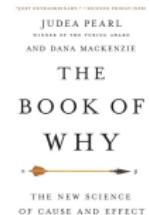
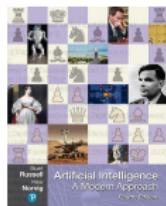


## 8.2: Causal Inference

**Instructor:** Dr. GP Saggese - [gsaggese@umd.edu](mailto:gsaggese@umd.edu)

**References:**

- AIMA (Artificial Intelligence: a Modern Approach)
- Pearl et al., The Book of Why, 2017



- ***Causal Networks***

- Causal DAGs and Structural Causal Models
- Variables
- Type of Variables in Causal AI
- Types of Paths in Causal AI
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- Front-door Adjustment
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# (Non-Causal) Bayesian Networks

- **Bayesian networks** represent a joint distribution function
  - The direction of the arrow represent *conditional dependence* (not causality)
  - $A \rightarrow B$  requires to estimate  $\Pr(A|B)$
- **Many possible Bayesian networks** with same nodes, different edges to explain the same phenomenon

## • Example

- A Bayesian network with *Fire* and *Smoke*, which are dependent
- $Fire \rightarrow Smoke$ 
  - Need  $\Pr(Fire)$  and  $\Pr(Smoke|Fire)$  to compute  $\Pr(Fire, Smoke)$
- $Smoke \rightarrow Fire$ 
  - Need  $\Pr(Smoke)$  and  $\Pr(Fire|Smoke)$



## • Different Bayesian networks:

- Are equivalent and convey the same information
  - Have different difficulties to be estimated
- ## • There is an **asymmetry in nature**
- Extinguishing fire stops smoke
  - Clearing smoke doesn't affect fire

# Causal (Bayesian) Networks

---

- Causal networks are Bayesian networks with only causal edges
  - Use judgment based on nature instead of just statistics
  - E.g., you need to go from
    - “Are random variables *Smoke* and *Fire* correlated?” to
    - “What causes what, *Smoke* or *Fire*? ”
- "Dependency in nature" is like assignment in programming
  - E.g., nature assigns *Smoke* based on *Fire*:
    - $\text{Smoke} := f(\text{Fire})$
    - $\text{Fire} := f(\text{Smoke})$
- Structural equations describe “assignment mechanism” in causal graphs

$$X_i := f(X_j) \iff X_j \rightarrow X_i$$

# Causal DAG

---

- **Causal DAG**

- *Directed*: Arrows show cause → effect
- *Ayclic*: No feedback loops
  - Causal relationships assume temporal order: cause before effect
  - A cycle implies a variable is both cause and effect of itself

- **Benefits**

- DAGs makes explicit *causal* links
- Support explainable AI models
- Stability in conditional probability estimation
- Reason about interventions and counterfactuals

- **Limitations**

- Requires domain knowledge for structure
- Assumes all relevant variables included (no hidden confounders)

# Causal Edges are Stable

---

- Causal edges reflect stable relationship
  - Mechanistic stability
    - Causal relationships show system function, not just behavior in one dataset
    - E.g., “Temperature → ice melting rate” holds true in Alaska and Arizona
  - Invariance under interventions
    - If  $X$  causes  $Y$ , intervening on  $X$  affects  $Y$  consistently, despite confounders or context changes
  - Easier estimation through causal modeling
    - Identifying causal direction focuses estimation on effect size (e.g., regression of  $Y$  on  $X$  under intervention)
- Example: study *Exercise* → *Health*:
  - Correlation may differ in young or elderly populations
  - Causal effect remains stable, as physiological mechanism doesn't change

# Causal DAG: Example

- **Explanatory variables**

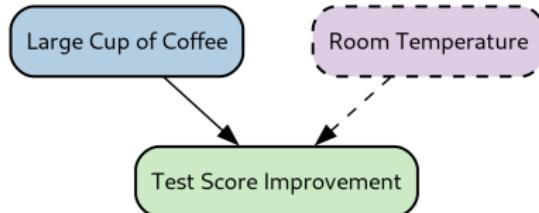
- You can manipulate or observe when changes are applied
- E.g., *"does a large cup of coffee before an exam help with a test?"*

- **Outcome variables**

- Result of the action
- E.g., *"by how much did the score test improve?"*

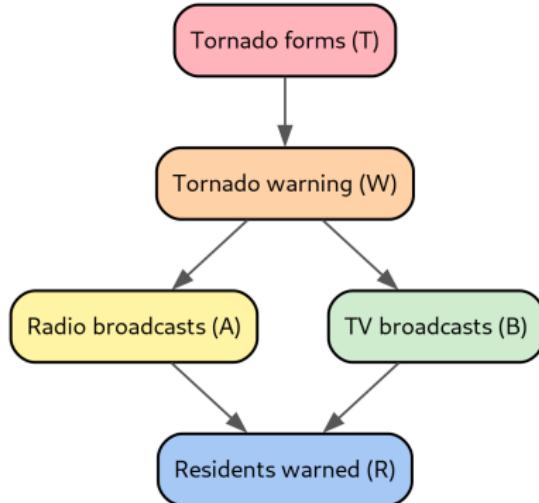
- **Unobserved variables**

- Not seen or more difficult to account
- E.g., *"temperature of the room makes students sleepy and less alert"*



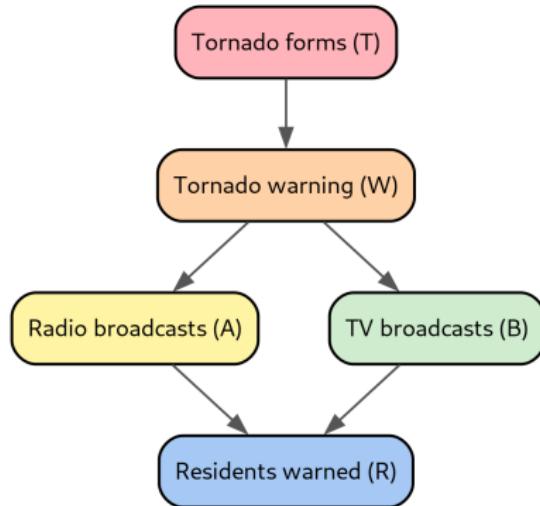
# Example: Tornado Warning

- **Example:** Severe weather alert to the public
- **Causal diagram:**
  - $T$ : Tornado forms
  - $W$ : National Weather Service issues tornado warning
  - $A, B$ : Radio and TV broadcast emergency
  - $R$ : Residents warned
- **Assumptions**
  - Channels broadcast only on official order
  - Delivery systems never fail
  - If either channel activates, residents warned



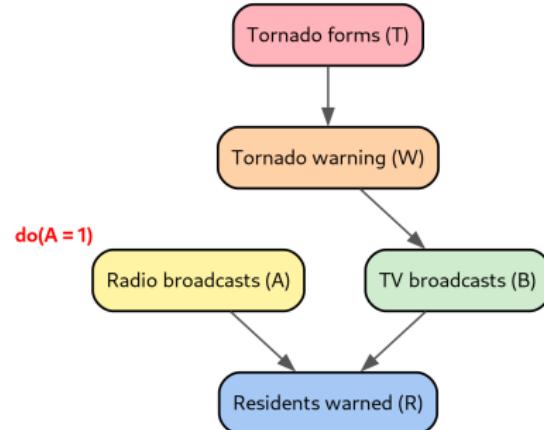
# Tornado Warning: Level 1 (Association)

- Use causal graph to understand how one fact informs another
  - Use predicate logic to answer
- If residents are warned, was a warning issued?
  - Does  $R \implies W$ ?
  - Yes
- If radio A broadcast, did TV B also broadcast?
  - Yes
  - True even though A doesn't cause B
  - Correlated through common cause W (confounder)



# Tornado Warning: Level 2 (Intervention)

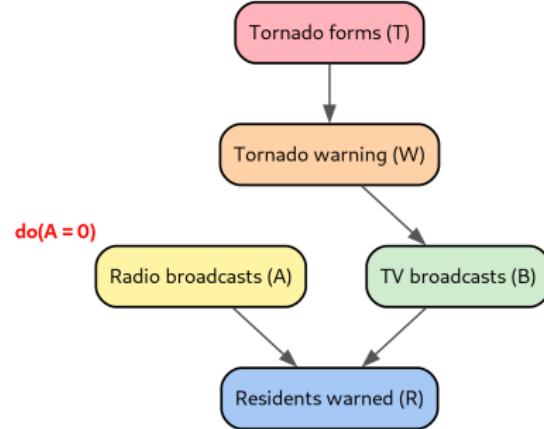
- If you make radio broadcast, will residents be warned?
  - Yes
  - Breaks “rules of nature” as  $A$  broadcasts only if  $W$  is issued
    - Like removing edge  $W \rightarrow A$  in graph
    - Set  $A$  to true:  $do(A = 1)$
  - Expect  $B$  did not broadcast
    - Forcing  $A$  shouldn’t affect  $B$  (no  $A \rightarrow B$  edge)



- Key difference between “seeing” and “doing”
  - If see  $A$  broadcast, conclude  $B$  broadcast too
  - If *make*  $A$  broadcast, expect  $B$  didn’t
- Observational data wouldn’t reveal causal structure
  - Either all variables  $T, W, A, B, R$  are true or all are false
  - Correlations alone are uninformative

# Tornado Warning: Level 3 (Counterfactual)

- Assume that residents are warned  $R = 1$ 
  - You know  $A$  and  $B$  broadcast because
  - ... the NWS issued a warning  $W$
  - ... after a tornado formed  $T$
- Would residents be warned if  $A$  had not broadcast?
  - Remove the edge into  $A$
  - Set  $do(A = 0)$
  - TV  $B$  still broadcasts (due to  $W$ ), and  $R = (A \vee B)$  remains true
  - Yes,  $R = 1$



# Structural Causal Model

---

- A **Structural Causal Model** (SCM) translates a causal DAG into mathematical equations to define how variables interact
- **Structure of SCMs**

- *Variables*  $X_1, X_2, \dots, X_n$  represent quantities in the system
- *Equations* model each variable as a function of its direct causes
- Formally,  $X_i$  is modeled as:

$$X_i = f_i(\text{Parents}(X_i), \varepsilon_i)$$

where:

- $\text{Parents}(X_i)$  are direct causes of  $X_i$
- $\varepsilon_i$  is an exogenous (external, unobserved) noise term

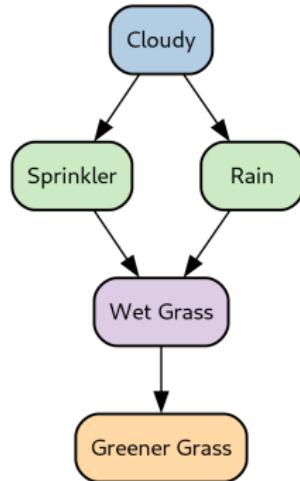
- **Properties**

- Same properties of causal networks
  - Explain causal relationships between variables
  - Provide a foundation for causal reasoning and simulation
  - ...
- Quantify effect
- Used in econometrics and genetics for a long time (even before theory of causality)

# Structural Causal Model: Sprinkler Example

- **Structural equations** for this causal DAG:

$$\begin{cases} C := f_C(\varepsilon_C) \\ R := f_R(C, \varepsilon_R) \\ S := f_S(C, \varepsilon_S) \\ W := f_W(R, S, \varepsilon_W) \\ G := f_G(W, \varepsilon_G) \end{cases}$$



- **Unmodeled variables**  $\varepsilon_x$  represent error terms
  - E.g.,  $\varepsilon_W$  is another source of wetness (e.g., *MorningDew*) besides *Sprinkler* and *Rain*
  - Assume unmodeled variables are exogenous, independent, with a certain distribution (prior)
- Express **joint distribution** of all variables as a product of conditional distributions using causal DAG topology:

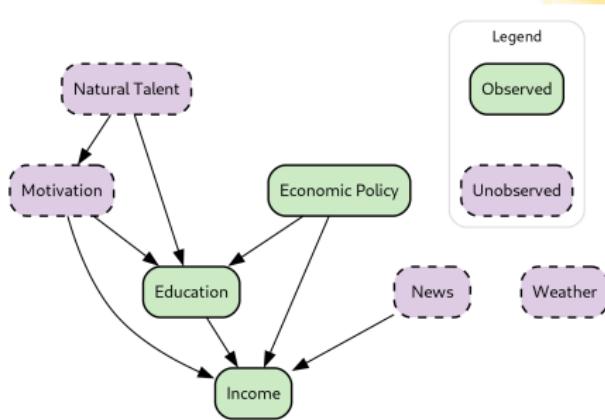
$$\Pr(C, R, S, W, G) = \Pr(W|R, S) \Pr(G|W) \Pr(S|C) \Pr(R|C) \Pr(C)$$

- Causal Networks
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# Observed Vs. Unobserved Variables

- **Observed variables**

- Aka “measurable” or “visible”
- Variables directly measured or collected in a dataset
- E.g.,
  - Education
  - Income



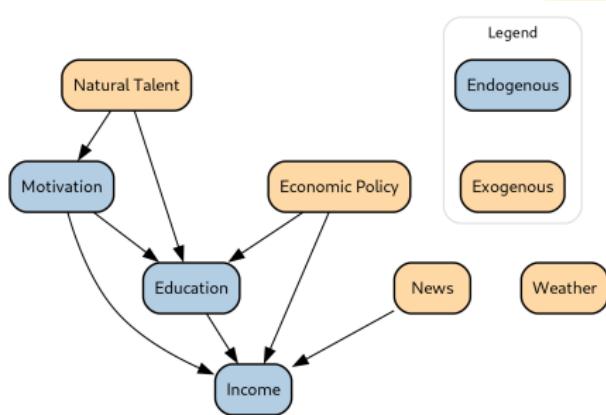
- **Unobserved variables**

- Aka “latent” or “hidden”
- Exist but not measured or included in data
- E.g.,
  - Natural talent
  - Motivation
- Ignoring unobserved variables leads to incorrect conclusions
  - E.g.,  $\text{IceCreamSales} \leftarrow \text{Temperature} \rightarrow \text{DrowningRates}$

# Endogenous Vs. Exogenous Variables

- **Endogenous variables**

- Values determined *within* the model
  - Dependent on other variables in the system
- Represent system's internal behavior and outcomes
- E.g.,
  - Motivation
  - Income



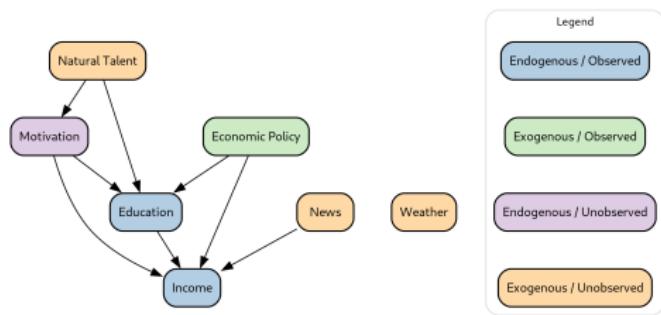
- **Exogenous variables**

- Originate *outside* the system being modeled
  - Not caused by other variables in the model
- Represent background conditions or external shocks
- E.g.,
  - Natural talent
  - Economic policy
  - Weather
  - News

# Endo / Exogenous, Observed / Unobserved Vars

- **Typically**

- *Exogenous / unobserved variables:* capture randomness or unknown external factors
- *Exogenous / observed variables:* potential intervention factors
- *Endogenous / observed variables:* focus for prediction and intervention



Variable Type	Observability	Example
Endogenous	Observed	Income
Exogenous	Observed	Education
Endogenous	Unobserved	Motivation
Exogenous	Unobserved	Natural Talent

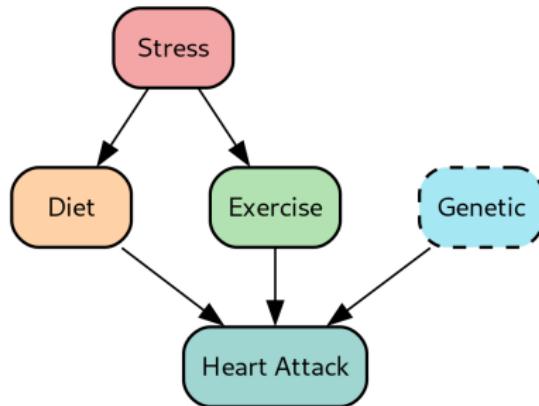
# Building a Causal DAG

---

- **Causal models** visually represent complex environments and relationships
  - Nodes are like “nouns”:
    - E.g., “price”, “sales”, “revenue”, “birth weight”, “gestation period”
    - Variables can be endogenous/exogenous and observed/unobserved
  - Relationships between variables are “verbs”:
    - Parents, children (direct relationships)
    - Descendants, ancestors (along the path)
    - Neighbors
- **Modeling as a Communication Tool:**
  - Shared language bridges gaps between technical and non-technical team members
- **Iterative Refinement:**
  - Continuously update models with new variables and insights

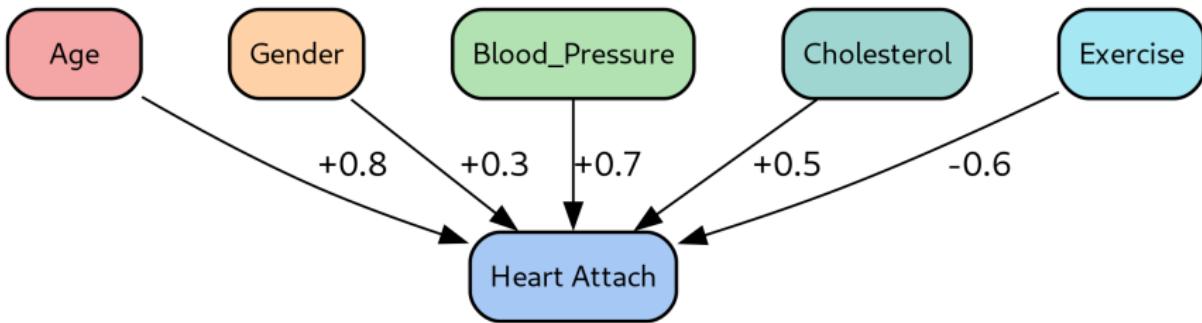
# Heart Attack: Example

- Research question: *What's the relationship between stress and heart attacks?*
- **Build a causal DAG**
  - Stress is the treatment
  - Heart attack is the outcome
  - Stress is *not a direct cause* of heart attack
    - E.g., a stressed person tends to have poor eating habits and tends not to exercise
  - Genetics is unobserved



# Weights

- Assign weights to paths to represent causal strength
- Sign indicates direction



- **How to estimate sign and weight**
  - Estimate using correlation
  - Use priors and then estimate using Bayesian approach

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# Mediator Variable

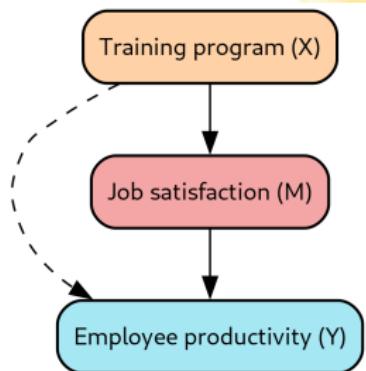
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- A **mediator variable**  $M$ 
  - Is an intermediate variable that *transmits* the causal effect from  $X$  (treatment) to  $Y$  (outcome)
  - Lies **on the causal path** between  $X$  and  $Y$
  - Captures the **mechanism or process** through which  $X$  influences  $Y$



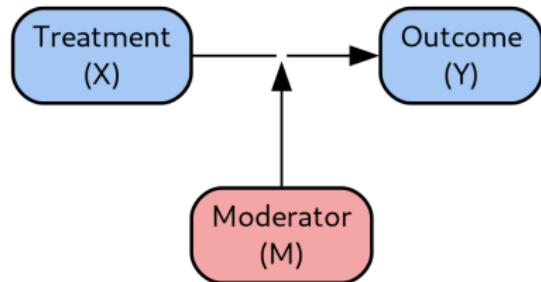
# Mediator Variable: Example

- **Research question:** “Does a training program increase employee productivity?”
- Causal effect may be indirect, through a **mediator**
  - Training might not immediately boost productivity
  - Could enhance job satisfaction, raising productivity
- **Causal interpretation**
  - $X$ : Training Program (cause)
  - $M$ : Job Satisfaction (mediator)
  - $Y$ : Employee Productivity (effect)
  - Path:  $X \rightarrow M \rightarrow Y$
- **Direct vs. Indirect effects**
  - *Indirect effect*:  $X$  affects  $Y$  through  $M$
  - *Direct effect*:  $X$  affects  $Y$  not through  $M$
  - Controlling for  $M$  separates effects, clarifying training impact



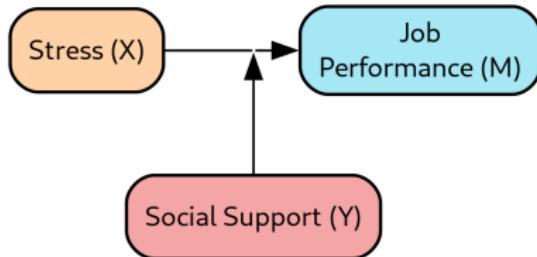
# Moderator Variable

- A **moderator variable**  $M$ 
  - Changes the *strength* or *direction* of the relationship between an independent variable ( $X$ ) and a dependent variable ( $Y$ )
  - Is not part of the causal chain but conditions the relationship



# Moderator Variable: Example

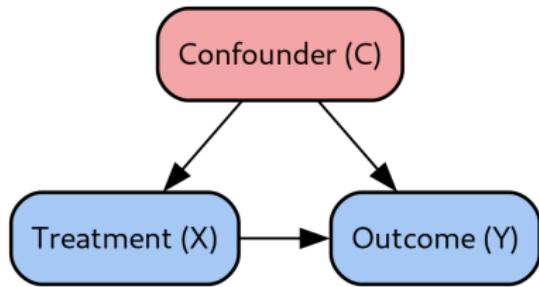
- **Research question:** “*Study relationship between stress and job performance*”
- **Social support M** as a moderator
  - High social support weakens stress's negative effect on performance
  - Low social support strengthens stress's negative effect on performance



# Confounder Variable

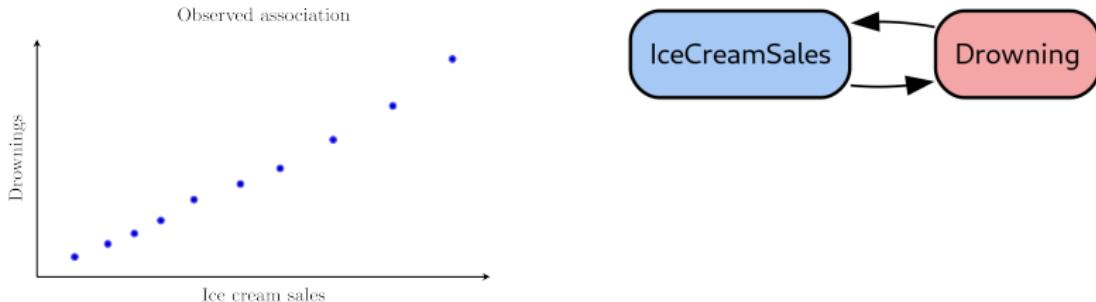
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- A **confounder**  $C$ 
  - Affects both treatment (cause) and outcome (effect)
  - Creates misleading association, if not controlled



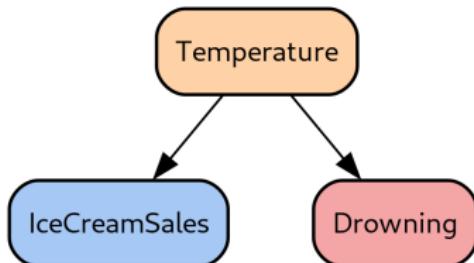
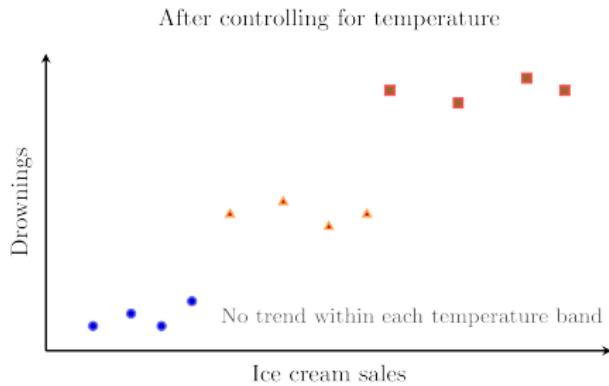
# Confounder Variable: Example

- *IceCreamSales* and *Drowning* **move together**
  - Correlation-based model claims association
  - Is it true?
  - You can always find an explanation (e.g., from ChatGPT)
    - Eating ice cream may distract children or guardians:
    - Cold food shock reflex causes hyperventilation in water
    - Sugar spike → hyperactivity near pools
- **How to use this relationship?**
  - Ban ice cream to prevent drowning?
  - Ice cream maker increase drowning to boost sales?



# Confounder Variable: Example

- In reality, **no cause-effect** between *IceCreamSales* and *Drowning*
  - *Temperature* is a confounder
- In fact, when control for temperature (in regression or intervention), association disappears

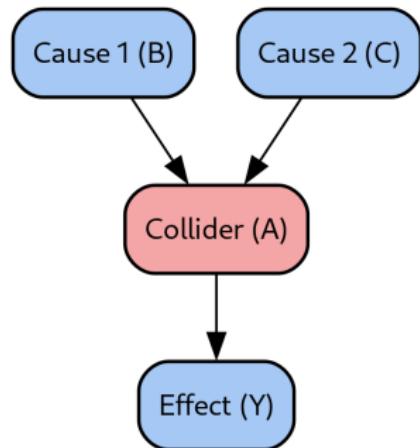


Flat: no cause-effect

# Collider

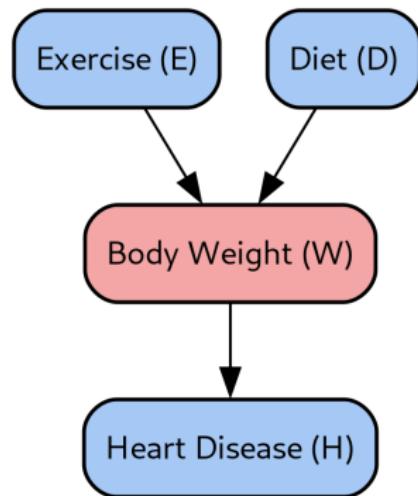
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- A **collider**  $A$ 
  - Is a variable influenced by multiple variables  $B, C$
  - Complicates understanding relationships between variables  $B, C$  and those it influences,  $Y$



# Collider: Examples

- Study the relationship between *Exercise* and *HeartDisease*
  - *Diet* and *Exercise* influence *BodyWeight*
  - *BodyWeight* influences *HeartDisease*
  - *BodyWeight* is a collider



# Collider Bias

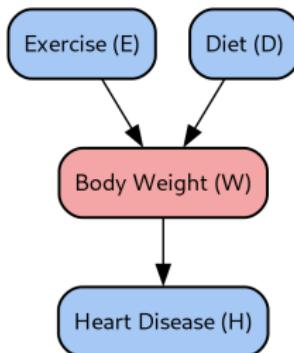
- Aka “Berkson’s paradox”
- **Conditioning on a collider** can introduce a spurious association between its parents by “*opening a path that is blocked*”

- **Example**

- Diet (D)
- Exercise (E)
- BodyWeight (W)
- HeartDisease (H)

- **Without conditioning on W**

- $E$  and  $D$  are independent
  - E.g., knowing exercise level  $E$  doesn't inform about diet  $D$ , and vice versa
- Collider  $W$  blocks association between  $E$  and  $D$



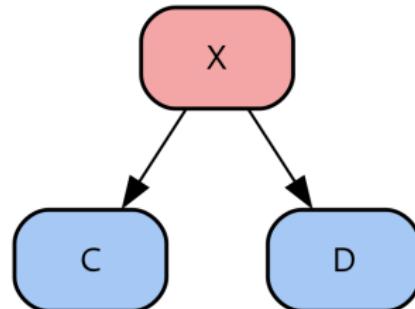
- **After conditioning on W**

- E.g., individuals with specific body weight
- Introduce dependency between  $E$  and  $D$
- With  $W$  fixed, changes in  $E$  balanced by changes in  $D$ , inducing spurious correlation between  $E$  and  $D$
- In Bayesian network it was called “*explaining away*”

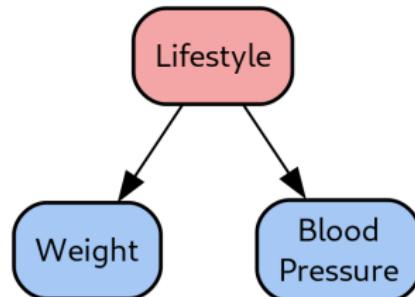
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# Fork Structure

- A **fork**  $D \leftarrow X \rightarrow C$  occurs when a single variable causally influences two or more variables
  - $X$  is a **confounder** (common cause) of  $C$  and  $D$
  - Forks induce statistical dependence between  $C$  and  $D$  even if  $C$  and  $D$  are not causally linked
- **Conditioning** on  $X$  blocks the path and removes spurious correlation

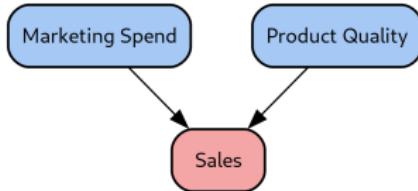
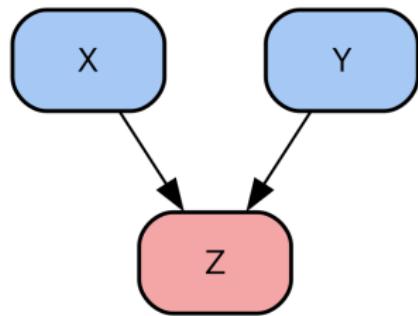


- **Example**
  - *Lifestyle* is a confounder that affects both *Weight* and *BloodPressure*
  - These outcomes may appear correlated due to shared cause



# Inverted Fork

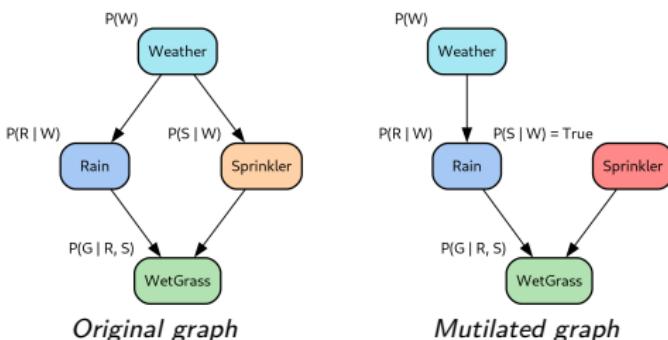
- An **inverted fork** occurs when two or more arrows converge on a common node
  - **Colliders** block associations unless the collider or its descendants are conditioned on
- **Conditioning on a collider** opens a path, inducing spurious correlations
  - This is the basis of selection bias
- **Example**
  - Sales influenced by multiple independent causes
  - *MarketingSpend* and *ProductQuality* both influence *Sales*
  - Conditioning on *Sales* can induce false dependence between *MarketingSpend* and *ProductQuality*



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# Interventions in Causal Networks

- **Causal Bayesian Networks** represent cause–effect relations between variables
  - E.g.,  $Rain \rightarrow WetGrass$
- **Interventions**
  - *Intervention* means setting a variable to a fixed value, overriding its causal mechanism
- E.g., “*Turning the sprinkler on manually*” regardless of cloudiness
  - Replace equation  $S = f_S(C, U_S)$  with  $S = \text{true}$
  - Causal link from *Cloudy* to *Sprinkler* is *cut*, forming a new “mutilated” model



# Interventions in Causal Networks

---

- **The do-operator**

- Denoted as  $\text{do}(X = x)$
- Represents performing an action that *sets*  $X$  to  $x$ , not *observing*  $X = x$
- $\text{do}(X_j = x_j^k)$  removes  $\Pr(x_j | \text{parents}(X_j))$  from the product and gives a new joint distribution:

$$P_{X_j=x_j^k}(x_1, \dots, x_n) = \begin{cases} \prod_{i \neq j} \Pr(x_i | \text{parents}(X_i)) & \text{if } X_j = x_j^k \\ 0 & \text{otherwise} \end{cases}$$

- **Difference between observation and intervention**

$$\Pr(Y | \text{do}(X = x)) \neq \Pr(Y | X)$$

- Observing  $S = \text{true}$  provides *information* about its causes (e.g., weather)
- Intervening with  $\text{do}(S = \text{true})$  *breaks* those causal dependencies (e.g., it doesn't inform about the weather)

# Intervention

---

- **Estimate causal effect** of  $X_j$  on  $X_i$  with adjustment formula:

$$\Pr(X_i = x_i | \text{do}(X_j = x_j^k)) = \sum_{\text{parents}(X_j)} \Pr(x_i | x_j^k, \text{parents}(X_j)) \Pr(\text{parents}(X_j))$$

- **Example**

- In the Sprinkler model)  $\text{do}(S = \text{true}) \rightarrow$  gives the new distribution:

$$\Pr(c, r, w, g | \text{do}(S = \text{true})) = \Pr(c) \Pr(r|c) \Pr(w|r, S = \text{true}) \Pr(g|w)$$

- Only descendants of *Sprinkler* (i.e., *WetGrass*) change
    - *Weather* and *Rain* remain unaffected

- **Intuition**

- Do-operator isolates *causal effects* by simulating external manipulation
  - Essential for answering “what if” questions: *What happens if you intervene and change X?*

# Counterfactuals

---

- A **counterfactual** describes what would have happened under a different scenario
  - “*What would the outcome have been if X had been different?*”
- **Business examples**
  - “*What if we had two suppliers instead of one? Would we have more sales?*”
  - “*Would customers be more satisfied if we shipped products in one week instead of three?*”
  - That’s what businesses want, but they can’t get it from correlation-based models!
- **Causal reasoning**
  - Goes beyond correlation and association
  - Requires a causal model to simulate alternate realities
  - E.g.,
    - Actual: “*A student received tutoring and scored 85%*”
    - Counterfactual: “*What if the student didn’t receive tutoring?*”
    - Causal model estimates the alternative outcome (e.g., 70%)
- **Challenges**
  - Requires strong assumptions and accurate models
  - Difficult to validate directly since counterfactuals are unobservable

# Causal Discovery

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- **Definition**
  - Causal discovery learns causal network structure from data
  - Identify which variables directly cause others (learn causal directions, not just correlations)
- **Approaches to causal discovery**
  - **Search-based methods**
    - Start with an empty or initial model and iteratively modify it (add, reverse, or delete links)
    - Evaluate each candidate network based on fit to data (e.g., likelihood)
    - Use search strategies while ensuring the network remains acyclic
  - **Constraint-based methods**
    - Infer causal directions from conditional independence tests among variables
    - If  $X$  and  $Y$  are independent given  $Z$ , this constrains possible arrows
- **Dealing with complexity**
  - Possible network structures grow superexponentially with the number of variables
  - Complexity penalties to avoid overfitting
- **Causality connection**
  - Causal discovery bridges Bayesian learning and causal inference
  - Under certain assumptions, infer causality from observational data not only



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  - Back-door Adjustment
  - Front-door Adjustment
  - Do-Calculus

# What is a Randomized Controlled Trial?

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- RCTs estimate causal effects by comparing treatment and control groups
  - Randomly assign treatment, not chosen by subjects
    - E.g., assign new drug vs placebo by lottery
  - Ensure groups are statistically equivalent except for treatment
  - Isolate treatment effect
- Goal: estimate  $\Pr(Y|do(X))$ 
  - Randomization simulates do-operator by removing incoming arrows to  $X$
  - Eliminate (known and unknown) confounding paths from background variables
  - Turn observational data into experimental data
    - Use  $\Pr(Y|X)$  to measure  $\Pr(Y|do(X))$
  - Allow causal inference without knowing the causal graph
- Pros
  - Implement intervention in a principled, unbiased way
  - Foundation for scientific experimentation and evidence-based policy
  - Gold standard of causal inference (when feasible)

# Randomized Controlled Trial: Example

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- **Research question:** “Does offering an after-school tutoring program increase the probability that a student passes the end-of-term exam?”
- **Population:** eligible students in a district with proper sample size  $n$
- **Treatment and control**
  - $X = 1$  (treatment): student is offered/assigned to tutoring.
  - $X = 0$  (control): student is not offered tutoring
- **Assignment mechanism (RCT)**
  - Students are randomly assigned to  $X \in \{0, 1\}$
  - Randomization ensures, in expectation, balance on prior GPA, motivation, parental income, etc
- **Outcome**
  - $\Pr(Y | do(X)) = \Pr(Y | X)$
  - $Y_{X=x} = I\{\text{pass exam}\}$  measured at term's end for treatment vs control
  - Measure and report  $Y_{X=1} - Y_{X=0}$

# Randomized Controlled Trial: Limits

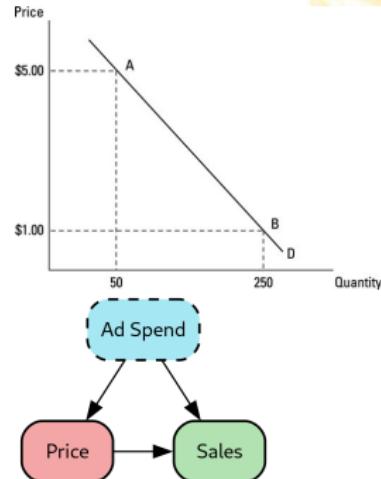
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- **May be unethical**
  - E.g., assigning harmful treatment
  - E.g., you want to verify if asbestos causes cancer
- Can be **expensive or impractical**
- **It doesn't always work**
  - Non-compliance: some participants may not follow assigned treatment
  - Attrition: dropout rates may differ between groups
  - May not generalize to broader populations
  - Requires careful implementation and monitoring
- **Blind RCT**: participants don't know which group they're in (e.g., placebo)
- **Double-blind RCTs**: participants and investigators/clinicians don't know assignments

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# Back-Door Paths: Example

- **Example**
  - A company wants to understand the causal effect of *price* on *sales*
  - Advertising spend *AdSpend* is a **confounder** since it can affect both:
    - The *price* the company can set
    - E.g., the cost increases to cover advertisement costs and the product is perceived as more valuable
    - The *sales* (directly)
- The **back-door path** is *Price* → *AdSpend* → *Sales*
- The company **needs to control** for *AdSpend* to estimate the causal effect of *Price* on *Sales* by:
  1. Using *AdSpend* as covariate in the regression
  2. Designing experiment holding *AdSpend* constant or randomized
  3. Using back-door criterion



# The Back-Door Adjustment

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- **Hypotheses**

- You have a (correct) causal graph
- You block the back-door paths (needs definition!) that satisfy the back-door criterion (needs definitions!)

- **Thesis**

- The *adjustment formula* holds

$$\Pr(Y \mid do(X)) = \sum_z \Pr(Y \mid X, Z = z) \Pr(Z = z)$$

- **Consequences**

- It allows you an intervention (level 2 of the causality ladder) only using observational data (level 1 of the causality ladder)
  - Correlation implies causation
- This is an alternative to randomized controlled experiments
  - Foundation of many empirical studies in epidemiology, economics, and social science
- **Mind blown!**

# Back-Door Criterion: Overview

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- A **back-door path** is any path from  $X$  to  $Y$  that starts with an arrow into  $X$  and ends into  $Y$ 
  - The direction of the arrow doesn't matter
  - If unblocked, back-door paths create spurious associations
- **Back-door criterion**
  - A set of variables  $Z$  satisfies the back-door criterion relative to  $(X, Y)$  if:
    - No variable in  $Z$  is a descendant of  $X$
    - $Z$  blocks every path from  $X$  to  $Y$  that starts with an arrow into  $X$

# Chains, Forks, and Colliders

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- In a **chain**  $X \rightarrow M \rightarrow Y$ 
  - Conditioning on  $M$  blocks causal effect
  - *Do not do it!*
- In a **fork**  $X \leftarrow Z \rightarrow Y$ 
  - Conditioning on  $Z$  removes confounding
  - *Need to do it!*
- In a **collider**  $X \rightarrow M \leftarrow Y$ 
  - Conditioning on  $M$  introduces bias
  - *Do not do it!*
- Back-door requires to block confounders / forks, not colliders / inverted forks
  - Colliders must remain unconditioned unless for specific causal queries
- It is essential to read and interpret graph structure correctly

# Common Mistakes

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- The back-door criterion tells you all and only what you need to condition on (i.e., block) to transform observation in intervention
- The solution that people use is to “condition on everything”
  - **This is incorrect!**
- **Common mistakes**
  - Conditioning on a descendant of  $X$  can bias the estimate
  - Controlling for too many variables can open colliders and introduce bias
  - Forgetting to block all back-door paths
  - Using variables that lie on the causal path (blocks the effect)
  - Ignoring unobserved confounders: can make causal effect unidentifiable
- **1000s of papers and their conclusions are wrong!**
  - In medicine, economics, social science that use observational study incorrectly

# When Back-Door Adjustment Fails

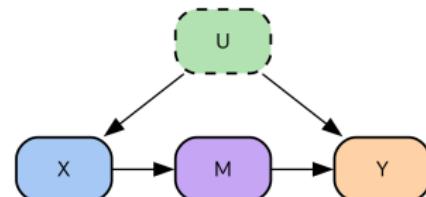
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- **Back-door is simple but not universally applicable**
  - No set of observable variables satisfies the back-door criterion
    - In order to condition on a variable it needs to be observable
    - E.g., an unobserved or unknown confounders
- **Alternatives:**
  - *Front-door criterion*: uses mediators
  - *Instrumental variables*: uses external variation
  - *Do-calculus*: symbolic transformations to eliminate  $do()$

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# Front-Door Adjustment in Causal Inference

- **Front-door criterion** identifies causal effects with unobserved confounders
  - Applies when a **mediator variable** transmits all causal influence from treatment to outcome
- Assume the causal graph looks like:
  - $X$ : treatment or cause
  - $M$ : mediator
  - $Y$ : outcome
  - $U$ : unobserved confounder
- **Hypotheses:**
  1. All directed paths from  $X$  to  $Y$  go through  $M$
  2. No unobserved confounder affects  $X$  and  $M$
  3. All backdoor paths from  $M$  to  $Y$  are blocked by  $X$
- **Thesis:** estimate the causal effect  $P(Y|do(X))$  despite unobserved  $U$

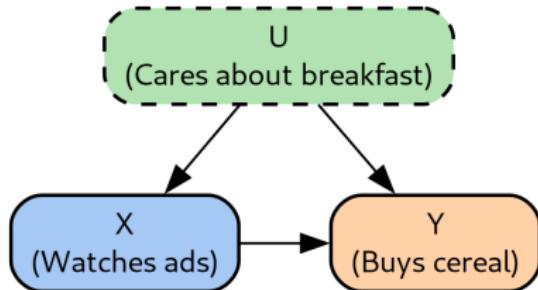


$$P(Y | do(X)) = \sum_m P(M | X) \sum_{x'} P(Y | M, X') P(X')$$

- Intuition: estimate observed link  $X \rightarrow M$  and  $M \rightarrow Y$

# Cereal and Ads: Example

- **Research question:** “Does watching ads ( $X$ ) make people buy more cereal ( $Y$ )?”
- **Of course!**
- **Hidden factor:** “Parents who care about breakfast ( $U$ )” might:
  - Let kids watch more TV and see ads
  - Buy more cereal anyway
- Hidden factor  $U$  confounds “watching ads” and “buying cereals”
  - Correlation exists even if ads don’t cause it
  - Observing  $X$  and  $Y$  without controlling for  $U$  leads to spurious association
  - Same of “ice cream” and “drowning”
- A spurious relationship is **terrible for the business!**
  - It means you spend money on ads and that doesn’t matter
  - Google and Facebook are worth \$3T and it’s all predicated on “buy ads” to “increase sales”



# Cereal and Ads: Solutions

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- **Strategy 1: Back-Door adjustment**

- If you know and can measure  $U$  “how much parents care about breakfast”, include  $U$  as a control variable in analysis
- Intuition:
  - Compare families with *the same* breakfast attitudes ( $U$  fixed)
  - See if ads ( $X$ ) still change cereal buying ( $Y$ )

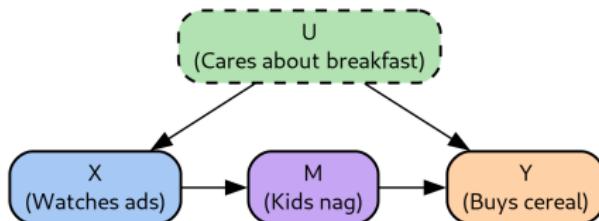
- **Strategy 2: Use randomization**

- Randomized experiments break link between  $X$  and  $U$ 
  - Randomly show ads to some families, not others
  - Randomization ignores parental breakfast attitude; differences in buying come from ads
- This is why controlled experiments are gold standard for causal inference

- **Strategy 3: Front-door Adjustment**

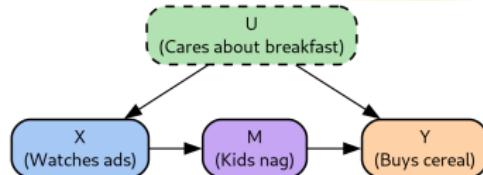
# Cereal and Ads: Finding a Mediator

- Imagine ads work by “*making kids ask for cereal*” (aka “nagging”)  $M$ 
  - This is a true advertisement strategy!
  - At the convenience store the candies are at the bottom of the desk
- There is a **mediator** variable
- So the **causal chain** is:
  - Ads ( $X$ ) → Kids Nagging ( $M$ ) → Parents Buy Cereal ( $Y$ )
- The hidden factor “*parents that care about breakfast*”  $U$ :
  - Affects how much cereal gets bought
  - Doesn’t affect how much kids nag (only ads do that)



# When Front-Door Works

- Is the front-door criterion verified for “kids’ nagging”  $M$ ?
  - Influence of ads on buying goes through nagging ( $X \rightarrow M \rightarrow Y$ )
  - No hidden confounders affect both ads and nagging (TV schedule is random, not linked to parents’ breakfast attitudes)
  - All confounding between nagging and buying is blocked by controlling for ads



- Yes! The front-door criterion is verified
- Instead of doing an intervention  $do(X)$ , just observe!

1. Observe how often ads make kids nag ( $\Pr(M|X)$ )
2. Observe nagging changes buying ( $\Pr(Y|M, X')$ )
3. Combine both to estimate what happens if you force more ads

$$\Pr(Y|do(X)) = \sum_m \Pr(M|X) \sum_{x'} \Pr(Y|M, X') \Pr(X')$$

- Intuition: “How ads cause nagging”  $\times$  “How nagging causes buying”

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# Do-Calculus

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- **Do-calculus** is a formal system for reasoning about causal effects in graphical models (Judea Pearl, 2000)
- Provides algebraic rules to transform intervention expressions (do-operator, e.g.,  $\text{do}(X = x)$ ) into expressions computable from observational data, given certain conditions
- Identify causal effects like:

$$\Pr(Y|\text{do}(X = x))$$

- I.e., distribution of  $Y$  if you intervene and set  $X$  to  $x$ , breaking causal links into  $X$
- Observational data provides:

$$\Pr(Y|X = x)$$

- This is generally **not equal** to  $\Pr(Y|\text{do}(X = x))$  due to confounding

# The Rules of Do-Calculus

- Do-calculus provides **three transformation rules** for manipulating expressions involving  $do()$ :
1. **Insertion/Deletion of Observations:** If  $Y \perp Z | X, W$  in  $G_{\overline{X}}$  (where incoming edges to  $X$  are removed), then:
$$P(Y | do(X), Z, W) = P(Y | do(X), W)$$
  2. **Action/Observation Exchange:** If  $Y \perp Z | X, W$  in  $G_{\overline{X}, \underline{Z}}$  (incoming edges to  $X$  removed, outgoing from  $Z$  removed), then:
$$P(Y | do(X), do(Z), W) = P(Y | do(X), Z, W)$$
  3. **Insertion/Deletion of Actions:** If  $Y \perp Z | X, W$  in  $G_{\overline{X}, \overline{Z(W)}}$  (incoming edges to  $X$  and to  $Z$  excluding those from  $W$  removed), then:
$$P(Y | do(X), do(Z), W) = P(Y | do(X), W)$$

- These rules allow the systematic reduction of expressions involving  $do()$  into observational terms if the causal graph permits



# Back-door and Front-door Adjustments and do-calculus

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- The **back-door** and **front-door** criteria are **specific applications** of do-calculus
- They are simpler, graphical conditions that allow  $P(Y | do(X))$  to be expressed using observational probabilities
- Back-door adjustment If a set of variables  $Z$  blocks all **back-door paths** from  $X$  to  $Y$  (paths that go into  $X$ ), then:

$$P(Y | do(X)) = \sum_z P(Y | X, Z)P(Z)$$

- Front-door adjustment If there exists a variable  $Z$  such that:
  1.  $Z$  is affected by  $X$ ,
  2.  $Z$  affects  $Y$ ,
  3. All back-door paths from  $X$  to  $Z$  are blocked, and
  4. All back-door paths from  $Z$  to  $Y$  are blocked by  $X$ , then:

$$P(Y | do(X)) = \sum_z P(Z | X) \sum_{x'} P(Y | Z, X')P(X')$$