

Causal Inference

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References:

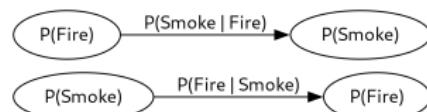
- Easy:
 - Hurwitz, Thompson: Causal Artificial Intelligence: The Next Step in Effective Business AI, 2024
- Medium / Difficult
 - AIMA
 - Facuce

- ***Causal Networks***

- Variables
- Intervention
- Type of Variables in Causal AI
- Paths

(Non-Causal) Bayesian Networks

- **Bayesian networks** represent a joint distribution function
 - The direction of the arrow represent **conditional dependence** (not causality)
 - $A \rightarrow B$ requires to estimate $\Pr(A|B)$
 - There are many possible **edges** and **node ordering** for the same Bayesian network
- E.g., a Bayesian network with *Fire* and *Smoke*, which are dependent
 - $Fire \rightarrow Smoke$
 - Need $\Pr(Fire)$ and $\Pr(Smoke|Fire)$ to compute $\Pr(Fire, Smoke)$
 - $Smoke \rightarrow Fire$
 - Need $\Pr(Smoke)$ and $\Pr(Fire|Smoke)$
 - Networks are equivalent and convey the same information
 - Different difficulties to estimate
- There is an **asymmetry in nature**
 - Extinguishing fire stops smoke
 - Clearing smoke doesn't affect fire



Causal (Bayesian) Networks

- Causal networks are Bayesian networks forbidding non-causal edges
- Use judgment based on nature instead of just statistics
 - E.g., from “Are random variables *Smoke* and *Fire* correlated?” to “What causes what, *Smoke* or *Fire*? ”
- "Dependency in nature" is like assignment in programming
 - E.g., nature assigns *Smoke* based on *Fire*:
 - $\text{Smoke} := f(\text{Fire})$
 - $\text{Fire} := f(\text{Smoke})$
 - Structural equations describe “assignment mechanism” in causal graphs

$$X_i := f(X_j) \iff X_j \rightarrow X_i$$

Causal DAG

- **Causal DAG**

- *Directed*: Arrows show direction of cause → effect
- *Ayclic*: No feedback loops
 - Causal relationships assume a temporal order: cause happens before effect
 - A cycle would imply a variable is both a cause and effect of itself (paradox)

- **Benefits**

- DAGs encode *causal* rather than *associative* links
- Enables reasoning about interventions and counterfactuals
- Supports explainable AI models
- Stability to conditional probability estimation

- **Limitations**

- Requires domain knowledge to specify structure
- Assumes all relevant variables are included (e.g., no hidden confounders)

Causal Edges are Stable

- **Causal edge** $X \rightarrow Y$ shows direct causal influence of X on Y , holding other variables constant
 - Captures how manipulating X changes Y , not just their covariance
- Causal edges reflect **stable relationship**
 - Mechanistic stability
 - Causal relationships show system function, not just behavior in one dataset
 - E.g., "Temperature \rightarrow ice melting rate" holds true in Alaska and Arizona
 - Invariance under interventions
 - If X causes Y , intervening on X affects Y consistently, despite confounders or context changes
 - Easier estimation through causal modeling
 - Identifying causal direction focuses estimation on effect size (e.g., regression of Y on X under intervention)
 - Reduced sensitivity to sampling and omitted variables
 - Correlations may change with confounder addition or removal
 - True causal edge persists, stable across model specifications
- **Example:** study *Exercise \rightarrow Health*:
 - Correlation may differ in young or elderly populations
 - Causal effect remains stable, as physiological mechanism doesn't change



Structural Causal Model

- A **Structural Causal Model** (SCM) translates a causal DAG into mathematical equations
 - DAGs show structure (variables and arrows)
 - SCMs use equations to define how variables interact
- **Structure of SCMs**
 - Variables X_1, X_2, \dots, X_n represent quantities in the system
 - Equations model each variable as a function of its direct causes
 - Formally, X_i is modeled as:

$$X_i = f_i(\text{Parents}(X_i), \varepsilon_i)$$

where:

- $\text{Parents}(X_i)$ are direct causes of X_i
- ε_i is an exogenous (external, unobserved) noise term

- **Properties**

- Explain causal relationships between variables
- Provide a foundation for causal reasoning and simulation
- Describe how the world works, not just variable correlations

Structural Causal Model: Example

- **Explanatory variables**

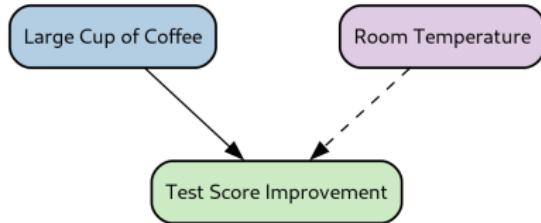
- You can manipulate or observe when changes are applied
- E.g., *"does a large cup of coffee before an exam help with a test?"*

- **Outcome variables**

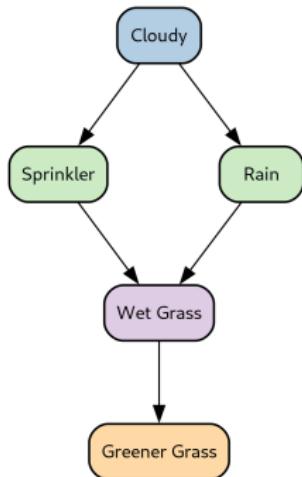
- Result of the action
- E.g., *"by how much did the score test improve?"*

- **Unobserved variables**

- Not seen or more difficult to account
- E.g., *"temperature of the room makes students sleepy and less alert"*



Structural Causal Model: Sprinkler Example



- Structural equations for this model:

$$\begin{cases} C := f_C(\varepsilon_C) \\ R := f_R(C, \varepsilon_R) \\ S := f_S(C, \varepsilon_S) \\ W := f_W(R, S, \varepsilon_W) \\ G := f_G(W, \varepsilon_G) \end{cases}$$

- Unmodeled variables ε_x represent error terms
 - E.g., ε_W is another source of wetness besides *Sprinkler* and *Rain* (e.g., *MorningDew*)
- Assume unmodeled variables are exogenous, independent, with a certain distribution (prior)
- Express joint distribution of five variables as a product of conditional distributions using causal DAG topology:

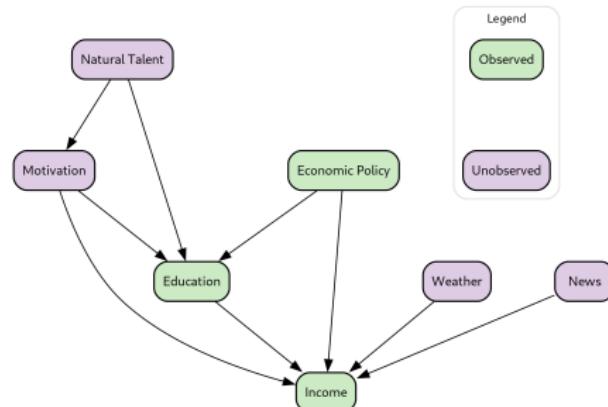
$$\Pr(C, R, S, W, G) = \Pr(C) \Pr(R|C) \Pr(S|C) \Pr(W|R, S) \Pr(G|W)$$

- Causal Networks
 - **Variables**
 - Intervention
 - Type of Variables in Causal AI
 - Paths

Observed Vs. Unobserved Variables

- **Observed variables**

- Aka “measurable” or “visible”
- Variables directly measured or collected in a dataset
- E.g.,
 - Education
 - Income
 - Blood pressure
 - Product price



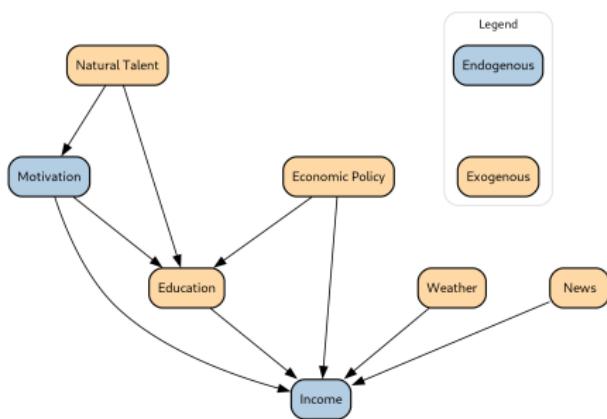
- **Unobserved variables**

- Aka “latent” or “hidden”
- Exist but not measured or included in data
- E.g.,
 - Natural talent
 - Motivation
 - Company culture
- Ignoring unobserved variables distorts causal relationships
 - Observed: *IceCreamSales* and *DrowningRates*
 - Unobserved: *Temperature*
 - Misleading conclusion: *IceCream* causes *Drowning*

Endogenous Vs. Exogenous Variables

- **Endogenous variables**

- Values determined *within* the model
 - Dependent on other variables in the system
- Represent system's internal behavior and outcomes
- E.g.,
 - Motivation
 - Income



- **Exogenous variables**

- Originate *outside* the system being modeled
 - Not caused by other variables in the model
- Represent background conditions or external shocks
- E.g.,
 - Natural talent
 - Economic policy
 - Weather
 - News

Endo / Exogenous, Observed / Unobserved Vars

- In **Structural Causal Models**

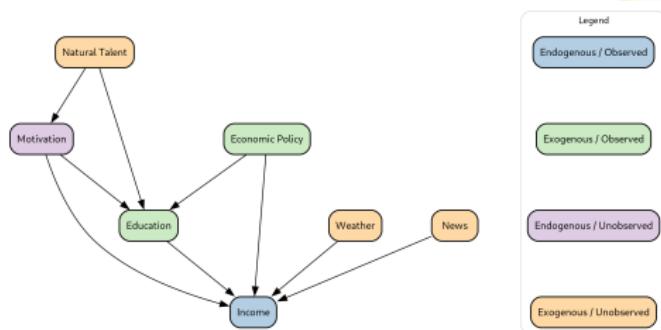
$$X_i = f_i(\text{Parents}(X_i), \varepsilon_i)$$

where:

- X_i : endogenous
- ε_i : exogenous noise

- Typically**

- Endogenous variables*: focus for prediction and intervention
- Exogenous variables*: capture randomness or unknown external factors

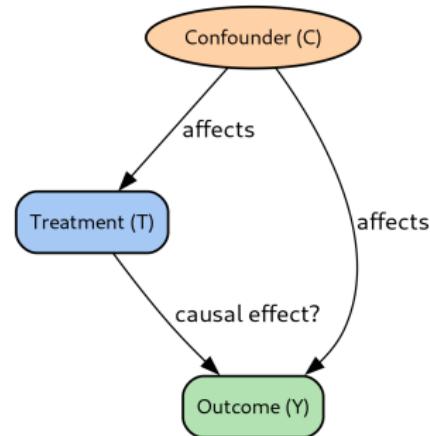


Variable Type	Observability	Example
Endogenous	Observed	Income
Exogenous	Observed	Education
Endogenous	Unobserved	Motivation
Exogenous	Unobserved	Natural Talent

- Causal Networks
 - Variables
 - *Intervention*
 - Type of Variables in Causal AI
 - Paths

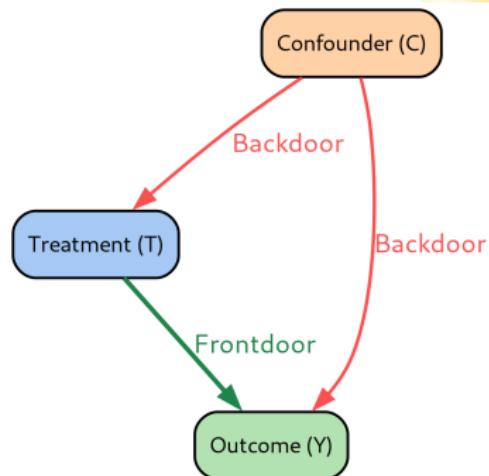
Estimating Causal Effects

- **Goal:** Determine the causal effect of a treatment variable (aka intervention) T on an outcome Y
- **Example:**
 - $T = \text{"takes drug"}$
 - $Y = \text{"recovers"}$
 - $C = \text{"overall health"}$
- Healthier people may take medicine and recover faster \implies correlation without causation
- In **observational data**
 - Confounding variable C affects both treatment T and outcome Y
 - C creates *spurious correlation* between T and Y
- **Problem**
 - There is a “backdoor path” $Treatment \leftarrow Confounder \rightarrow Outcome$



Frontdoor and Backdoor Paths: Intuition

- A **backdoor path** is any path from T to Y starting with an arrow into T
 - E.g., $T \leftarrow C \rightarrow Y$
 - Interpretation:
 - C is a common cause of T and Y , confounding their relationship
 - Controlling (conditioning) for C blocks the backdoor path, identifying the causal effect of T on Y



- A **frontdoor path** goes directly or indirectly from T to Y through mediators, following causal flow
 - E.g., $T \rightarrow Y$
 - Interpretation:
 - Direct causal path of interest
 - No mediators, so front-door path is direct causal effect of T on Y

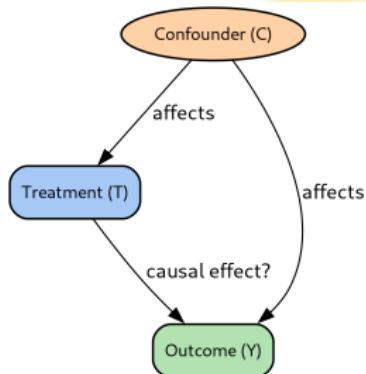
Randomized Controlled Trials (RCTs)

- **Randomized Controlled Trial** is an experimental study to assess causal effect of an intervention or treatment
 - Determine whether an intervention causes an effect, not just associated with it
 - Eliminate selection bias and confounding variables through randomization
- **Key Components**
 - *Randomization*: ensures groups are statistically equivalent at baseline
 - *Control Group*: receives a placebo or standard treatment
 - *Blinding*: participants and/or researchers do not know the assignment to avoid bias
 - *Outcome Measurement*: pre-defined metrics assess the intervention's effect
- **Example**: testing a new drug
 - Treatment group receives the new drug
 - Control group receives a placebo
 - Compare recovery rates after a fixed period
- **Pros**
 - Provides clear causal inference due to randomization
- **Cons**
 - Expensive and time-consuming
 - Ethical or practical constraints may prevent randomization

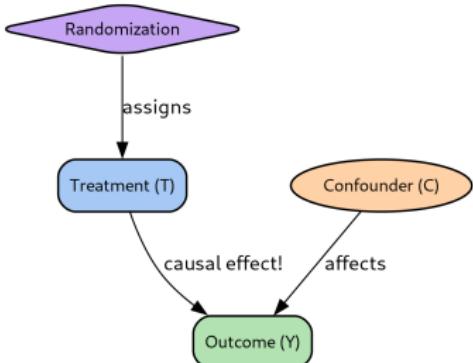


RCTs Solve the Problem of Confounders

- In **observational data**
 - Confounding variable C affects both treatment T and outcome Y
 - C creates *spurious correlation* between T and Y



- In **experimental settings**
 - Randomization (R) breaks link between C and T
 - Random assignment prevents influence on both treatment and outcome
 - T is independent of C : $T \perp C$
 - Only open path between T and Y is causal path $T \rightarrow Y$



Causal Graphs and Interventions

- **Observing correlations** between variables *does not reveal causality*
 - $\Pr(Y|T)$ confounds direct and indirect influences
- **Randomized Controlled Trials** provide the *gold standard* for causal inference
 - Randomization breaks all back-door (confounding) paths
 - RCTs are expensive, slow, or ethically impossible
- **Alternative solution**
 - Can we estimate the *causal effect* from *observational data alone*?
 - Under *what conditions* and using *which variables*?
- **Idea:** Identify and condition on the right *confounders* to:
 - Block spurious associations between T and Y
 - Recover the true causal effect $\Pr(Y|do(T))$

Intervention in Structural Equations

- **Purpose of Structural Equations**

- Capture causal mechanisms among variables
- Predict impact of external interventions

- **Effect of Intervention** $do(X_j = x_j)$

- Original equation:

$$X_j = f_j(\text{Parents}(X_j), \varepsilon_j)$$

- Modified by intervention:

$$X_j = x_j \text{ (fixed value)}$$

- “Mutilate” causal network by *removing incoming edges* to X_j
- Recompute joint distribution of all variables using modified structure

- **Intuition**

- do -operator enforces variable's value externally, breaking causal dependencies
- Enables reasoning about “what would happen if...?” scenarios

Adjustment Formula in Causal Networks

- **Goal**

- Estimate causal effect of intervention $do(X_j = x_{jk})$ on another variable X_i

- **The Adjustment Formula**

- Derived from the post-intervention joint distribution:

$$\Pr(X_i = x_i | do(X_j = x_j^*)) = \sum_{\text{Parents}(X_j)} \Pr(x_i | x_j^*, \text{Parents}(X_j)) \Pr(\text{Parents}(X_j))$$

- The mechanism for X_j is *removed*: it is treated as a fixed cause, not a random variable

- **Interpretation**

- Computes a *weighted average* of effects of X_j and its parents on X_i
- Weights come from prior probabilities of the parents' values

- **Back-Door Criterion**

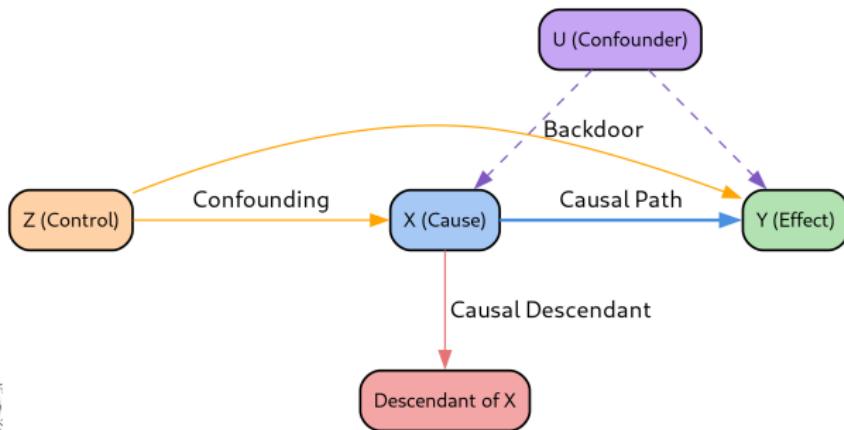
- A set Z is a valid adjustment set if it blocks *all back-door paths* from X_j to X_i
- Ensures $X_i \perp \text{Parents}(X_j) | X_j, Z$

- **Why It Matters**

- Enables causal inference from observational data
- Estimate treatment and policy effects *without randomized trials*

Backdoor Criterion: Definition

- A set of variables Z satisfies the **backdoor criterion** for variables X (cause) and Y (effect) in a causal graph if:
 - No element of Z is a descendant of X**
 - Ensures Z does not “block” part of the causal effect of X on Y
 - Descendants of X may carry information about the causal effect and should not be controlled for
 - Z blocks every path between X and Y containing an arrow into X**
 - These paths are *backdoor paths*, representing potential confounding influences
 - Blocking them ensures any remaining association between X and Y is causal, not spurious



Backdoor Criterion: Intuition

- **Intuition:**

- The goal is to isolate the causal effect of X on Y by eliminating *confounding bias*
- Controlling for an appropriate set Z makes the relationship between X and Y as if X were randomly assigned

- **Application:**

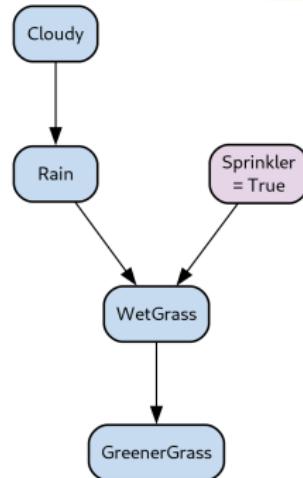
- When Z satisfies the backdoor criterion, we can estimate causal effects from **observational data** (without experiments)
- The causal effect can be computed using:

$$\Pr(Y|do(X)) = \sum_z \Pr(Y|X, Z = z)P(Z = z)$$

Intervention: Sprinkler Example

- “Intervene” by turning the sprinkler on
 - In do-calculus $do(Sprinkler = T)$
 - Sprinkler variable s is independent of cloudy day
- Structural equations after intervention:

$$\begin{cases} C := f_C(\varepsilon_C) \\ R := f_R(C, \varepsilon_R) \\ S := \text{True} \\ W := f_W(R, S, \varepsilon_W) \\ G := f_G(W, \varepsilon_G) \end{cases}$$



- $\Pr(S|C) = 1$ and $\Pr(W|R, S) = \Pr(W|R, S = T)$ and the joint probability becomes:

$$\Pr(C, R, W, G|do(S = \text{True})) = \Pr(C) \Pr(R|C) \Pr(W|R, S = \text{True}) \Pr(G|W)$$

- Only descendants of manipulated variable *Sprinkler* are affected

Intervention vs. Observation in Causal Models

- **Intervention** conceptually *breaks* normal causal dependencies
 - Intervening on *Sprinkler* removes causal link from *Weather* to *Sprinkler*
 - After intervention, causal graph excludes arrow $\text{Weather} \rightarrow \text{Sprinkler}$
 - *Weather* and *Sprinkler* become independent under intervention
- **Observation vs. Intervention**
 - **Observation:** seeing *Sprinkler* = T
 - Expressed as $\Pr(\cdot | \text{Sprinkler} = T)$
 - Reflects *passive observation* — sprinkler on provides information about weather
 - Since *Weather* influences *Sprinkler*, observing *Sprinkler* = T makes it *less likely* *Weather* is cloudy
 - **Intervention:** forcing *Sprinkler* = T
 - Expressed as $\Pr(\cdot | \text{do}(\text{Sprinkler} = T))$
 - *Active manipulation* — set sprinkler on regardless of weather
 - Causal link from *Weather* to *Sprinkler* is cut, weather distribution remains unchanged
- **Key intuition**
 - Observation → correlation (information flows along causal links)
 - Intervention → causation (links into manipulated variable are removed)
 - Thus, $\Pr(\text{Weather} | \text{Sprinkler} = T) \neq \Pr(\text{Weather} | \text{do}(\text{Sprinkler} = T))$

Controlling for a Variable in Causal Analysis

- **Definition**

- To *control* a variable means to hold it constant (statistically or experimentally) to isolate the causal effect of another variable

- **Example**

- Does exercise (X) cause weight loss (Y)?
- Confounder: Diet (Z) affects both exercise and weight
- By controlling for diet (e.g., comparing people with similar diets), you can estimate the effect of exercise more accurately

- In **regression analysis**

- Include Z as an additional independent variable
- E.g., in $Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon$
 - β_1 measures the effect of X *controlling for Z*
 - Coefficient $\beta_1 = \text{change in } Y \text{ with a one-unit change in } X_1, \text{ holding } X_2 \text{ constant}$
 - Isolates X_1 's unique contribution
 - Compares individuals with the same X_2 but different X_1

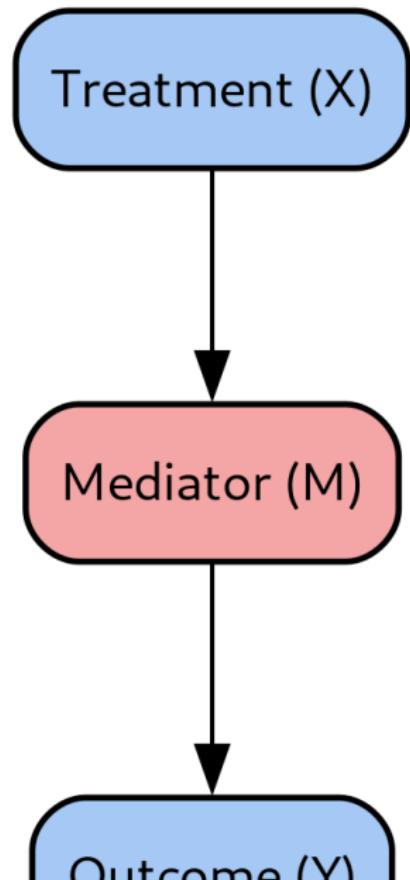
- In **experiments**

- Keep Z constant or randomize it

- Causal Networks
 - Variables
 - Intervention
 - ***Type of Variables in Causal AI***
 - Paths

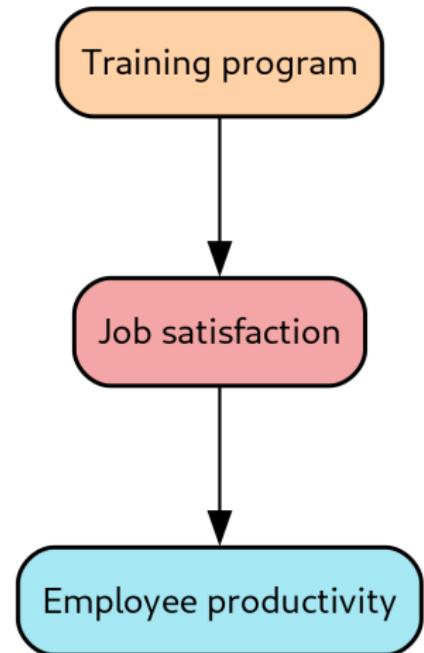
Mediator Variable

- A **mediator variable** M is an intermediate variable that *transmits* the causal effect from X (treatment) to Y (outcome)
 - Lies **on the causal path** between X and Y
 - Captures the **mechanism or process** through which X influences Y



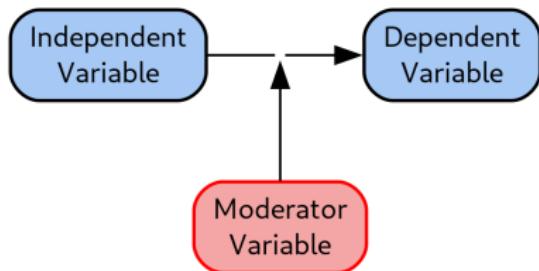
Mediator Variable: Example

- Research question: does a *training program* increase *employee productivity*?
- The causal effect may be **indirect**, operating through a **mediator**
 - The training program might not immediately boost productivity
 - Instead, it could enhance **job satisfaction**, which in turn raises productivity
- **Causal interpretation**
 - X : Training Program (cause)
 - M : Job Satisfaction (mediator)
 - Y : Employee Productivity (effect)
 - Path: $X \rightarrow M \rightarrow Y$
- **Direct vs. Indirect effects**
 - *Indirect effect* X affects Y through M
 - *Direct effect* X affects Y not through M
 - Controlling for M separates these two effects, clarifying *how* training impacts outcomes



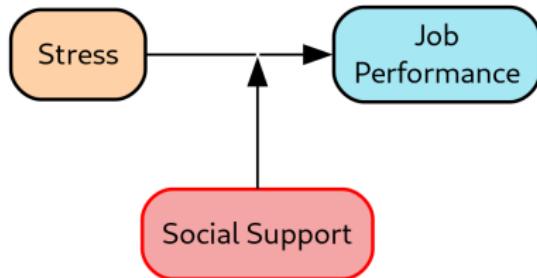
Moderator Variable

- A **moderator variable** changes the *strength or direction* of the relationship between an independent variable (X) and a dependent variable (Y)
 - Moderator is not part of the causal chain but conditions the relationship



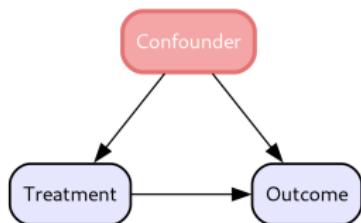
Moderator Variable: Example

- Research question: study relationship between stress X and job performance Y
- Social support M as a moderator
 - High social support weakens stress's negative effect on performance
 - Low social support strengthens stress's negative effect on performance



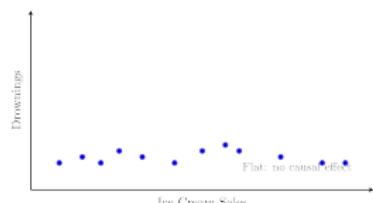
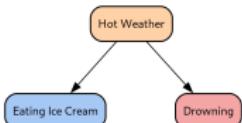
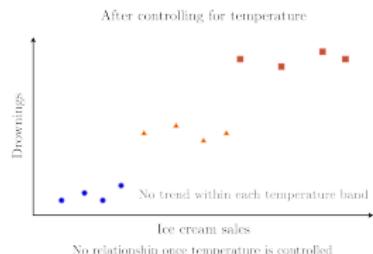
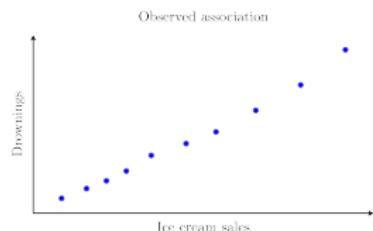
Confounder Variable

- A **confounder**
 - Influences multiple variables in a causal graph
 - Affects both treatment (cause) and outcome
 - Creates misleading association if not controlled



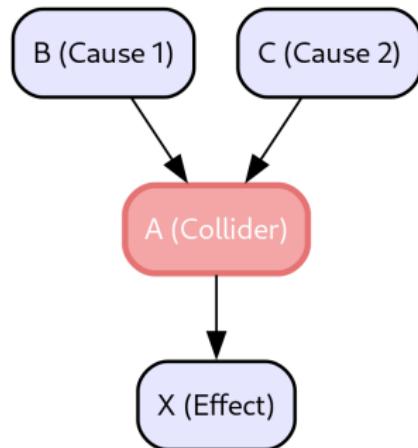
Confounder Variable: Example

- *IceCreamSales* and *Drowning* move together
 - Correlation-based model claims association, but how to use this relationship?
 - Ban ice cream to prevent drowning?
 - Ice cream maker increase drowning to boost sales?
- In reality, no cause-effect between *IceCreamSales* and *Drowning*
 - *Temperature* is a confounder
 - When controlling for season in regression or intervention, association disappears



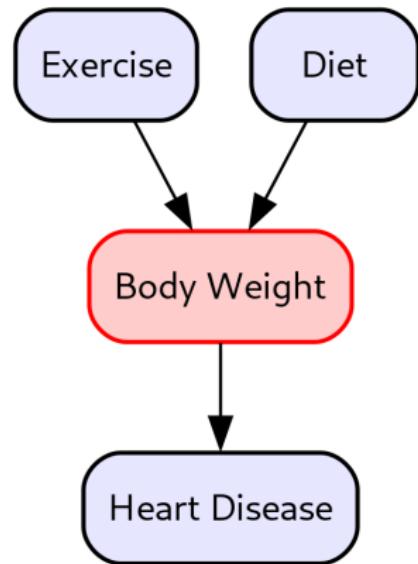
Collider

- A **collider** is a variable A influenced by multiple variables
 - In a causal graph A with incoming edges from variables B, C
- A collider complicates understanding relationships between variables B, C and those it influences, X



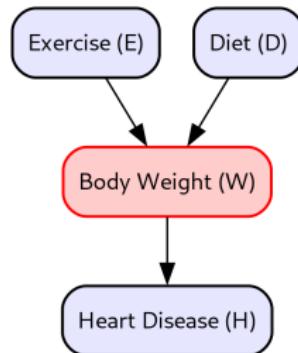
Collider: Examples

- Study the relationship between *Exercise* and *HeartDisease*
 - *Diet* and *Exercise* influence *BodyWeight*
 - *BodyWeight* influences *HeartDisease*
 - *BodyWeight* is a collider



Collider Bias

- Aka “Berkson’s paradox”
- Conditioning on a collider can introduce a spurious association between its parents by “*opening a path that is blocked*”
- Consider the variables:
 - Diet (D)
 - Exercise (E)
 - BodyWeight (W)
 - HeartDisease (H)
- **Without conditioning on W**
 - E and D are independent
 - E.g., knowing someone's exercise level E doesn't give information about diet D , and vice versa
 - The collider W blocks any association between E and D

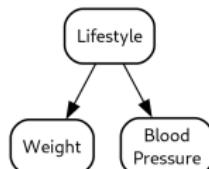
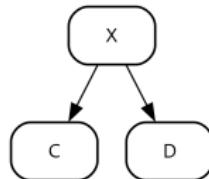


- **After conditioning on W**
 - E.g., looking for individuals with specific body weight
 - You introduce a dependency between E and D

- Causal Networks
 - Variables
 - Intervention
 - Type of Variables in Causal AI
 - **Paths**

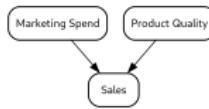
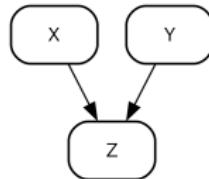
Fork Structure

- A **fork** occurs when a single variable causally influences two or more variables
 - Formally: $X \rightarrow C$ and $X \rightarrow D$
- X is a common cause (confounder) of C and D
- Forks induce statistical dependence between C and D
 - Even if C and D are not causally linked
- Conditioning on X blocks the path and removes spurious correlation
- Example:
 - Lifestyle factors as confounders
 - *Lifestyle* affects both *Weight* and *BloodPressure*
 - These outcomes may appear correlated due to shared cause



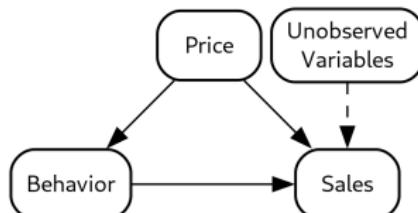
Inverted Fork

- An **inverted fork** occurs when two or more arrows converge on a common node
 - Also known as a **collider**
- Colliders block associations unless the collider or its descendants are conditioned on
- Conditioning on a collider “opens” a path, inducing spurious correlations
- Example:
 - Sales influenced by multiple independent causes
 - *MarketingSpend* and *ProductQuality* both influence *Sales*
 - Conditioning on *Sales* can induce false dependence between *MarketingSpend* and *ProductQuality*



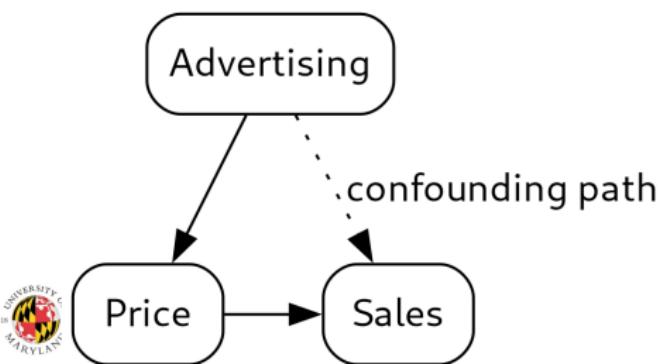
Path connecting unobserved variables

- **Unobserved variables** affect the model but we don't have a direct measure of it
- E.g., consider the causal DAG
 - A retailer does market research, expecting *Price* to influence *Sales* in a predictable way
 - A retailer sets the *Price* of a new product based on market research
 - The retailer can observe and measure *Behavior*, e.g.,
 - Discounts
 - Promotional campaign
 - There are unobserved vars that influence the model, e.g.,
 - Social media buzz
 - Word-of-mouth recommendation



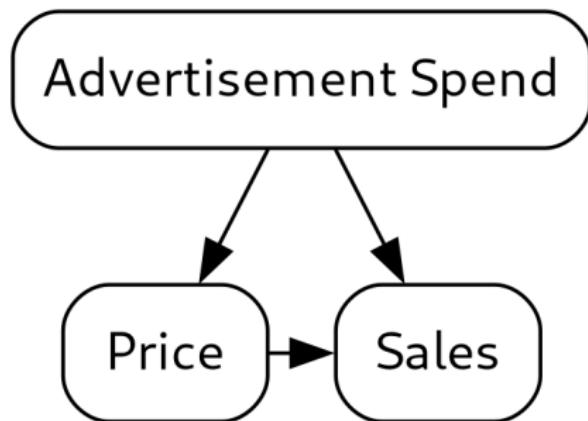
Front-door Paths in Causal Inference

- A front-door path reveals causal influence through an observable mediator
 - The causal effect flows: $A \rightarrow P \rightarrow S$
- Requirements for identifiability:
 - All confounders of $A \rightarrow P$ and $P \rightarrow S$ are observed and controlled
 - There are no back-door paths from A to S through unobserved variables
- Enables causal estimation when back-door adjustment is infeasible
- Example:
 - Advertising impacts sales through customer perception of price
 - A : Advertising, P : Price perception, S : Sales
- Pearl's front-door criterion provides a formal method for adjustment
 - Estimate $P(P|A)$, $P(S|P, A)$, and $P(A)$ from data to compute causal effect



Back-Door Paths

- A company wants to understand the causal effect of price on sales



- Price → Sales is the front-door path
- A confounder is Advertising spend since it can affect both:
 - The price the company can set (e.g., the cost increases to cover advertisement costs and the product is perceived as more valuable)
 - The sales (directly)

The back-door path goes from Sales to Price via Advertising spend

Frontdoor and Backdoor Paths

- Question: *Will increasing our customer satisfaction increase our sales?*
- Assume that the Causal DAG is



- **Front-door path** (i.e., a direct causal relationship):
 $CustomerSatisfaction \rightarrow Sales$

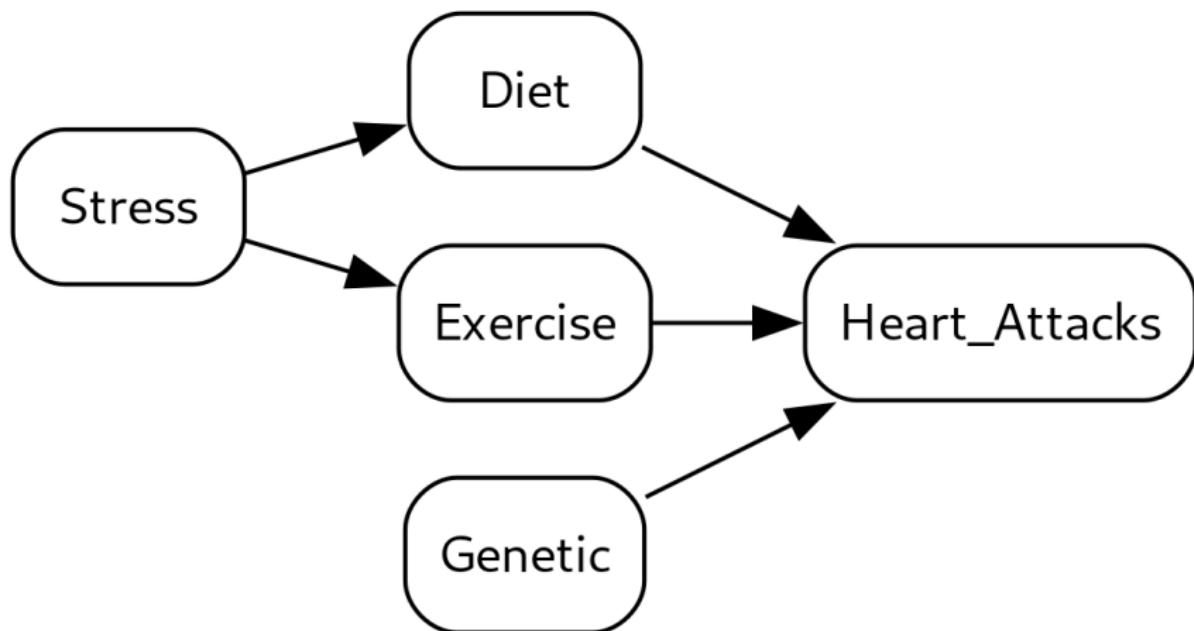
- **Backdoor path:**
ProductQuality is a common cause (confounder) of both *CustomerSatisfaction* and *Sales*
- To analyze the relationship between customer satisfaction and sales, we need to:
 - Control for *ProductQuality* to close the backdoor path
 - Eliminate the confounding effect
- In reality there are more confounding effects (e.g., price)

Building a DAG

- **Causal models** visually represent complex environments and relationships
- Nodes are like “nouns” in the model:
 - E.g., “price”, “sales”, “revenue”, “birth weight”, “gestation period”
 - Variables can be endogenous/exogenous and observed/unobserved
 - Complex relationships between variables:
 - Parents, children (direct relationships)
 - Descendants, ancestors (along the path)
 - Neighbors
- **Iterative Refinement:**
 - Models are continuously updated with new variables and insights
- **Modeling as a Communication Tool:**
 - A shared language that bridges gaps between technical and non-technical team members
- **Unobservable Variables:**
 - Supports inclusion of variables not empirically observed but known to exist
 - E.g., trust or competitor activity can be modeled despite lack of direct data

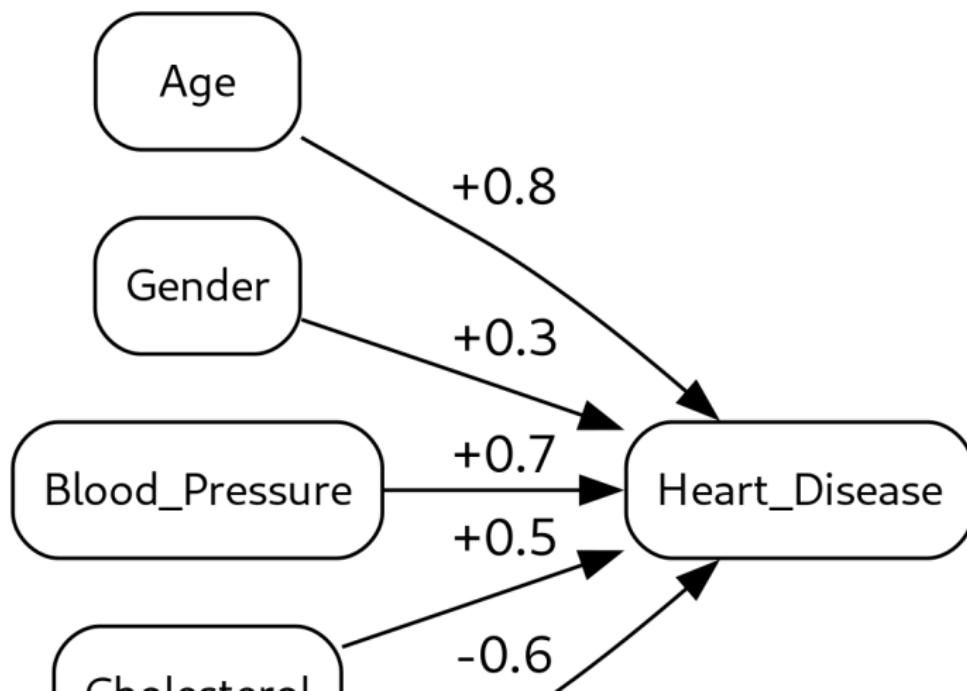
Heart Attack: Example

- What's the relationship between stress and heart attacks?
 - Stress is the treatment
 - Heart attack is the outcome
 - Stress is not a direct cause of heart attack
 - E.g., a stressed person tends to have poor eating habits



Weights

- Weights can be assigned to paths to represent the strength of the causal relationship
 - Weights can be estimated using statistical methods
- Sign represents the direction



Counterfactuals

- A **counterfactual** describes what would have happened under a different scenario
 - “*What would the outcome have been if X had been different?*”
 - “*If kangaroos had no tails, they would topple over*”
 - “*What if we had two suppliers of our product, rather than one? Would we have more sales?*”
 - “*Would customers be more satisfied if we could ship products in one week, rather than three weeks?*”
- **Causal reasoning:**
 - Goes beyond correlation and association
 - Requires a causal model (like an SCM) to simulate alternate realities
 - E.g.,
 - Actual: A student received tutoring and scored 85%
 - Counterfactual: What if the student didn't receive tutoring?
 - Causal model estimates the alternative outcome (e.g., 70%)
- **Challenges:**
 - Requires strong assumptions and accurate models
 - Difficult to validate directly since counterfactuals are unobservable