UNIVERSITY OF MARYLAND

MSML610: Advanced Machine Learning

8.2: Causal Inference

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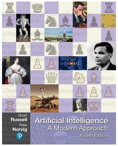
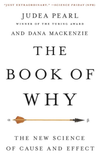
References:

- AIMA (Artificial Intelligence: a Modern Approach)
- Pearl et al., The Book of Why, 2017





SCIENCE ACADEMY

- ***Causal Networks***
  - Causal DAGs and Structural Causal Models
  - Variables
  - Type of Variables in Causal AI
  - Paths
  - Back-door Adjustment
  - Front-door Adjustment
  - Do-calculus
  - Intervention and Counterfactuals
  - Randomized Controlled Treatment

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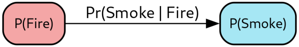
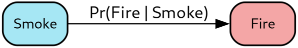
(Non-Causal) Bayesian Networks

- **Bayesian networks** represent a joint distribution function
  - The direction of the arrow represent *conditional dependence* (not causality)
  - $A \rightarrow B$ requires to estimate $\Pr(A|B)$
- **Many possible Bayesian networks** with same nodes, different edges to explain the same phenomenon
- **Example**
  - A Bayesian network with *Fire* and *Smoke*, which are dependent
  - $Fire \rightarrow Smoke$
    - Need $\Pr(Fire)$ and $\Pr(Smoke|Fire)$ to compute $\Pr(Fire, Smoke)$
  - $Smoke \rightarrow Fire$
    - Need $\Pr(Smoke)$ and $\Pr(Fire|Smoke)$





- **Different Bayesian networks**:
  - Are equivalent and convey the same information
  - Have different difficulties to be estimated
- There is an **asymmetry in nature**
  - Extinguishing fire stops smoke
  - Clearing smoke doesn't affect fire

Causal (Bayesian) Networks

- **Causal networks are Bayesian networks with only causal edges**
  - Use judgment based on nature instead of just statistics
  - E.g., you need to go from
    - *"Are random variables Smoke and Fire correlated?"* to
    - *"What causes what, Smoke or Fire?"*
- **"Dependency in nature"** is like assignment in programming
  - E.g., nature assigns *Smoke* based on *Fire*:
    - ✅ $Smoke := f(Fire)$
    - ❌ $Fire := f(Smoke)$
- **Structural equations** describe "assignment mechanism" in causal graphs

$$X_i := f(X_j) \iff X_j \to X_i$$

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Causal DAG

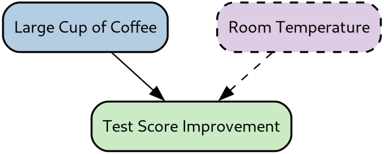
- **Causal DAG**
  - *Directed*: Arrows show cause $\rightarrow$ effect
  - *Acyclic*: No feedback loops
    - Causal relationships assume temporal order: cause before effect
    - A cycle implies a variable is both cause and effect of itself
- **Benefits**
  - DAGs makes explicit *causal* links
  - Support explainable AI models
  - Stability in conditional probability estimation
  - Reason about interventions and counterfactuals
- **Limitations**
  - Requires domain knowledge for structure
  - Assumes all relevant variables included (no hidden confounders)

Causal Edges are Stable

- **Causal edges reflect stable relationship**
  - *Mechanistic stability*
    - Causal relationships show system function, not just behavior in one dataset
    - E.g., "*Temperature* $\rightarrow$ *ice melting rate*" holds true in Alaska and Arizona
  - *Invariance under interventions*
    - If X causes Y, intervening on X affects Y consistently, despite confounders or context changes
  - *Easier estimation through causal modeling*
    - Identifying causal direction focuses estimation on effect size (e.g., regression of Y on X under intervention)
- **Example**: study *Exercise* $\rightarrow$ *Health*:
  - Correlation may differ in young or elderly populations
  - Causal effect remains stable, as physiological mechanism doesn't change

Causal DAG: Example

- **Explanatory variables**
  - You can manipulate or observe when changes are applied
  - E.g., *"does a large cup of coffee before an exam help with a test?"*
- **Outcome variables**
  - Result of the action
  - E.g., *"by how much did the score test improve?"*
- **Unobserved variables**
  - Not seen or more difficult to account
  - E.g., *"temperature of the room makes students sleepy and less alert"*



Structural Causal Model

- A **Structural Causal Model** (SCM) translates a causal DAG into mathematical equations to define how variables interact
- **Structure of SCMs**
  - *Variables* $X_1, X_2, ..., X_n$ represent quantities in the system
  - *Equations* model each variable as a function of its direct causes
  - Formally, X_i is modeled as:

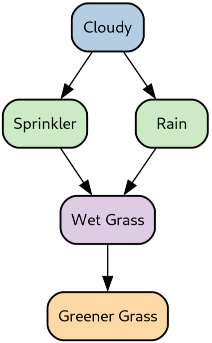
$$X_i = f_i(Parents(X_i), \varepsilon_i)$$

    where:
    - $Parents(X_i)$ are direct causes of X_i
    - ε_i is an exogenous (external, unobserved) noise term
- **Properties**
  - Same properties of causal networks
    - Explain causal relationships between variables
    - Provide a foundation for causal reasoning and simulation
    - ...
  - Quantify effect
- Used in econometrics and genetics for a long time (even before theory of causality)

Structural Causal Model: Sprinkler Example

- **Structural equations** for this causal DAG:

$$\begin{cases} C := f_C(\varepsilon_C) \\ R := f_R(C, \varepsilon_R) \\ S := f_S(C, \varepsilon_S) \\ W := f_W(R, S, \varepsilon_W) \\ G := f_G(W, \varepsilon_G) \end{cases}$$



- **Unmodeled variables** ε_x represent error terms
  - E.g., ε_W is another source of wetness (e.g., *MorningDew*) besides *Sprinkler* and *Rain*
  - Assume unmodeled variables are exogenous, independent, with a certain distribution (prior)
- Express **joint distribution** of all variables as a product of conditional distributions using causal DAG topology:

$$\Pr(C, R, S, W, G) = \Pr(W|R, S)\Pr(G|W)\Pr(S|C)\Pr(R|C)\Pr(C)$$

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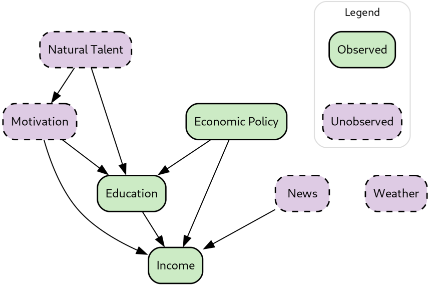
Observed Vs. Unobserved Variables

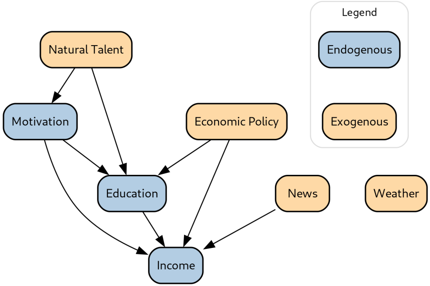
- **Observed variables**
  - Aka "measurable" or "visible"
  - Variables directly measured or collected in a dataset
  - E.g.,
    - Education
    - Income



- **Unobserved variables**
  - Aka "latent" or "hidden"
  - Exist but not measured or included in data
  - E.g.,
    - Natural talent
    - Motivation
  - Ignoring unobserved variables leads to incorrect conclusions
    - E.g., $IceCreamSales \leftarrow Temperature \rightarrow DrowningRates$

Endogenous Vs. Exogenous Variables

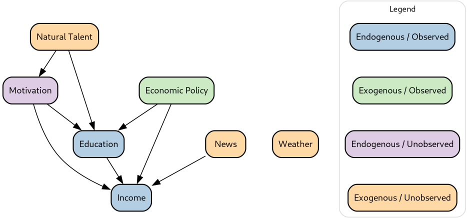
- **Endogenous variables**
  - Values determined *within* the model
    - Dependent on other variables in the system
  - Represent system's internal behavior and outcomes
  - E.g.,
    - Motivation
    - Income



- **Exogenous variables**
  - Originate *outside* the system being modeled
    - Not caused by other variables in the model
  - Represent background conditions or external shocks
  - E.g.,
    - Natural talent
    - Economic policy
    - Weather
    - News

Endo / Exogenous, Observed / Unobserved Vars

- **Typically**
  - *Exogenous / unobserved variables*: capture randomness or unknown external factors
  - *Exogenous / observed variables*: potential intervention factors
  - *Endogenous / observed variables*: focus for prediction and intervention



Variable Type	Observability	Example
Endogenous	Observed	Income
Exogenous	Observed	Education
Endogenous	Unobserved	Motivation
Exogenous	Unobserved	Natural Talent

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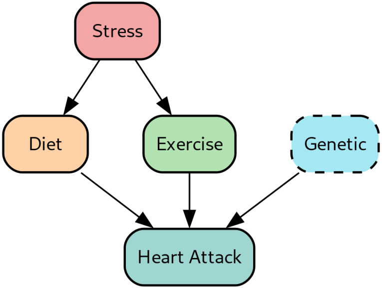
Building a Causal DAG

- **Causal models** visually represent complex environments and relationships
  - Nodes are like "nouns":
    - E.g., "price", "sales", "revenue", "birth weight", "gestation period"
    - Variables can be endogenous/exogenous and observed/unobserved
  - Relationships between variables are "verbs":
    - Parents, children (direct relationships)
    - Descendants, ancestors (along the path)
    - Neighbors
- **Modeling as a Communication Tool**:
  - Shared language bridges gaps between technical and non-technical team members
- **Iterative Refinement**:
  - Continuously update models with new variables and insights

Heart Attack: Example

- Research question: *What's the relationship between stress and heart attacks*?
- **Build a causal DAG**
  - *Stress* is the treatment
  - *Heart attack* is the outcome
  - Stress is *not a direct cause* of heart attack
    - E.g., a stressed person tend to have poor eating habits and tends not to exercise
  - *Genetics* is unobserved



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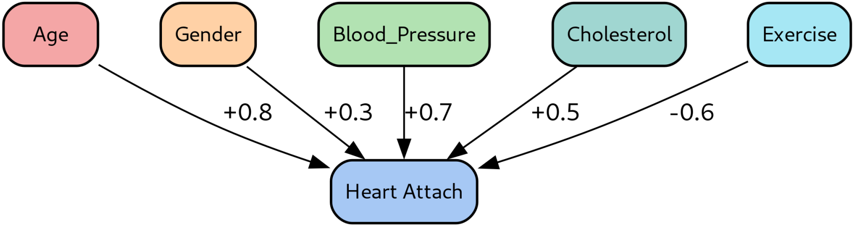
Weights

- Assign weights to paths to represent causal strength
- Sign indicates direction



- **How to estimate sign and weight**
  - Estimate using correlation
  - Use priors and then estimate using Bayesian approach

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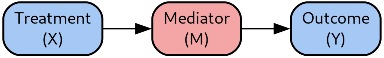
Mediator Variable

- A **mediator variable** M
  - Is an intermediate variable that *transmits* the causal effect from X (treatment) to Y (outcome)
  - Lies **on the causal path** between X and Y
  - Captures the **mechanism or process** through which X influences Y



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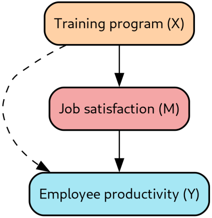
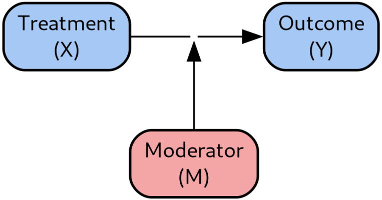
Mediator Variable: Example

- **Research question**: *"Does a training program increase employee productivity?"*
- Causal effect may be indirect, through a **mediator**
  - Training might not immediately boost productivity
  - Could enhance job satisfaction, raising productivity
- **Causal interpretation**
  - X: Training Program (cause)
  - M: Job Satisfaction (mediator)
  - Y: Employee Productivity (effect)
  - Path: $X \to M \to Y$
- **Direct vs. Indirect effects**
  - *Indirect effect*: X affects Y through M
  - *Direct effect*: X affects Y not through M
  - Controlling for M separates effects, clarifying training impact



Moderator Variable

- A **moderator variable** M
  - Changes the *strength* or *direction* of the relationship between an independent variable (X) and a dependent variable (Y)
  - Is not part of the causal chain but conditions the relationship



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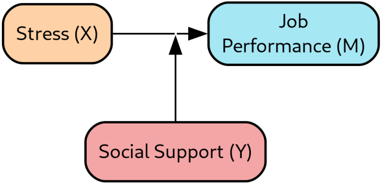
Moderator Variable: Example

- **Research question**: *"Study relationship between stress and job performance"*
- **Social support** M as a moderator
  - High social support weakens stress's negative effect on performance
  - Low social support strengthens stress's negative effect on performance



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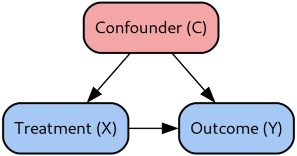
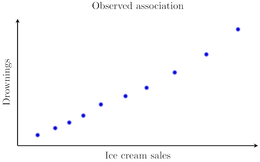
Confounder Variable

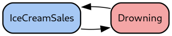
- A **confounder** C
  - Affects both treatment (cause) and outcome
  - Creates misleading association if not controlled



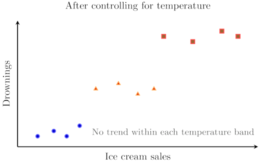
Confounder Variable: Example

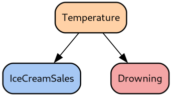
- *IceCreamSales* and *Drowning* move together
  - Correlation-based model claims association, but how to use this relationship?
  - Ban ice cream to prevent drowning?
  - Ice cream maker increase drowning to boost sales?

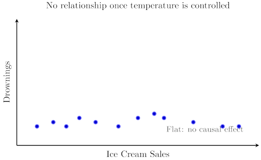




- No cause-effect between *IceCreamSales* and *Drowning*
  - *Temperature* is a confounder
  - Control for season in regression or intervention, association disappears

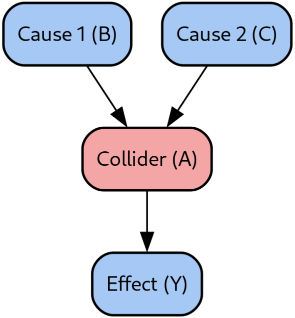






Collider

- A **collider** A
  - Is a variable influenced by multiple variables B, C
  - Complicates understanding relationships between variables B, C and those it influences, Y



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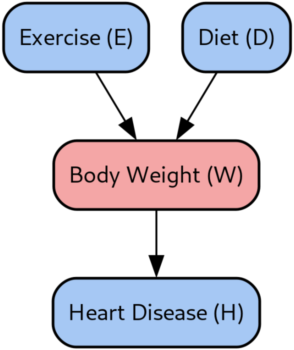
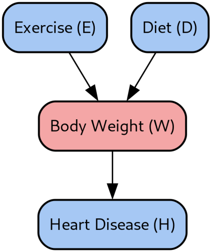
Collider: Examples

- Study the relationship between *Exercise* and *HeartDisease*
  - *Diet* and *Exercise* influence *BodyWeight*
  - *BodyWeight* influences *HeartDisease*
  - *BodyWeight* is a collider



Collider Bias

- Aka "Berkson's paradox"
- **Conditioning on a collider** can introduce a spurious association between its parents by *"opening a path that is blocked"*
- Consider the variables:
  - *Diet* (D)
  - *Exercise* (E)
  - *BodyWeight* (W)
  - *HeartDisease* (D)
- **Without conditioning on W**
  - E and D are independent
    - E.g., knowing exercise level E doesn't inform about diet D, and vice versa
  - Collider W blocks association between E and D
- **After conditioning on W**
  - E.g., individuals with specific body weight
  - Introduce dependency between E and D
  - With W fixed, changes in E balanced by changes in D, inducing spurious correlation between E and D



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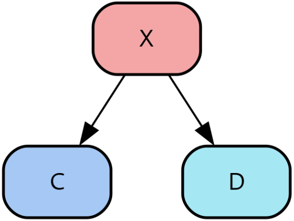
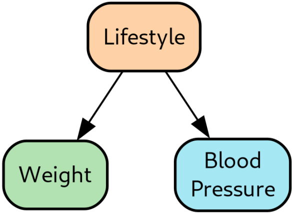
Fork Structure

- A **fork** $D \leftarrow X \rightarrow C$ occurs when a single variable causally influences two or more variables
    - X is a **confounder** (common cause) of C and D
    - Forks induce statistical dependence between C and D even if C and D are not causally linked
- **Conditioning** on X blocks the path and removes spurious correlation

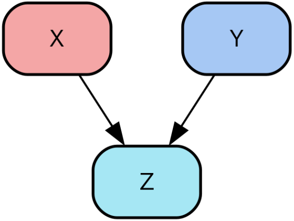


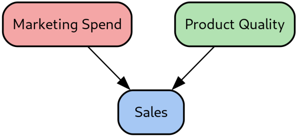
- **Example**
    - *Lifestyle* is a confounder that affects both *Weight* and *BloodPressure*
    - These outcomes may appear correlated due to shared cause



Inverted Fork

- An **inverted fork** occurs when two or more arrows converge on a common node
  - **Colliders** block associations unless the collider or its descendants are conditioned on
- **Conditioning on a collider** *opens a path*, inducing spurious correlations
  - This is the basis of selection bias
- **Example**
  - Sales influenced by multiple independent causes
  - *MarketingSpend* and *ProductQuality* both influence *Sales*
  - Conditioning on *Sales* can induce false dependence between *MarketingSpend* and *ProductQuality*





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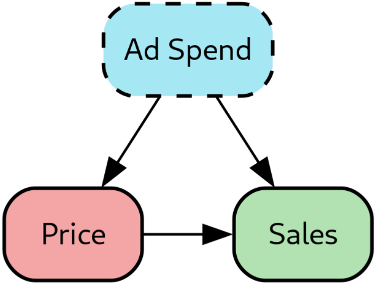
Back-Door Paths

- A company wants to understand the causal effect of price on sales
- Advertising spend *AdSpend* is a confounder since it can affect both:
  - The price the company can set (e.g., the cost increases to cover advertisement costs and the product is perceived as more valuable)
  - The sales (directly)
- The back-door path goes from *Price* to *Sales* via *AdSpend*



- The company needs to control for *AdSpend* to estimate the causal effect of *Price* on *Sales* by:
  - Using *AdSpent* as covariate in the regression
  - Designing experiment holding *AdSpend* constant or randomized
  - Using back-door criterion

Back-Door Criterion: Overview

- A method to control for confounding in observational studies
- Relies on a causal graph (DAG) representing relationships
  - Ensures identification of causal effect from X to Y
  - Blocks all non-causal (back-door) paths from X to Y
  - Allows estimation of $\Pr(Y \mid do(X))$ from observational data

$$\Pr(Y \mid do(X)) = \sum_{z} \Pr(Y \mid X, Z = z) \Pr(Z = z)$$

- Essential when randomization is not possible
- Key to understanding when correlation can imply causation
- Foundation of many empirical studies in epidemiology, economics, and social science

What is a Back-Door Path?

- Any path from X to Y that starts with an arrow into X
- Represents potential confounding, not causal influence
  - If unblocked, back-door paths create spurious associations
- Goal: block these paths without disturbing the causal effect
  - Blocking done by conditioning on variables (adjusting for them)
  - Not all paths need to be blocked—only the back-door ones
  - Conditioning improperly can introduce bias (e.g., colliders)

Back-Door Criterion: Formal Definition

- A set of variables Z satisfies the back-door criterion relative to (X, Y) if:
  - No variable in Z is a descendant of X
  - Z blocks every path from X to Y that starts with an arrow into X
- If Z satisfies this, then:

$$\Pr(Y \mid do(X)) = \sum_z \Pr(Y \mid X, Z = z) \Pr(Z = z)$$

- This is called the *adjustment formula*
- It allows us to use observational data to compute causal effect

Chains, Forks, and Colliders

- In a **chain** $X \to M \to Y$
  - Conditioning on M blocks causal effect (avoid)
- In a **fork** $X \leftarrow Z \to Y$
  - Conditioning on Z removes confounding (good)
- In a **collider** $X \to M \leftarrow Y$
  - Conditioning on M introduces bias (avoid)
- Back-door sets block forks, not colliders
  - Colliders must remain unconditioned unless for specific causal queries
- Essential to read and interpret graph structure correctly

Common Mistakes

- Conditioning on a descendant of X can bias the estimate
- Controlling for too many variables can open colliders (introduce bias)
- Forgetting to block all back-door paths
- Using variables that lie on the causal path (blocks the effect)
- Ignoring unobserved confounders: can make causal effect unidentifiable

When Back-Door Adjustment Fails

- No set of observable variables satisfies the back-door criterion
- Unobserved confounders make $\Pr(Y \mid do(X))$ non-identifiable
- Alternatives:
  - **Front-door criterion**: uses mediators
  - **Instrumental variables**: uses external variation
  - **Do-calculus**: symbolic transformations to eliminate $do()$
- Graph structure determines whether causal effect can be estimated
- Back-door is simple but not universally applicable

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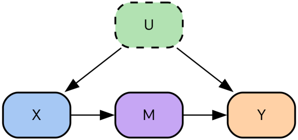
Front-Door Adjustment in Causal Inference

- **Front-door criterion** identifies causal effects with unobserved confounders
  - Applies when a **mediator variable** transmits all causal influence from treatment to outcome
- Basic Setup
  - X: treatment or cause
  - M: mediator
  - Y: outcome
  - U: unobserved confounder



- Hypotheses:
  1. *All directed paths* from X to Y go through M
  2. *No unobserved confounder* affects X and M
  3. *All backdoor paths* from M to Y are blocked by X
- Thesis: estimate the causal effect $P(Y|do(X))$ despite unobserved U

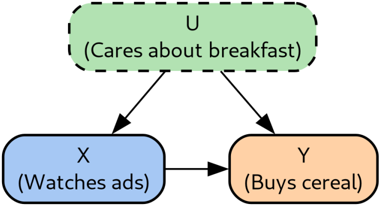
$$P(Y \mid do(X)) = \sum_{m} P(M \mid X) \sum_{x'} P(Y \mid M, X')P(X')$$

  - Intuition: estimate observed link $X \rightarrow M$ and $M \rightarrow Y$

SCIENCE ACADEMY

Cereal and Ads: Example

- **Research question**: *"Does watching ads (X) make people buy more cereal (Y)?"*
- **Hidden factor**: *"Parents who care about breakfast (U)"* might
  - Let kids watch more TV (see ads)
  - Buy more cereal anyway



- Hidden factor U confounds ads and buying
  - Correlation exists even if ads don't cause it
  - Observing X and Y without controlling for U leads to spurious association

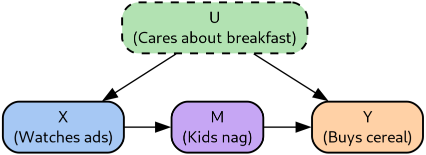
Cereal and Ads: Solutions

- **Strategy 1: Back-Door adjustment**
  - If you *know* and can *measure* U "how much parents care about breakfast", include U as a control variable in analysis
  - Intuition:
    - Compare families with *the same* breakfast attitudes (U fixed)
    - See if ads (X) still change cereal buying (Y)
- **Strategy 2: Use randomization**
  - Randomized experiments break link between X and U
    - Randomly show ads to some families, not others
    - Randomization ignores parental breakfast attitude; differences in buying come from ads
  - This is why controlled experiments are gold standard for causal inference
- **Strategy 3: Front-door Adjustment**

Cereal and Ads: Finding a Mediator

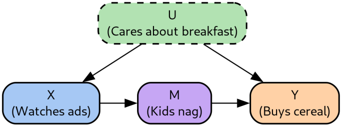
- Imagine ads work by *"making kids ask for cereal"* (aka "nagging") M
  - This is a true advertisement strategy!
  - At the convenience store the candies are at the bottom of the desk
- There is a **mediator** variable

- So the **causal chain** is:
  - Ads (X) $\rightarrow$ Kids Nagging (M) $\rightarrow$ Parents Buy Cereal (Y)
- The hidden factor *"parents' breakfast attitude"* U:
  - Affects how much cereal gets bought
  - But doesn't affect how much kids nag (only ads do that)



When Front-Door Works

- You can use *"kids' nagging"* M to "see inside" the causal path if:
  - Influence of ads on buying goes through nagging ($X \to M \to Y$)
  - No hidden confounders affect both ads and nagging (TV schedule is random, not linked to parents' breakfast attitudes)
  - All confounding between nagging and buying is blocked by controlling for ads



- Instead of doing an intervention $do(X)$, just observe!
  1. Measure how often ads make kids nag ($\Pr(M|X)$)
  2. Measure how nagging changes buying ($\Pr(Y|M, X')$)
  3. Combine both to estimate what happens if you *force* more ads

$$\Pr(Y|do(X)) = \sum_{m} \Pr(M|X) \sum_{x'} \Pr(Y|M, X') \Pr(X')$$

- Intuition: "How ads cause nagging" $\times$ "How nagging causes buying"

SCIENCE ACADEMY

- Causal Networks
  - Causal DAGs and Structural Causal Models
  - Variables
  - Type of Variables in Causal AI
  - Paths
  - Back-door Adjustment
  - Front-door Adjustment
  - ***Do-calculus***
  - Intervention and Counterfactuals
  - Randomized Controlled Treatment

Do-Calculus

- Do-calculus is a formal system for reasoning about causal effects in graphical models (Judea Pearl, 2000)
- Provides algebraic rules to transform intervention expressions (do-operator, e.g., do($X = x$)) into expressions computable from observational data, given certain conditions
- Identify causal effects like:

$$\Pr(Y|do(X = x))$$

  Distribution of Y if you intervene and set X to x, breaking causal links into X
- Observational data provides:

$$\Pr(Y|X = x)$$

  Generally **not equal** to $\Pr(Y|do(X = x))$ due to confounding

The Rules of Do-Calculus

- Do-calculus provides **three transformation rules** for manipulating expressions involving $do()$:

1. **Insertion/Deletion of Observations:** If $Y \perp Z \mid X, W$ in $G_{\overline{X}}$ (where incoming edges to X are removed), then:

$$P(Y \mid do(X), Z, W) = P(Y \mid do(X), W)$$

2. **Action/Observation Exchange:** If $Y \perp Z \mid X, W$ in $G_{\overline{X},\underline{Z}}$ (incoming edges to X removed, outgoing from Z removed), then:

$$P(Y \mid do(X), do(Z), W) = P(Y \mid do(X), Z, W)$$

3. **Insertion/Deletion of Actions:** If $Y \perp Z \mid X, W$ in $G_{\overline{X},\overline{Z(W)}}$ (incoming edges to X and to Z excluding those from W removed), then:

$$P(Y \mid do(X), do(Z), W) = P(Y \mid do(X), W)$$

- These rules allow the systematic reduction of expressions involving $do()$ into observational terms if the causal graph permits

SCIENCE ACADEMY

Back-door and Front-door Adjustments and do-calculus

- The **back-door** and **front-door** criteria are **specific applications** of do-calculus
- They are simpler, graphical conditions that allow $P(Y \mid do(X))$ to be expressed using observational probabilities
- Back-door adjustment If a set of variables Z blocks all **back-door paths** from X to Y (paths that go into X), then:

$$P(Y \mid do(X)) = \sum_{z} P(Y \mid X, Z)P(Z)$$

- Front-door adjustment If there exists a variable Z such that:
  1. Z is affected by X,
  2. Z affects Y,
  3. All back-door paths from X to Z are blocked, and
  4. All back-door paths from Z to Y are blocked by X, then:

$$P(Y \mid do(X)) = \sum_{z} P(Z \mid X) \sum_{x'} P(Y \mid Z, X')P(X')$$

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The do-operator and interventions in causal inference

- **Causal Bayesian Networks**
  - Represent cause–effect relations between variables (e.g., `Rain` $\rightarrow$ `WetGrass`)
- **Interventions**
  - *Intervention* means setting a variable to a fixed value, overriding its causal mechanism
  - E.g., *Turning the sprinkler on manually* regardless of cloudiness
  - Replace equation $S = f_S(C, U_S)$ with $S = \text{true}$
  - Causal link from `Cloudy` to `Sprinkler` is *cut*, forming a new "mutilated" model
- **The do-operator**
  - Denoted as $\text{do}(X = x)$
  - Represents performing an action that *sets* X to x, not *observing* $X = x$
  - After $\text{do}(X_j = x_j^k)$, new joint distribution:

$$P_{x_j^k}(x_1, \ldots, x_n) = \begin{cases} \prod_{i \neq j} \Pr(x_i | \text{parents}(X_i)) & \text{if } x_j = x_j^k, \\ 0 & \text{otherwise.} \end{cases}$$

  - Removes $\Pr(x_j | \text{parents}(X_j))$ from the product
- **Difference between observation and intervention**

Intervention

- **Effect estimation**
  - Causal effect of X_j on X_i:

$$\Pr(X_i = x_i | \text{do}(X_j = x_j^k)) = \sum_{\text{parents}(X_j)} \Pr(x_i | x_j^k, \text{parents}(X_j)) \Pr(\text{parents}(X_j))$$

  - Known as the *adjustment formula*
- **Example (Sprinkler model)**
  - $\text{do}(S = \text{true}) \to$ New distribution:

$$\Pr(c, r, w, g | \text{do}(S = \text{true})) = \Pr(c) \Pr(r|c) \Pr(w|r, S = \text{true}) \Pr(g|w)$$

    - Only descendants of `Sprinkler` (WetGrass, GreenerGrass) change
    - `Cloudy` and `Rain` remain unaffected
- **Intuition**
  - Do-operator isolates *causal effects* by simulating external manipulation
  - Essential for answering "what if" questions: *What happens if you intervene and change X?*

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Counterfactuals

- A **counterfactual** describes what would have happened under a different scenario
  - *"What would the outcome have been if X had been different?"*
  - *"If kangaroos had no tails, they would topple over"*
  - *"What if we had two suppliers instead of one? Would we have more sales?"*
  - *"Would customers be more satisfied if we shipped products in one week instead of three?"*
- **Causal reasoning**:
  - Goes beyond correlation and association
  - Requires a causal model to simulate alternate realities
  - E.g.,
    - Actual: A student received tutoring and scored 85%
    - Counterfactual: What if the student didn't receive tutoring?
    - Causal model estimates the alternative outcome (e.g., 70%)
- **Challenges**:
  - Requires strong assumptions and accurate models
  - Difficult to validate directly since counterfactuals are unobservable

Causal Discovery

- **Definition**
  - Causal discovery learns the structure of a causal network from data
  - Identify which variables directly cause others (learn the direction of causal arrows, not just correlations)
- **Approaches to causal discovery**
  - **Search-based methods**
    - Start with an empty or initial model and iteratively modify it (adding, reversing, or deleting links)
    - Evaluate each candidate network based on its fit to data (likelihood or score-based approach)
    - Use search strategies like hill climbing or simulated annealing while ensuring the network remains acyclic
  - **Constraint-based methods**
    - Infer causal directions from conditional independence tests among variables
    - If X and Y are independent given Z, this constrains which arrows are possible
    - E.g., the PC algorithm uses statistical independence tests to infer structure
- **Dealing with complexity**
  - The number of possible network structures grows superexponentially with the number of variables
  - Complexity penalties help avoid overfitting by discouraging unnecessary links
- **Causality connection**

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What is a Randomized Controlled Treatment?

- A method to estimate causal effects by comparing treatment and control groups
- Treatment is assigned randomly, not chosen by the subjects
- Ensures the two groups are statistically equivalent in all respects except for the treatment
- Goal is to isolate the effect of the treatment itself
- Fundamental question: $\Pr(Y \mid do(X))$ vs $\Pr(Y \mid X)$
- Randomization simulates the do-operator by removing all incoming arrows to X
- Eliminates confounding paths from background variables
- Allows causal inference without knowing all confounders

Why is Randomization Important?

- Removes influence of hidden biases and confounders
- Ensures independence between treatment and background variables
- Guarantees that observed differences in outcomes are due to the treatment
- Makes $\Pr(Y \mid X) = \Pr(Y \mid do(X))$
- Randomization turns observational data into experimental data
- Enables fair comparisons between groups
- Example: Assigning a new drug vs placebo to patients by lottery
- Prevents self-selection bias (e.g., healthier people choosing treatment)

Limits of RCT

- May be unethical (e.g., assigning harmful treatment)
- Can be expensive or impractical
- Non-compliance: some participants may not follow assigned treatment
- Attrition: dropout rates may differ between groups
- May not generalize to broader populations
- Requires careful implementation and monitoring

RCT vs Observational Studies

- Observational: treatment not randomized, subject to confounding
- RCT: removes confounding by design
- Observational data gives $\Pr(Y \mid X)$
- Causal effect requires $\Pr(Y \mid do(X))$
- In observational studies, need methods like back-door adjustment, instrumental variables, or front-door criterion
- RCTs bypass need for model assumptions if implemented correctly
- RCT is a design strategy, not a statistical formula
- Key distinction: intervention (doing) vs passive observation (seeing)

Summary: Why RCT is Powerful

- Implements intervention in a principled, unbiased way
- Severs all back-door paths to isolate treatment effect
- Translates the abstract $do(X)$ into concrete randomized assignments
- Gold standard of causal inference (when feasible)
- Empowers data to answer causal, not just correlational, questions
- Supported by formal tools like causal diagrams and do-calculus
- Bridges the gap between statistics and cause-effect reasoning
- Foundation for scientific experimentation and evidence-based policy