic})}{K_c}\sum_{k'\ne k}^{K_c}\mathbb{E}\left|\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_{k'}}^*}{||\widehat{\boldsymbol{h}}_{c_{k'}}||}\right|^2\mathbb{E}|q_{k'}|^2,\end{aligned}$$

$$= \frac{\mathbb{E}(P_{ic})}{K_c} \sum_{k' \neq k}^{K_c} \mathbb{E}\left[\frac{\widehat{\boldsymbol{h}}_{c_{k'}}^{\mathrm{T}} \mathbb{E}\left(\boldsymbol{h}_{c_k}^{*} \boldsymbol{h}_{c_k}^{\mathrm{T}}\right) \widehat{\boldsymbol{h}}_{c_{k'}}^{*}}{||\widehat{\boldsymbol{h}}_{c_{k'}}||^2} \right],$$
$$= \frac{\psi_{ic}}{K_c}(K_c - 1)\gamma_k. \quad (43)$$

The variance of the fourth term arising due to beamforming uncertainty is given by

$$\mathbb{V}\mathrm{ar}(\mathrm{term}④) = \frac{s_s^2}{K_c} \mathbb{E}\left| \sqrt{P_{ic}} ||\widehat{\boldsymbol{h}}_{c_k}|| - \mathbb{E}\left(\sqrt{P_{ic}}\right) \mathbb{E}(||\widehat{\boldsymbol{h}}_{c_k}||) \right|^2 \mathbb{E}|q_k|^2,$$
$$= \frac{s_s^2}{K_c}\left[\psi_{ic} N \gamma_k - \left(\zeta_{ic} \sqrt{\gamma_k} \frac{\Gamma\left(N + \frac{1}{2}\right)}{\Gamma(N)} \right)^2 \right]. \quad (44)$$

To compute the variance of term ⑤, we exploit the facts that $\mathbb{E}\{s_i s_j^*\} = 0$ for $i \neq j$ and that the channel from the primary BS to any PU is uncorrelated to the channel from the primary BS to any CU. Thus, the variance of the fifth term arising due to interference from PUs is given by

$$\mathbb{V}\mathrm{ar}(\mathrm{term}⑤) = \mathbb{E}\left| \sqrt{\frac{P_p}{K_p}} \sum_{j=1}^{K_p} \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^{*} s_j}{||\boldsymbol{h}_{p_j}||} \right|^2,$$
$$= \frac{P_p}{K_p} \sum_{j=1}^{K_p} \mathbb{E}\left| \frac{\boldsymbol{g}_{p_k}^{\mathrm{T}} \boldsymbol{h}_{p_j}^{*}}{||\boldsymbol{h}_{p_j}||} \right|^2 \mathbb{E}|s_j|^2 = \frac{P_p}{K_p} \sum_{j=1}^{K_p} \phi_k = P_p \phi_k. \quad (45)$$

And the variance of noise equals $\mathbb{E}(|n_k|^2) = 1$.

Appendix C
Proof of Theorem 3

We can re-write (18) as

$$(1 - P_o) = \Pr\left(Z_1 \leq \eta \max_{k \in \mathcal{S}}\{Z_k\} \ldots Z_{K_p} \leq \eta \max_{k \in \mathcal{S}}\{Z_k\} \right), \quad (46)$$

where, $Z_k = \sum_{i=1}^{K_c} |\sqrt{\frac{1}{K_c}} \frac{\boldsymbol{g}_{c_k}^{\mathrm{T}} \boldsymbol{h}_{c_i}^{*}}{||\boldsymbol{h}_{c_i}||}|^2$. Since we know the channel to $|\mathcal{S}|$ out of the K_p PUs, we can modify (46) as follows:

$$(1 - P_o) = \Pr\left(\left(\max_{k \in \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k) \right) \cap \left(\max_{k' \notin \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k) \right) \right),$$
$$= \Pr\left(\max_{k \in \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k) \right)$$
$$\times \Pr\left(\max_{k' \notin \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k) \right). \quad (47)$$

Note that the event $\{\max_{k \in \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k)\}$ will always be true, i.e., it is sure event.

Therefore, $\Pr(\max_{k \in \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k)) = 1$. And, $(1-P_o)$ above can be written down in a further simplified form as $(1 - P_o) = \Pr(\max_{k' \notin \mathcal{S}}(Z_k) \leq \eta \max_{k \in \mathcal{S}}(Z_k))$. Now in order to evaluate this probability, let $Y_1 = \max_{k' \notin \mathcal{S}}(Z_{k'})$ and $Y_2 = \eta \max_{k \in \mathcal{S}}(Z_k)$. Therefore, we are interested in evaluating $(1-P_o) = \Pr(Y_1 \leq Y_2)$. Since, Y_1 is independent of Y_2, therefore,

$$(1 - P_o) = \Pr(Y_1 \leq Y_2) = \int_0^{\infty} F_{Y_1}(y_2) f_{Y_2}(y_2) dy_2. \quad (48)$$

To simplify this further, we need to find the CDF of Y_1 and the pdf of Y_2. Based on the analysis in Appendix A, it can be shown that the CDF of Y_1 equals

$$F_{Y_1}(y_2) = \Pr(Y_1 \leq y_2) = \frac{\prod_{k' \notin \mathcal{S}} \gamma\left(K_c, \frac{K_c y_2}{\alpha_{k'}}\right)}{(\Gamma(K_c))^{K_p - S}}, \quad (49)$$

and the CDF of Y_2 equals $F_{Y_2}(y_2) = \Pr(Y_2 \leq y_2) = \frac{1}{(\Gamma(K_c))^S} \prod_{k \in \mathcal{S}} \gamma(K_c, \frac{K_c y_2}{\alpha_k \eta})$. The pdf of Y_2 can be found by taking the derivative of $F_{Y_2}(y_2)$ w.r.t. y_2 and simplified further to obtain [45]

$$f_{Y_2}(y_2) = \frac{(K_c)^{K_c} y_2^{K_c - 1}}{(\Gamma(K_c))^S \eta^{K_c}} \sum_{k \in \mathcal{S}} \frac{e^{-\frac{K_c y_2}{\alpha_k \eta}}}{\alpha_k^{K_c}} \prod_{\substack{i \in \mathcal{S} \\ i \neq k}} \gamma\left(K_c, \frac{K_c y_2}{\alpha_i \eta}\right). \quad (50)$$

By substituting (49) and (50) in (48), we get

$$(1 - P_o) = \int_0^{\infty} \frac{K_c^{K_c} y^{K_c - 1}}{(\Gamma(K_c))^{K_p} \eta^{K_c}} \prod_{k' \notin \mathcal{S}} \gamma\left(K_c, \frac{K_c y}{\alpha_{k'}}\right)$$
$$\times \left(\sum_{k \in \mathcal{S}} \frac{e^{-\frac{K_c y}{\alpha_k \eta}}}{\alpha_k^{K_c}} \left[\prod_{\substack{i \in \mathcal{S} \\ i \neq k}} \gamma\left(K_c, \frac{K_c y}{\alpha_i \eta}\right) \right] \right) dy. \quad (51)$$

Using Gauss Laguerre integration, this can be further simplified to obtain (19).

A. Proof of Theorem 4

We are interested in simplifying the SINR expression in (22) obtained from (21). To this end, we compute the variance of each of the terms in (21). The variance of term① equals

$$\mathbb{V}\mathrm{ar}(\mathrm{term}①) = \frac{1}{K_c}\left(\mathbb{E}\left(\sqrt{P_{pi}}\right)\right)^2 (\mathbb{E}(||\boldsymbol{h}_{c_k}||))^2,$$
$$= \frac{1}{K_c}\left(\mathbb{E}\left(\sqrt{P_{pi}}\right) \sqrt{\gamma_k} \frac{\Gamma\left(N + \frac{1}{2}\right)}{\Gamma(N)} \right)^2,$$
$$= \frac{1}{K_c}\left(\zeta_{pi} \frac{\sqrt{\gamma_k}\Gamma\left(N + \frac{1}{2}\right)}{\Gamma(N)} \right)^2. \quad (52)$$

The first equality above follows from the fact that $\mathbb{E}(|q_k|^2) = 1$ and the fact that P_{pi} being a function of $\boldsymbol{g}_{c_k}$ is independent and uncorrelated to $\boldsymbol{h}_{c_k}$. Note that $\zeta_{pi} = \mathbb{E}(\sqrt{P_{pi}}) = \mathbb{E}(\sqrt{\frac{\overline{I}_p}{\eta X}}) \approx \sqrt{\frac{\overline{I}_p}{\eta}}(\frac{1}{\mathbb{E}(\sqrt{X})} + \frac{\mathbb{V}\mathrm{ar}(\sqrt{X})}{(\mathbb{E}(\sqrt{X}))^3})$, where $X = \max_{k \in \mathcal{S}}(Z_k)$, and its CDF $F_X(x) = \frac{1}{(\Gamma(K_c))^S} \prod_{i \in \mathcal{S}} \gamma(K_c, \frac{K_c x}{\alpha_i})$. Furthermore, $\mathbb{E}(\sqrt{X}) = \int_0^{\infty} \frac{1}{2\sqrt{x}}(1 - F_X(x))dx$, and $\mathbb{V}\mathrm{ar}(\sqrt{X}) = \int_0^{\infty}(1 - F_X(x))dx - (\mathbb{E}(\sqrt{X}))^2$ [39]. Also, let $U = ||\boldsymbol{h}_{c_k}||^2$, where $h_{c_{k_i}} \sim \mathcal{CN}(0, \gamma_k)$, it can be shown that $f_U(u) = \frac{1}{(\gamma_k)^N} \frac{1}{\Gamma(N)} (u)^{N-1} \mathrm{e}^{-\frac{u}{\gamma_k}}$, $\mathbb{E}(U) = N\gamma_k$ and $\mathbb{E}(\sqrt{U}) = \sqrt{\gamma_k} \frac{\Gamma(N + \frac{1}{2})}{\Gamma(N)}$ [39].

The variance of the second term is given by

$$\mathbb{V}\mathrm{ar}(\mathrm{term}②) = \frac{1}{K_c} \mathbb{E}\left| \sqrt{P_{pi}} ||\boldsymbol{h}_{c_k}|| - \mathbb{E}\left(\sqrt{P_{pi}}\right) \mathbb{E}(||\boldsymbol{h}_{c_k}||) \right|^2 \mathbb{E}|q_k|^2,$$

$$= \frac{1}{K_c}\left[\psi_{pi} N\gamma_k - \left(\zeta_{pi}\frac{\sqrt{\gamma_k}\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)}\right)^2\right]. \tag{53}$$

The first equality above is due to the fact that P_{pi} is independent and uncorrelated to $\boldsymbol{h}_{c_k}$. Note that $\psi_{pi} = \mathbb{E}(P_{pi}) = \mathbb{E}(\frac{\overline{I}_p}{\eta X}) \approx \frac{\overline{I}_p}{\eta}(\frac{1}{\mathbb{E}(X)} + \frac{\mathbb{V}\mathrm{ar}(X)}{(\mathbb{E}(X))^3})$, $\mathbb{E}(X) = \int_0^\infty (1 - F_X(x))dx$ and $\mathbb{V}\mathrm{ar}(X) = \int_0^\infty 2x(1 - F_X(x))dx - (\mathbb{E}(X))^2$ [39].

To compute the variance of the third term, we use the facts that P_{pi}, $\boldsymbol{h}_{c_k}$ and $\boldsymbol{h}_{c_{k'}}$ are mutually independent for any k or k' and that the symbols transmitted by different users are uncorrelated to each other to simplify as follows:

$$\begin{aligned}\mathbb{V}\mathrm{ar}(\mathrm{term}③) &= \mathbb{E}\left|\sqrt{\frac{P_{pi}}{K_c}}\sum_{k'\neq k}^{K_c}\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\boldsymbol{h}_{c_{k'}}^{*}q_{k'}}{||\boldsymbol{h}_{c_{k'}}||}\right|^2,\\ &= \frac{\mathbb{E}(P_{pi})}{K_c}\sum_{k'\neq k}^{K_c}\mathbb{E}\left|\frac{\boldsymbol{h}_{c_k}^{\mathrm{T}}\boldsymbol{h}_{c_{k'}}^{*}}{||\boldsymbol{h}_{c_{k'}}||}\right|^2\mathbb{E}|q_{k'}|^2,\\ &= \frac{\psi_{pi}}{K_c}(K_c - 1)\gamma_k. \end{aligned}\tag{54}$$

The variance of term④ in (21) is already derived in Appendix B. And the variance of noise equals $\mathbb{E}(|n_k|^2) = 1$.

APPENDIX E
PROOF OF THEOREM 5

We can rewrite (27) as

$$(1 - P_o) = \Pr\Big(Z_1 \leq \eta \max_{k\in S}\{\widehat{Z}_k\}\ldots Z_{K_p} \leq \eta \max_{k\in S}\{\widehat{Z}_k\}\Big), \tag{55}$$

where, $Z_k = \sum_{i=1}^{K_c}|\sqrt{\frac{1}{K_c}}\frac{\boldsymbol{g}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||}|^2$ and $\widehat{Z}_k = \sum_{i=1}^{K_c}|\sqrt{\frac{1}{K_c}}\frac{\widehat{\boldsymbol{g}}_{c_k}^{\mathrm{T}}\widehat{\boldsymbol{h}}_{c_i}^{*}}{||\widehat{\boldsymbol{h}}_{c_i}||}|^2$. Since we know the channel estimates to $|\mathcal{S}|$ out of the K_p PUs, we can modify (55) as follows:

$$\begin{aligned}(1-P_o) &= \Pr\left(\left(\max_{k\in S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right)\cap\left(\max_{k'\notin S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right)\right),\\ &= \Pr\left(\max_{k\in S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right)\Pr\left(\max_{k'\notin S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right).\end{aligned}\tag{56}$$

Using Bonferroni's inequality [39], a lower bound on $\Pr(\max_{k\in S}(Z_k) \leq \eta\max_{k\in S}(\widehat{Z}_k))$ is

$$\begin{aligned}\Pr\left(\max_{k\in S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right) &\geq \sum_{i\in\mathcal{S}}\Pr\left(Z_i \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right) - S + 1,\\ &= 1 - \sum_{i\in\mathcal{S}}\Pr\left(Z_i \geq \eta\max_{k\in\mathcal{S}}\{\widehat{Z}_k\}\right).\end{aligned}\tag{57}$$

We can use the joint pdf of Z_i and $\widehat{Z}_i$ to simplify and write down the above equation as

$$\begin{aligned}\Pr\left(\max_{k\in S}(Z_k) \leq \eta\max_{k\in S}\left(\widehat{Z}_k\right)\right) &\geq 1 - \sum_{i\in\mathcal{S}}\int_{z_i=0}^{\infty}\frac{f_{Z_i}(z_i)}{\Gamma(K_c)^{S-1}}\\ &\times \prod_{\substack{j\in\mathcal{S}\\ j\neq i}}\gamma\left(K_c, \frac{K_c z_i}{\eta\alpha_j}\right)\left(1 - Q_{K_c}\left(\sqrt{\frac{2K_c\rho z_i}{\alpha_i(1-\rho)}}, \sqrt{\frac{2K_c z_i}{\eta\alpha_i(1-\rho)}}\right)\right)dz_i.\end{aligned}\tag{58}$$

Now in order to evaluate the probability $\Pr(\max_{k'\notin S}(Z_k) \leq \eta\max_{k\in S}(\widehat{Z}_k))$, let $W_1 = \max_{k'\notin\mathcal{S}}(Z_{k'})$ and $W_2 = \eta\max_{k\in\mathcal{S}}(\widehat{Z}_k)$. Since, W_1 is independent of W_2, therefore,

$$\Pr(W_1 \leq W_2) = \int_0^\infty F_{W_1}(w_2)f_{W_2}(w_2)dw_2. \tag{59}$$

To simplify this further, we need to find CDF of W_1 and pdf of W_2. The CDF of W_1 equals

$$F_{W_1}(w_2) = \Pr(W_1 \leq W_2) = \frac{\prod_{k'\notin\mathcal{S}}\gamma\left(K_c, \frac{K_c w_2}{\alpha_{k'}}\right)}{(\Gamma(K_c))^{K_p - S}}, \tag{60}$$

and the CDF of W_2 equals $F_{W_2}(w_2) = \Pr(W_2 \leq w_2) = \frac{1}{(\Gamma(K_c))^S}\prod_{k\in\mathcal{S}}\gamma(K_c, \frac{K_c w_2}{\alpha_k\eta})$. The pdf of W_2 is found by taking derivative of $F_{W_2}(w_2)$ w.r.t. w_2 and simplified further to obtain [45]

$$f_{W_2}(w_2) = \frac{(K_c)^{K_c}w_2^{K_c-1}}{(\Gamma(K_c))^S\eta^{K_c}}\sum_{k\in\mathcal{S}}\frac{e^{-\frac{K_c w_2}{\alpha_k\eta}}}{\alpha_k^{K_c}}\prod_{\substack{i\in\mathcal{S}\\ i\neq k}}\gamma\left(K_c, \frac{K_c w_2}{\alpha_i\eta}\right). \tag{61}$$

By substituting (60) and (61) in (59), and also using (58), we get

$$\begin{aligned}(1 - P_o) &\geq P_{L_{(ii)}}\\ &= \left[1 - \sum_{k\in\mathcal{S}}\int_{z_k=0}^{\infty}\frac{f_{Z_k}(z_k)}{(\Gamma(K_c))^{S-1}}\prod_{\substack{j\in\mathcal{S}\\ j\neq k}}\gamma\left(K_c, \frac{K_c z_k}{\eta\alpha_j}\right)\right.\\ &\quad\left.\times\left(1 - Q_{K_c}\left(\sqrt{\frac{2K_c\rho z_k}{\alpha_k(1-\rho)}}, \sqrt{\frac{2K_c z_k}{\eta\alpha_k(1-\rho)}}\right)\right)dz_k\right]\\ &\quad\times\left[\int_0^\infty\frac{K_c^{K_c}y^{K_c-1}}{(\Gamma(K_c))^{K_p}\eta^{K_c}}\prod_{k'\notin\mathcal{S}}\gamma\left(K_c, \frac{K_c y}{\alpha_{k'}}\right)\right.\\ &\quad\left.\times\left(\sum_{k\in\mathcal{S}}\frac{e^{-\frac{K_c y}{\alpha_k\eta}}}{\alpha_k^{K_c}}\left[\prod_{\substack{i\in\mathcal{S}\\ i\neq k}}\gamma\left(K_c, \frac{K_c y}{\alpha_i\eta}\right)\right]\right)dy\right].\end{aligned}\tag{62}$$

Using Gauss Laguerre integration, this can be further simplified to obtain (28).

REFERENCES

[1] R. Gupta, S. Kashyap, and E. V. Pothan, "Interference violation probability constrained underlay cognitive massive MIMO network under imperfect channel knowledge," in *Proc. IEEE Nat. Conf. Commun.*, Feb. 2019, pp. 1–6.

[2] *Unlicensed Use of the 6 GHz Band; Expanding Flexible Use in Mid-Band Spectrum Between 3.7 and 24 GHz*, document FCC-20-51, FCC, Washington, DC, USA, 2020.

[3] P. K. Sangdeh, H. Pirayesh, A. Quadri, and H. Zeng, "A practical spectrum sharing scheme for cognitive radio networks: Design and experiments," *IEEE/ACM Trans. Netw.*, vol. 28, no. 4, pp. 1818–1831, Aug. 2020.

[4] S. Kashyap and N. B. Mehta, "SEP-optimal transmit power policy for peak power and interference outage probability constrained underlay cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6371–6381, Dec. 2013.

[5] L. Sboui, Z. Rezki, and M. S. Alouini, "A unified framework for the ergodic capacity of spectrum sharing cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 877–887, Feb. 2013.

[6] E. G. Larsson, F. Tufvesson, O. Edfors, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[7] E. Björnson, E. G. Larsson, and T. L. Marzetta, "Massive MIMO: Ten myths and one critical question," *IEEE Commun. Mag.*, vol. 54, no. 2, pp. 114–123, Feb. 2016.

[8] G. Geraci, A. G. Rodriguez, D. L. Perez, A. Bonfante, G. Giordano, and H. Claussen, "Operating massive MIMO in unlicensed bands for enhanced coexistence and spatial reuse," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1282–1293, Jun. 2017.

[9] S. Chaudhari and D. Cabric, "Downlink transceiver beamforming and admission control for massive MIMO cognitive radio networks," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Nov. 2015, pp. 1257–1261.

[10] L. Wang, H. Q. Ngo, M. Elkashlan, T. Q. Duong, and K.-K. Wong, "Massive MIMO in spectrum sharing networks: Achievable rate and power efficiency," *IEEE Syst. J.*, vol. 11, no. 1, pp. 20–31, Mar. 2017.

[11] H. Al-Hraishawi and G. A. A. Baduge, "Wireless energy harvesting in cognitive massive MIMO systems with underlay spectrum sharing," *IEEE Commun. Lett.*, vol. 6, no. 1, pp. 134–137, Feb. 2017.

[12] M. Cui, B. J. Hu, X. Li, H. Chen, S. Hu, and Y. Wang, "Energy-efficient power control algorithms in massive MIMO cognitive radio networks," *IEEE Access*, vol. 5, pp. 1164–1177, 2017.

[13] G. Amarasuriya and Y. Li, "Cognitive massive MIMO relay networks," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2017, pp. 1–6.

[14] M. Cui, B.-J. Hu, J. Tang, and Y. Wang, "Energy-efficient joint power allocation in uplink massive MIMO cognitive radio networks with imperfect CSI," *IEEE Access*, vol. 5, pp. 27611–27621, 2017.

[15] S. Kusaladharma and C. Tellambura, "Massive MIMO based underlay networks with power control," in *Proc. IEEE Int. Conf. Commun.*, May 2016, pp. 1–6.

[16] A. G.-Rodriguez and C. Masouros, "Exploiting the tolerance of massive MIMO to incomplete CSI for low-complexity transmission," in *Proc. IEEE Global Commun. Conf.*, Dec. 2015, pp. 1–6.

[17] S. Silva, M. Ardakani, and C. Tellambura, "Relay selection for cognitive massive MIMO two-way relay networks," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2017, pp. 1–6.

[18] P. Das, N. B. Mehta, and P. N. Arya, "Cognitive relay selection with incomplete channel state information of interference links," in *Proc. IEEE Int. Conf. Commun.*, May 2017, pp. 1–6.

[19] Y. Li, D. Kudathanthirige, and G. A. A. Baduge, "Massive MIMO relay networks with underlay spectrum sharing," *IEEE Trans. Cogn. Commun. Netw.*, vol. 4, no. 4, pp. 677–691, Dec. 2018.

[20] H. Al-Hraishawi, G. A. A. Baduge, H. Q. Ngo, and E. G. Larsson, "Multi-cell massive MIMO uplink with underlay spectrum sharing," *IEEE Trans. Cogn. Commun. Netw.*, vol. 5, no. 1, pp. 119–137, Mar. 2019.

[21] S. Kusaladharma and C. Tellambura, "Secondary user interference characterization for spatially random underlay networks with massive MIMO and power control," *IEEE Trans. Veh. Technol.*, vol. 66, no. 9, pp. 7897–7912, Sep. 2017.

[22] K. Hosseini, W. Yu, and R. S. Adve, "Large-scale MIMO versus network MIMO for multicell interference mitigation," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 930–941, May 2014.

[23] S. Chaudhari and D. Cabric, "QoS aware power allocation and user selection in massive MIMO underlay cognitive radio networks," *IEEE Trans. Cogn. Commun. Netw.*, vol. 4, no. 2, pp. 220–231, Jun. 2018.

[24] Z. Shi, X. Xie, and H. Lu, "Deep reinforcement learning based intelligent user selection in massive MIMO underlay cognitive radios," *IEEE Access*, vol. 7, pp. 110884–110894, 2019.

[25] W. Hao, O. Muta, H. Gacanin, and H. Furukawa, "Power allocation for massive MIMO cognitive radio networks with pilot sharing under SINR requirements of primary users," *IEEE Trans. Veh. Technol.*, vol. 67, no. 2, pp. 1174–1186, Feb. 2018.

[26] H. Zhang, J. Feng, Z. Shi, S. Ma, and G. Yang, "Rate maximization of wireless powered cognitive massive MIMO systems," *IEEE Internet Things J.*, vol. 8, no. 7, pp. 5632–5644, Apr. 2021.

[27] R. Sarvendranath and N. B. Mehta, "Exploiting power adaptation with transmit antenna selection for interference-outage constrained underlay spectrum sharing," *IEEE Trans. Commun.*, vol. 68, no. 1, pp. 480–492, Jan. 2020.

[28] R. Sarvendranath and N. B. Mehta, "Transmit antenna selection for interference-outage constrained underlay CR," *IEEE Trans. Commun.*, vol. 66, no. 9, pp. 3772–3783, Sep. 2018.

[29] M. Vu, N. Devroye, and V. Tarokh, "On the primary exclusive region of cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3380–3385, Jul. 2009.

[30] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, Feb. 2013.

[31] B. Nosrat-Makouei, J. G. Andrews, and R. W. Heath, "MIMO interference alignment over correlated channels with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2783–2794, Jun. 2011.

[32] C. Wang, E. K. Au, R. D. Murch, W. H. Mow, R. S. Cheng, and V. Lau, "On the performance of the MIMO zero-forcing receiver in the presence of channel estimation error," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 805–810, Mar. 2007.

[33] L. Musavian, M. R. Nakhai, M. Dohler, and A. H. Aghvami, "Effect of channel uncertainty on the mutual information of MIMO fading channels," *IEEE Trans. Veh. Technol.*, vol. 56, no. 5, pp. 2798–2806, Sep. 2007.

[34] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO achievable rates with downlink training and channel state feedback," *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2845–2866, Jun. 2010.

[35] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*. New York, NY, USA: Academic, 2000.

[36] M. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed. Hoboken, NJ, USA: Wiley-Intersci., 2005.

[37] T. L. Marzetta, E. G. Larsson, H. Yang, and H. Q. Ngo, *Fundamentals of Massive MIMO*. Cambridge, U.K.: Cambridge Univ. Press, 2016.

[38] A. Papoulis and S. Pillai, *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill Electrical and Electronic Engineering Series). New York, NY, USA: McGraw-Hill, 2002.

[39] D. P. Bertsekas and J. N. Tsitsiklis, *Introduction to Probability*, 2nd ed. London, U.K.: Athena Sci., 2008.

[40] E. Björnson, D. Hammarwall, and B. Ottersten, "Exploiting quantized channel norm feedback through conditional statistics in arbitrarily correlated MIMO systems," *IEEE Trans. Signal Process.*, vol. 57, no. 10, pp. 4027–4041, Jun. 2009.

[41] E. Björnson, E. Jorswieck, and B. Ottersten, "Impact of spatial correlation and precoding design in OSTBC MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3578–3589, Oct. 2010.

[42] L. Zhang *et al.*, "Primary channel gain estimation for spectrum sharing in cognitive radio networks," *IEEE Trans. Commun.*, vol. 65, no. 10, pp. 4152–4162, Oct. 2017.

[43] E. Björnson, J. Hoydis, and L. Sanguinetti, *Massive MIMO Networks: Spectral, Energy, and Hardware Efficiency*. Hanover, MA, USA: Now, 2017.

[44] T. Izawa, "Two or multi-dimensional gamma-type distribution and its application to rainfall data," *Meteorol. Geophys.*, vol. 15, nos. 3–4, pp. 167–200, Mar. 1965.

[45] M. H. Protter and C. B. Morrey, *Intermediate Calculus*, 2nd ed. New York, NY, USA: Springer, 2012.

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