



White Horizon PDR

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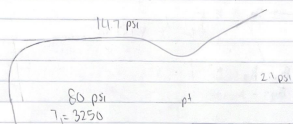
Calculations and Critical Values

| | A | B | C | D | E | F |
|----|-------------------------------|------------|--------------|----------|-------|-------------|
| 1 | Parameter | Metric | Units | Imperial | Units | Notes |
| 2 | Optomized Specific Impulse | 274 | s | | | |
| 3 | Specific Impulse at sea level | 92.3359 | s | | | |
| 4 | Optimal Thrust | 35.0 | kgm/s^2 | 7.864 | lbf | |
| 5 | Sea Level Thrust | 11.8 | kgm/s^2 | 2.647 | lbf | |
| 6 | Exit Velocity (V2) | 2690.95842 | m/s | 8829.035 | ft/s | |
| 7 | Critical Pressure Ratio | 0.58 | unitless | | | |
| 8 | Pressure Value | 319.06 | kPa | 46.276 | psi | @ Mach 1 |
| 9 | Area Ratio | 0.153 | no units lol | | | |
| 10 | Throat Area | 0.410 | cm^2 | 0.064 | in^2 | circular |
| 11 | Throat Diameter | 0.722 | cm | 0.284 | in | |
| 12 | Exit Area | 2.67 | cm^2 | 0.414 | in^2 | A2 |
| 13 | Exit Diameter | 1.84 | cm | 0.726 | in | |
| 14 | Nozzle Length | 1.6749 | cm | 0.659 | in | Bell Nozzle |

Calculations

Exit Velocity

$$V_2 = \sqrt{\frac{2(1.13)}{1.13-1} \cdot \frac{(3741.56887) \cdot (7)}{3250} \left[1 - \left(\frac{2.1 \text{ psi}}{60 \text{ psi}} \right)^{\frac{1.13}{1.13-1}} \right]}$$



$$V_2 = 2,690.958542 \text{ m/sec}$$

optimal throat

$$V_2 = C$$

exit velocity = relative throat velocity

$$F = \dot{m} V_2 = \frac{139}{\text{sec}} = \frac{0.13 \text{ kg}}{\text{sec}} = 34.98246 \text{ N}$$

optimal throat

throat at sea level

$$F = 34.98246 + (14.470446 \cdot 101,325) A_2$$

$$F = 11.775 \text{ N}$$

no source decrease

px to point

$$P = 6.84476 \text{ psia} = 14.478996 \text{ Pa} = 21 \text{ psi}$$

Respecter values

$$F(\text{kg}) = 34.98246 (0.693 \text{ kg/sec} \cdot 9.81 \text{ m/sec}^2) = 275.9457 \text{ sec}$$

$$F(\text{kg}) = 11.756 (0.013 \text{ kg/sec} \cdot 4.013) = 92.359 \text{ sec}$$

(gross) section Area:

$$A_2 = \dot{m} V_2 / \rho_2$$

pressure at the exit of the nozzle is the same as the optimization pressure

$$P_2/P_1 = 2.207 \text{ m}^3 / \text{kg} \quad V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1.13}{1.13-1}} = 55.3133 \text{ m}^3/\text{kg}$$

$$\text{exit velocity} = 2690.958542 \text{ m/s}$$

$$A_2 = 2.672 \times 10^{-4} \text{ m}^2$$

$$\frac{0.13 \text{ kg}}{\text{sec}} \cdot \frac{1}{\text{m}^3/\text{kg}} = \text{m}^2$$

$$A_1 = \frac{\dot{m}}{P_1} \sqrt{\frac{T \cdot R}{k \cdot 2 \cdot \pi \cdot \gamma \cdot k \cdot (1.13-1)}}$$

$$A_1 = \frac{0.13 \text{ kg/sec}}{551,580.8 \text{ Pa}} \sqrt{\frac{3250 \cdot 374.56887 \cdot \frac{2}{\text{kg} \cdot \text{m}^3}}{10 \cdot 272.13 \cdot 2.13}}$$

$$A_1 = 4.09798 \times 10^{-6} \text{ m}^2$$

$$A_1 = 4.09798 \text{ cm}^2$$

throat nozzle

$$\frac{A_1}{A_2} = \left(\frac{1.13}{2} \right)^{\frac{1.13}{1.13-1}} \left(\frac{2.1 \text{ psi}}{60 \text{ psi}} \right)^{\frac{1.13}{1.13-1}} \sqrt{\frac{2.13}{1.13} \left[1 - \left(\frac{2.1}{60} \right)^{\frac{1.13}{1.13-1}} \right]}$$

$$0.0471$$

$$2.4871$$

$$1.933$$

$$A_2 = 6.5207 \text{ mm}^2$$

Maximum gas flow rate

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} = \left(\frac{2}{2.13} \right)^{\frac{1.13}{1.13-1}}$$

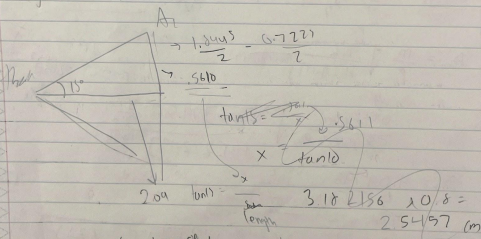
$$\text{critical pressure ratio} = 0.57845$$

$$\text{critical pressure ratio} = \frac{P_2}{P_1} = 0.57845$$

$$\frac{80 \text{ psi}}{P_2} = 0.57845$$

$$P_2 = 46.276 \text{ psi}$$

length of nozzle

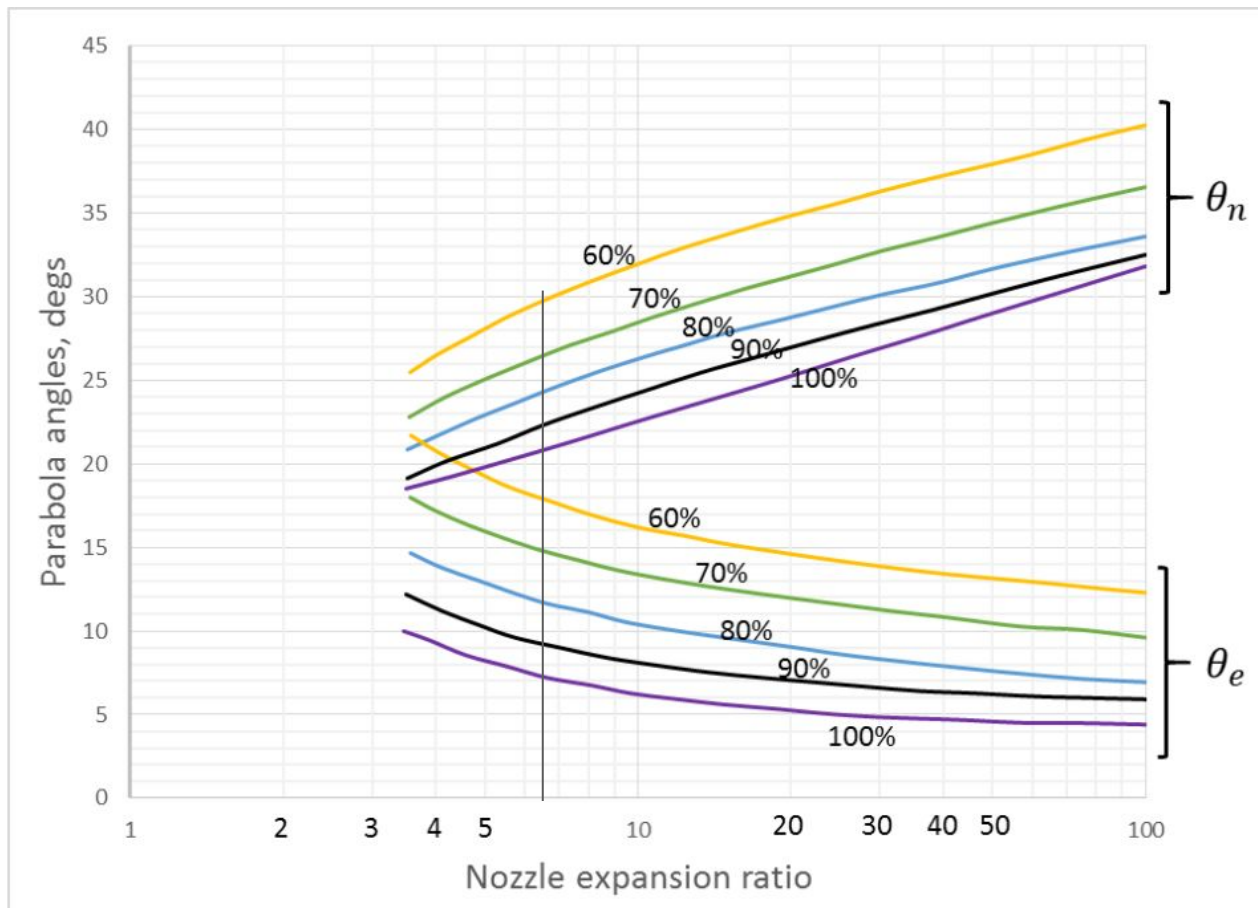


$$274 \text{ optimum } \text{px} \text{ ch.}$$

$$350 \text{ kg}$$

$$\frac{3.18 \cdot 2.136}{2.54 \cdot 97} = 2.09366 \times 0.8 = 1.674944 \text{ cm}$$

Creating a Rao Nozzle and Choosing Approximation Angles



Our Expansion Ratio is:
6.51

Thus we can choose
approximation angles
of:

Exit Angle: 11.6
degrees

Initial Angle: 24.2
degrees

The Math Behind the Points Found



Technical papers

The mathematical approach

The throat

The equations of the above circular arcs defining the throat are defined trigonometrically, defining the origin of the coordinates as the centre of the narrowest part of the throat:

For the entrant section:

$$\begin{aligned} x &= 1.5 R_t \cos \theta \\ y &= 1.5 R_t \sin \theta + 1.5 R_t + R_t \quad \text{equ.s 4} \end{aligned}$$

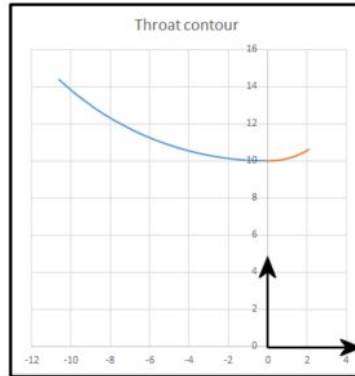
where: $-135 \leq \theta \leq -90$

(The initial angle isn't defined and is up to the combustion chamber designer, -135 degrees is typical.)

For the exit section:

$$\begin{aligned} x &= 0.382 R_t \cos \theta \\ y &= 0.382 R_t \sin \theta + 0.382 R_t + R_t \quad \text{equ.s 5} \end{aligned}$$

where: $-90 \leq \theta \leq (\theta_n - 90)$



The bell

The bell is a quadratic Bézier curve, which has equations (see Wikipedia):

$$\begin{aligned} x(t) &= (1-t)^2 N_x + 2(1-t)t Q_x + t^2 E_x & 0 \leq t \leq 1 \\ y(t) &= (1-t)^2 N_y + 2(1-t)t Q_y + t^2 E_y & 0 \leq t \leq 1 \end{aligned} \quad \text{equ.s 6}$$

Selecting equally spaced divisions between 0 and 1 produces the points described earlier in the graphical method, for example 0.25, 0.5, and 0.75.

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Equations 6 are defined by points N, Q, and E (see the graphical method earlier for the locations of these points).

Point N is defined by equations 5 setting the angle to $(\theta_n - 90)$.

Coordinate E_x is defined by equation 3, and coordinate E_y is defined by equation 2.

Point Q is the intersection of the lines: $\overline{NQ} = m_1 x + C_1$ and $\overline{QE} = m_2 x + C_2$ **equ.s 7**

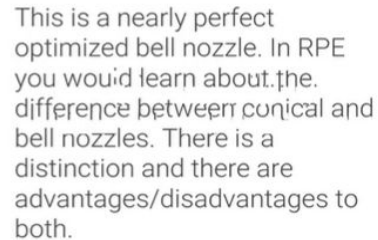
where: gradient $m_1 = \tan(\theta_n)$, gradient $m_2 = \tan(\theta_e)$ **equ.s 8**

and: intercept $C_1 = N_y - m_1 N_x$, intercept $C_2 = E_y - m_2 E_x$ **equ.s 9**

The intersection of these two lines (at point Q) is given by:

$$Q_x = \frac{(C_2 - C_1)}{(m_1 - m_2)}, Q_y = \frac{(m_1 C_2 - m_2 C_1)}{(m_1 - m_2)} \quad \text{equ.s 10}$$

✓ Simplification of the bell nozzle

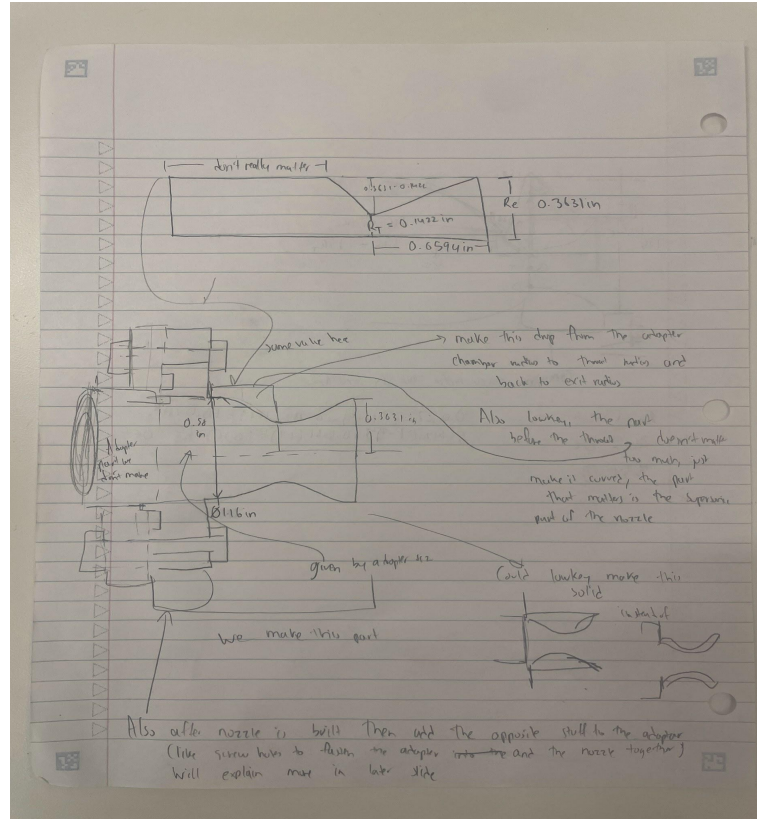


With a bell nozzle, the geometry follows this contour.

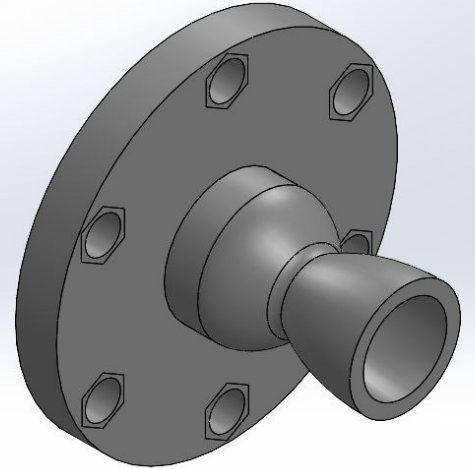
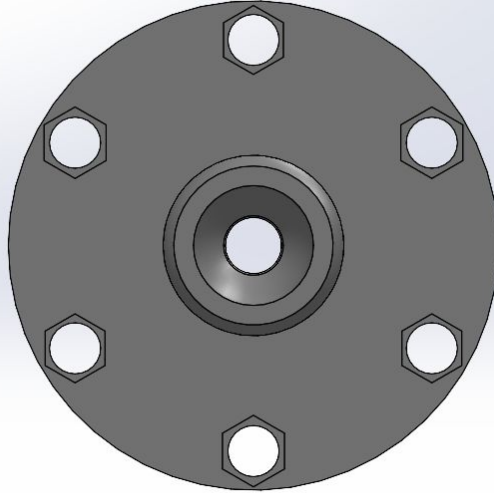
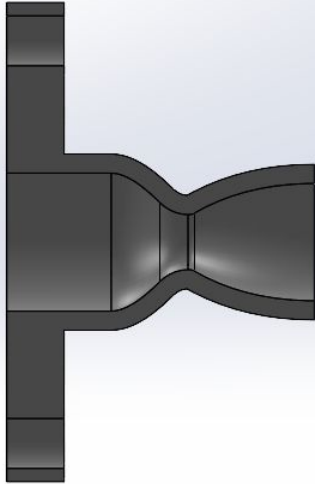
Angles at nozzle and exit as well as the radius of exit, throat, and the length of the nozzle were specified in the spreadsheet above and slide 6

Simple Nozzle Design

CAD video help



CAD Model of Nozzle



Material Selection and Properties

Properties Tables & Curves Appearance CrossHatch Custom Application Data Favorites Sheet Metal

Material properties

Materials in the default library can not be edited. You must first copy the material to a custom library to edit it.

Model Type: ☐ Save model type in library

Units:

Category:

Name:

Default failure criterion:

Description:

Source:

Sustainability:

| Property | Value | Units |
|-------------------------------|------------|----------|
| Elastic Modulus | 2000000000 | N/m^2 |
| Poisson's Ratio | 0.394 | N/A |
| Shear Modulus | 318900000 | N/m^2 |
| Mass Density | 1195 | kg/m^3 |
| Tensile Strength | 36000000 | N/m^2 |
| Compressive Strength | | N/m^2 |
| Yield Strength | 25200000 | N/m^2 |
| Thermal Expansion Coefficient | | /K |
| Thermal Conductivity | 0.2256 | W/(m·K) |
| Specific Heat | 1386 | J/(kg·K) |
| Material Damping Ratio | | N/A |

Resin Parameters:

- Resin Type: ABS-like resin V2.0
- Hardness: 84 D; Shrinkage: 7.1 %
- Viscosity (25°C): 150-200 mPa.s
- Liquid Density: 1.100 g/cm³
- Solid Density: 1.195 g/cm³
- Flexure Strength: 59-70 Mpa
- Extension Strength: 36-53 Mpa
- Elongation at Break: 14.2
- Shelf Life: 2 year

Applied Materials

Simulation Mass Properties (-Static 2-)

| | | Item | Mass (kg) |
|---|--|---------------------------|-----------|
| 1 | | nozzle (-RPG_Plastic-) | 0.03014 |

Decimal Places: 5 ☐ Scientific notation

☐ Show center of mass Unit system: SI (MKS)

Report coordinate values relative to: --Default--

Mass properties of "Static 2"

Mass = 0.03014 kg

Volume = 0.00003 m³

Surface Area = 0.01195 m²

Center of mass: (m)

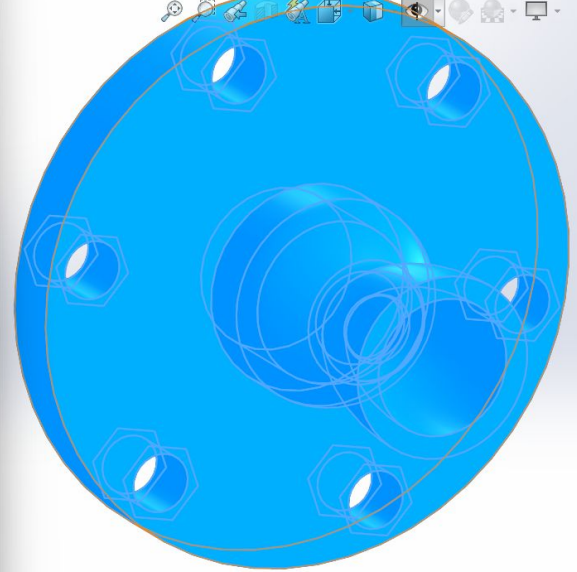
X = -0.01672
Y = 0.00000
Z = 0.00000

Principal axes of inertia and principal moments of inertia: (kg.m²)
Taken at the center of mass.

| | |
|-----------------------------------|--------------|
| Ix = (0.00000, 1.00000, 0.00000) | Px = 0.00001 |
| Iy = (0.00000, 0.00000, 1.00000) | Py = 0.00001 |
| Iz = (1.00000, 0.00000, -0.00000) | Pz = 0.00001 |

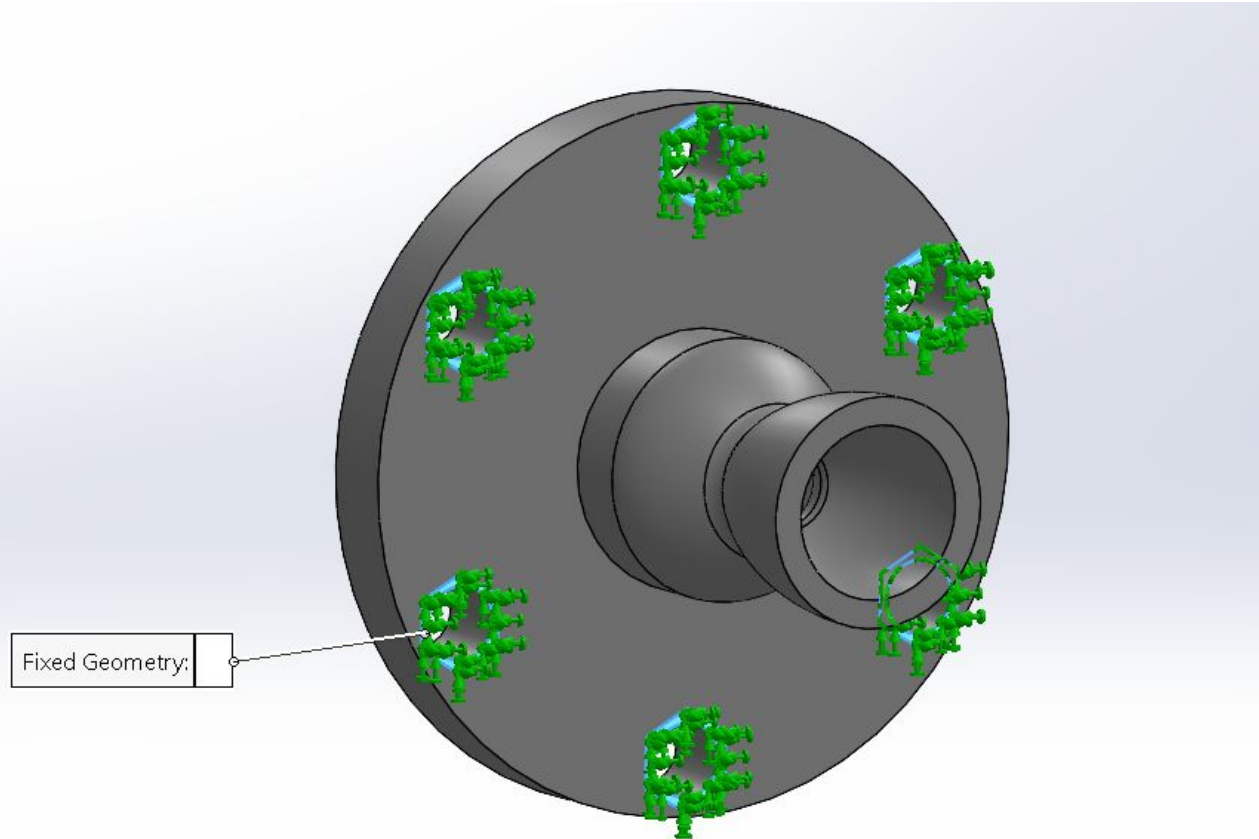
nozzle *

Simulation



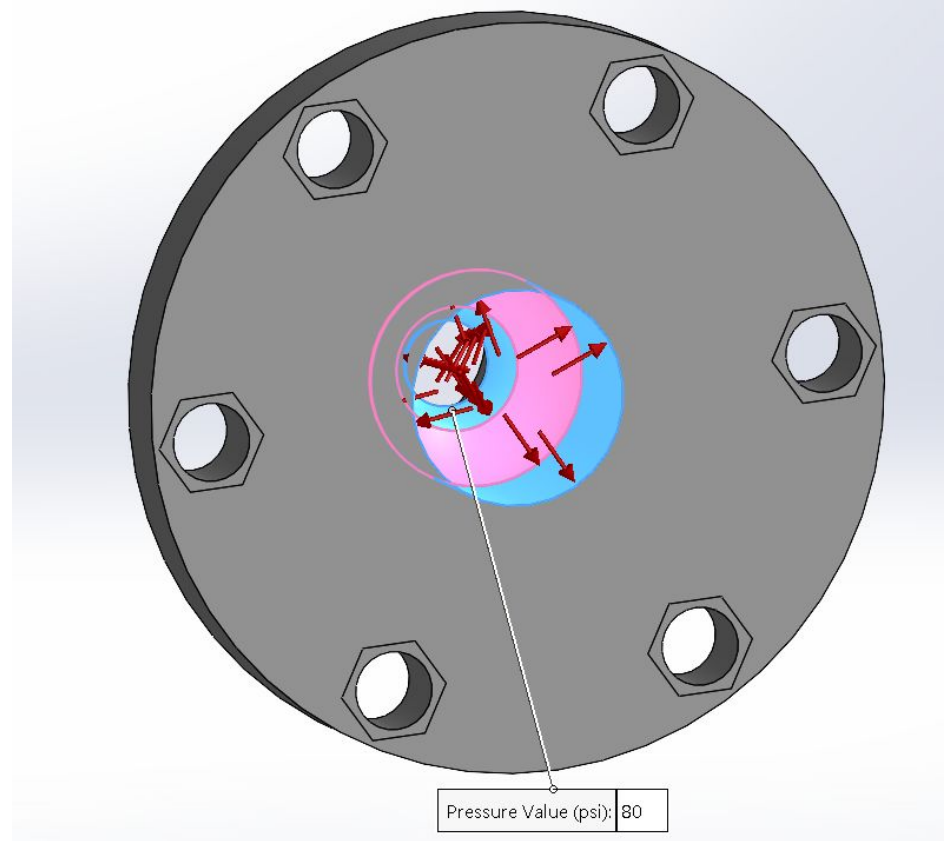
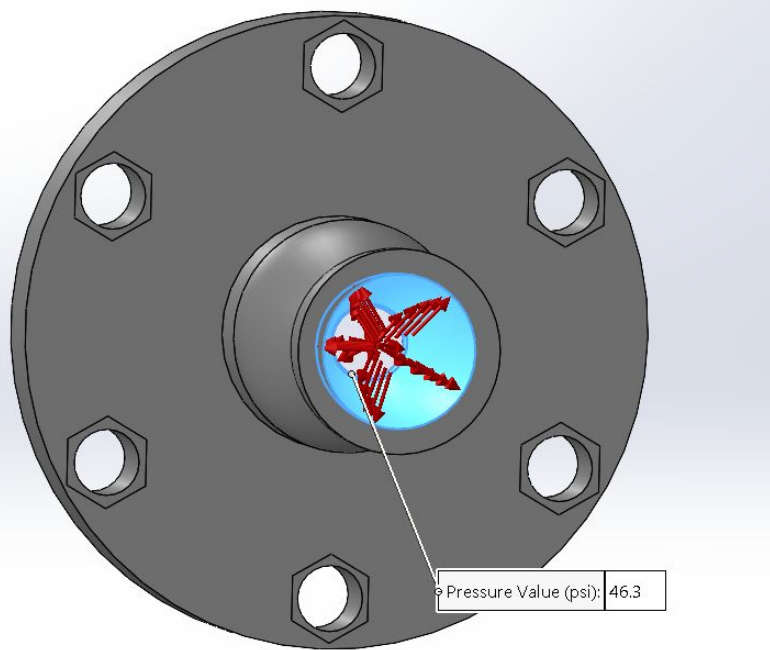


Fixed Geometry





Loads

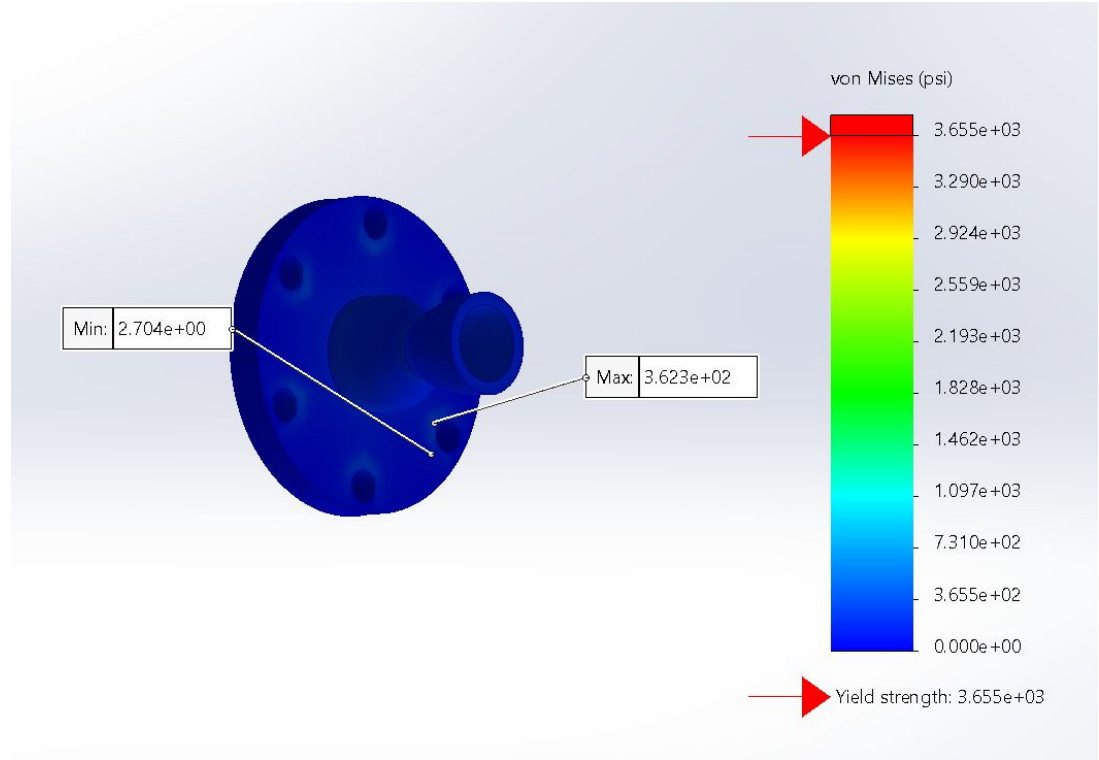


Stress Testing

Yield Strength: 25.2 Mpa or 3654.951 psi

Ultimate Tensile Strength: 36Mpa or 5221.36 psi

Went for the lowest possible in the strength ranges of the material and made sure that the forces on it would not exceed that strength minimum





FOS Plot

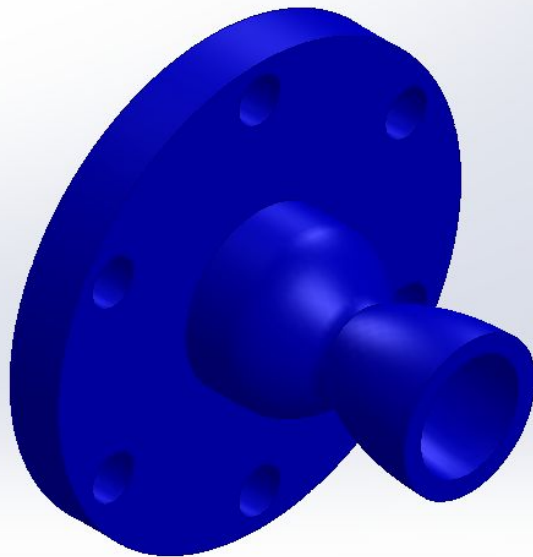
Model name: nozzle

Study name: Static2(-Default-)

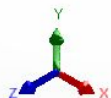
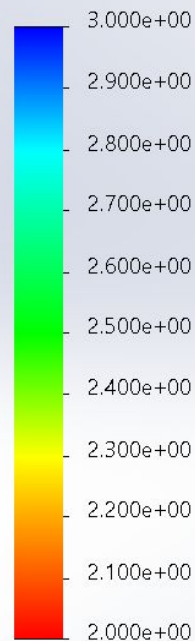
Plot type: Factor of Safety Factor of Safety2

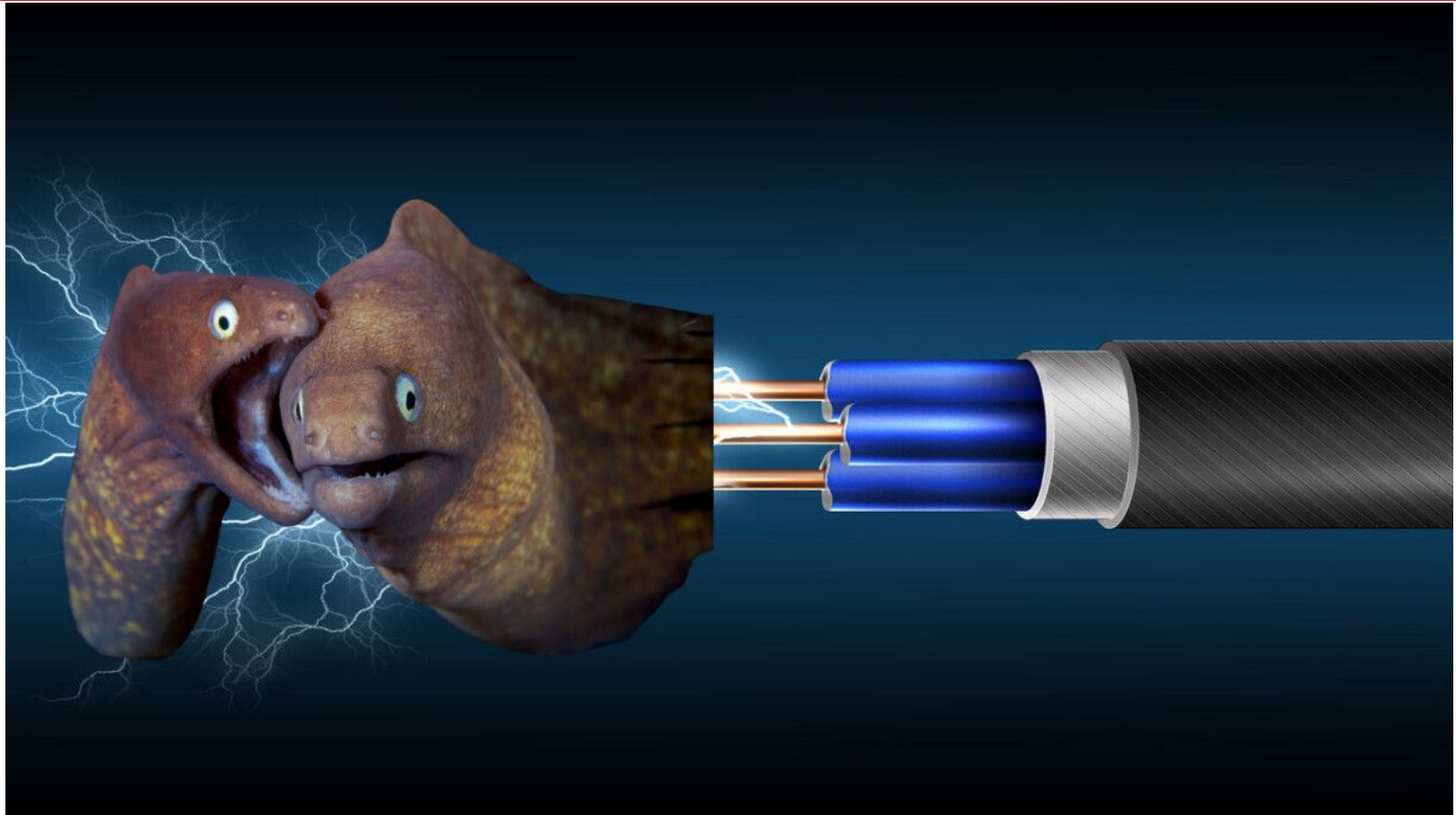
Criterion : Automatic

Factor of safety distribution: Min FOS = 10

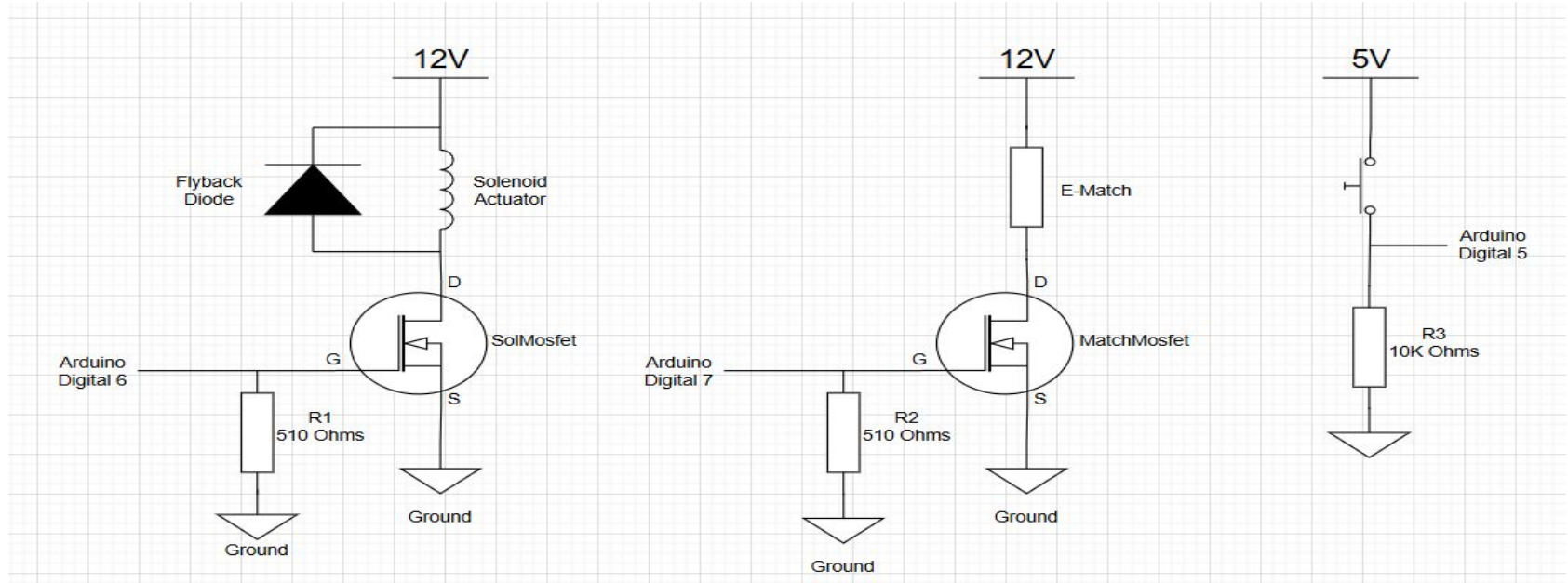


FOS

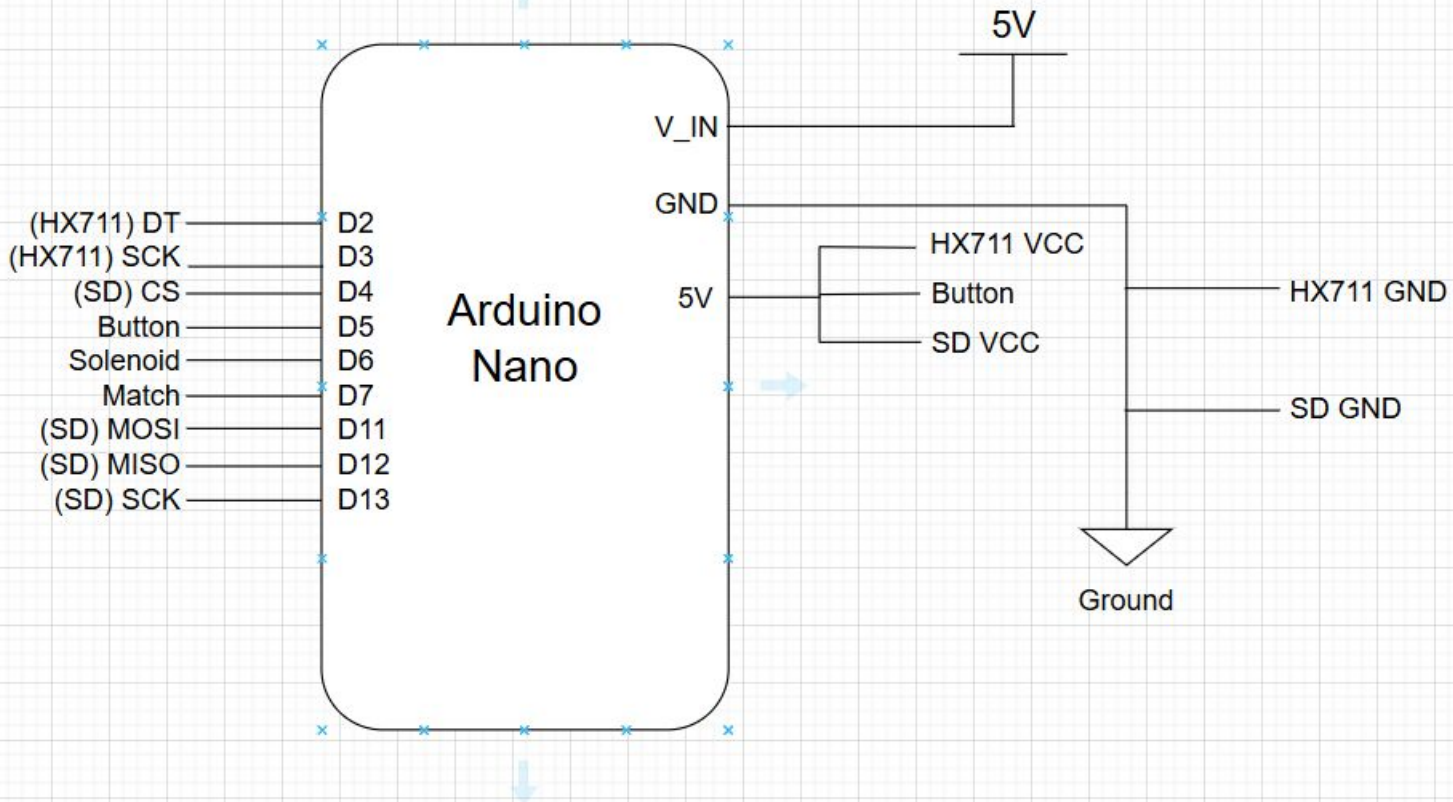




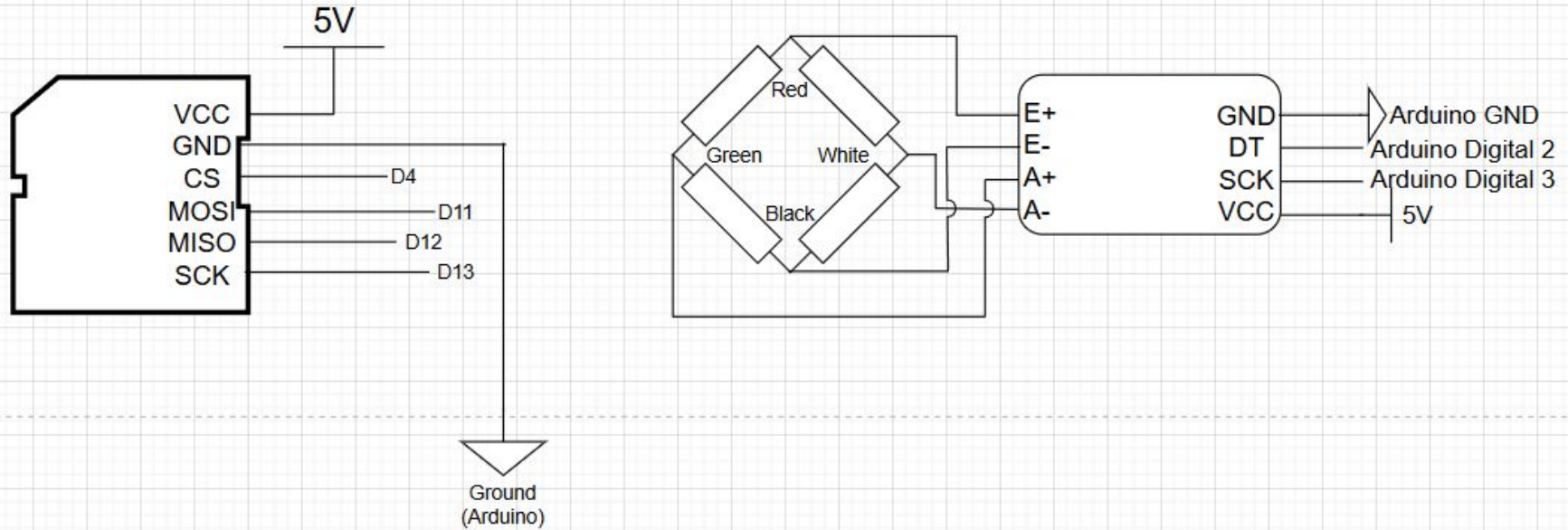
Schematic (Solenoid, E-match and Button)



Schematic- Arduino



Schematic (SD Card and HX711)





Code

```
#include "HX711.h" //Add library of HX711 so can use
#include <SPI.h>    //Includes so pins 4,11,12 and 13 work as intended
#include <SD.h>     //Add library of SD so can open file

//Pin Designations
const int buttonPin = 5;           //Set button pin to pin 5
const int SolMosfetPin = 6;        //Set connector pin to MOSFET gate of solenoid path to pin 6
const int MatchMosfetPin = 7;      //Set connector pin to MOSFET gate of e-match path to pin 7
const int DTPin = 2;               // set pin for Data from HX711 Amp to pin 2
const int SCKPin = 3;              //set pin for clock from HX711 Amp to pin 3
const int chip = 4;                //set pin which decides which to communicate from to SD Card module,
used 4 cuz uncreative
const long interval = 200;         //sets the interval length to 200 milliseconds
unsigned long previousReadingTime = 0; //begins the program with having read at 0 seconds
```



Code

```
//Initializing Scale
HX711 scale;    //Initializes HX711
File dataFile; //Declares dataFile globally so it works in function and loop
void InitializeSDCard() {
    // Initializes the SD card and makes sure it works
    Serial.print("Initializing SD Card...");
    if (!SD.begin(chip)) {                //checks to see if the SD Card is communicating properly is
functional
        Serial.println("Initializing Failed"); //prints that something occurred and Pin 4 is malfunctioning
        return;                               //Stops process from going any further, halting program
    dataFile = SD.open("data.txt", FILE_WRITE); //initializes data file in setup (open once)
    if (dataFile) {
        Serial.println("File has successfully opened"); //prints that the file opened successfully
    } else {
        Serial.println("data.txt did not open successfully"); //prints that the file failed to open
        return;                                                //stops the process
    } Serial.println("Initializing Complete"); //if nothing happens, it prints that the sd card has initialized
}
```




Code

```
void InitializePins() {
    //Sets the pins up and turns them to low automatically
    pinMode(buttonPin, INPUT);          //Sets button as input
    pinMode(SolMosfetPin, OUTPUT);       //sets MOSFET pin on Solenoid path as output
    pinMode(MatchMosfetPin, OUTPUT);     //sets MOSFET pin on Ematch path as output
    Serial.println("The Pin settings are complete");
    //Setting pins to low so it starts out off
    digitalWrite(SolMosfetPin, LOW);     //Sets Mosfet pin on solenoid path to 0, so starts at 0
    digitalWrite(MatchMosfetPin, LOW);   //same as above, but for the one on the ematch
    Serial.println("The Output Pins have been set to Low");
} enum State { //declares variable state and the 3 different kinds states possible, IDLE, SOLENOID_ON and
MATCH_ON, used enum bc easier to read name than look at boolean
    IDLE,
    SOLENOID_ON,
    MATCH_ON,
};
State currentState = IDLE;              //initiates program with state IDLE
unsigned long stateStartTime = 0;       //Sets the initial start time of the first IDLE section to 0
```



Code

```
void setup() {  
    Serial.begin(9600); //Begin card comms at baud rate 9600, cuz arduino nano can't get much higher  
    while (!Serial) {    // waits for the serial port to connect (I.E computer)  
        ;  
    }  
    Serial.println("USB Connection is Good");  
  
    //Running Initialization Functions for SD Card Module and Nano Pins  
    InitializeSDCard(); // Runs the SD Card Initialization Function  
    InitializePins();    // Runs the Pin Initialization Function  
  
    //Starting the loadcell up  
    scale.begin(DTPin, SCKPin); //initializes the Loadcell, DT pin signals which pin the data comes from and  
    SCK controls the timing of data collection  
    Serial.println("The Loadcell is Initialized");  
}
```



Code

```
void loop() {  
    // Deals with button mechanism turning on and off with button  
    unsigned long currentTime = millis(); //Checks how many milliseconds have passed since the loop began each  
    time the loop is run  
  
    switch (currentState) { //sets up a switch() case: statement so the program swaps when currentState changes  
        case IDLE:  
            if (digitalRead(buttonPin) == HIGH) {  
                delay(50); //Debounce  
                if (digitalRead(buttonPin) == HIGH) {  
                    digitalWrite(SolMosfetPin, HIGH);  
                    stateStartTime = currentTime; //sets the stateStartTime for the SOLENOID_ON to whatever time it is  
                    currentState = SOLENOID_ON; //swaps case to SOLENOID_ON  
                }  
            }  
            break; //prevents falling-through (moving to next case)
```



Code

```
case SOLENOID_ON:
    if (currentTime - stateStartTime >= 200) { //if 1/5 of a second has passed since the solenoid mosfet
turned on
        digitalWrite(MatchMosfetPin, HIGH); //sets ematch to ignite
        stateStartTime = currentTime; //sets the start time for MATCH_ON
        currentState = MATCH_ON; //sets current state to MATCH_ON
    }
    break;
case MATCH_ON:
    if (currentTime - stateStartTime >= 5000) { //after 5 seconds since the ematch turned on
        digitalWrite(SolMosfetPin, LOW); //Turns off solenoid MOSFET
        digitalWrite(MatchMosfetPin, LOW); //Turns off match MOSFET
        dataFile.close(); //Saves it and closes file
        Serial.print("File Closed"); //Communicates that the data has been filed and saved
        currentState = IDLE; //Resets to IDLE state
    }
    break;
}
```



Code

```
if (currentTime - previousReadingTime >= interval) { //Checks if 1/5 of a second has passed since the data
was last read
    previousReadingTime = currentTime; //sets the previous reading time to the current time
as it is being checked
    int reading = scale.read(); //reads whatever loadcell says and stores as reading
    Serial.print("Timestamp: ");
    Serial.print(currentTime);
    Serial.print(" Value: "); // prints opening
    Serial.println(reading); // prints value loadcell reads on the previous line
    if (dataFile) {
        dataFile.print("Timestamp: ");
        dataFile.print(currentTime);
        dataFile.print(", HX711 Reading: ");
        dataFile.println(reading); // writes into data file
        dataFile.flush(); //pushes value from buffer to file if value gets stuck
        Serial.println("Data saved in SD Card");
    } else {
        Serial.println("Error opening data.txt"); //prints that file opening failed if it did
    }
}
```



Shopping List

- 1 Arduino Nano + A perfboard
 - Kinda necessary, otherwise the last 7 slides were for no reason, need the perfboard to put stuff on
- 2 N-Channel MOSFETs Low Capacitance, Form channel w/ gate mean fast switch time
- 1 Solenoid Actuator
 - To open and close fuel line for nozzle to work
- 1 Flyback Diode
 - Flyback Voltage Bad
- 1 E-match
 - someone could pull out a Bic and light it. might work.
- Red Button
- 3 10K ohm resistors (to prevent floating MOSFETs and button)(Dunno how to do the math to find exact value, but according to the HIP chat, this is good)
- SD Card Module and SD Card itself
 - Collect data for this
- Loadcell and HX711 Loadcell amp
 - arduino calibrates the loadcell, loadcell collects data, amp sends data to arduino
- 12V battery
 - Drugs